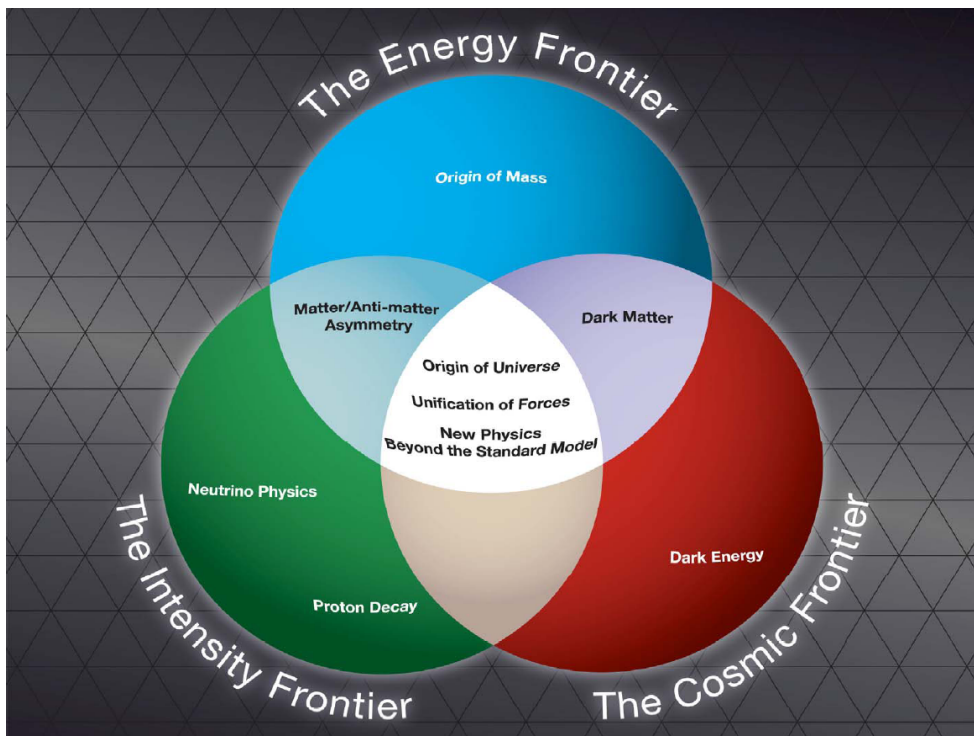


**General Seminar Miniworkshop Congiunto INFN LNF - INFN Roma:  
First results from the Muon  $g-2$  Experiment at Fermilab, 15 Apr. 2021**

*Theoretical implications of the  
leptonic  $g-2$  measurements*

**Antonio Masiero  
INFN and Univ. of Padova**



During the long sequel of restless attempts of finding experimental evidences or at least hints of **NEW PHYSICS** beyond the SM along the **traditional High-Energy (HE) and High-Intensity (HI) paths**, several 3 or even 4  $\sigma$  signals at variance w.r.t. the SM expectations **have shown up**, but they have also (rather sooner than later) **invariably faded away**.

A remarkable exception is represented by

**the anomalous magnetic moment of the muon**

which has been for **several years now** and **still** represents a **major observational evidence along the HI frontier of the possible presence of NEW PHYSICS**

The other more recent hint of NEW PHYSICS along these two roads is again in the HI frontier, namely the possible **violation of lepton flavour universality in some B-meson semileptonic decays**.

# What the SM does not account for...

neutrino masses  
dark matter  
baryogenesis  
inflation



**OBSERVATIONAL REASONS**  
of **New Physics**:  
all of them along  
the **ASTROPARTICLE frontier**

$M_{\text{HIGGS}} / M_{\text{PLANCK}} \sim 10^{-16}$   
 $E_{\text{VACUUM}} (\text{DE}) / M_{\text{HIGGS}} \sim 10^{-14}$   
 $\Theta_{\text{CPV in STRONG INTERAC.}} < 10^{-9}$



**THEORETICAL REASONS:**  
the unbearable **fine-tuning** of  
fundamental parameters  
+ other “**aesthetic**” **puzzles**  
the **flavour “problem”**, the  
barely **missed true unification**  
of fundamental interactions,  
the inclusion of **gravity**  
in a QM consistent framework, ...

