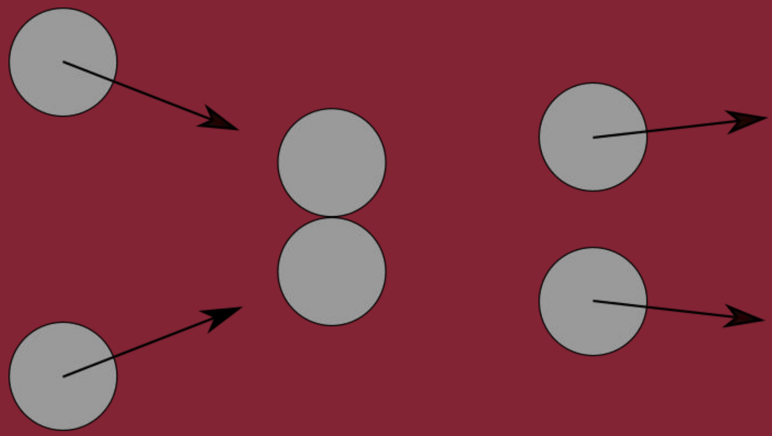
Getting hotter by heating less: How driven granular materials dissipate energy in excess



"One might even say that the study of granular materials gives one a chance to reinvent statistical mechanics in a new context." (Leo P. Kadanoff Rev. Mod. Phys. 71, 435 , 1999)

Istituto dei Sistemi Complessi

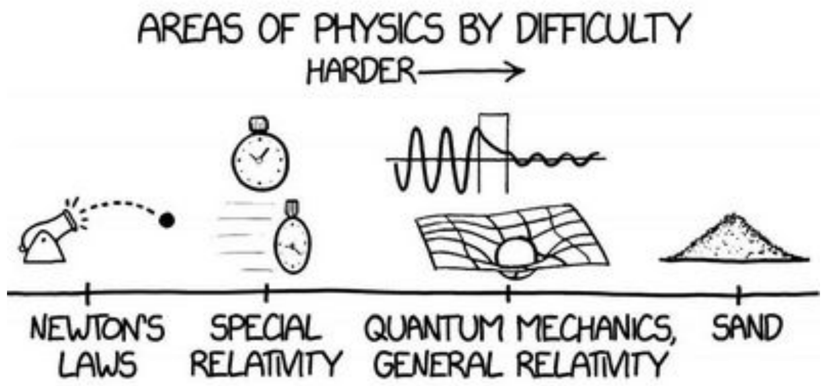
SAPIENZA UNIVERSITÀ DI ROMA

Andrea Plati - PhD Seminars Sapienza - 5th May 2021

Università degli Studi della Campania *Luigi Vanvitelli*

AP, L. de Arcangelis, A. Gnoli, E. Lippiello, A. Puglisi, and A. Sarracino Phys. Rev. Research **3,** 013011 (2021)

Overview



- Introduction
  - Granular Materials
  - Systems driven out of equilibrium
- Getting hotter by heating less
  - Experimental and numerical evidence of non-monotonic energy transfer to a dense granular system
  - Simple model to rationalize

Granular Matter

Granular materials are macroscopic systems made of macroscopic fundamental units (i.e. the grains)

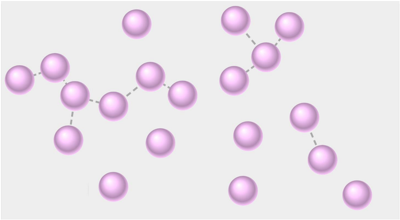
macroscopic systems:

- A large number of degrees of freedom are considered
  - Large enough for a statistical description
  - Not large enough to neglect fluctuations

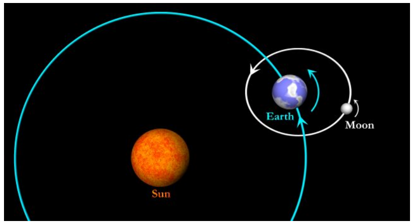
macroscopic grains:

- The internal degrees of freedom of the grains are neglected
  - Inelastic interactions
  - Athermal systems $k_b T \ll mRg$

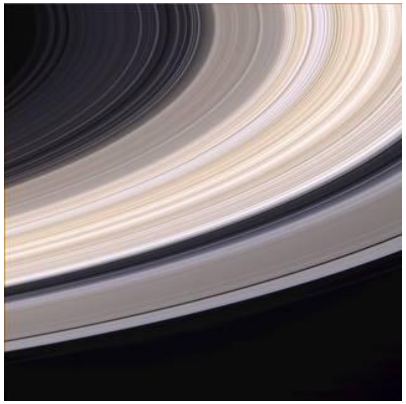
Granular Matter



molecular gas: macroscopic system of microscopic units → not granular!



planetary system: few macroscopic objects interacting→ not granular!



Sand, raw materials, cereals, powders, saturn rings → granular!

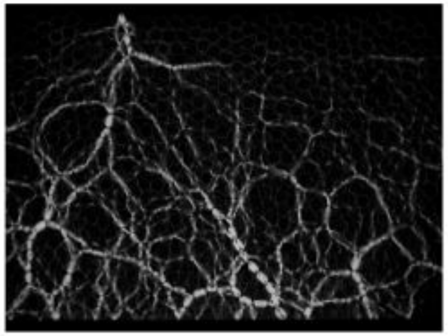
(sizes from tenths of micron without in principle an upper limit)

Why studying granular materials?

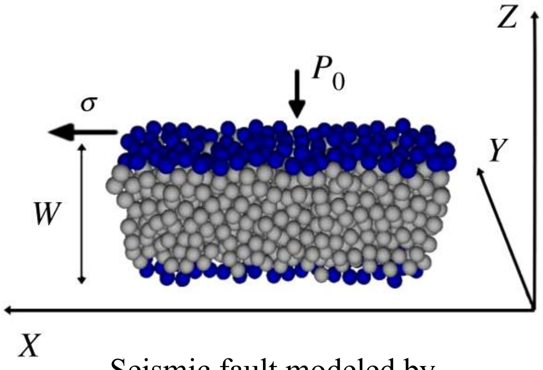
Applications:

- Industry
- Seismology

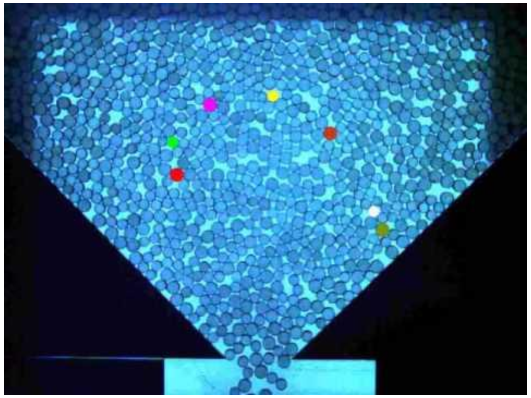
Silos collapse



Force chains



Seismic fault modeled by sheared granular materials

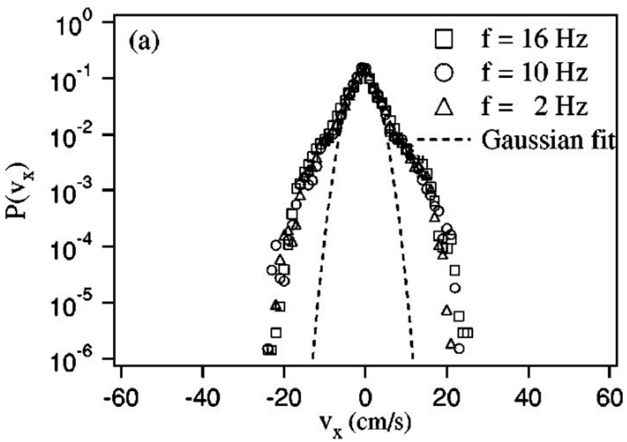


Grains jamming in a hopper

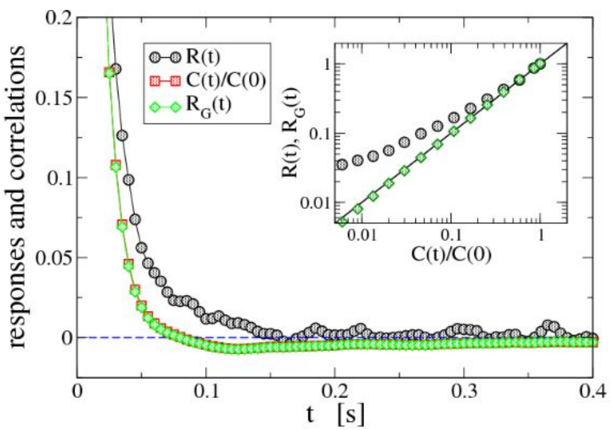
Why studying granular materials?

Fundamentals:

- Violation of equilibrium properties (equipartition, Maxwell-Boltzmann, FDT)
- Stochastic thermodynamics
- Fluctuating hydrodynamic



A. Kudrolli et al. Phys. Rev. E **62** R1489 (2000)
A. Puglisi et al. Phys. Rev. Lett. **81** 3848 (1998)

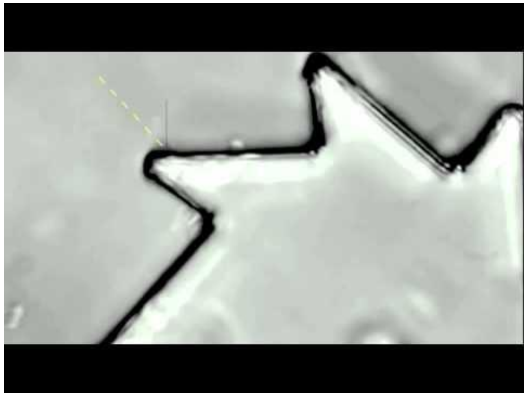


A. Gnoli et al. PLOS ONE 9(4): e93720 (2015)

$$\sout{e^{-\beta H}}$$

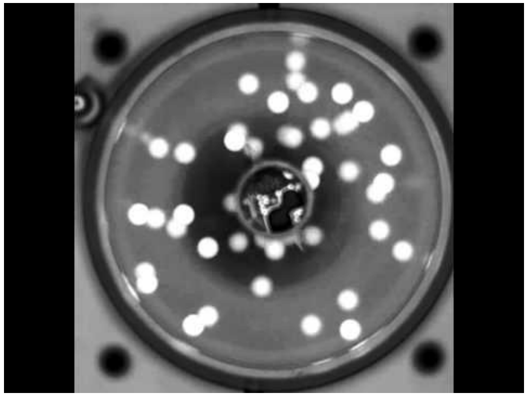
Standard equilibrium tools are usually inappropriate (we have no Hamiltonian nor temperature)

