



Measuring the muon magnetic anomaly a_μ with the *Muon g-2* experiment at Fermilab

INFN Pisa Seminar – 14 April 2021

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What is “g-2” ?



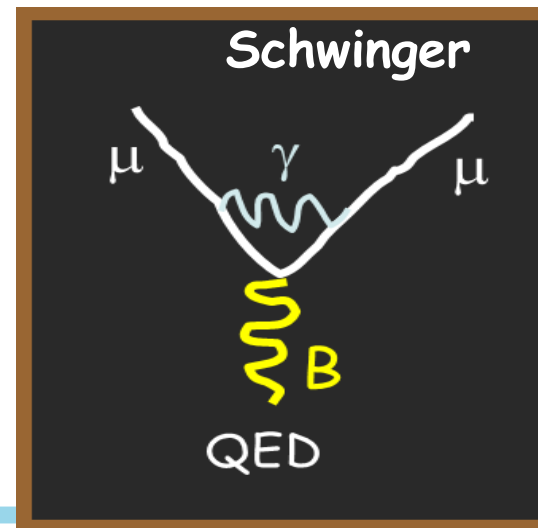
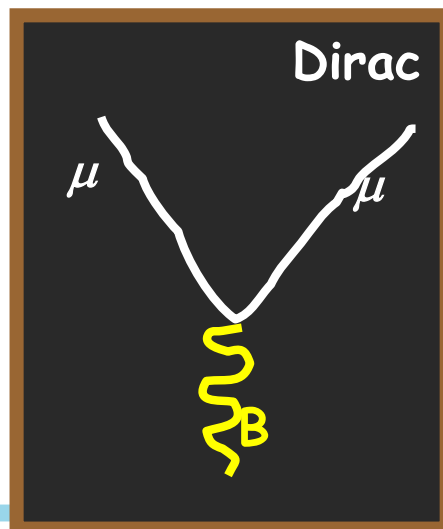
$$\vec{\mu}_p = -g_p \frac{e}{2m_p} \vec{S}$$

$$a_p = \frac{g_p - 2}{2}$$

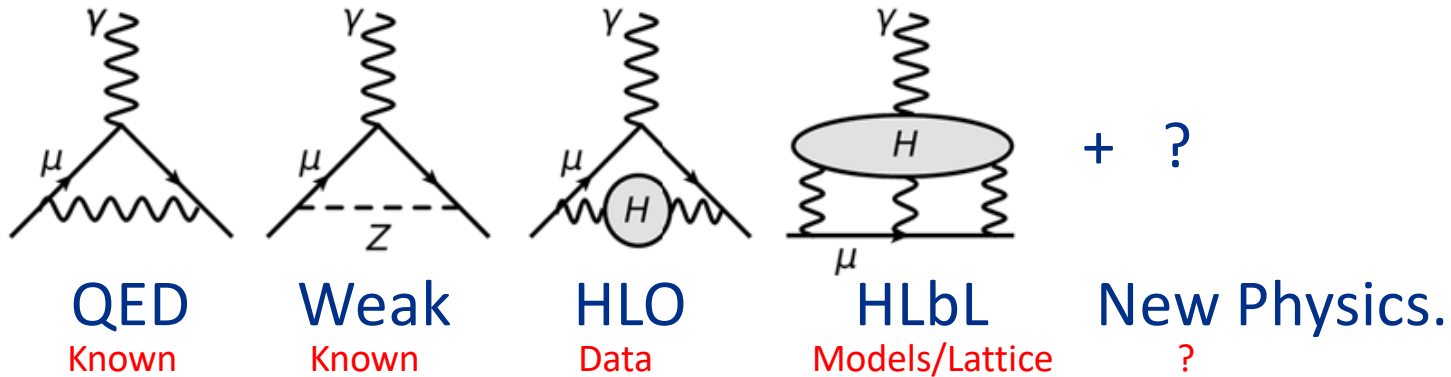
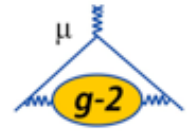
- g_p : proportionality between spin and magnetic moment for particle P
- a_p : magnetic anomaly
- $a_p = 0$ at tree level (purely “Dirac particle”)

- The term $(g-2)/2$ reflects the magnitude of the Feynmann diagrams beyond leading order

$$a = 0 + \alpha/2\pi + \dots$$



Contributions to a_μ



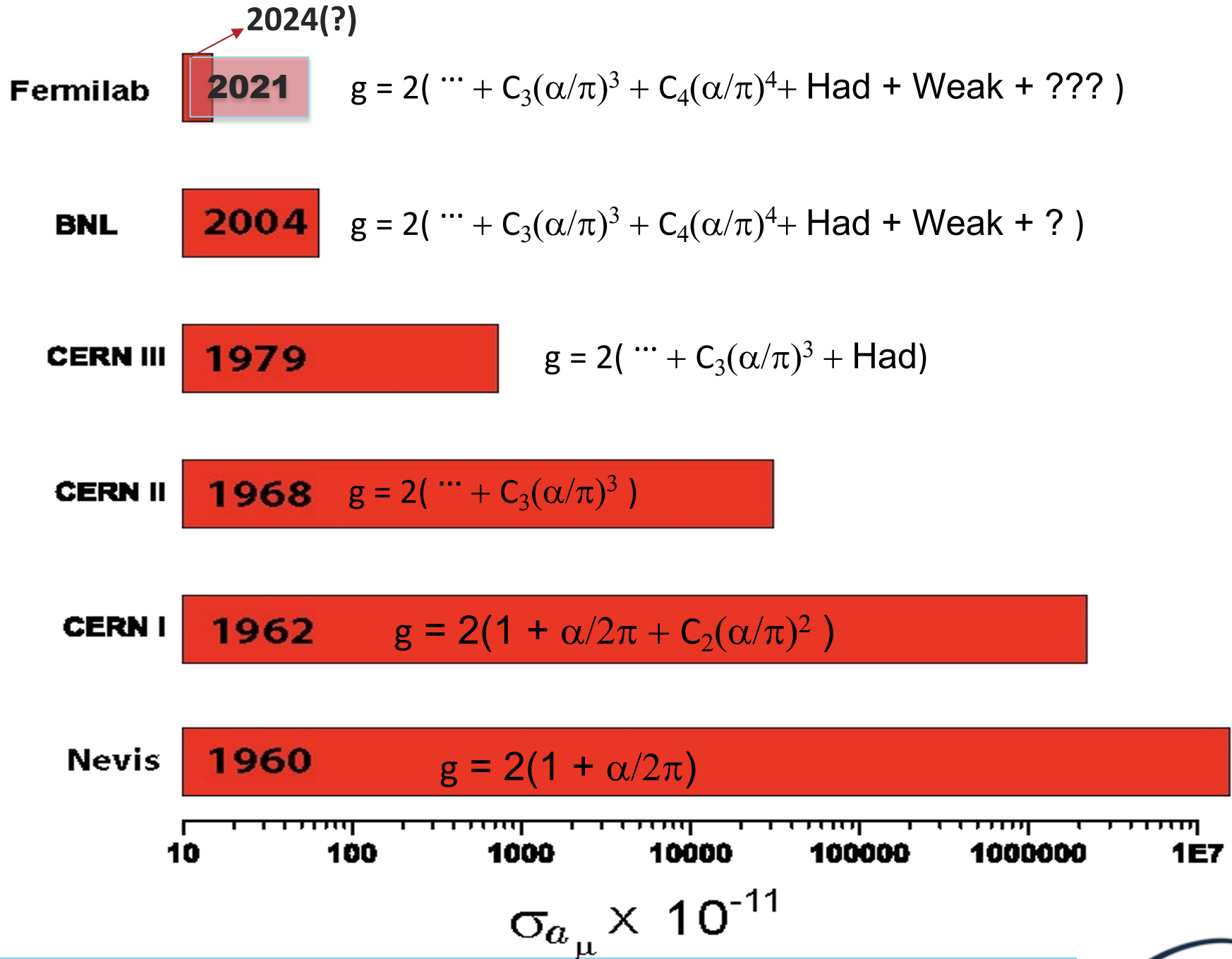
HLbL = Hadronic Light by Light = Hadronic higher order

	VALUE ($\times 10^{-10}$) UNITS
QED ($\gamma + \ell$)	$11\,658\,471.8951 \pm 0.0009 \pm 0.0019 \pm 0.0007 \pm 0.0077_\alpha$
HVP(lo) Davier17	692.6 ± 3.33
HLbL Glasgow	10.5 ± 2.6
EW	15.4 ± 0.1
Total SM Davier17	$11\,659\,181.7 \pm 4.2$

686 ppt !!!

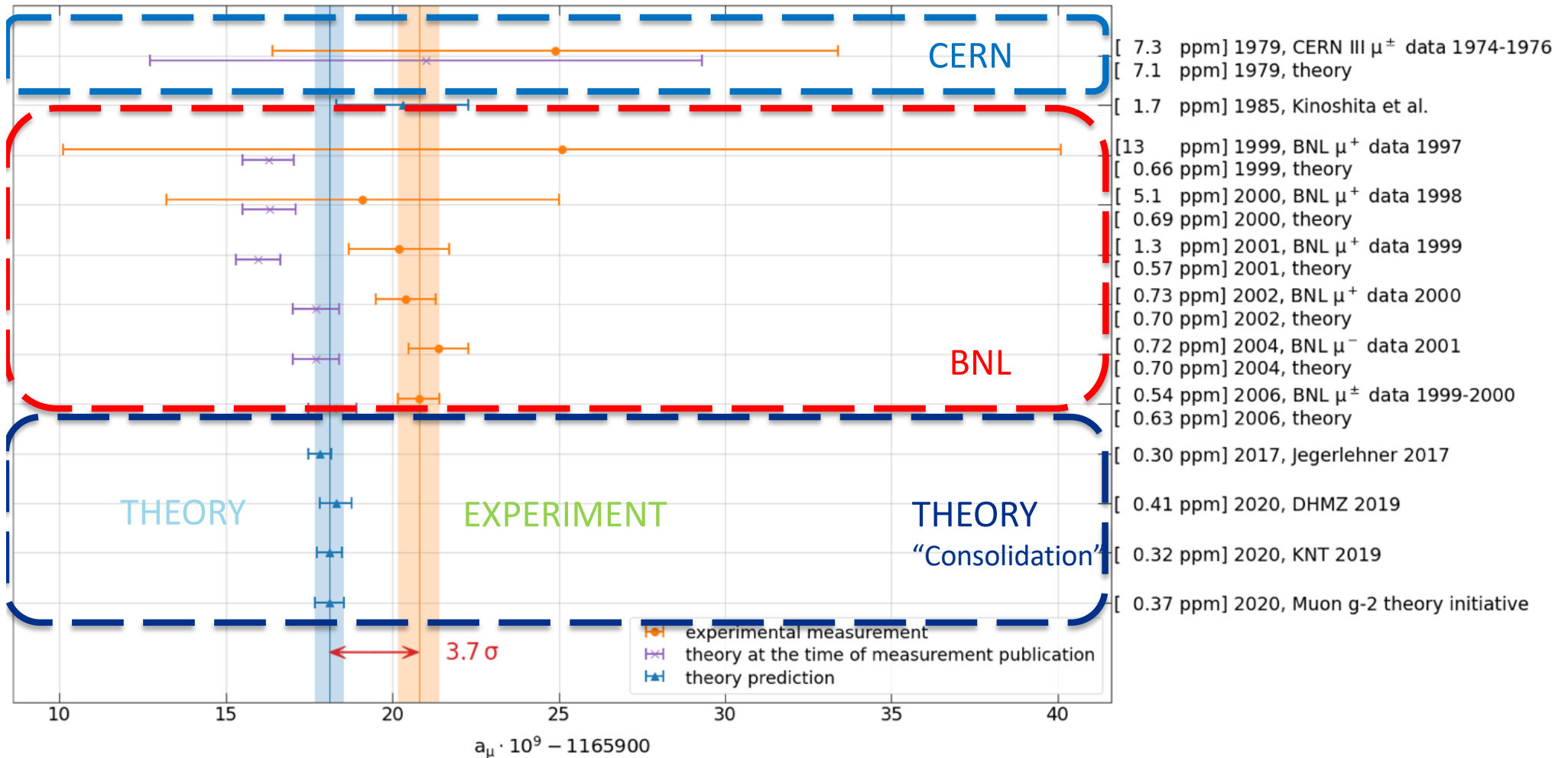
Theory initiative (June 2020) $11659181.0 \pm 4.3 \rightarrow 370 \text{ ppb}$