- Uhlenbeck and Goudsmit in 1925 proposed for electrons

$$\begin{aligned}\vec{\mu} &= g \frac{e}{2m} \vec{s} \\ g &= \underline{2} \text{ (not 1!)}\end{aligned}$$

- Dirac 1928:

$$(i\partial_\mu - eA_\mu) \gamma^\mu \psi = m\psi$$

- A Pauli term in Dirac's eq would give a deviation...

$$a \frac{e}{2m} \sigma_{\mu\nu} F^{\mu\nu} \psi \quad \rightarrow \quad g = 2(1 + a)$$

...but there was no need for it!  $g=2$  stood for ~20 yrs.

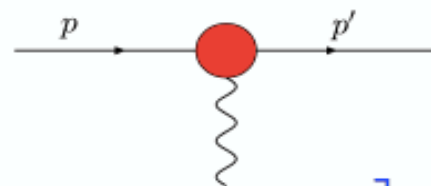
- **Kusch and Foley 1948:**

$$\left(\frac{g_e}{2}\right)^{\text{exp}} \equiv 1 + a_e^{\text{exp}} = 1.00119 \pm 0.00005$$

- **Schwinger 1948 (triumph of QED!):**

$$\left(\frac{g_e}{2}\right)^{\text{th}} \equiv 1 + a_e^{\text{th}} = 1.00116 \dots$$

- **We keep studying the lepton- $\gamma$  vertex:**



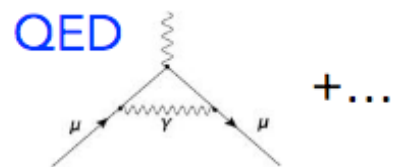
$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \dots\right]u(p)$$

$$F_1(0) = 1 \quad F_2(0) = a_l$$

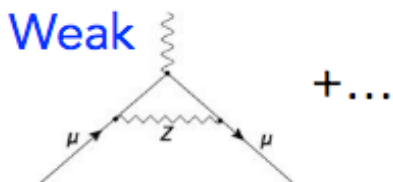
A pure "quantum correction" effect!

The 4 classes of SM contributions: **uncertainty largely dominated** by the **hadronic contributions** in **Vacuum Polarization (HVP)** and **Light-by-Light (HLbL)**

$$a_{\mu}(\text{SM}) = a_{\mu}(\text{QED}) + a_{\mu}(\text{Weak}) + a_{\mu}(\text{Hadronic})$$



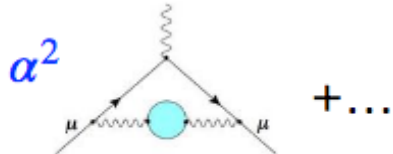
$$116\,584\,718.9(1) \times 10^{-11} \quad 0.001 \text{ ppm}$$



$$153.6(1.0) \times 10^{-11} \quad 0.01 \text{ ppm}$$

Hadronic...

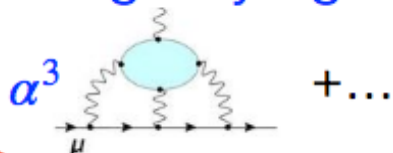
...Vacuum Polarization (HVP)



$$6845(40) \times 10^{-11} \quad 0.37 \text{ ppm}$$

[0.6%]

...Light-by-Light (HLbL)