Systems driven out of thermodynamic equilibrium

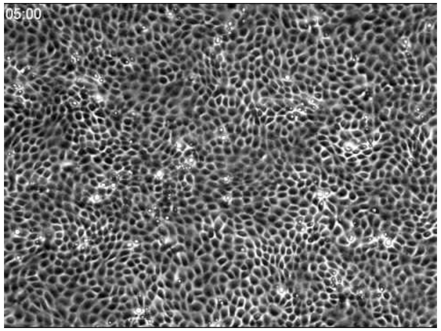


Bacteria

Animals



Grains

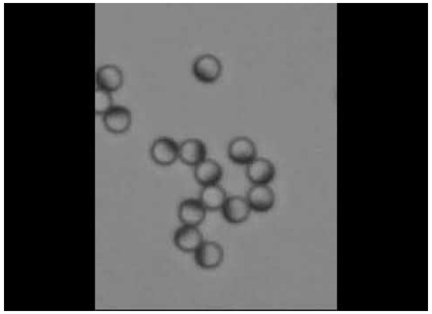


Cells monolayer

Energy constantly flows in the system
(with different mechanisms)

↓

Non-equilibrium steady states (NESS)

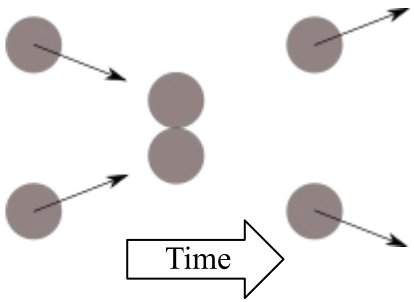


Active colloids

Systems driven out of thermodynamic equilibrium

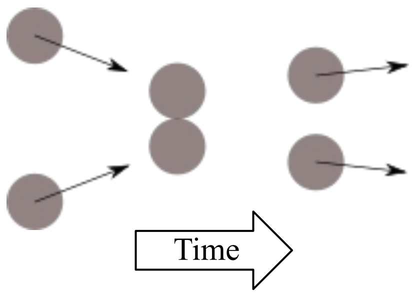
(a granular perspective)

Elastic collision



- Symmetric under time (and velocity) reversal
- Detailed balance (microscopic reversibility)

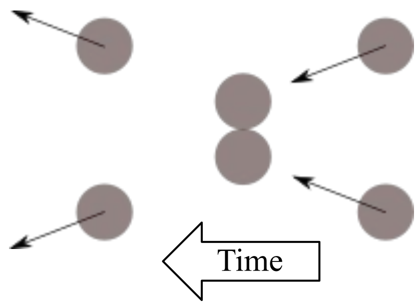
Inelastic collision



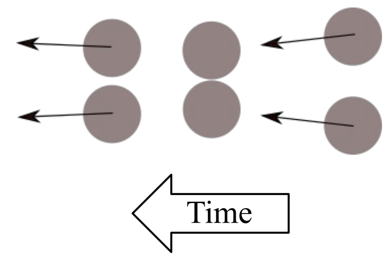
- Non-symmetric under time (and velocity) reversal
- No detailed balance (microscopic irreversibility)

Systems driven out of thermodynamic equilibrium
(a granular perspective)

Elastic collision



Inelastic collision



- Symmetric under time (and velocity) reversal
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- Non-symmetric under time (and velocity) reversal
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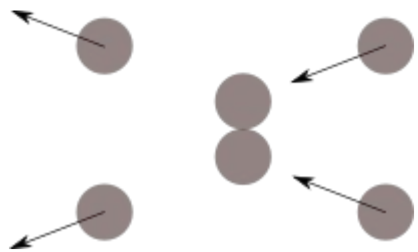
Systems driven out of thermodynamic equilibrium

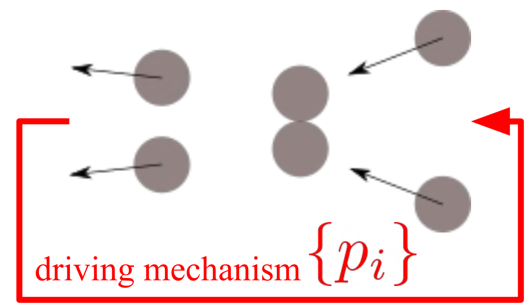
(a granular perspective)

Elastic collision



- Symmetric under time (and velocity) reversal
- Detailed balance (microscopic reversibility)
- Equilibrium is enough for stationarity

Inelastic collision

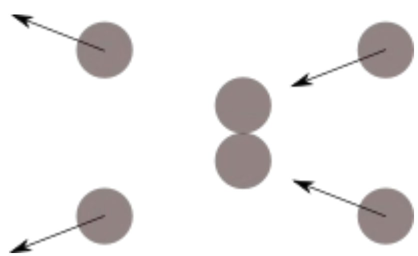


- Non-symmetric under time (and velocity) reversal
- No detailed balance (microscopic irreversibility)
- The system needs a driving mechanism to be stationary

Systems driven out of thermodynamic equilibrium

(a granular perspective)

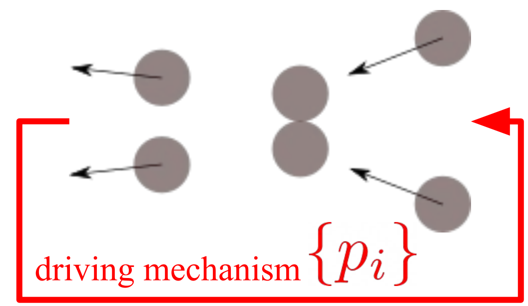
Elastic collision



- Symmetric under time (and velocity) reversal
- Detailed balance (microscopic reversibility)
- Equilibrium is enough for stationarity

$$O^{\mathrm{EQ}}(T)$$

Inelastic collision



- Non-symmetric under time (and velocity) reversal
- No detailed balance (microscopic irreversibility)
- The system needs a driving mechanism to be stationary

$$O^{\mathrm{NESS}}(\{p_i\})$$

A new question arises:

How the properties of the NESS are related to the specific driving mechanism?

Vibro-fluidized granular matter

Vertical sinusoidal shaking: $z_p(t) = A\sin(2\pi f t) \quad \{p_i\} = A, f$

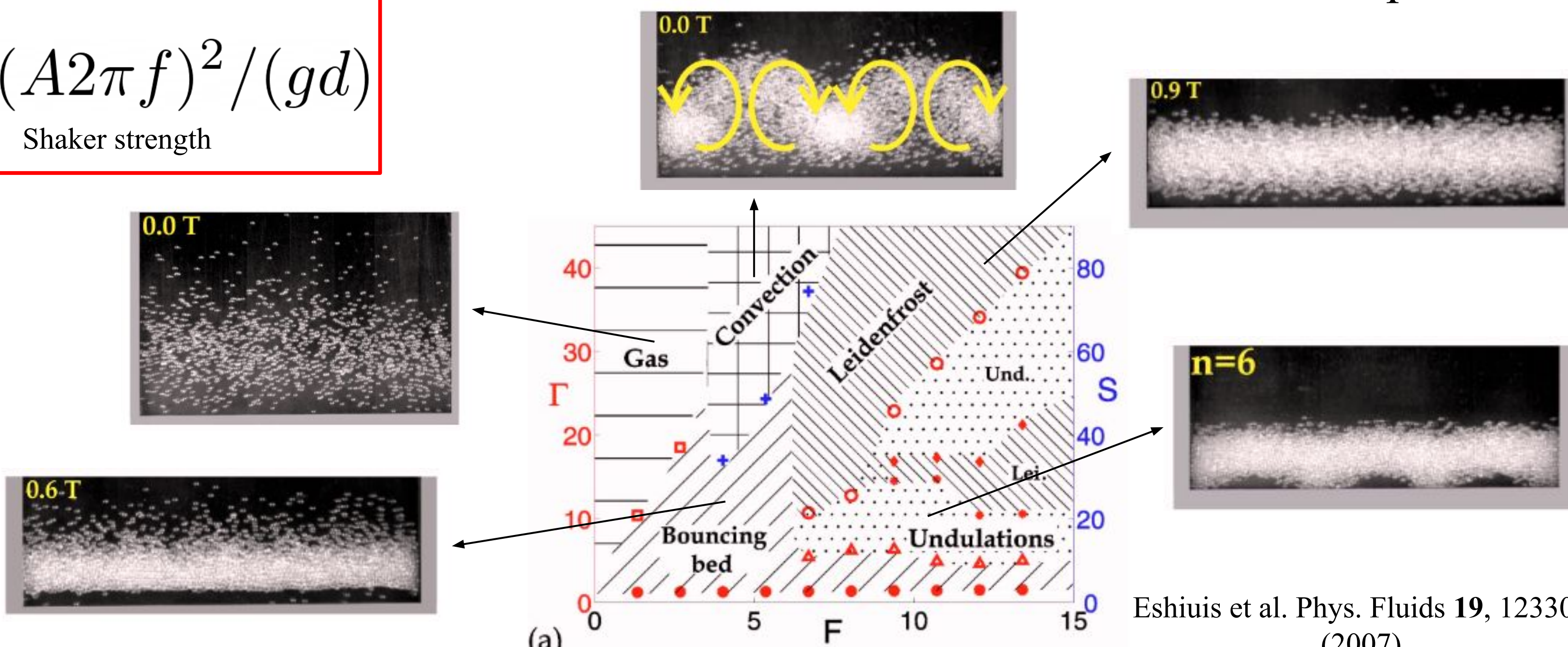
$$\Gamma = A(2\pi f)^2/g$$

Maximum rescaled acceleration

$$S = (A2\pi f)^2/(gd)$$

Shaker strength

Granular phases



Eshiuis et al. Phys. Fluids **19**, 123301 (2007)