Experiment



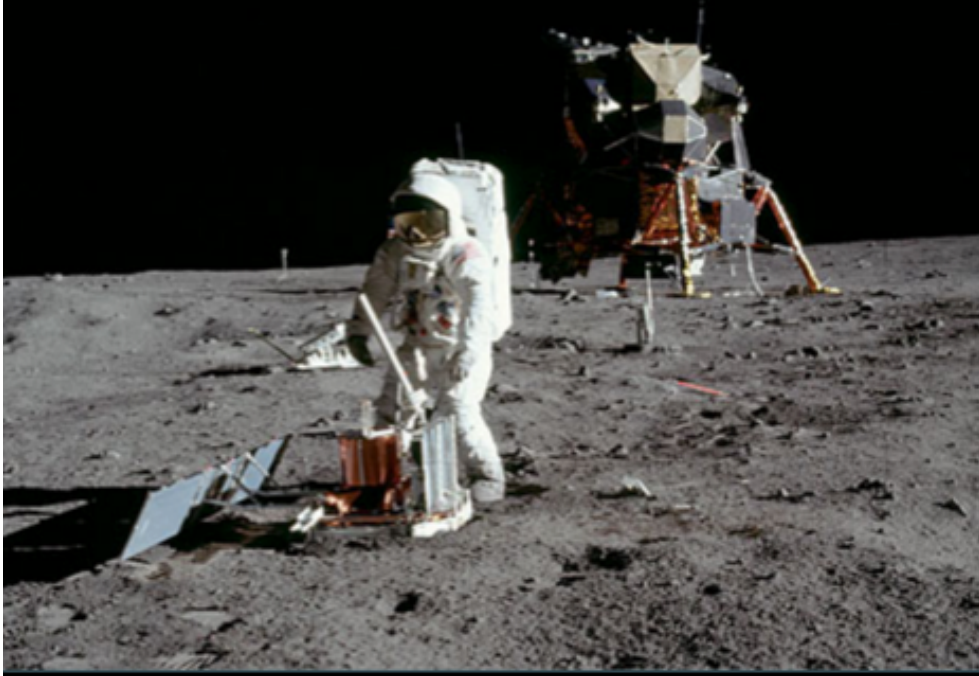
A rich history of g-2 Theory and measurements

History of muon anomaly measurements and predictions

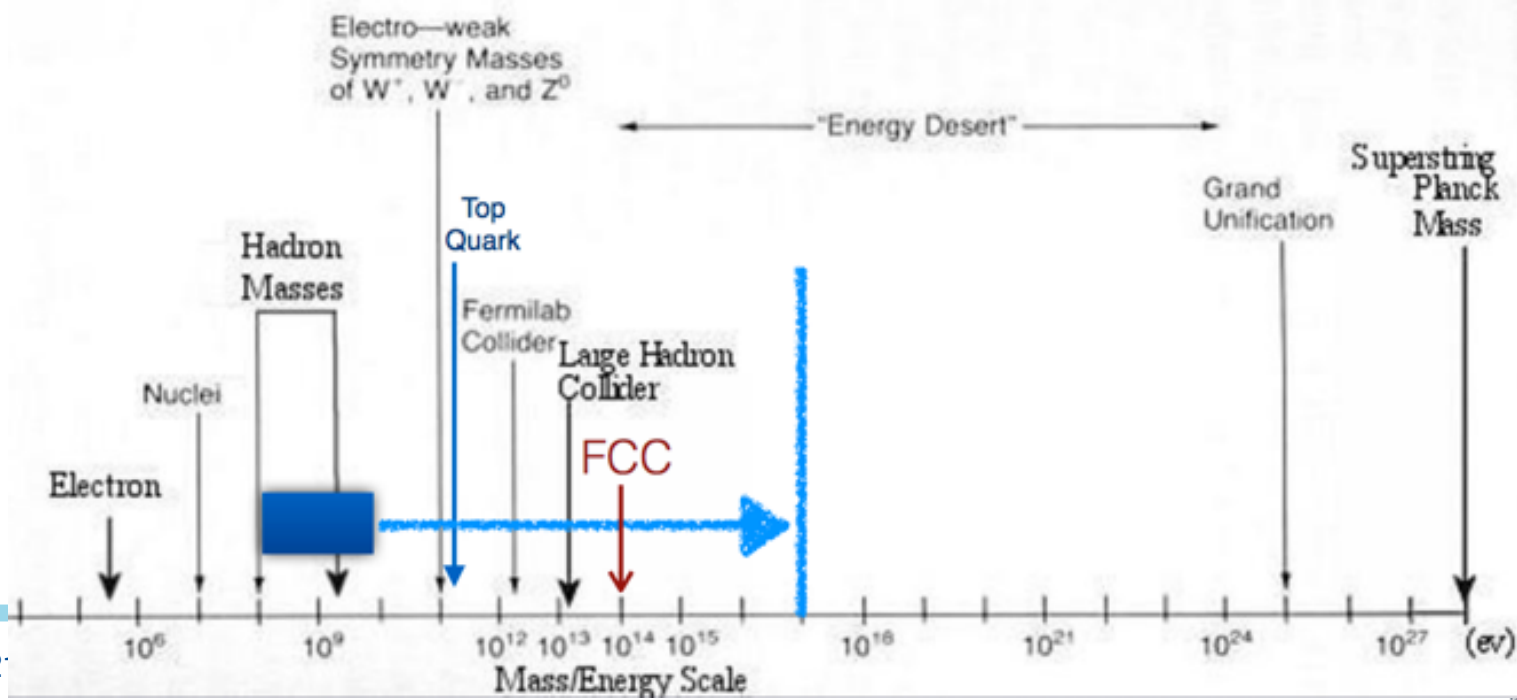
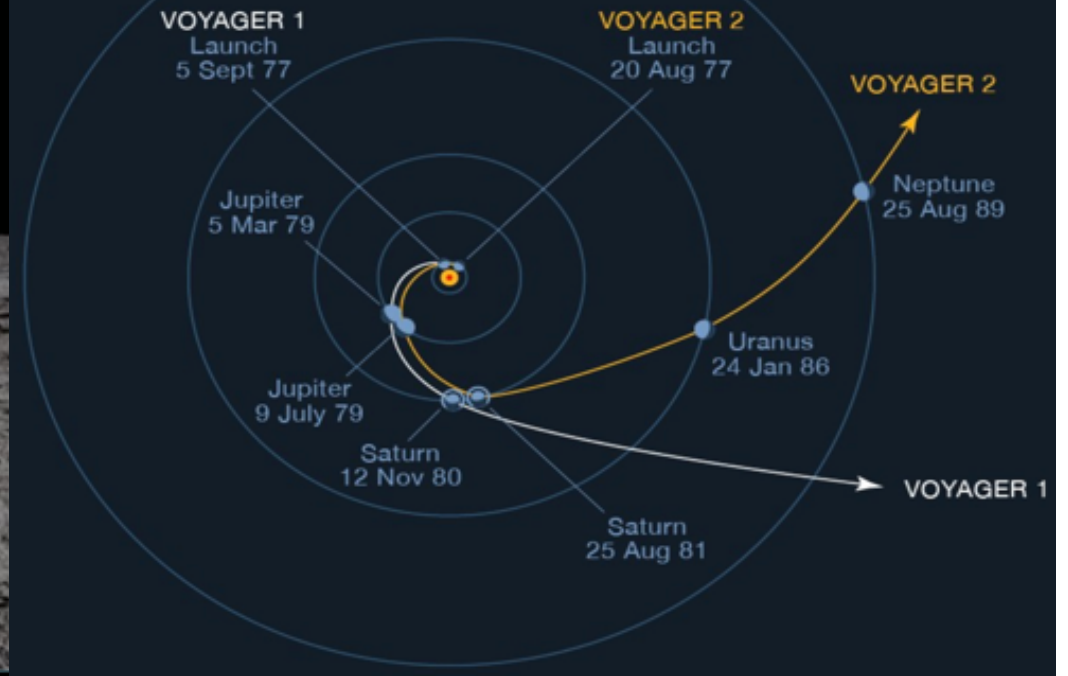


Tension between theory and experiment

HIGH ENERGY: systematic search



HIGH PRECISION: launching probes



...stituto Nazionale di Fisica Nucleare

The Fundamental Experimental Principle

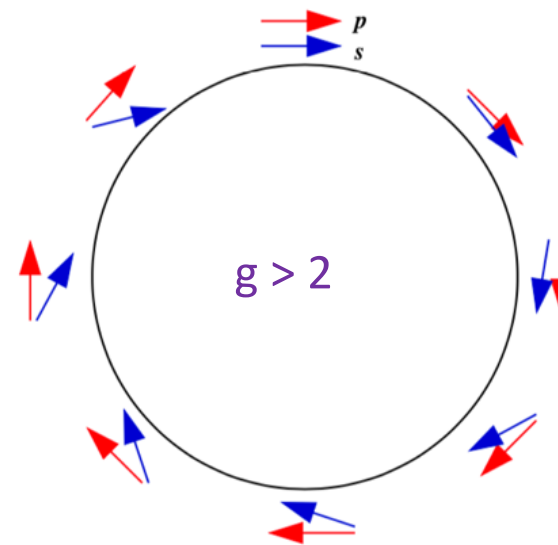
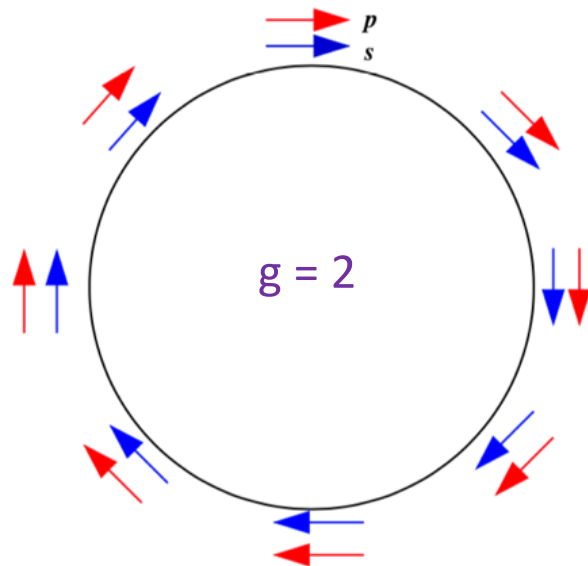


- Difference between **spin precession** and **cyclotron revolution** for a muon (charged particle with spin) in a magnetic field*:

$$\omega_a = \omega_s - \omega_c = g \frac{e}{2m} B - \frac{e}{m} B = \frac{g - 2}{2} \frac{e}{m} B = a_\mu \frac{e}{m} B$$

***s** and **p** are assumed to be in a plane perpendicular to **B**

- simple classical calculation
- the relativistic approach provides the same result



From particle to beam



- It is not possible to follow the spin of a single muon, we can only have an *information of the spin direction when the muon is produced and when it decays* (see next slide)
- Need a **beam of muons** →
 - focusing elements: using *electrostatic quadrupoles*
 - betatron oscillations around ideal trajectory
- Additional terms in the muon precession complex

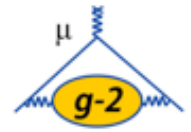
$$\vec{\omega}_a = -\frac{e}{mc} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} - a_\mu \left(\frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} \right]$$

Term due to ElectroStatic
Quadrupoles (ESQ)

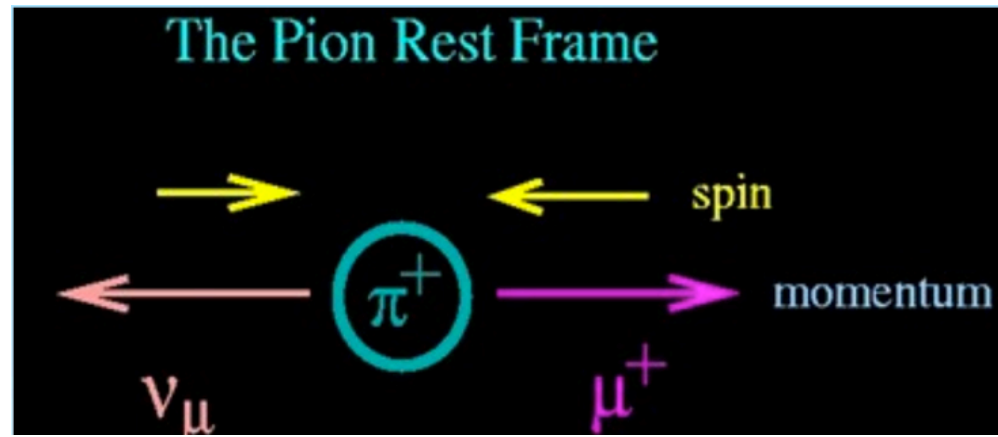
Term due to beam vertical
oscillations: pitch correction

Becomes ~ 0 at “magic γ ” ~ 29.3
or $p \sim 3.1$ GeV/c

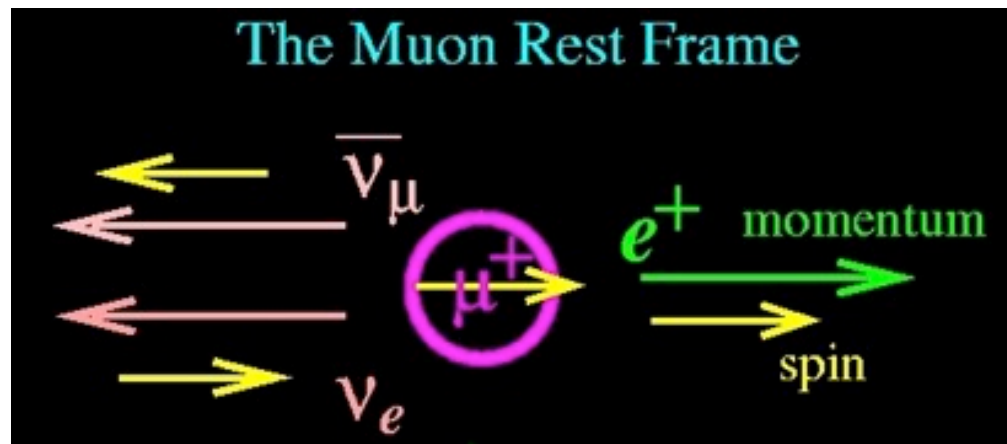
How do we measure the spin direction?