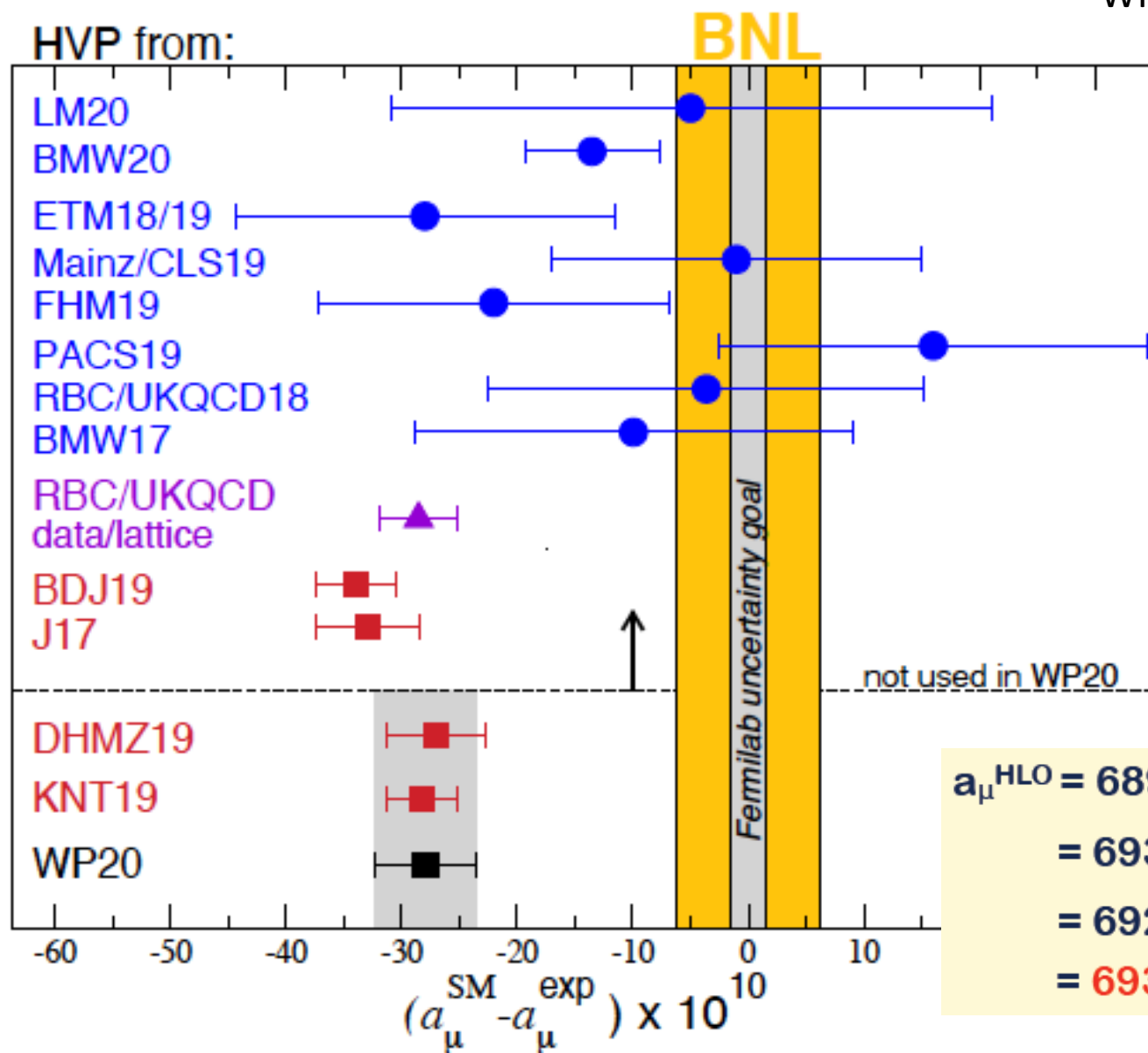
$$92(18) \times 10^{-11} \quad 0.15 \text{ ppm}$$

[20%]

# HADRONIC VACUUM POLARIZATION CONTRIBUTION

WP20: White Paper of the Muon g-2 Theory Initiative:

arXiv:2006.04822



Ab-initio lattice calculations

Dispersive relations,  
 $e^+e^- \rightarrow \text{hadrons exps.}$

$$a_\mu^{\text{HLO}} = 6895 (33) \times 10^{-11}$$

F. Jegerlehner, arXiv:1711.06089

$$= 6939 (40) \times 10^{-11}$$

Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

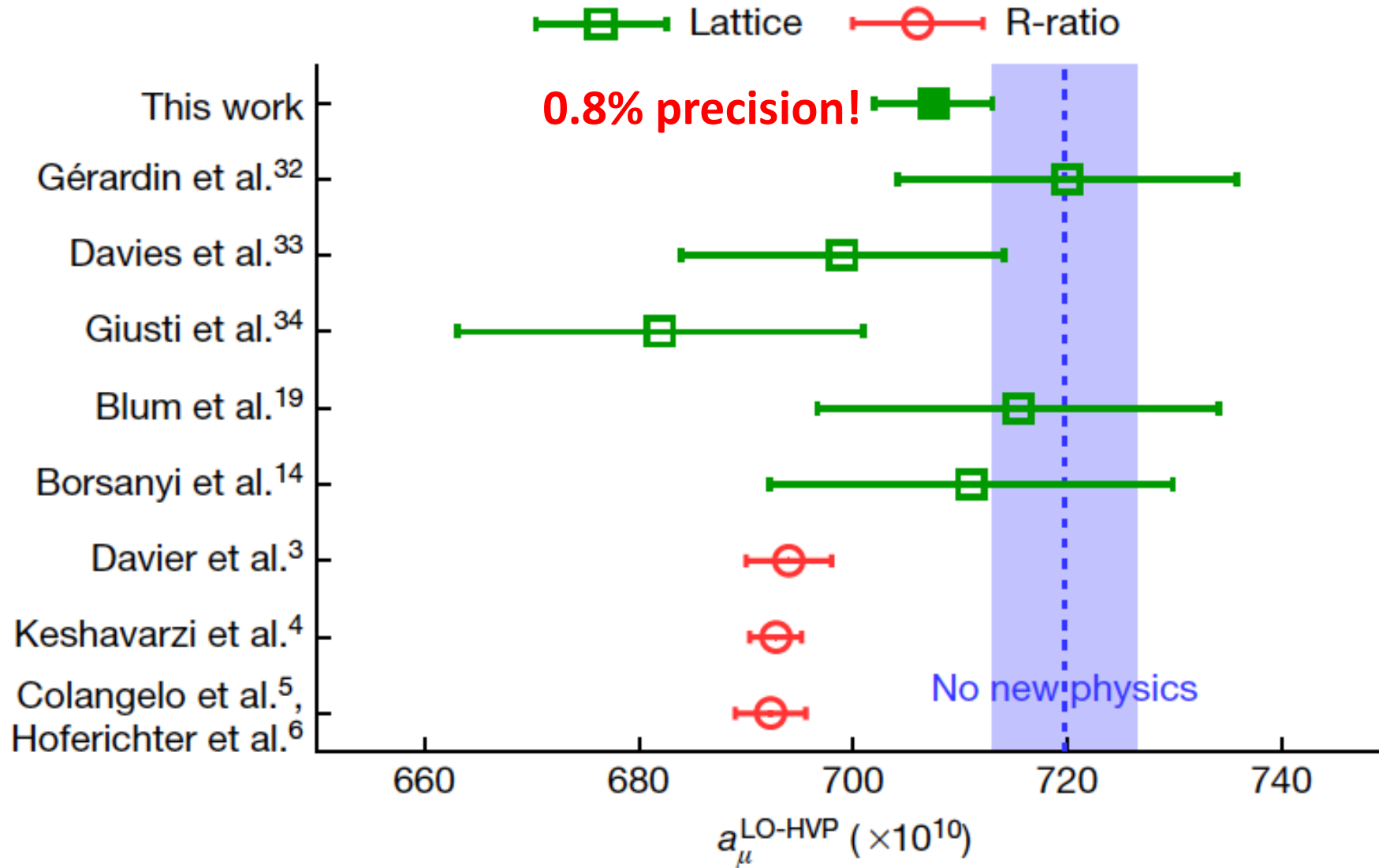
$$= 6928 (24) \times 10^{-11}$$

Keshavarzi, Nomura, Teubner, arXiv:1911.00367

$$= 6931 (40) \times 10^{-11} (0.6\%)$$

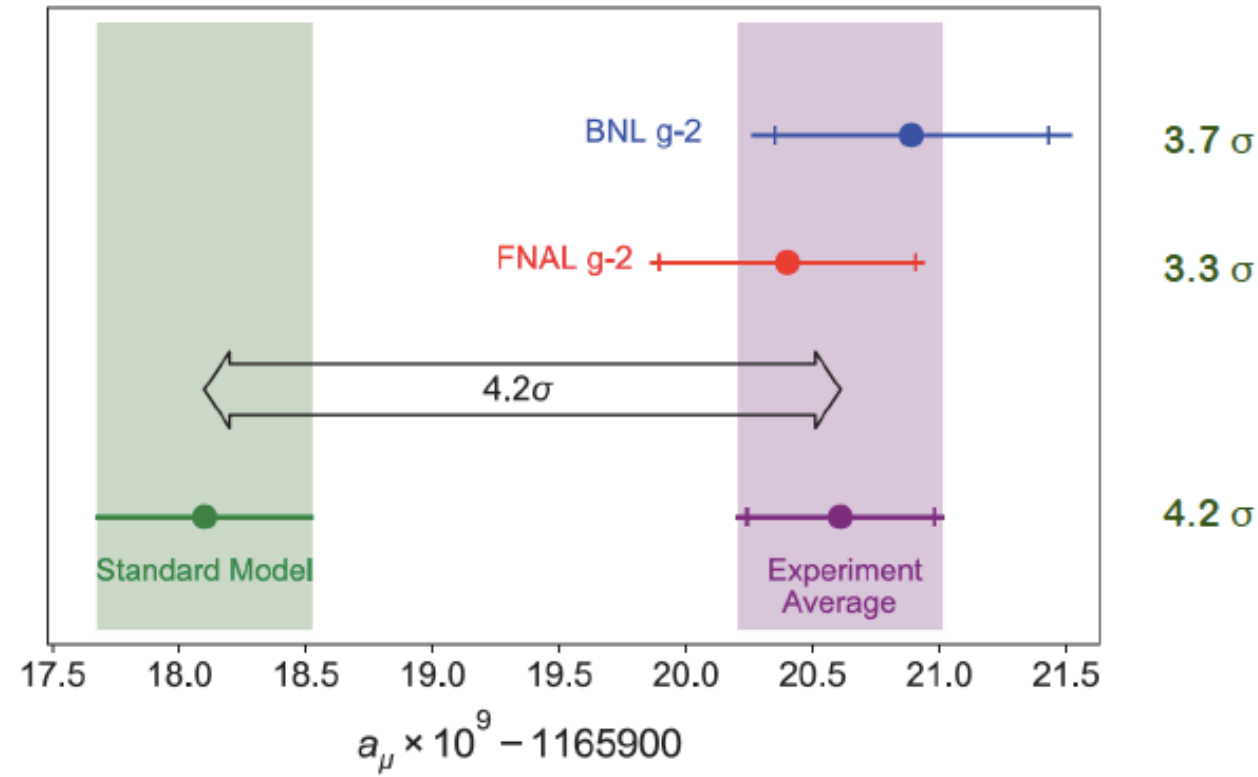
Muon g-2 TI WP: arXiv:2006.04822

BMW20: S. Borsanyi et al. 2002.12347, published on Nature, April 7, 2021  
first published lattice result with sub-percent precision!





## Muon g-2: FNAL confirms BNL



$$a_\mu^{\text{EXP}} = (116592089 \pm 63) \times 10^{-11} [0.54\text{ppm}] \quad \text{BNL E821}$$

$$a_\mu^{\text{EXP}} = (116592040 \pm 54) \times 10^{-11} [0.46\text{ppm}] \quad \text{FNAL E989 Run 1}$$

$$a_\mu^{\text{EXP}} = (116592061 \pm 41) \times 10^{-11} [0.35\text{ppm}] \quad \text{WA}$$

- FNAL aims at  $16 \times 10^{-11}$ . First 3 runs completed, 4th in progress.