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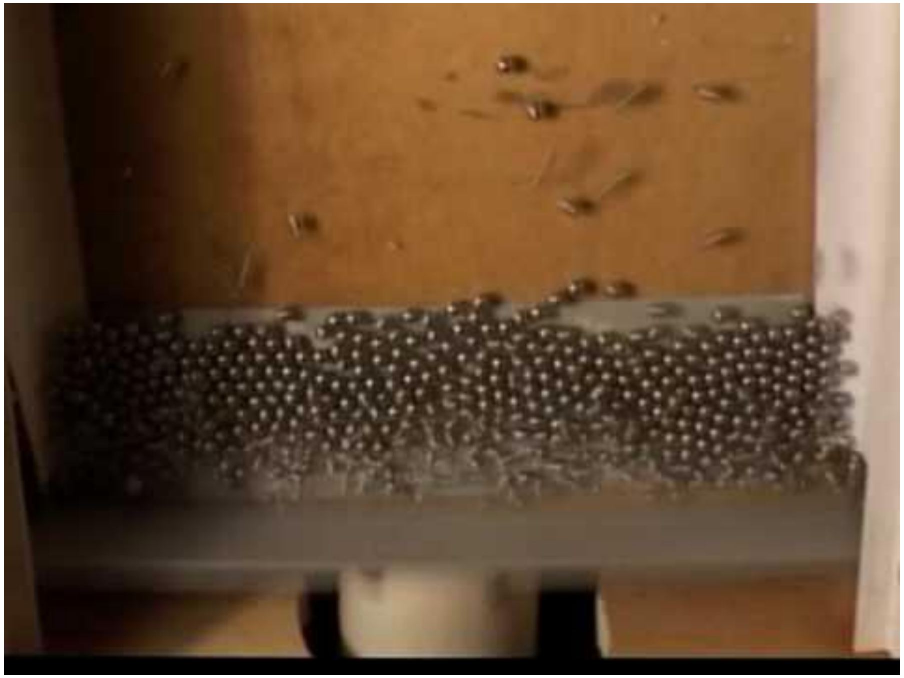


The case study

Investigating how the kinetic energy acquired by a dense granular system driven by an external vibration depends on the input energy

$$K^{\mathrm{NESS}}(A, f)$$

Usually one has:

$$K \sim S^{\nu} \quad \nu > 0$$

S. McNamara et al. Phys. Rev. E **58**, 813 (1998)

Remind:

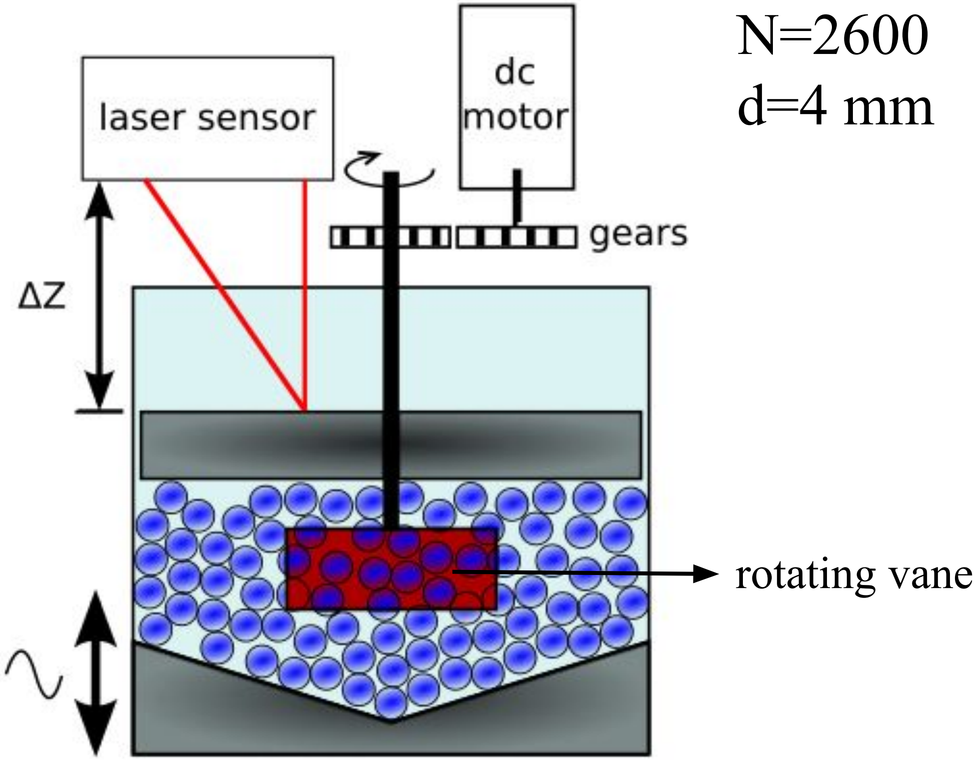
$$S = (A 2\pi f)^2/(gd)$$

We find regimes where: $\nu < 0$

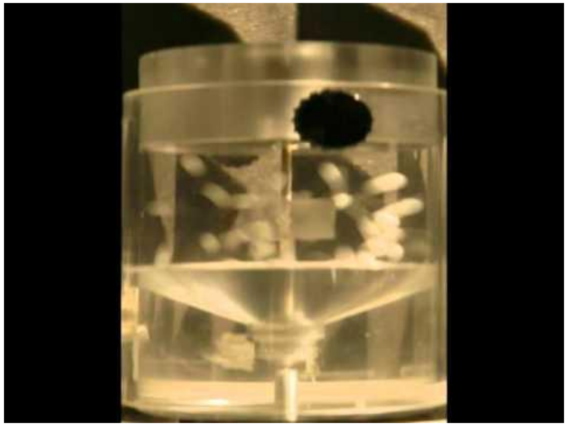
AP et al. Phys. Rev. Research **3,** 013011 (2021)

Experimental apparatus

(we can study it also with numerical simulations)



N=2600
d=4 mm



$$z_p(t) = A \sin(2\pi f t)$$

Varying:

$$\begin{matrix} A \\ f \end{matrix} \rightarrow \Gamma, S$$

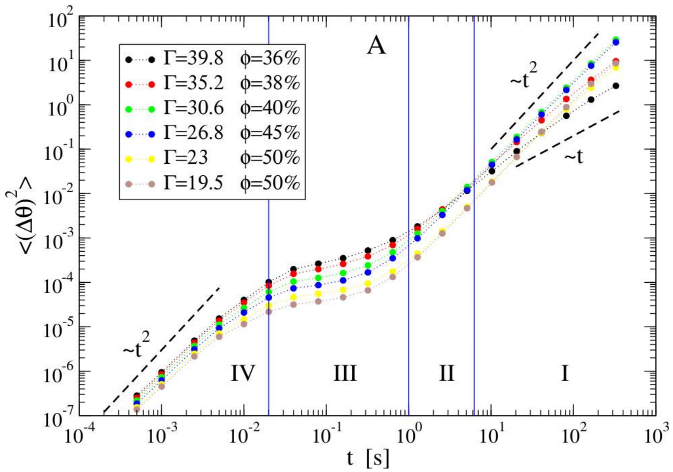
Measuring:

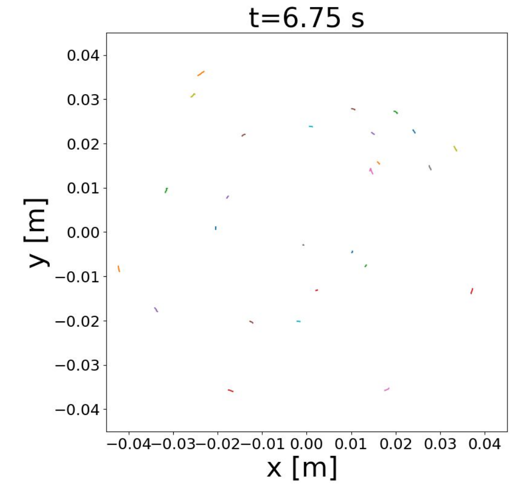
$$\Omega(t)$$

$$K_v = I \left[\langle \Omega^2 \rangle - \langle \Omega \rangle^2 \right] / 2$$

Emerging slow time scales

In the same experimental/numerical setup we also studied anomalous diffusion and slow collective dynamics





See:

C. Scalliet et al. Rev. Lett. **114**, 198001 (2015)
AP et al. Phys. Rev. Lett. **123**, 038002 (2019)
AP et al. Phys. Rev. E **102**, 012908 (2020)
AP et al. arXiv:2101.09516 (submitted)
Recorded Talk: http://denali.phys.uniroma1.it/twiki/bin/view/TNTgroup/TNTeaTime

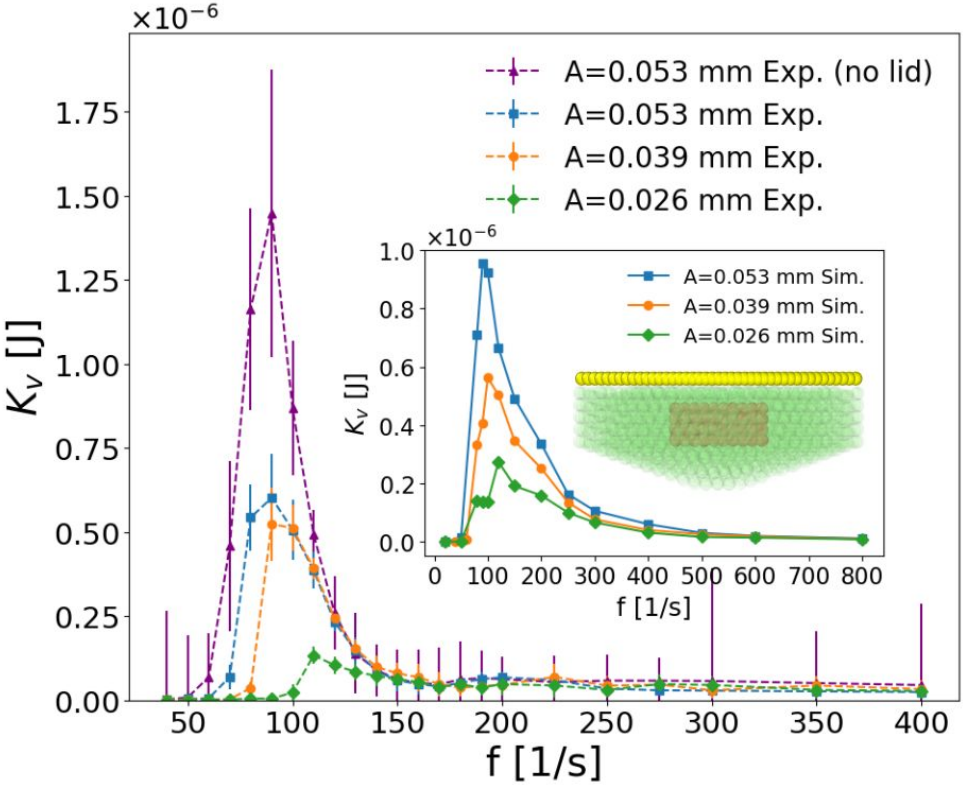
Experimental and numerical results

(kinetic energy of the vane)