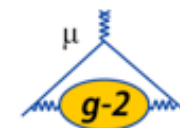


- Use V-A structure of weak decays to build a polarized beam...



- ... and to measure the muon polarization looking for energetic positrons

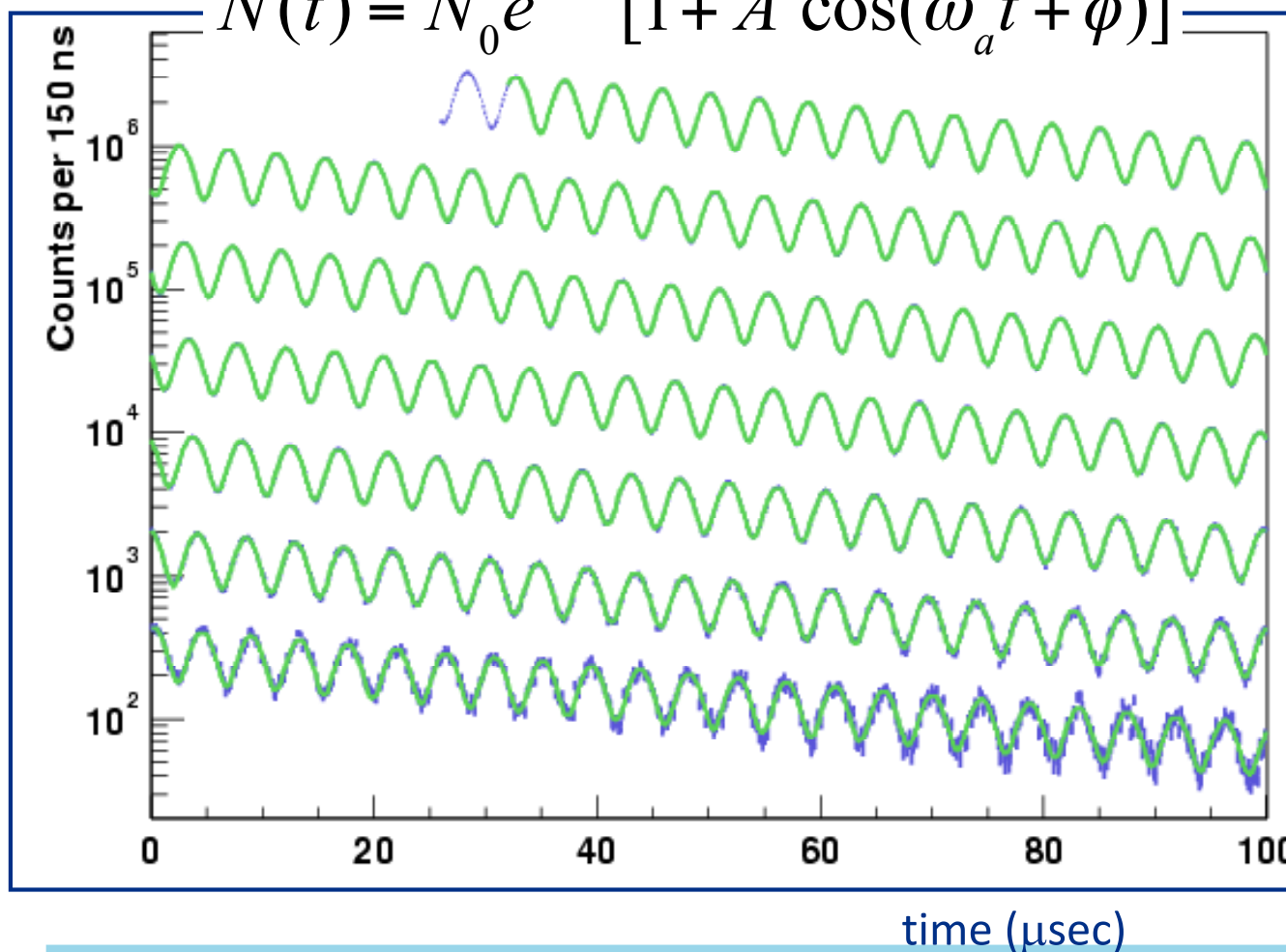




Measuring the spin precession

- The number of observed positrons above a threshold energy oscillates with the $\omega_a/2\pi$ frequency due to spin precession

$$N(t) = N_0 e^{-t/\tau} [1 + A \cos(\omega_a t + \phi)]$$



- exponential decay modulated by spin precession
- note that the x-axis "wraps up" every 100 μsec for a total of $\sim 700 \mu\text{s} \rightarrow \sim 11$ muon lifetimes

Extracting a_μ (simplified)

$$\omega_a = a_\mu (e/m)B \rightarrow a_\mu = \omega_a (m/eB)$$

by expressing B in terms of the (shielded) proton precession frequency ($B = \hbar\omega'_p/2\mu'_p$):

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \cdot \frac{\mu'_p m_\mu g_e}{\mu_e m_e 2} = R'_\mu \cdot \frac{\mu'_p m_\mu g_e}{\mu_e m_e 2}$$

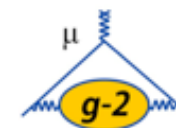
What we measure External (precise) data

$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p}$$

ratio of muon to proton precession in the same magnetic dipole field

$\tilde{\omega}'_p =$ (shielded) Proton Larmor angular velocity **weighted for the muon distribution**

Extracting a_μ (more precise) left as reference



$$\frac{\mu_e(H)}{\mu'_p(T)}$$

Measured to 10.5 ppb accuracy
at *reference temp. $T_r = 34.7^\circ\text{C}$*
Metrologia **13**, 179 (1977)

$$\frac{m_\mu}{m_e}$$

Known to 22 ppb from
muonium hyperfine splitting
Phys. Rev. Lett. **82**, 711 (1999)

$$\frac{\mu_e}{\mu_e(H)}$$

Bound-state QED (exact)
Rev. Mod. Phys. **88** 035009 (2016)

$$\frac{g_e}{2}$$

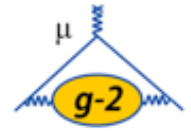
Measured to 0.28 ppt
Phys. Rev. A **83**, 052122 (2011)

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p}$$

Total uncertainty of 25 ppb

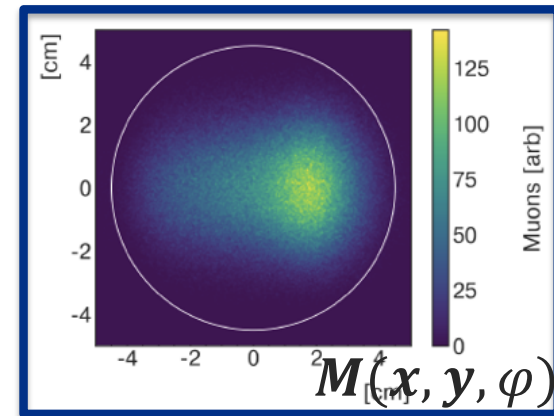
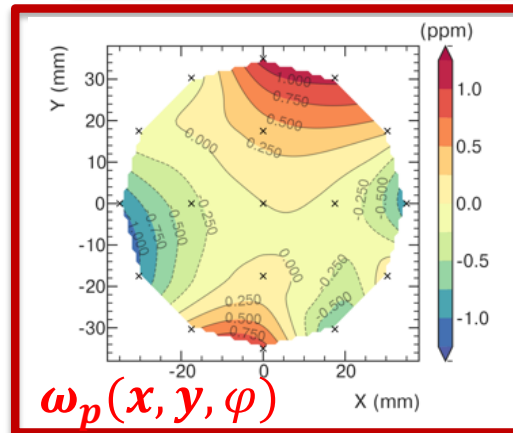
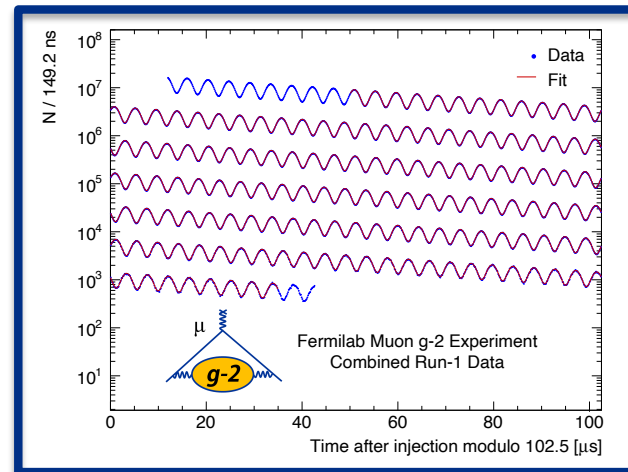
The key ingredients



ω_a =muon spin precession respect to momentum (in B field)

$$\omega_a$$

$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \sim$$



$$\tilde{\omega}'_p = \omega'_p(x, y, \varphi) \cdot M(x, y, \varphi)$$

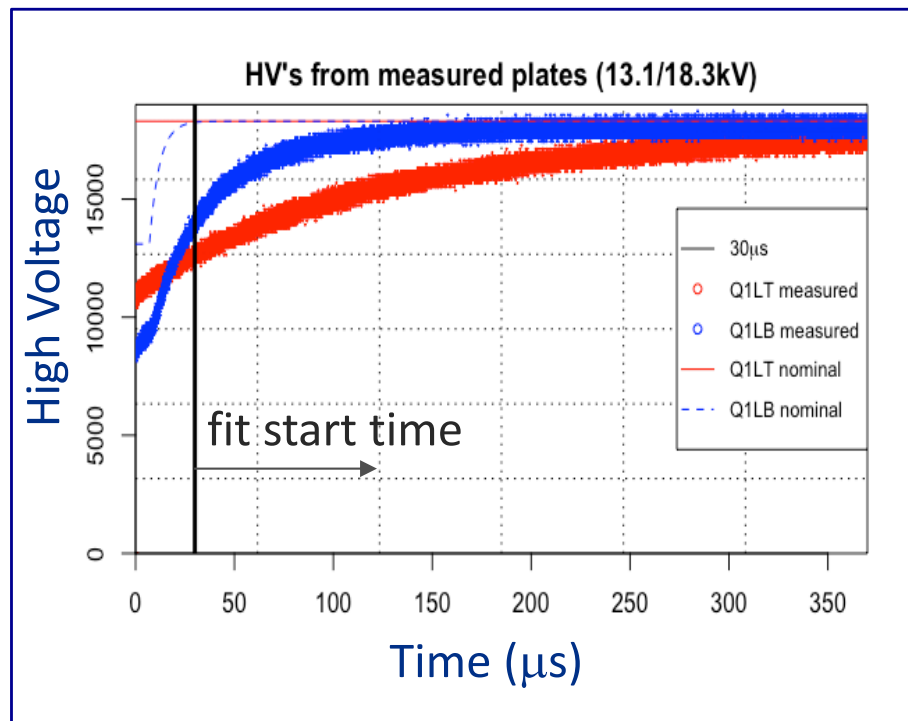
ω_p =proton precession frequency

M=muon spatial distribution

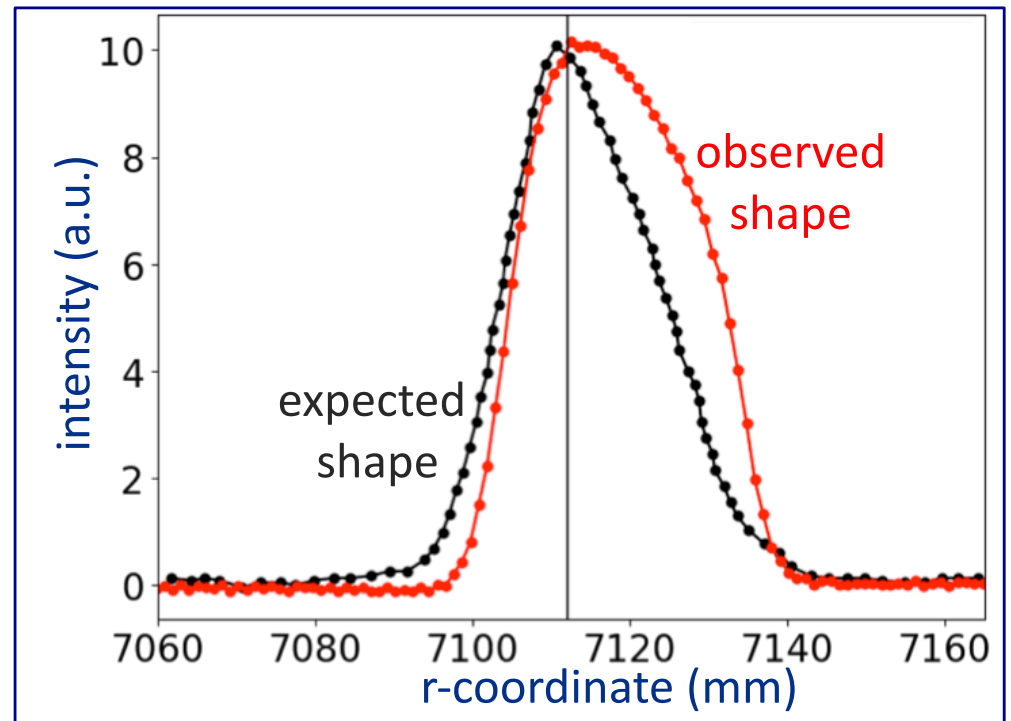
Problems related to Storage Ring in Run1



- Two main problems observed in Run-1, fixed in Run-2:
 - bad resistors (QUADRUPOLES)
 - low current (KICKER)



Two resistors providing HV to the vertical plates of a QUADRUPOLE had a slow recovery time
The beam slowly moved vertically during fill.