- Muon g-2 proposal at J-PARC: Phase-1 with  $\sim$  BNL precision.

Comparing the SM prediction with the measured muon g-2 value:

$$a_{\mu}^{\text{EXP}} = 116592061 (41) \times 10^{-11}$$

BNL+FNAL

$$a_{\mu}^{\text{SM}} = 116591810 (43) \times 10^{-11}$$

WP20

$$\Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = 251 (59) \times 10^{-11}$$

4.2  $\sigma$

Is  $\Delta a_{\mu}$  due to new physics beyond the SM?

# Could $\Delta a_\mu$ (using DRs for $a_\mu^{\text{SM}}$ ) be due to some **missing contributions** in the hadronic cross section?

Yes, possible to increase the hadronic cross section to account for  $\Delta a_\mu$ , BUT:

- If such increase occurs because of new (so far missed) contributions **ABOVE  $\sim 1$  GeV**  $\rightarrow$  **conflict with the EW precision fit** arises  
Marciano, Passera, Sirlin 2008 & 2010;  
Keshavarzi, Marciano, Passera, Sirlin 2020
- If such increase occurs **BELOW  $\sim 1$  GeV**  $\rightarrow$  entails a **conflict with the precision of the measured hadronic cross section**

(KMPS 2020, updated 2021)