Remind:

$$\Gamma = A(2\pi f)^2/g$$

$$S = (A2\pi f)^2/(gd)$$



Varying frequency at fixed amplitude

Fluidization frequency

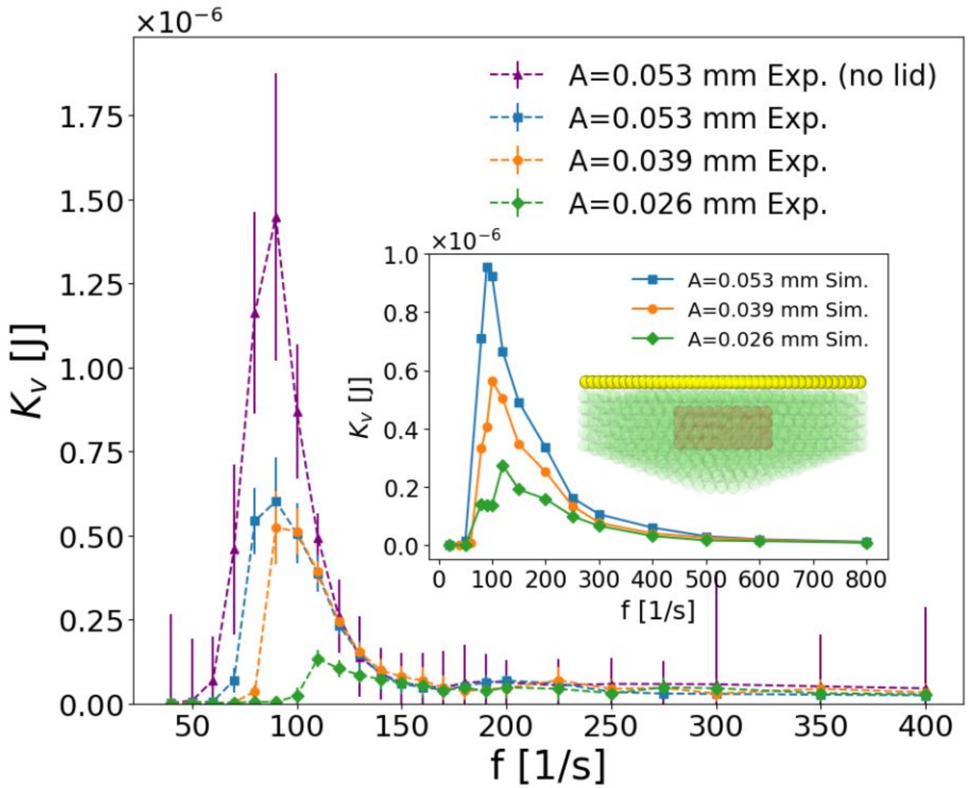
$$f_1 = \frac{\sqrt{g/A}}{2\pi} \qquad (\Gamma = 1)$$

Friction-recovery frequency

$$f_2 > f_1$$

Experimental and numerical results
(kinetic energy of the vane)

Remind:
$\Gamma = A(2\pi f)^2/g$
$S = (A2\pi f)^2/(gd)$



Varying frequency at fixed amplitude

Fluidization frequency

$$f_1 = \frac{\sqrt{g/A}}{2\pi} \qquad (\Gamma = 1)$$

Friction-recovery frequency

$$f_2 > f_1$$

related to frictional properties of the material

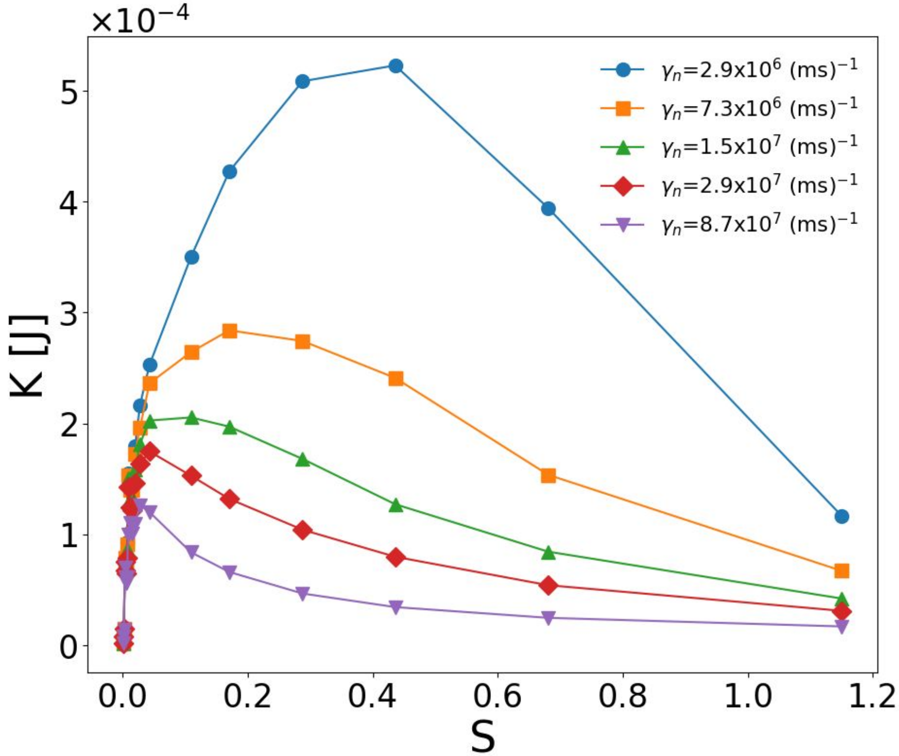
A. Gnoli et al. Phys. Rev. Lett. **120**, 138001 (2018)

Numerical results
(kinetic energy of the granular medium)

Remind:
$S = (A2\pi f)^2/(gd)$

$$K = \sum_{i=1}^{N} \frac{m}{2} \langle \boldsymbol{v}_i^2 \rangle$$

Simulations
without the vane



Varying S at fixed A

Increasing dissipation anticipates
the friction-recovery

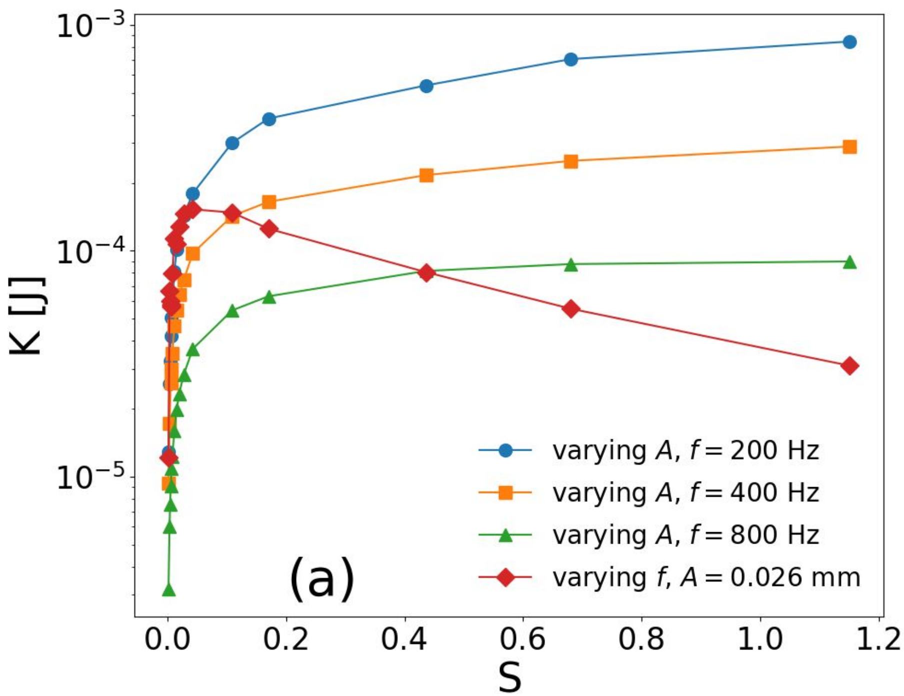
Numerical results
(kinetic energy of the granular medium)

Remind:

$$S = (A2\pi f)^2/(gd)$$

$$K = \sum_{i=1}^{N} \frac{m}{2} \langle \boldsymbol{v}_i^2 \rangle$$

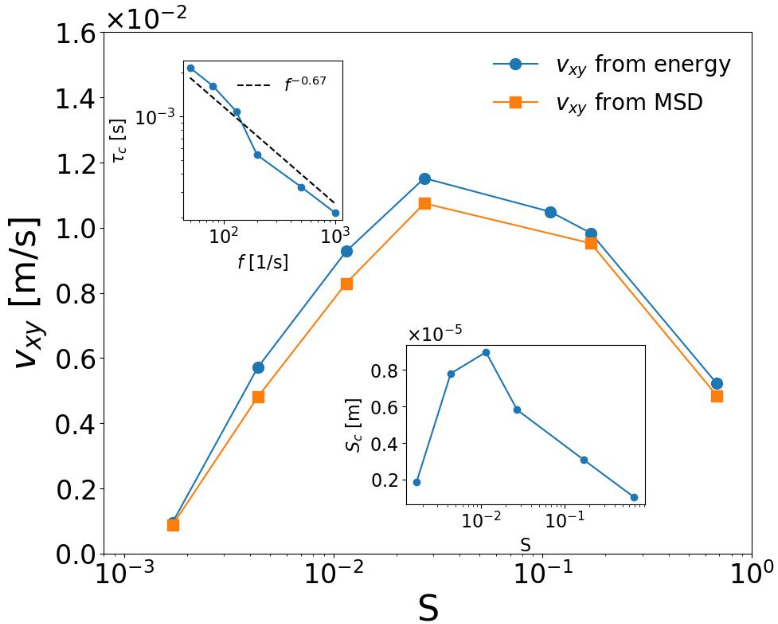
Simulations without the vane



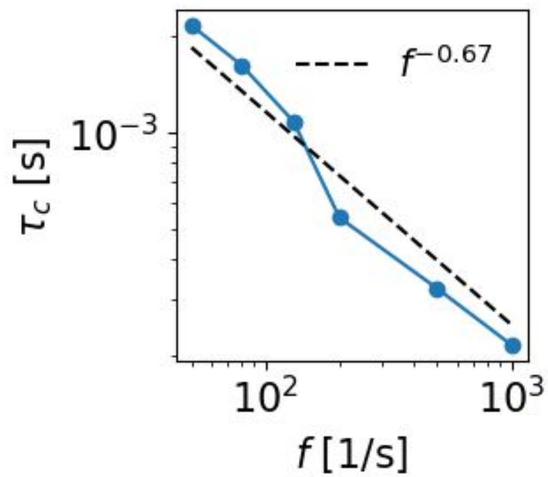
Varying S at fixed f

Raising the amplitude does not trigger the friction-recovery

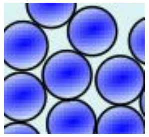
Single particle scale



Non-monotonic behavior originates at the single particle scale



S_c Typical size of a cage



τ_c Typical trapping time inside a cage

$$v_{XY} = S_c/\tau_c$$

$$1/\tau_c \sim f^{\alpha} \quad \alpha \simeq 2/3$$

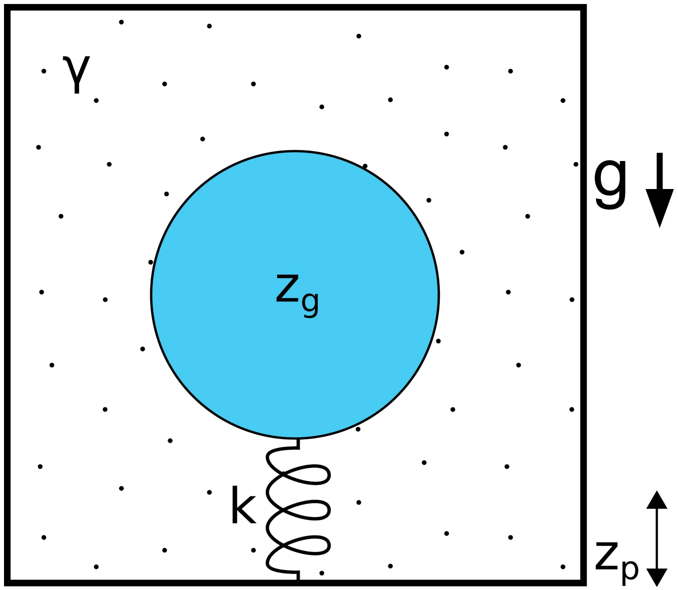
Representative for the amount of dissipation in the system (it grows with the driving frequency)

The generalized driven-damped oscillator

$$\ddot{z}_g(t) + 4\pi f_s \dot{z}_g(t) + (2\pi f_k)^2 (z_g(t) - z_p(t) - l_0) = 0$$

$$2\pi f_s = \gamma/(2m) \qquad 2\pi f_k = \sqrt{(k/m)}$$

Viscous prop. Elastic prop.



$$f_s = a f^\alpha \qquad \alpha > 0$$

The driving mechanism modifies the effective properties of the material

Remind:

$$z_p(t) = A\sin(2\pi f t)$$

$$z_g(t) - z_g^{eq} = \frac{f_k^2 A}{\sqrt{(f_k^2 - f^2)^2 + 4a^2 f^{2(\alpha+1)}}} \cos(2\pi f t - \phi)$$

The generalized driven-damped oscillator

Remind:

$$S = (A2\pi f)^2/(gd)$$

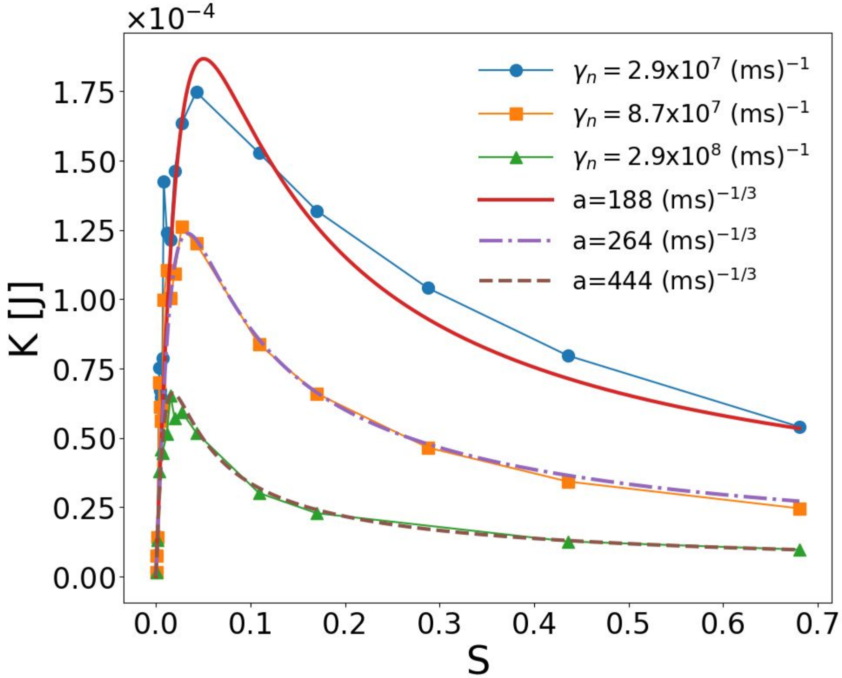
$$\langle \dot{z_g}^2 \rangle = \frac{\frac{1}{2} A^2 f_k^4 (2\pi f)^2}{4a^2 f^{2(\alpha+1)} + (f_k^2 - f^2)^2}$$

$$K = m_{\text{eff}} \langle \dot{z_g}^2 \rangle / 2 \quad \alpha = 2/3$$

Taken from $1/\tau_c$

A Fixed to the real value

m_{eff}, f_k Fitted values compatible with realistic ones



Good agreement between model and simulation for large dissipation

Beyond a simple resonance

All this can remind a resonant behaviour but...

$$f_k^4 - f_{\max}^4 - 4\alpha a^2 f_{\max}^{2(\alpha+1)} = 0$$

$\alpha = 0$

$$f_{\max} = f_k$$

In a resonance the maximum energy transfer occurs at the characteristic frequency

$\alpha > 0$

$$f_{\max}(a) \ll f_k$$

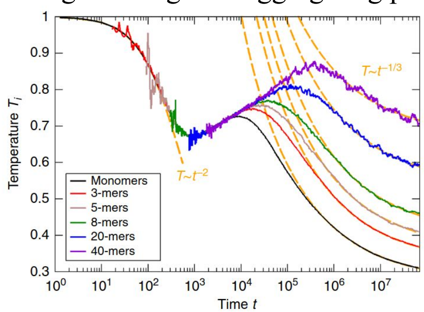
Our model predicts a frictional-dependent optimal frequency much smaller than the characteristic one

Beyond a simple resonance

Negative specific heat?

(let's talk about that, there is always a trick)

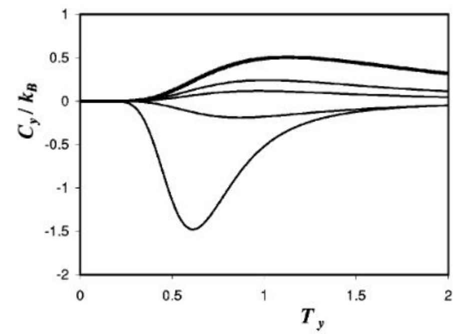
Free granular gas of aggregating particles



Reduction of effective degrees of freedom (aggregation faster than energy loss)

N. V. Brillantov et al. Nat. Commun. 9, 797 (2018).

System with 3 energy levels



Forbidden transitions

R. K. P. Zia et al. Am. J. Phys.70, 384 (2002)

What's the trick in our system?

$$K \propto A^2 f^2 \left(1 - \frac{a^2 f^{2(\alpha+1)}}{b_k + a^2 f^{2(\alpha+1)}}\right)$$

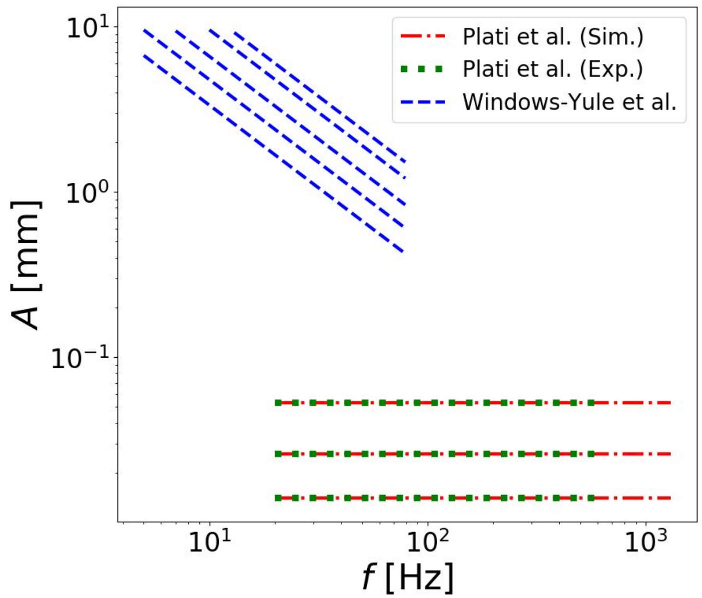
$$K = K_{\text{in}}(f)(1 - D(f))$$

Both increase with the driving frequency

We understand non-monotonic energy transfer through competing effects of forcing and dissipation

What's next ?