Beam radial profile: ring acceptance $\Delta p = \pm 0.5\%$
Asymmetry due to not perfect KICK

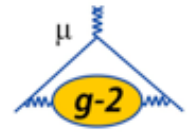
Additional (important) corrections



- On top of the *key ingredients* additional corrections due to beam dynamics and transient fields have to be included

$$R'_\mu = \left(\frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + \underbrace{C_e + C_p + C_{ml} + C_{pa}}_{\text{Corrections due to beam dynamics}})}{f_{calib} \cdot \omega'_p(x, y, \varphi) \cdot M(x, y, \varphi) \cdot (1 + \underbrace{B_k + B_q}_{\text{Corrections due to transient magnetic fields}})} \right)$$

Measuring the ratio

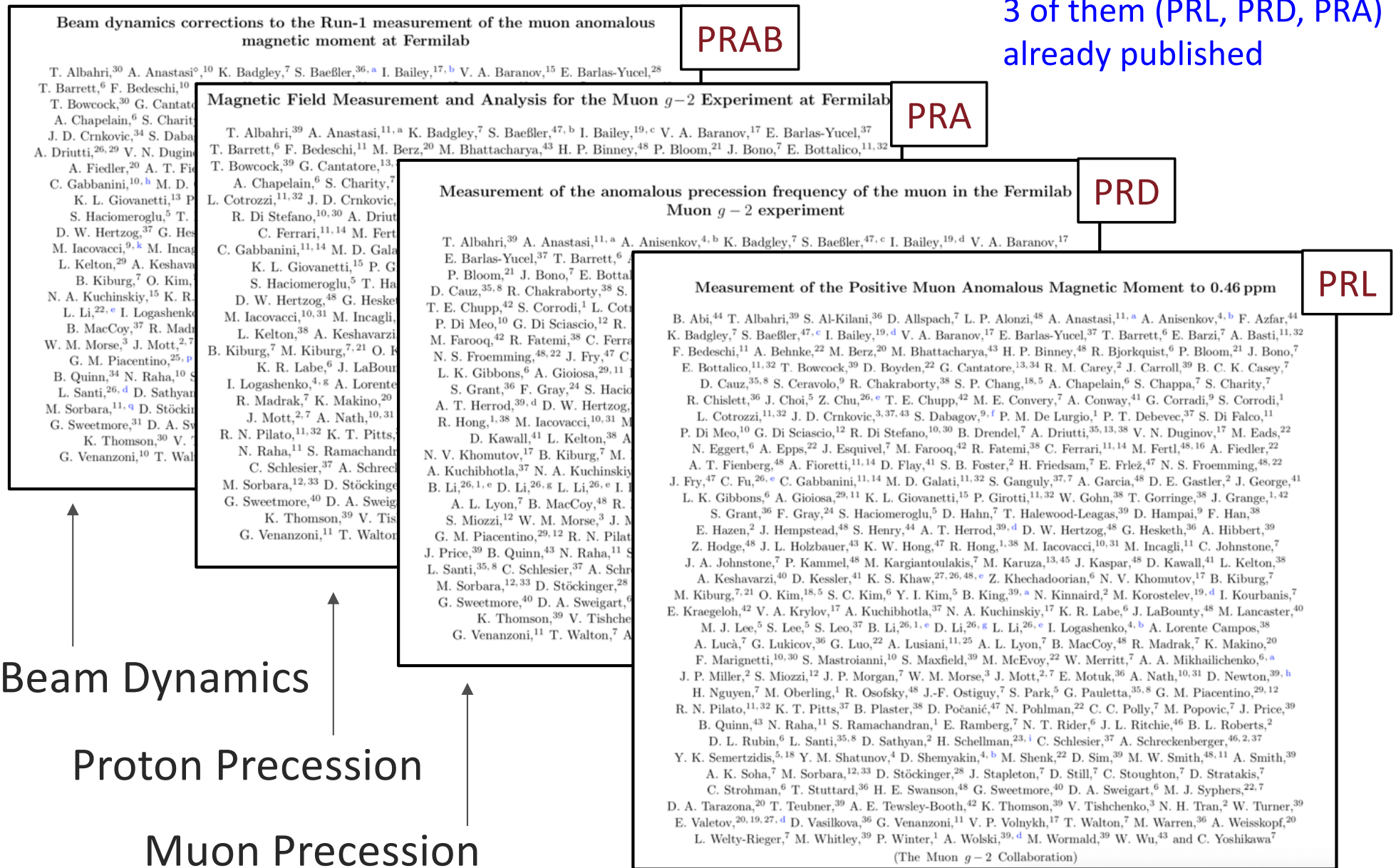


- Contributo specifico dei gruppi italiani

$$R'_\mu = \left(\frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib} \cdot \omega'_p(x, y, \varphi) \cdot M(x, y, \varphi) \cdot (1 + B_k + B_q)} \right)$$

Four articles on arXiv for details

3 of them (PRL, PRD, PRA) already published



APRIL 2017

BEAM DYN.

FIELD

DETECTORS

muons

Inflector

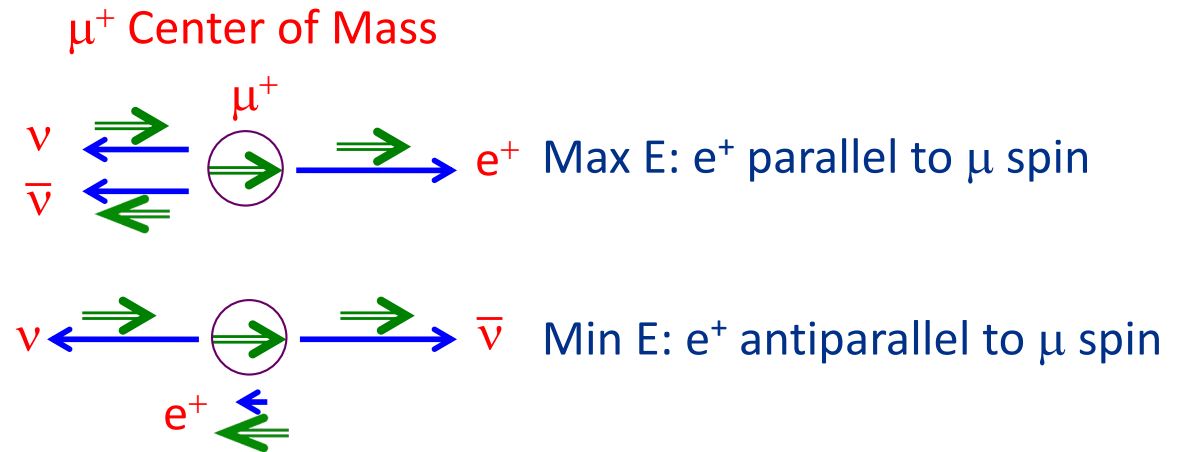
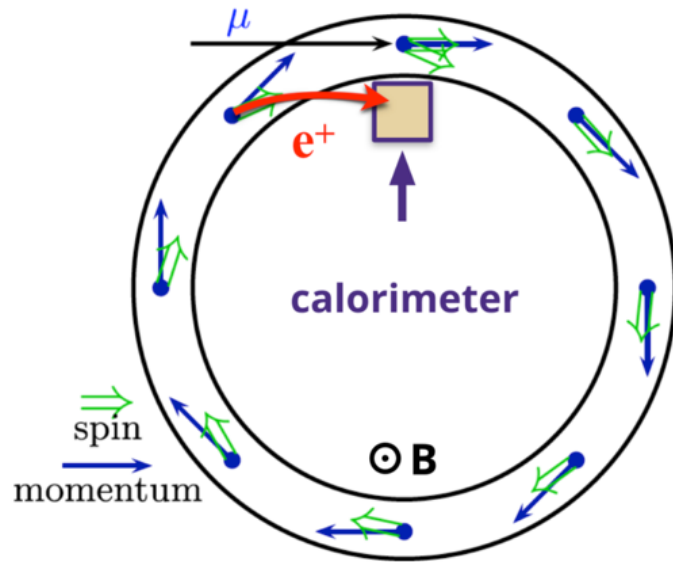
QUADS

24 Calorimeters + 2 trackers located all around the ring

NMR probes and electronics located all around the ring

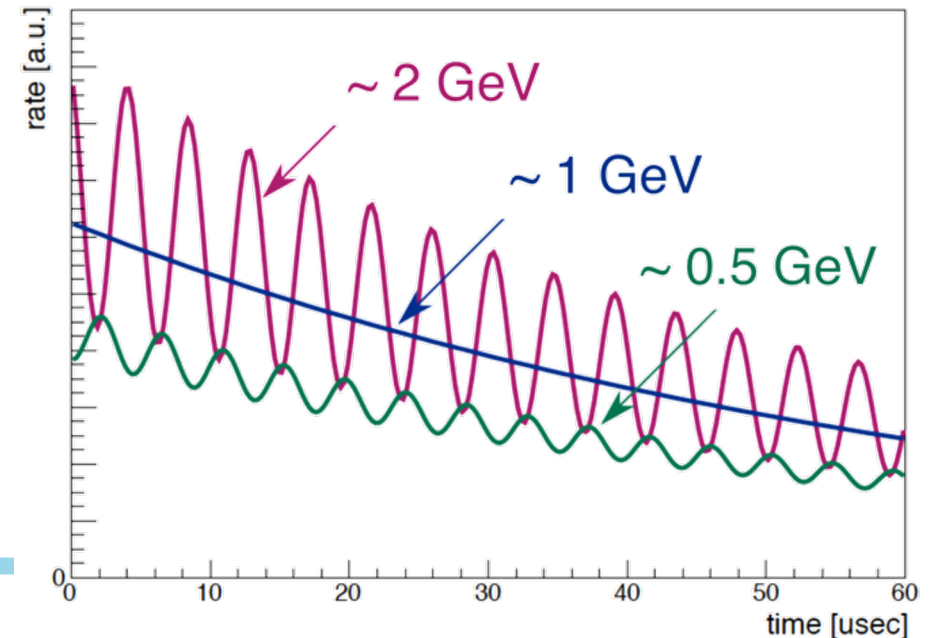
Kicker

ω_a principle of measurement

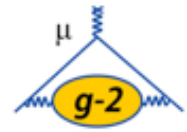


$$N(t) = N_0 e^{-t/\tau} (1 + A \cos(\omega_a t + \varphi))$$

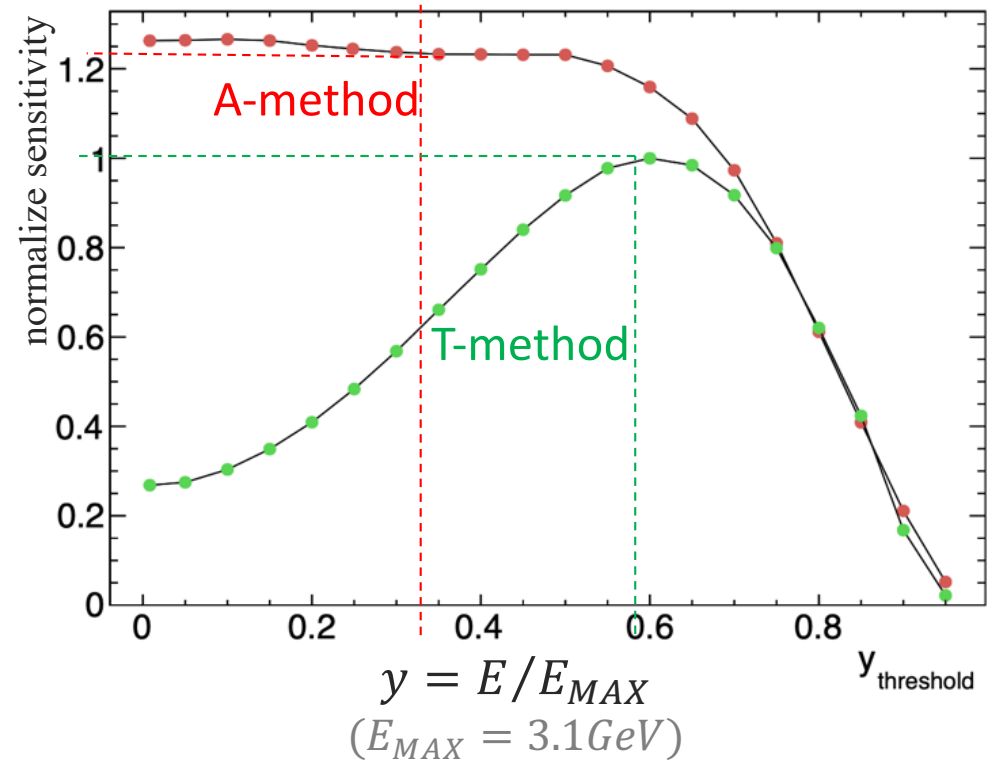
- positron direction correlated with muon spin direction
- correlation depends on *positron energy*: the Asymmetry $A(E)$ can be positive, null or negative