M. Passera, talk at the IAS, Apr. 13, 2021

- Crivellin, Hoferichter, Manzari and Montull, “Hadronic vacuum polarization:  $(g-2)_\mu$  versus global electroweak fits,” arXiv:2003.04886.
- Eduardo de Rafael, “On Constraints Between  $\Delta\alpha_{\text{had}}(M_Z^2)$  and  $(g_\mu-2)_{\text{HVP}}$ ,” arXiv:2006.13880.
- Malaescu and Schott, “Impact of correlations between  $a_\mu$  and  $\alpha_{\text{QED}}$  on the EW fit,” arXiv:2008.08107.
- Colangelo, Hoferichter and Stoffer, “Constraints on the two-pion contribution to hadronic vacuum polarization,” arXiv:2010.07943.

# NEW PHYSICS for the muon g-2: at which scale?

$$\Delta a_\mu \equiv a_\mu^{\text{NP}} \approx (a_\mu^{\text{SM}})_{\text{weak}} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$


- ▶ A weakly interacting NP at  $\Lambda \approx v$  can naturally explain  $\Delta a_\mu \approx 2 \times 10^{-9}$
- ▶  $\Lambda \approx v$  favoured by the *hierarchy problem* and by a WIMP DM candidate.

On the other hand, HE experiments (LEP, Tevatron, LHC) have NOT provided any clue for the presence of new (charged) particles at the ELW. scale

- ▶ NP is very light ( $\Lambda \lesssim 1$  GeV) and feebly coupled to SM particles.
- ▶ NP is very heavy ( $\Lambda \gg v$ ) and strongly coupled to SM particles.

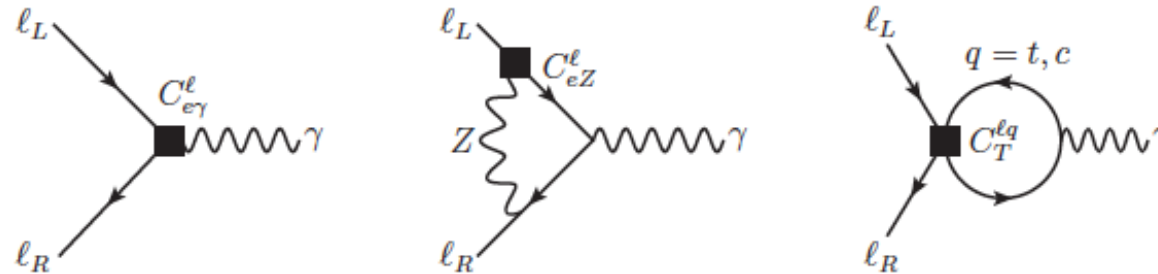
P. Paradisi, La Thuile 2021

$$\mathcal{L}_{\text{EFT}}(\Lambda \gg v) = \frac{C_{e\gamma}^\ell}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H F_{\mu\nu} + h.c. \quad \Rightarrow \quad \Delta a_\mu = \frac{4m_\mu v}{e\Lambda^2} C_{e\gamma}^\mu$$

- What is the NP scale  $\Lambda$  probed by  $\Delta a_\mu \equiv a_\mu^{\text{NP}} = (2.79 \pm 0.76) \times 10^{-9}$ ?

- SMEFT Lagrangian relevant for  $\Delta a_\ell$**

$$\mathcal{L} = \sum_{V=B,W} \frac{C_{eV}^\ell}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H V_{\mu\nu} + \sum_{q=c,t} \frac{C_T^{\ell q}}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R) (\bar{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$



$$\Delta a_\ell \simeq \frac{4m_\ell V}{e\Lambda^2} \left( C_{e\gamma}^\ell - \frac{3\alpha}{2\pi} \frac{c_W^2 - s_W^2}{s_W c_W} C_{eZ}^\ell \log \frac{\Lambda}{m_Z} \right) - \sum_{q=c,t} \frac{4m_\ell m_q}{\pi^2} \frac{C_T^{\ell q}}{\Lambda^2} \log \frac{\Lambda}{m_q},$$

$$\frac{|\Delta a_\mu|}{3 \times 10^{-9}} \approx \left( \frac{250 \text{ TeV}}{\Lambda} \right)^2 |C_{e\gamma}^\mu|$$

$$\frac{|\Delta a_\mu|}{3 \times 10^{-9}} \approx \left( \frac{50 \text{ TeV}}{\Lambda} \right)^2 |C_{eZ}^\mu|$$

$$\frac{|\Delta a_\mu|}{3 \times 10^{-9}} \approx \left( \frac{100 \text{ TeV}}{\Lambda} \right)^2 |C_T^{\mu t}|$$

$$\frac{|\Delta a_\mu|}{3 \times 10^{-9}} \approx \left( \frac{10 \text{ TeV}}{\Lambda} \right)^2 |C_T^{\mu c}|$$