- Phase diagram of energy transfer
- Thermodynamic uncertainty relations
- Applications: Optimal energy transfer, friction weakening



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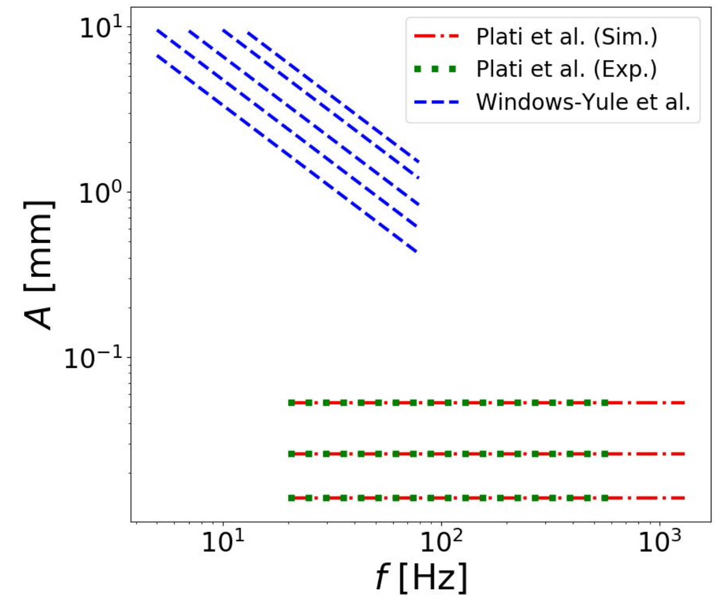
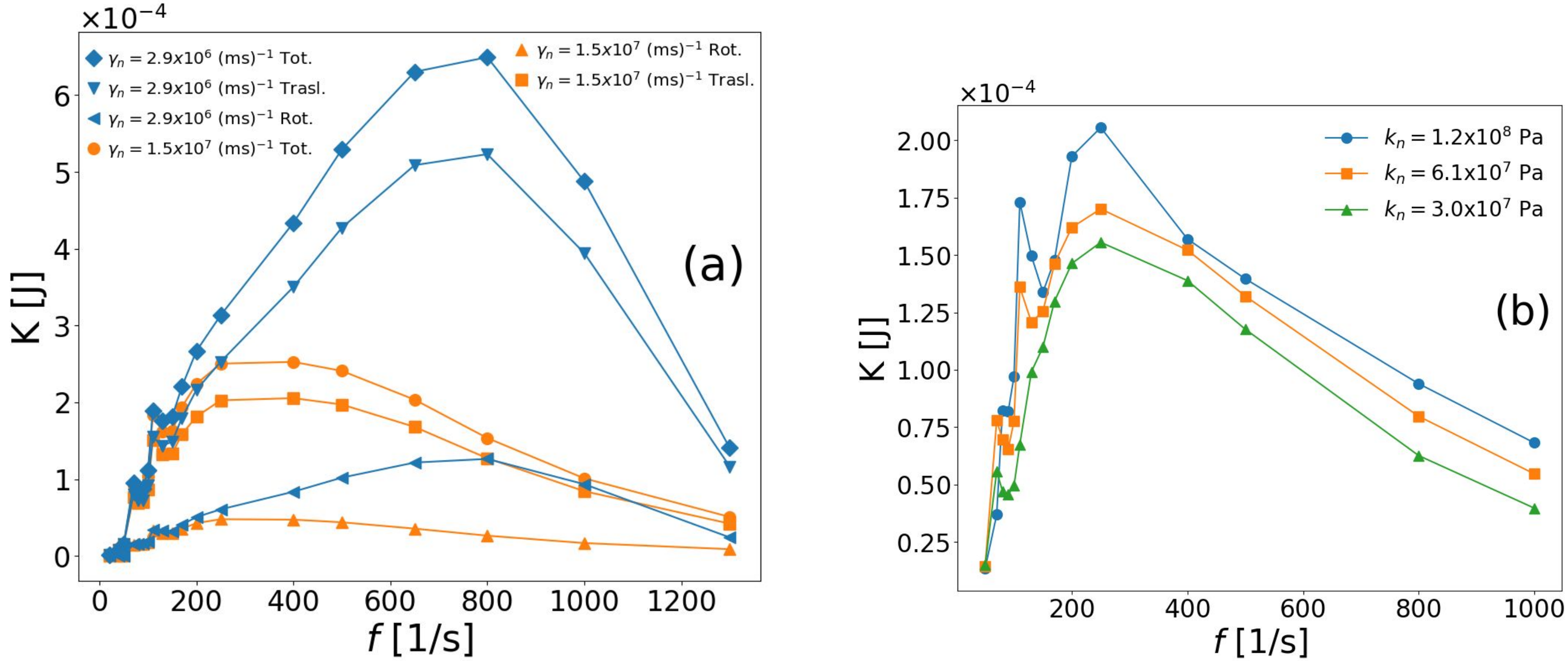


Thanks for your attention!

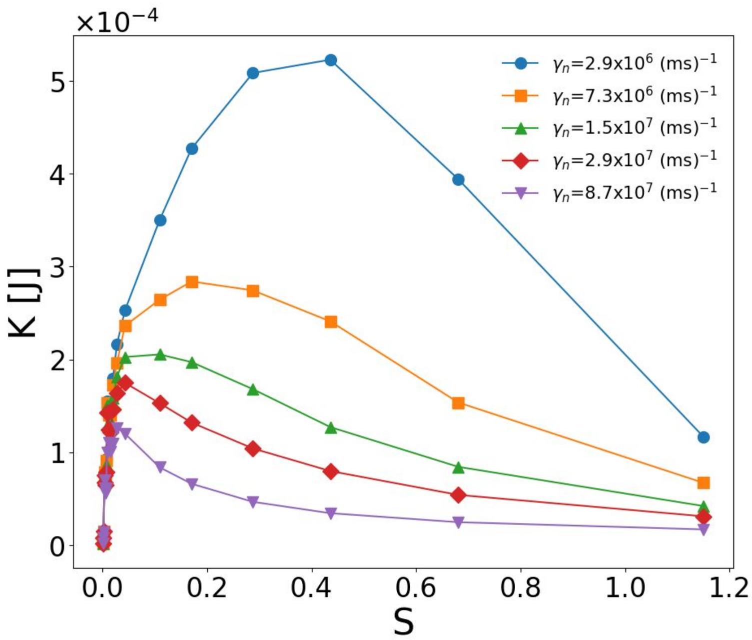

×10⁻⁴
$\gamma_n = 2.9x10^6$ (ms)$^{-1}$ Tot.
$\gamma_n = 2.9x10^6$ (ms)$^{-1}$ Trasl.
$\gamma_n = 2.9x10^6$ (ms)$^{-1}$ Rot.
$\gamma_n = 1.5x10^7$ (ms)$^{-1}$ Tot.
$\gamma_n = 1.5x10^7$ (ms)$^{-1}$ Rot.
$\gamma_n = 1.5x10^7$ (ms)$^{-1}$ Trasl.
(a)
K [J]
f [1/s]
×10⁻⁴
$k_n = 1.2x10^8$ Pa
$k_n = 6.1x10^7$ Pa
$k_n = 3.0x10^7$ Pa
(b)
K [J]
f [1/s]

Numerical results
(kinetic energy of the granular medium)

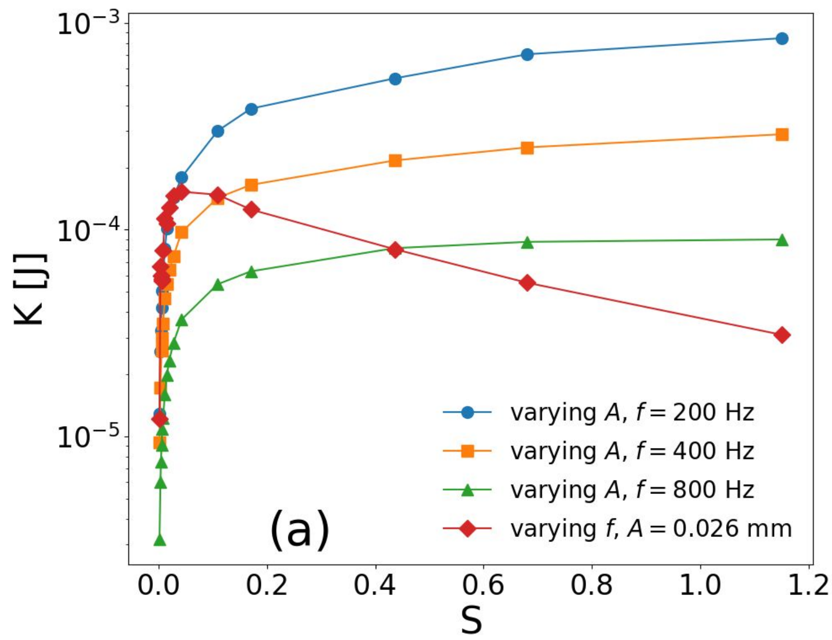
Remind:

$$S = (A 2\pi f)^2/(gd)$$

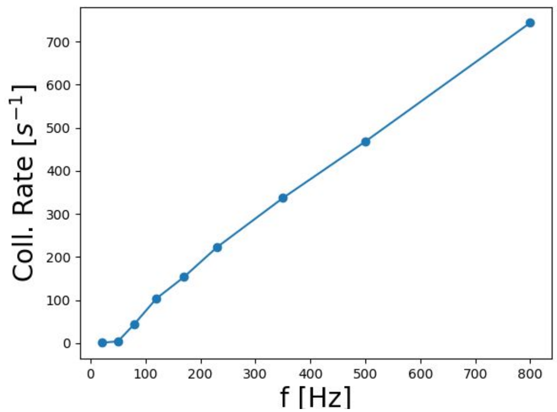
$$K = \sum_{i=1}^{N} \frac{m}{2} \langle \boldsymbol{v}_i^2 \rangle$$