Optimizing the statistical sensitivity



- **T-method** = counting positrons above E_{thr} vs time
 - by decreasing the threshold the asymmetry decreases but the number of events increases \rightarrow *max sensitivity for $E_{thr} \sim 1.7$ GeV*
- **A-method** = each positron is weighted by the value of its asymmetry $A(E)$
 - optimize statistical sensitivity
- In theory the A-method can use *all decay positrons*, in practice, due to calorimeter acceptance and to low $A(E)$ value at low energies, only positrons with $E_{thr} > 1.1$ GeV ($y > 0.3$) are used



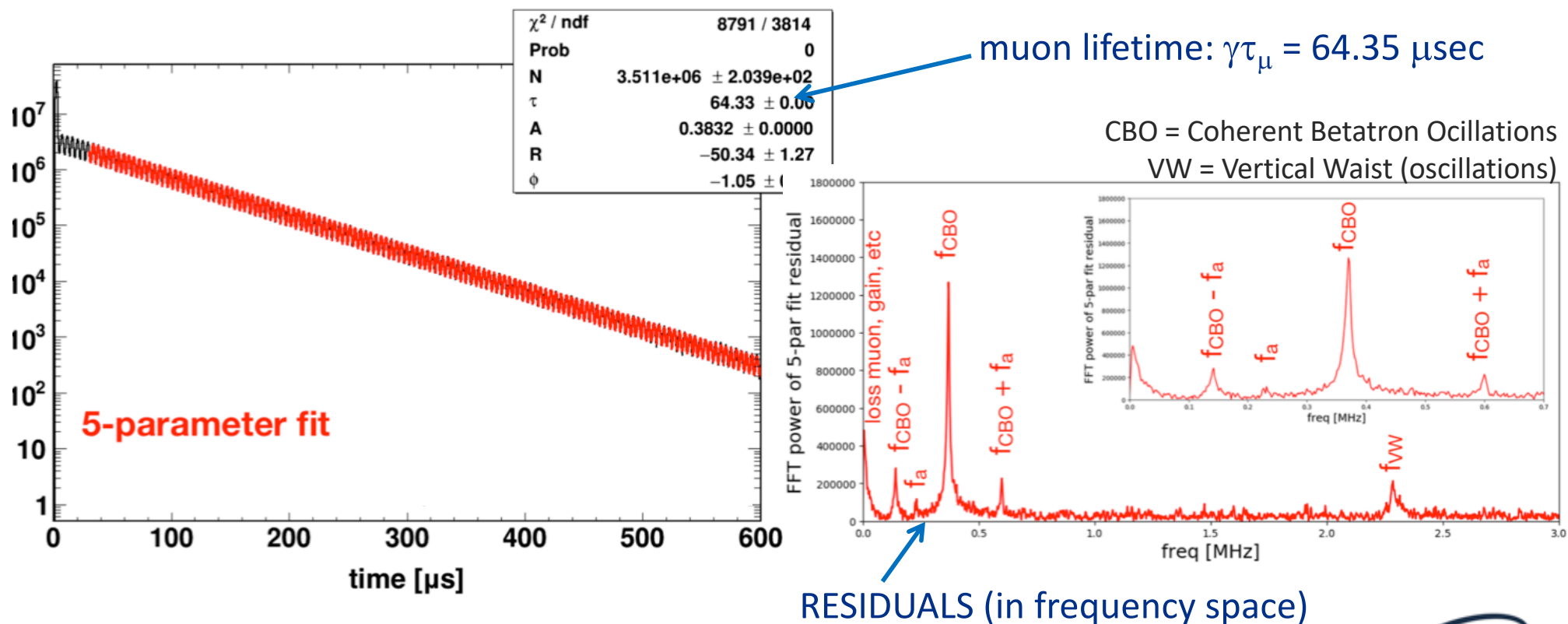


Measuring ω_a : 5 parameters fit function

- Fit with simple positron oscillation:

$$N_{ideal}(t) = N_0 \exp(-t/\tau_\mu) [1 + A \cos(\omega_a t + \varphi)]$$

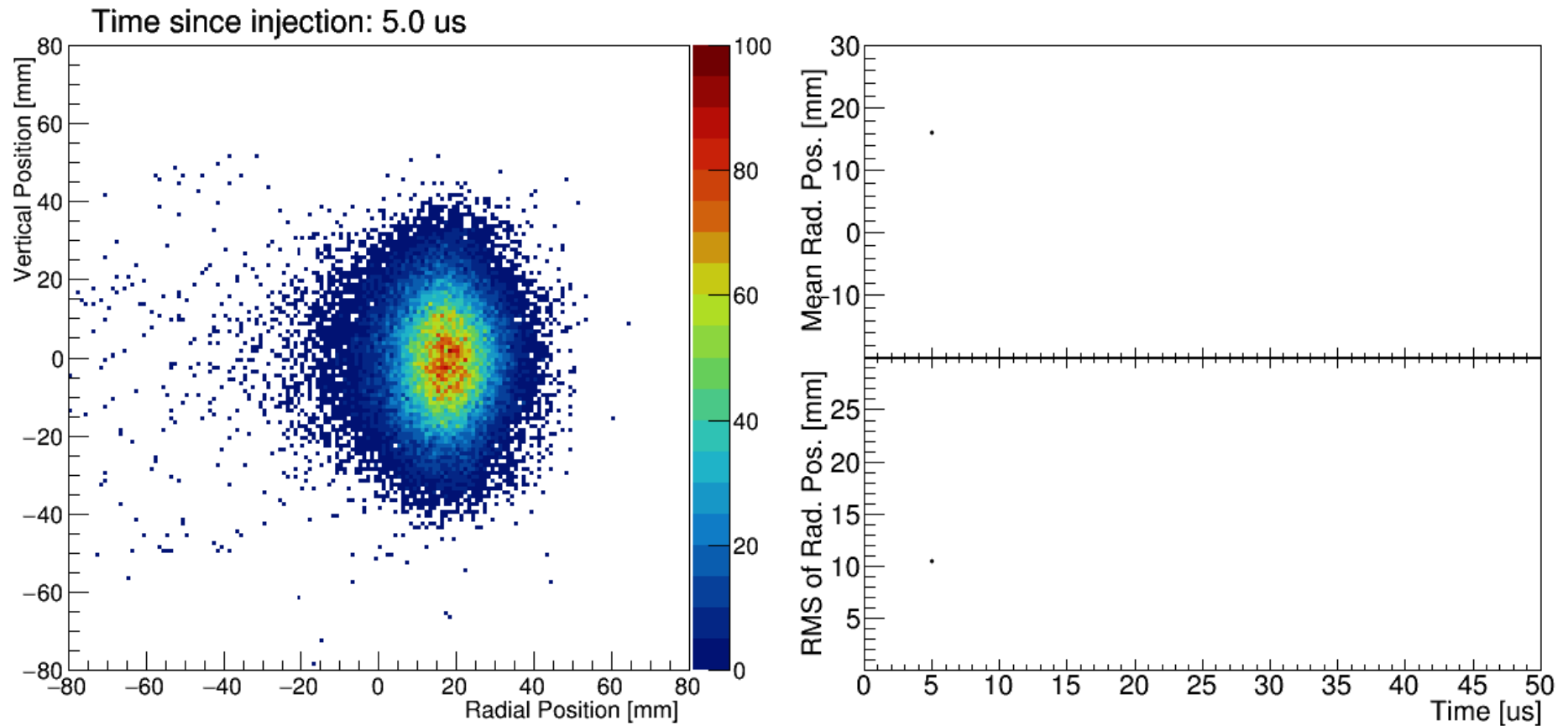
- This simple fit is clearly not sufficient and well defined resonances are observed in the residuals



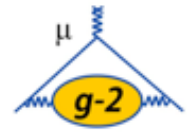
Beam oscillations



- Beam oscillations are accurately measured by the tracker and folded into the fit function



The complete 22 parameters fit function



ω_y, ω_{VW} vertical oscillations

$\omega_{CBO}, \omega_{2CBO}$ radial oscillations

$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_a t + \phi \cdot \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{2\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t)t + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t)t + \phi_y) e^{-\frac{t}{\tau_y}}$$

Red = free parameters
Blue = fixed parameters

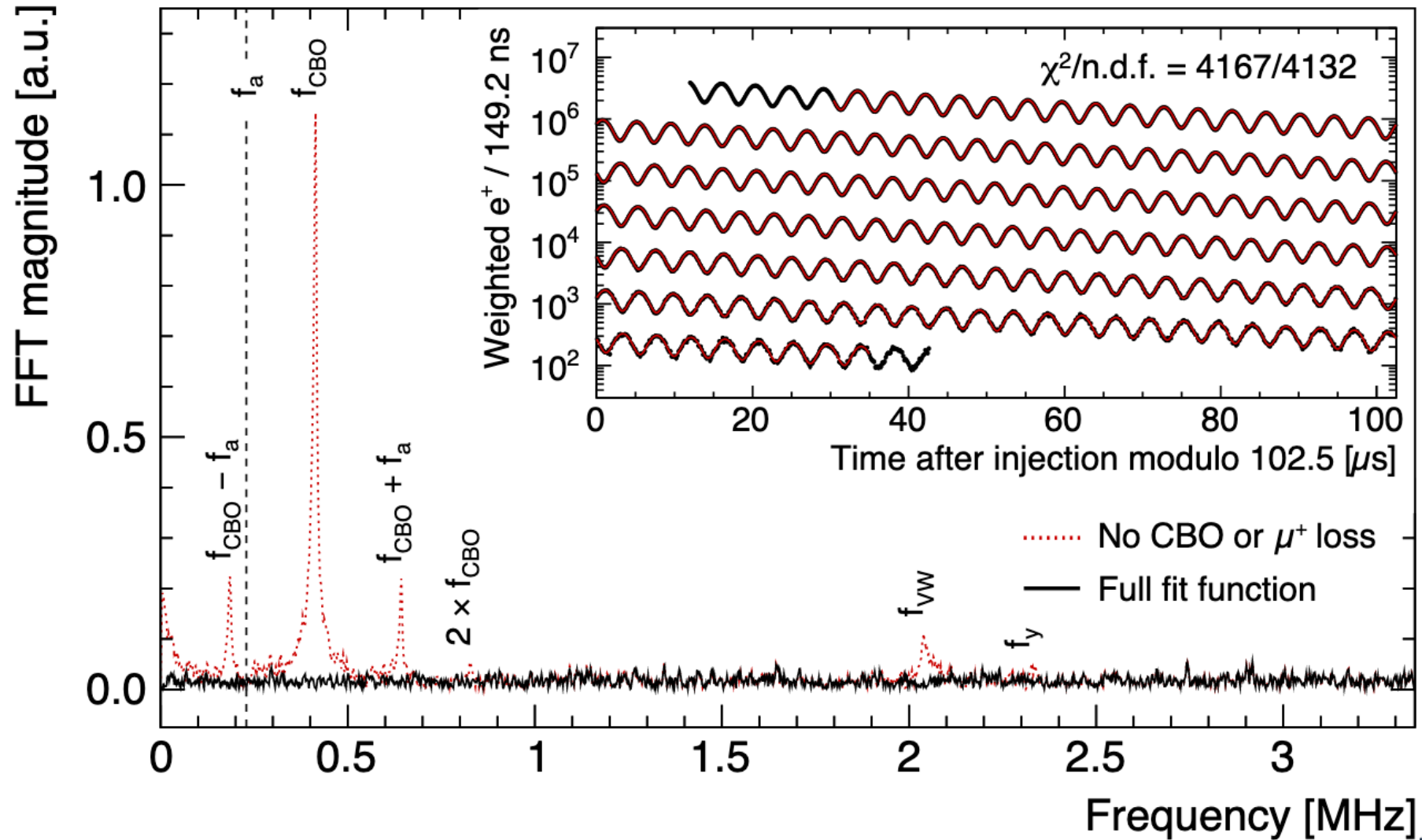
$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons } (\mu \text{ hitting collimators})$$

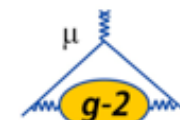
$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