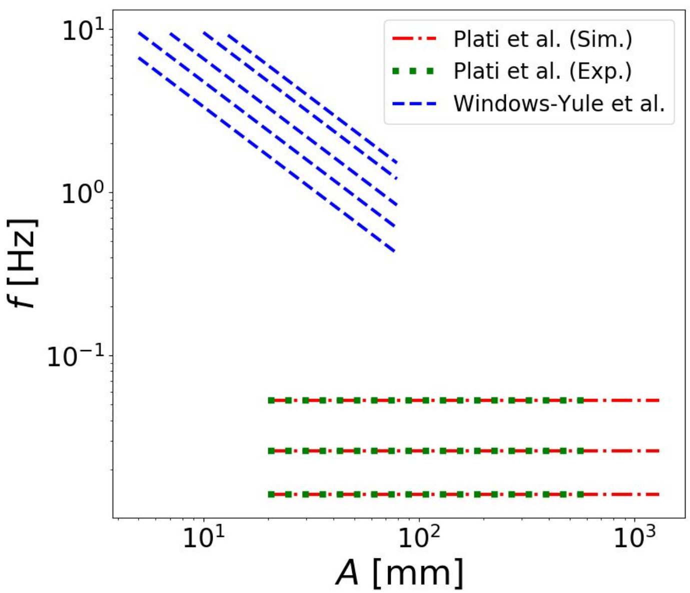


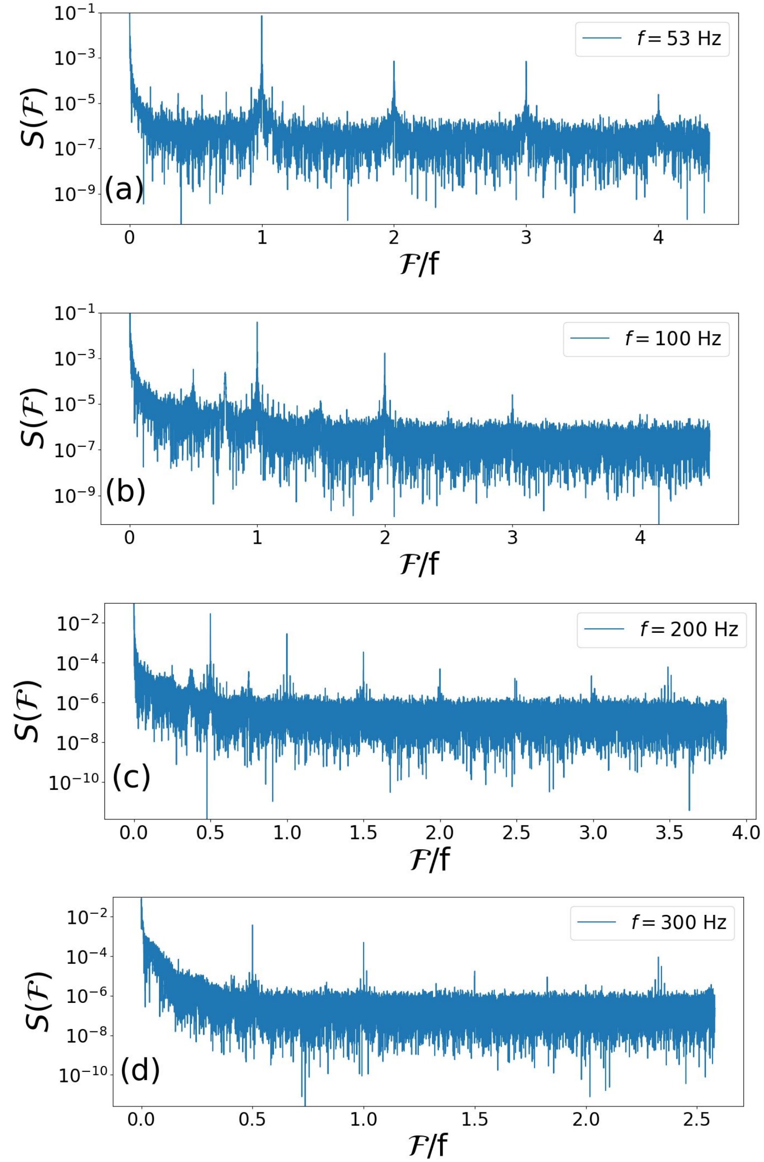
Increasing dissipation anticipates the friction-recovery



Raising the amplitude does not trigger the friction-recovery


Coll. Rate [s^{-1}]
f [Hz]

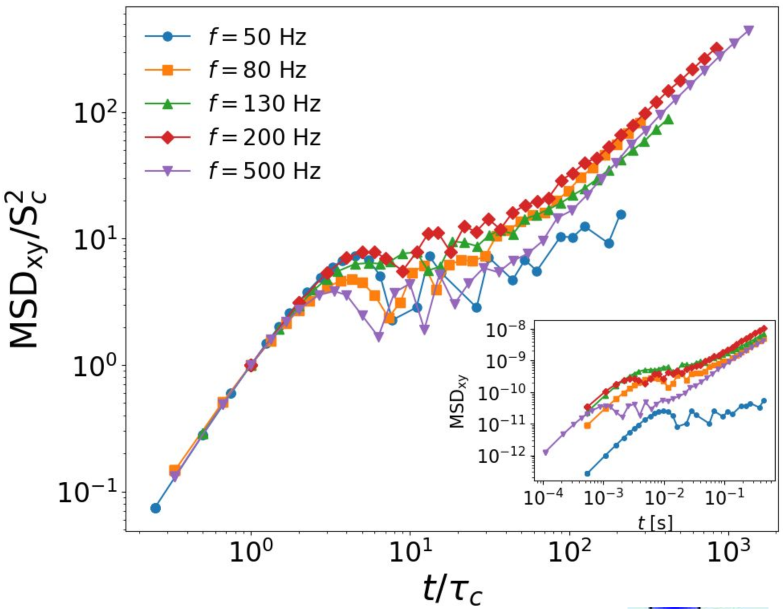

Plati et al. (Sim.)
Plati et al. (Exp.)
Windows-Yule et al.
f [Hz]
A [mm]

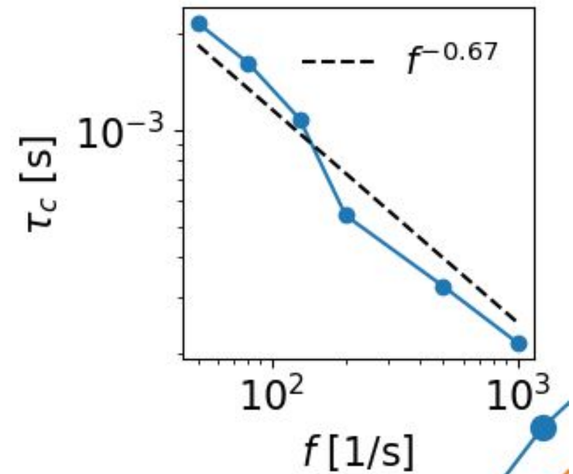

$f = 53$ Hz
$f = 100$ Hz
$f = 200$ Hz
$f = 300$ Hz
$S(\mathcal{F})$
(a)
(b)
(c)
(d)
$\mathcal{F}$/f

Single particle scale

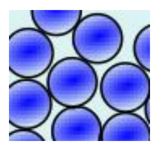
Remind:

$$S = (A2\pi f)^2/(gd)$$





S_c Typical size of a cage



τ_c Typical trapping time inside a cage

$$v_{XY} = S_c/\tau_c$$

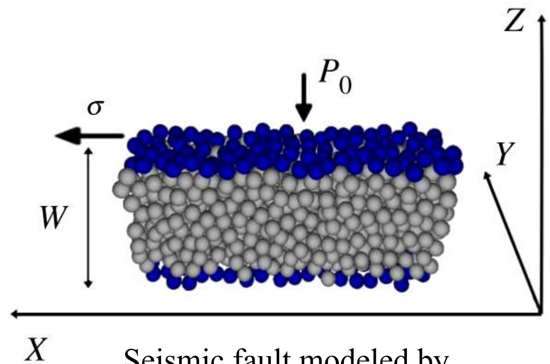
Non-monotonic behavior originates at the single particle scale

$1/\tau_c$ is related to the amount of dissipation in the system and it grows with the driving frequency

Why studying granular materials?

Applications:

- Industry
- Seismology
- Rheology



Seismic fault modeled by sheared granular materials

Silos collapse

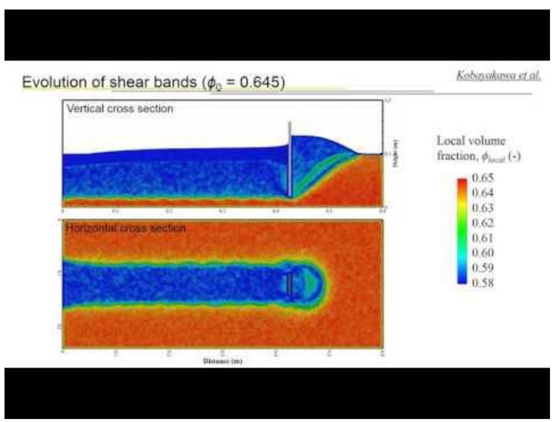
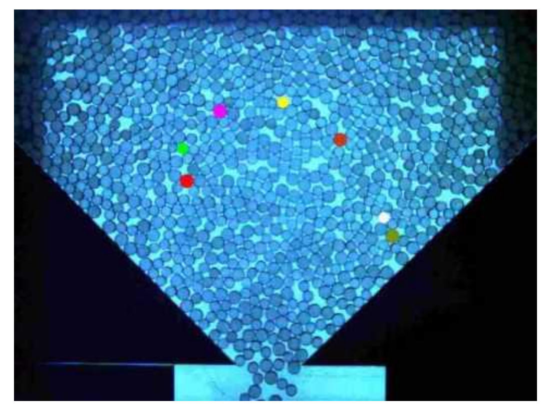
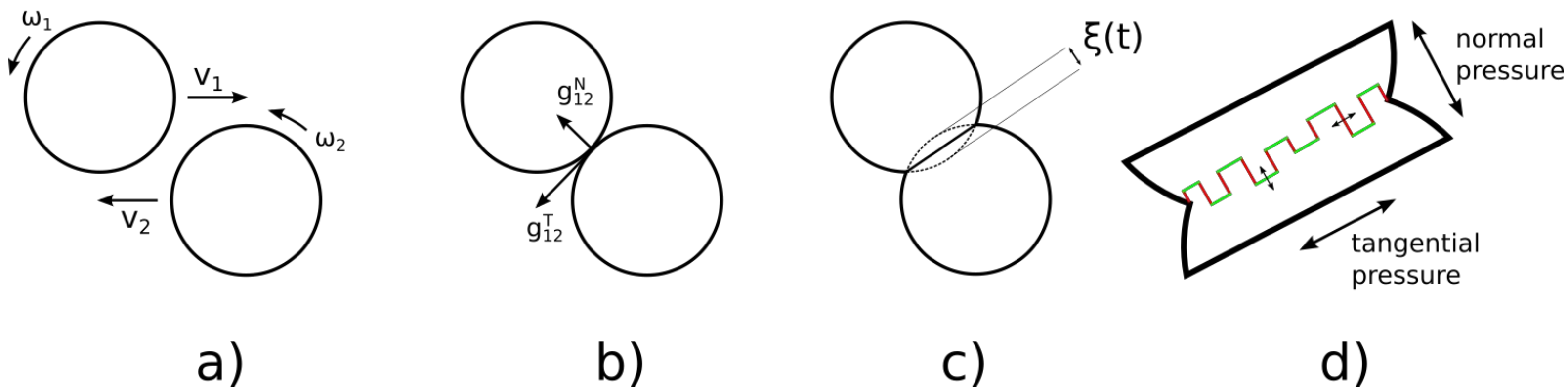