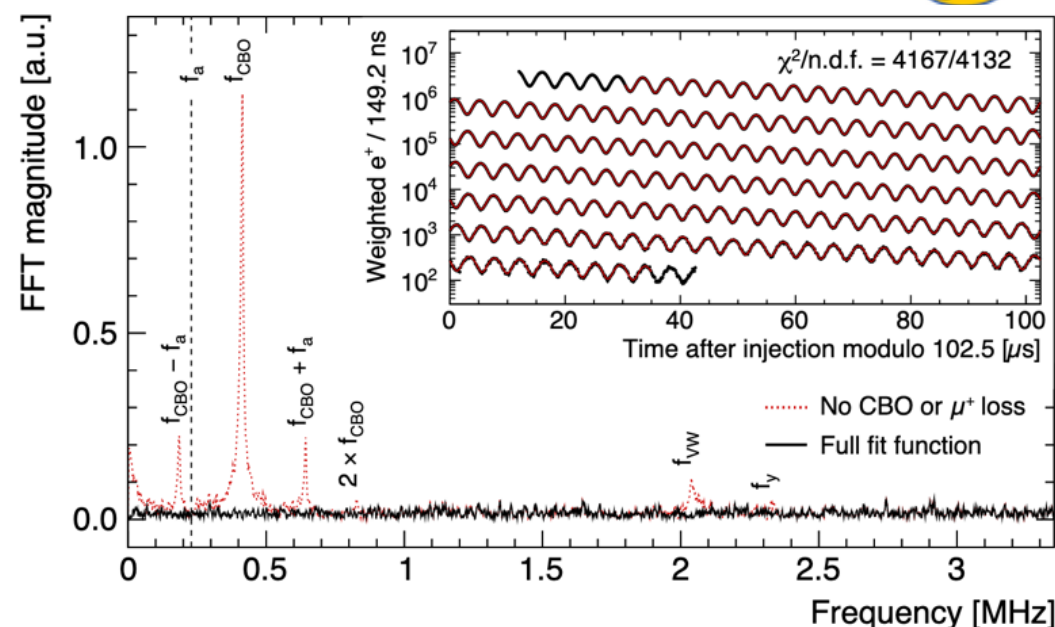
Final fit





Final fit

- Four datasets in Run-1
- ω_a value expressed in terms of a “blinded” (see later) parameter R
- Four independent analyses used for the final result
- The analyses are strongly correlated, although not identical, and contribute to the robustness of the final result



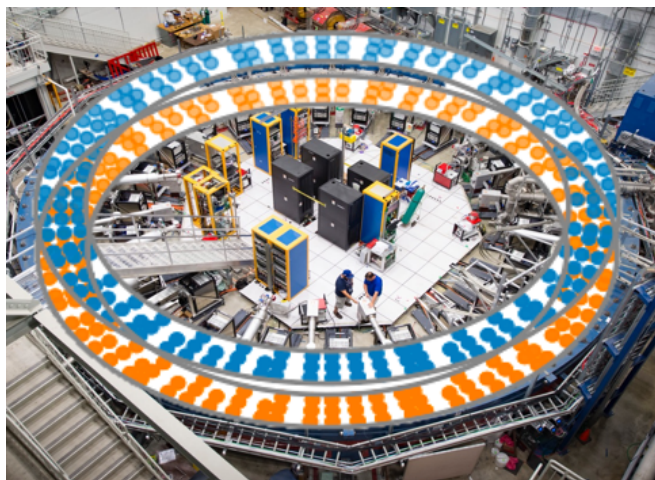
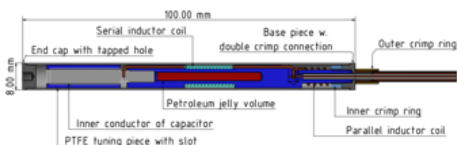
Recon. Method	Pileup	R [ppm] for each dataset			
		Run-1a	Run-1b	Run-1c	Run-1d
global A	empirical	-82.98 ± 1.21	-81.70 ± 1.03	-82.30 ± 0.82	-82.34 ± 0.68
local A	shadow	-83.23 ± 1.20	-81.77 ± 1.02	-82.35 ± 0.82	-82.48 ± 0.67
local A	shadow	-83.17 ± 1.21	-81.84 ± 1.03	-82.50 ± 0.83	-82.45 ± 0.68
local A	pdf	-83.39 ± 1.22	-81.72 ± 1.04	-82.32 ± 0.83	-82.42 ± 0.68

We also need B to determine a_μ

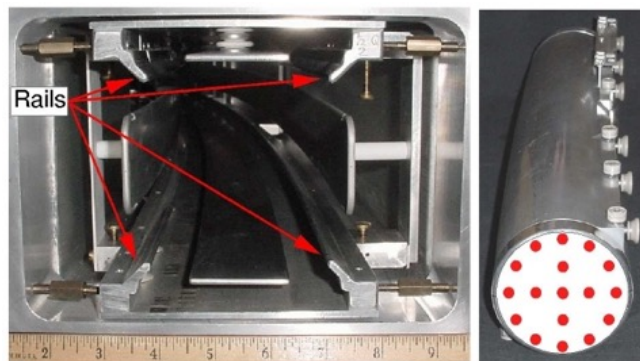
$$\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{mc}$$

- Use NMR to find B-field in terms of proton precession frequency ω_p (comagnetometer)

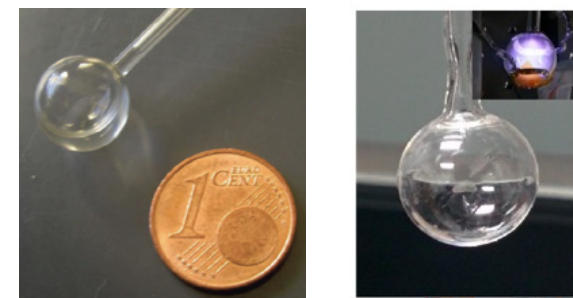
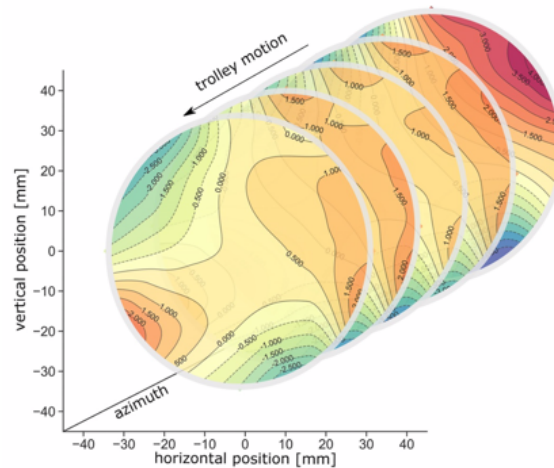
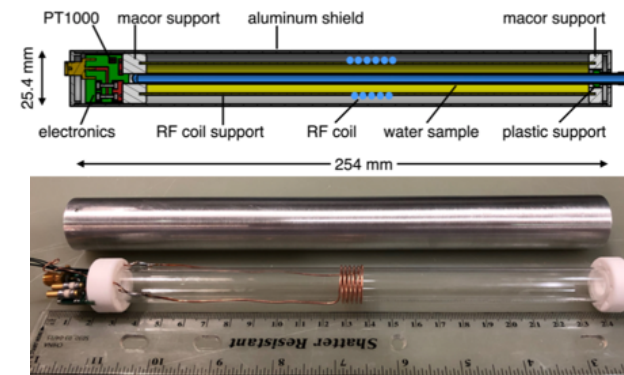
378 fixed probes monitor 24/7



NMR trolley maps field every 3 days



Trolley cross-calibrated to absolute probes



Absolute probes all cross-calibrated at ANL test magnet

$\omega'_p \rightarrow \tilde{\omega}'_p$: muon weighted average

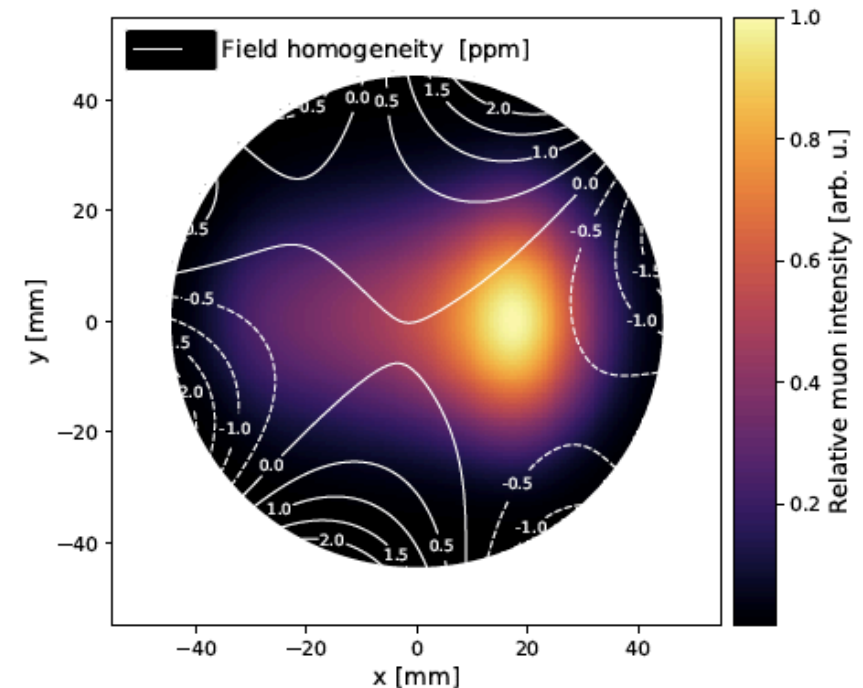


- Need field experienced by muons
- Use tracker to reconstruct e+ track and muon vertex in storage ring
- Use beam dynamics models, tuned to the tracker data, to extrapolate the distribution all around the ring
- Systematic uncertainties mostly due to Beam Dynamics models used for extrapolation, to field map and to tracker alignment

Muon's view of a tracker



$$\delta_{\tilde{\omega}'_p} \sim 56 \text{ ppb}$$



Final uncertainties from Run 1

Quantity	Correction Terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	–	434
ω_a^m (systematic)	–	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{\text{calib}} \langle \omega_p(x, y, \phi) \times M(x, y, \phi) \rangle$	–	56
B_k	-27	37
B_q	-17	92
$\mu'_p(34.7^\circ)/\mu_e$	–	10
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	157
Total fundamental factors	–	25
Totals	544	462

- 462 ppb overall error
 - 434 ppb statistical
 - 157 ppb systematic
 - 22 ppb external inputs
- Results for Run 1 are vastly dominated by statistical error
- At 157 ppb systematic error
 - Nearly half of BNL
 - Not quite to 100 ppb goal

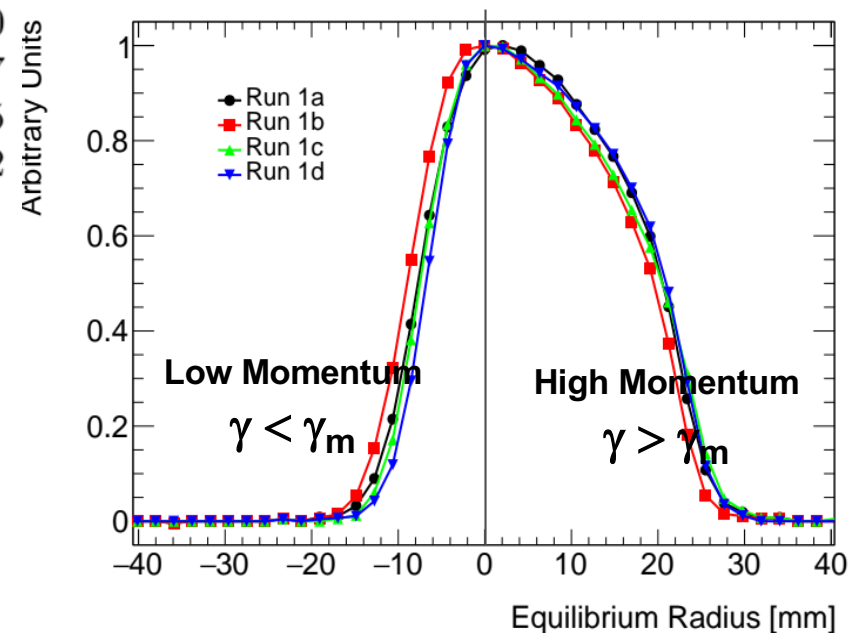
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- Correction terms larger than total error
- Dominated by Electric Field corrections C_E
- Related to non-centered radial distribution
- Fixed in Run-2

$$\vec{\omega}_a = -\frac{e}{mc} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right]$$