Plate drag in granular materials



Grains jamming in a hopper

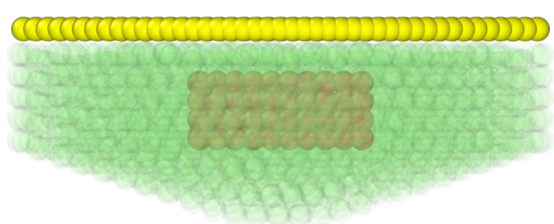
Numerical Simulations



Molecular dynamics simulations with the discrete element method (DEM)

- Suitable for high densities
- Tunable material properties (viscosity)

$$\gamma_n$$

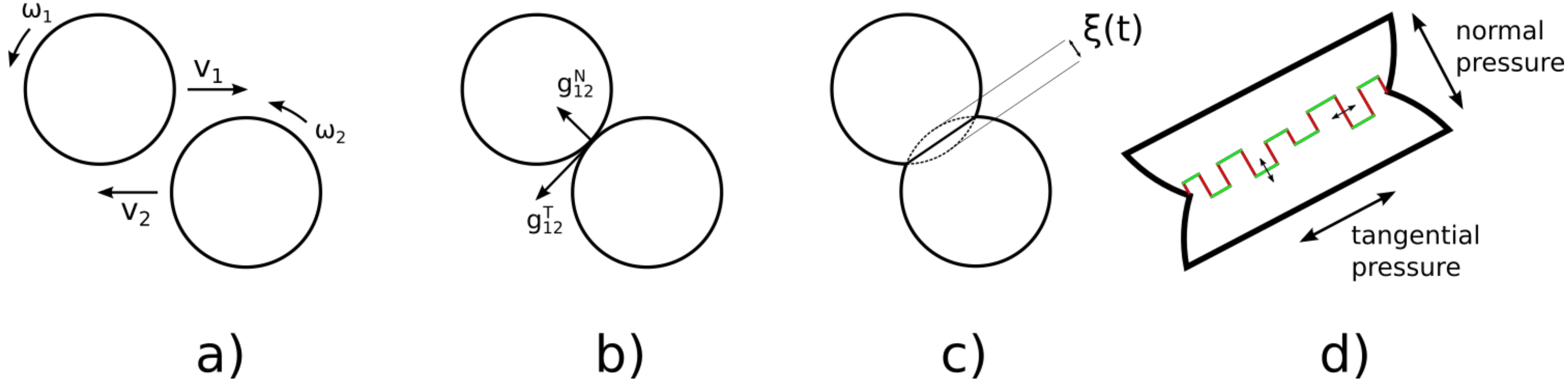


Numerical setup

Realistic spherical grains:

- Rotations
- Compressions
- Superficial asperities
- Dissipation

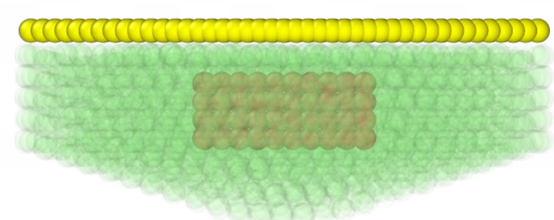
Numerical model for granular interactions



Realistic spherical grains:

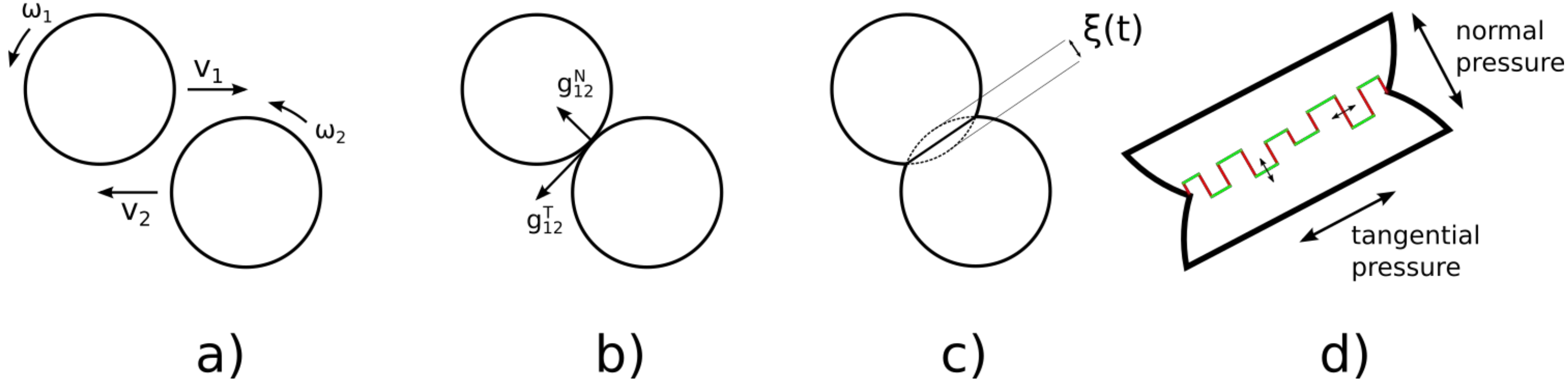
- Rotations
- Compressions
- Superficial asperities
- Dissipations

$$\begin{cases} \vec{F}_{ij}^{N} = \sqrt{R_{ij}^{\text{eff}}}\sqrt{\xi_{ij}(t)}\left[(k_n\xi_{ij}(t) - m^{\text{eff}}\gamma_n\dot{\xi}_{ij}(t))\vec{n}(t)\right] \\ \vec{F}_{ij}^{T} = -\sqrt{R_{ij}^{\text{eff}}}\left[\vec{F}_{ij}^{\text{hist}} + m_{\text{eff}}\gamma_t\sqrt{\xi_{ij}(t)}\vec{g}_{ij}^{T}(t)\right] \quad \text{if} \quad |\vec{F}_{ij}^{\text{hist}}| \leq |\mu\vec{F}_{ij}^{N}| \\ \vec{F}_{ij}^{T} = -\frac{|\mu\vec{F}_{ij}^{N}|}{|\vec{g}_{ij}^{T}(t)|}\cdot\vec{g}_{ij}^{T}(t) \quad \text{otherwise} \\ \vec{F}_{ij}^{\text{hist}} = k_t\int_{\text{s(t)}}\sqrt{\xi_{ij}(t')}\vec{ds}(t') \end{cases}$$



Numerical setup

Numerical model for granular interactions



Realistic spherical grains:

- Rotations
- Compressions
- Superficial asperities
- Dissipations

$$
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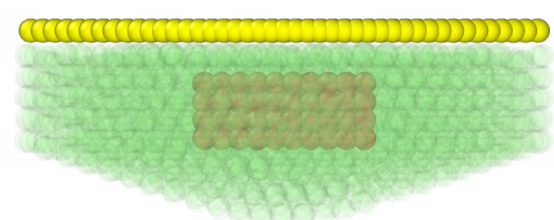
\vec{F}_{ij}^{\text{hist}} = k_t \int_{\text{s(t)}} \sqrt{\xi_{ij}(t')}\vec{ds}(t')
\end{cases}
$$

Main control parameter for dissipation



Numerical setup

What’s next ?

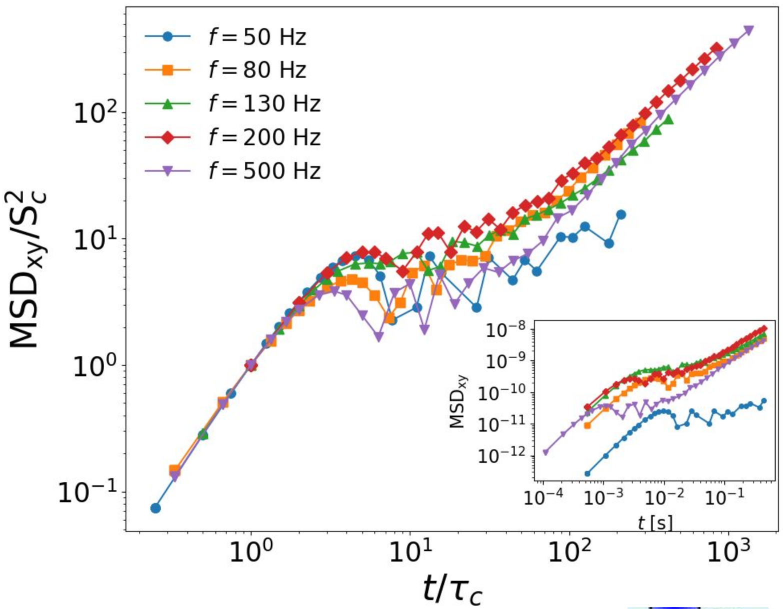
- Phase diagram of energy transfer
- Thermodynamic uncertainty relations

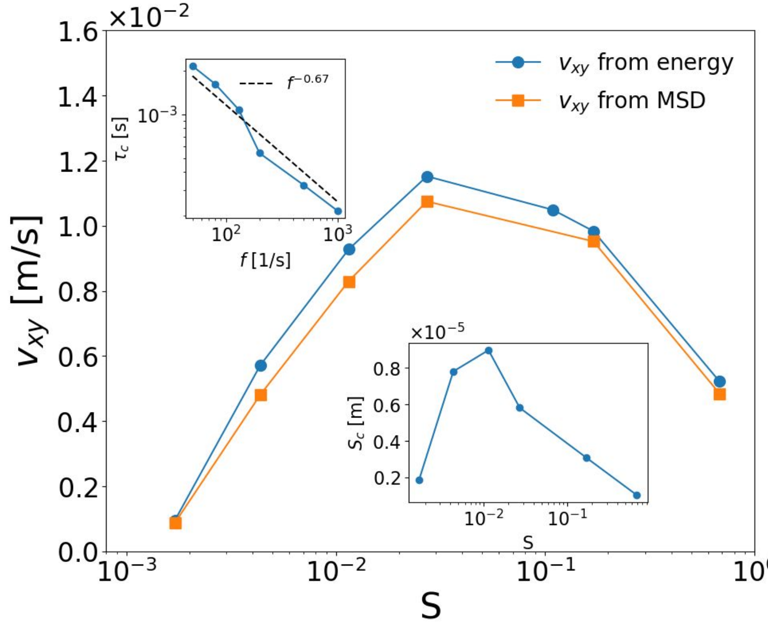
Thanks for your attention!

Single particle scale

Remind:

$$S = (A2\pi f)^2/(gd)$$





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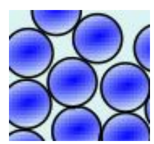

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