(..) = 0 for $\gamma = \gamma_{\text{magic}}$

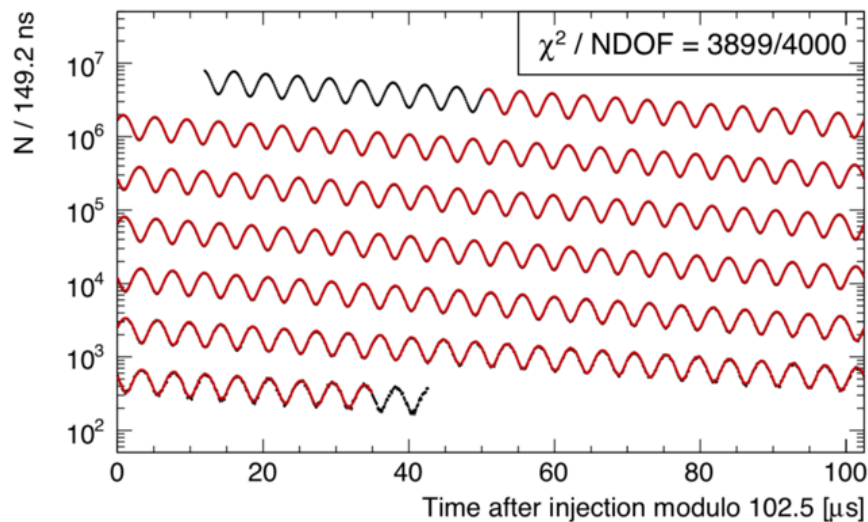


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$g_e/2$	–	0
Total systematic	–	157
Total fundamental factors	–	25
Totals	544	462

- Two largest systematics:
 - *phase acceptance*
 - *quadrupole field*

C_{PA} – Phase acceptance error

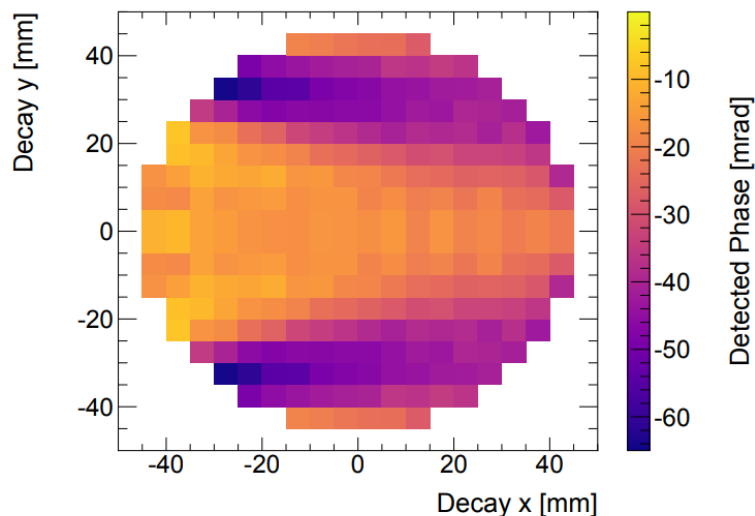


$$f(t) \simeq N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)]$$

But what if the phase of the muon population changes in time $\phi(t)$?

$$\begin{aligned} \cos(\omega_a t + \phi(t)) &= \cos(\omega_a t + \phi_0 + \phi' t + \dots) \\ &= \cos((\omega_a + \phi')t + \phi_0 + \dots) \end{aligned}$$

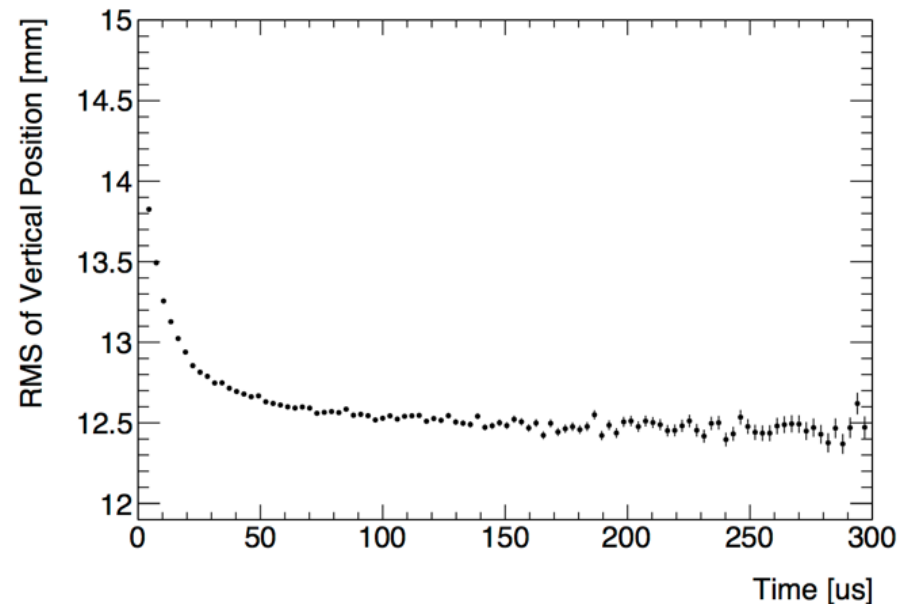
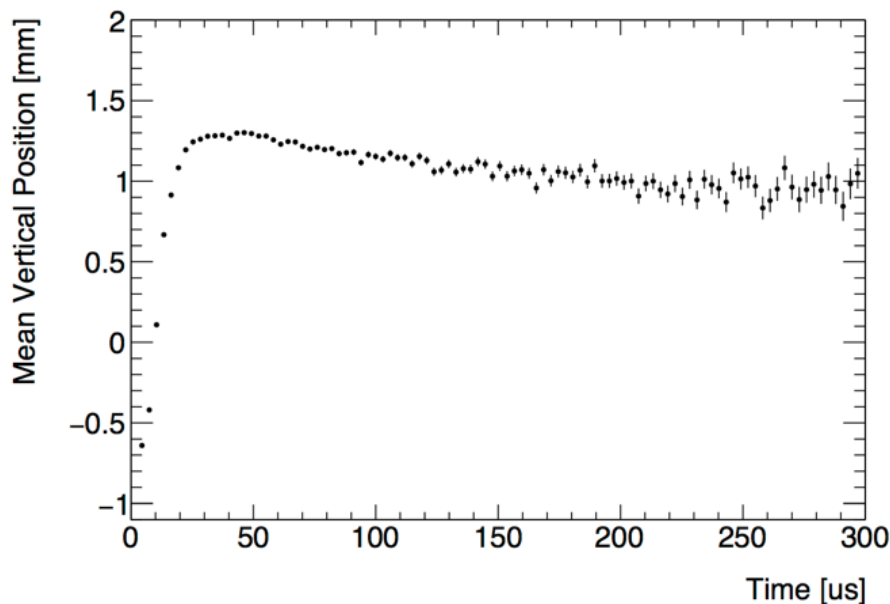
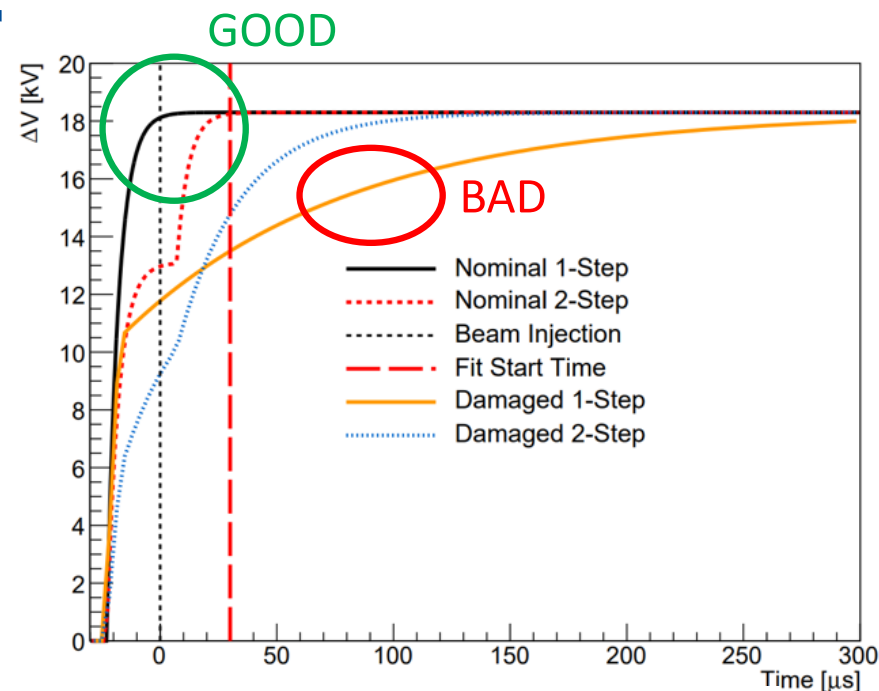
The extracted ω_a is shifted by ϕ' 😬



- The decay positrons we detect carry a particular phase
- That phase depends on muon decay position (x,y) and energy E
- Not a big issue if the muon distribution remains stable in the gap

C_{PA} – Phase acceptance error

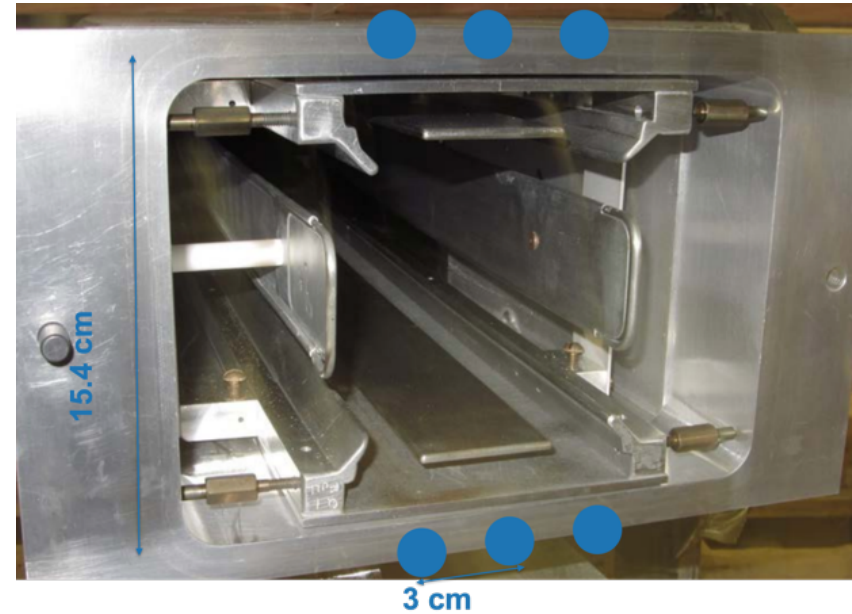
- HV resistors failed \rightarrow changing E-field \rightarrow beam vertical mean and width changed
- $C_{PA} = -158 \text{ ppb}$, $\delta_{PA} = 75 \text{ ppb}$
- Faulty resistors fixed before Run-2



ElectroStatic Quadrupoles transient field B_q



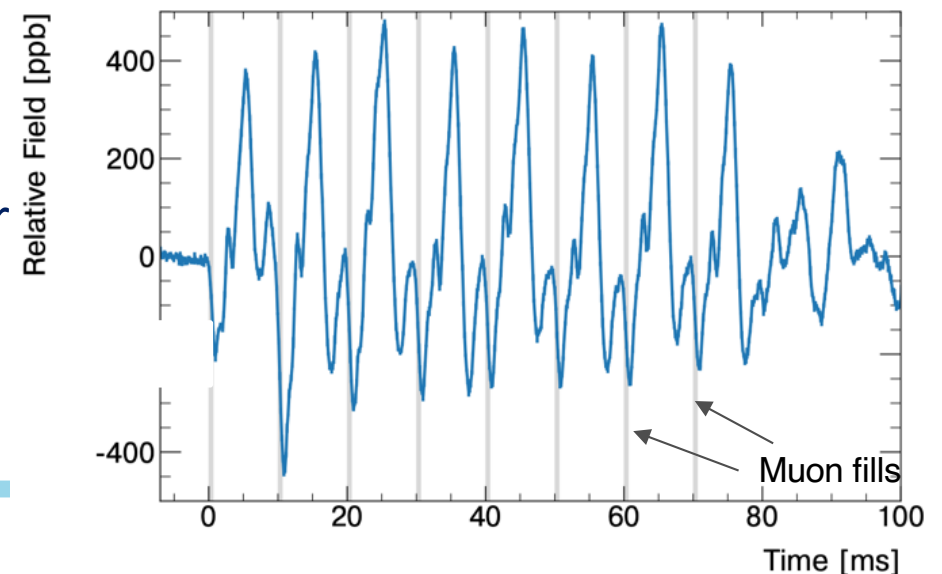
- The ESQ are charged/discharged every muon fill (700ms)
- The electric pulse induces mechanical vibrations in the plates which generate magnetic perturbations
- Special NMR probes measured B_q at several positions



Quad Plates inside Vacuum Chamber

$$B_q \sim 20 \text{ ppb}, \delta_{B_q} \sim 90 \text{ ppb}$$

- Uncertainty dominated by limited number of measurements in Run-1, reduced in Run-2 by more measurements



The blinding



$$R'_{\mu} = \left(\frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib} \cdot \omega'_p(x, y, \varphi) \cdot M(x, y, \varphi) \cdot (1 + B_k + B_q)} \right)$$

- Clock frequency f_{clock} uncalibrated by Joe Lykken and Greg Bock (FNAL Directorate) Feb 22 2018
 - stop in each week to check clock and sealing
- Secret envelopes kept until physics analysis complete and ready to be revealed Feb 25 2021

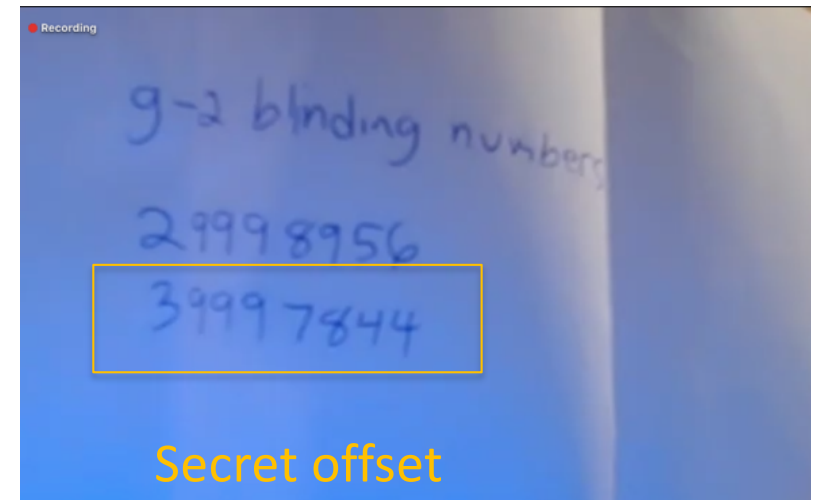


a_μ : Unblinding



On February 25 the collaboration met for the unblinding:

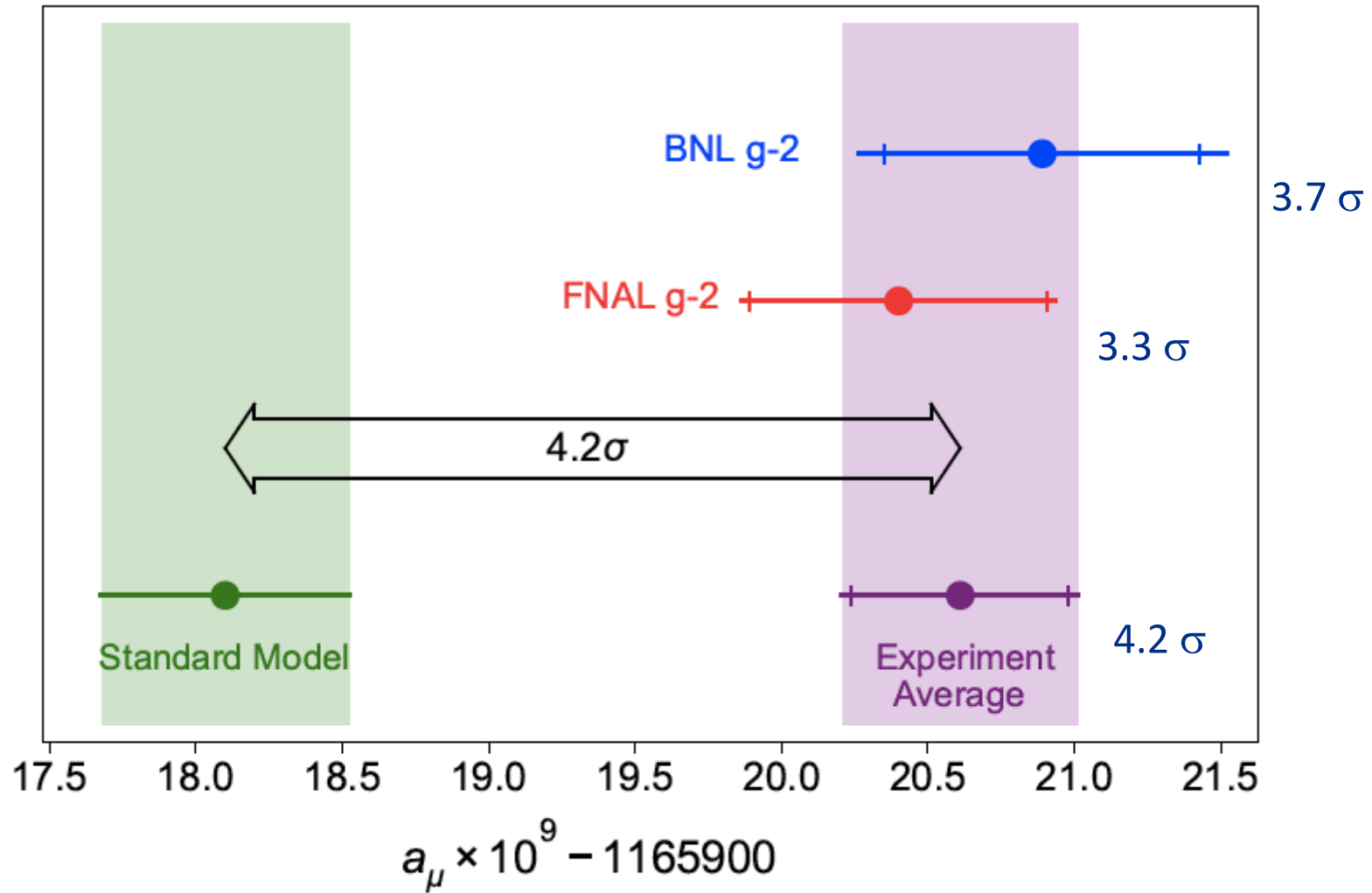
- 1) The *box* (envelope) was opened
- 2) The number was plugged into two independent programs
- 3) And the result was....



Are we ready to unblind?



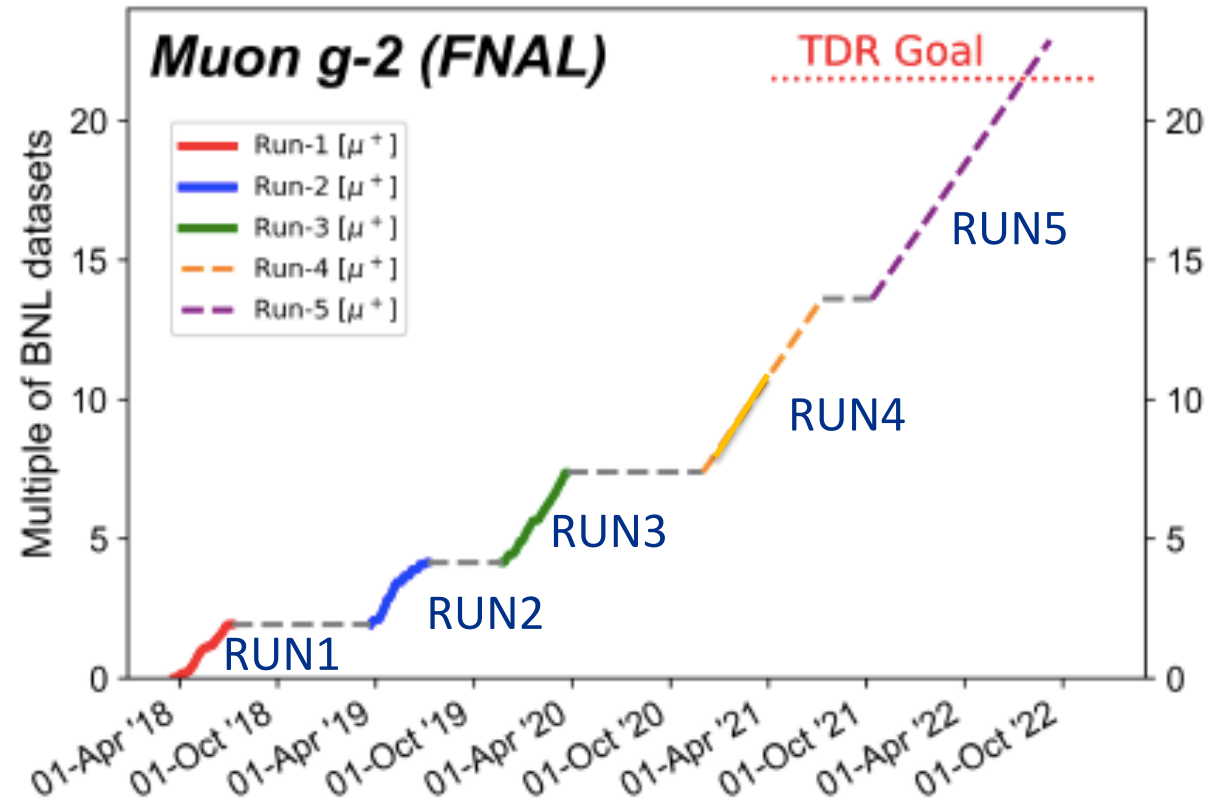
a_μ : Unblinding



The future of E989 *Muon g-2* (Fnal)



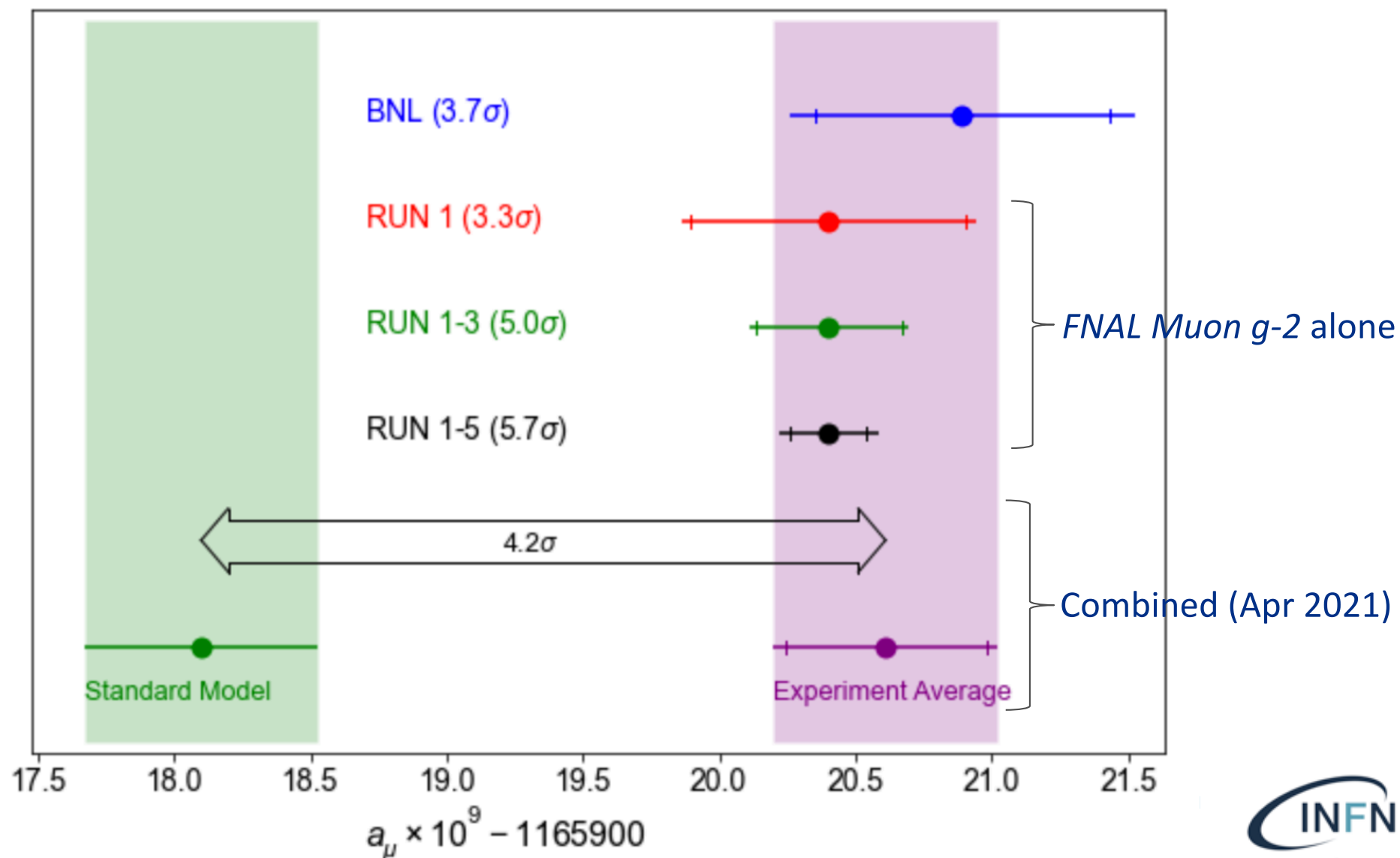
- RUN-1 is only 6% of the final dataset
- Analysis of RUN-2/3 in progress (factor ~ 2 in precision)
- RUN-4 (November 2020-July 2021) is expected to bring the statistics to ~ 12 times the Run-1 dataset
- RUN-5 in 2021-2022 should allow to achieve the project goal which will allow to reduce by a factor ~ 4 current total error



Future



- Discovery potential of *FNAL Muon g-2*, for same central value as Run-1 and same SM prediction



Conclusions



- We have presented the first measurement of a_μ at 0.46 ppm
- Our result is consistent with the BNL one (within one standard deviation) with slightly better precision

$$a_\mu(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11} \quad (0.46 \text{ ppm})$$

- The discrepancy with the Standard Model prediction of the $g-2$ by the Theory Initiative is 4.2σ
- We expect an improvement in precision of a factor 2 from the RUN2/3 data and more from Run4 and 5.