- Strongly coupled NP:**  $C_{e\gamma}^\mu, C_T^{\mu t} \sim g_{\text{NP}}^2/16\pi^2 \lesssim 1$  implying  $\Lambda \lesssim \text{few} \times 100 \text{ TeV}$ , beyond the direct production reach of any foreseen collider.
- Weakly coupled NP:**  $C_{e\gamma}^\mu, C_T^{\mu t} \lesssim 1/16\pi^2$  implying  $\Lambda \lesssim 20 \text{ TeV}$  maybe within the direct production reach of a very high-energy Muon Collider

# Minimal extensions of the SM to account for the $(g-2)_\mu$ anomaly

Addition of a **SINGLE NEW FIELD**:

i) The addition of a **single fermion** cannot explain this anomaly ;

(C. Biggio 2008; Freitas, Lykken, Kell, Westhoff 2014; Biggio, Bordone 2014)

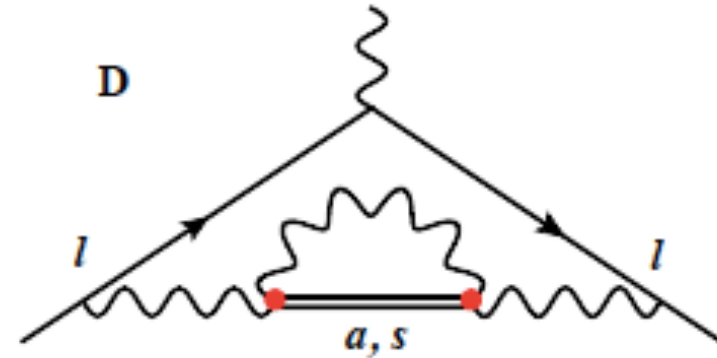
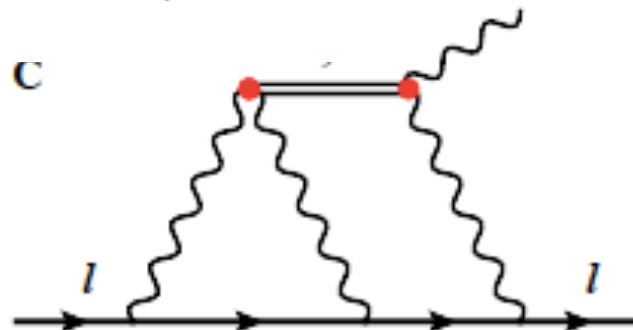
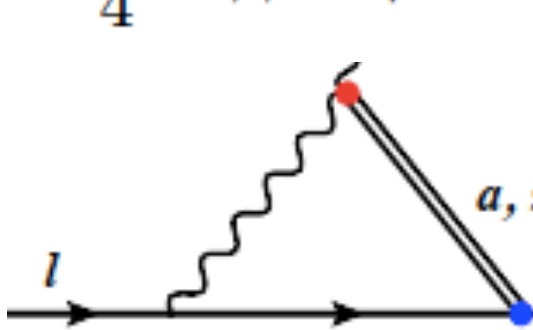
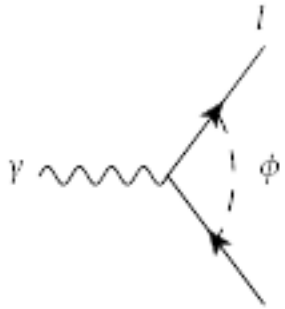
ii) The addition of a **single scalar** can account for the discrepancy if the new scalar is:

a **new Higgs doublet**; (Freitas, Lykken, Kell, Westhoff 2014; Broggio, Chun, Passera, Patel, Vempati 2014; Biggio, Bordone 2014; Cherchiglia, Kneschke, Stockinger, Stockinger-Kim 2017)

one of the two **leptoquarks**:  $S^{1/3}(3, 1, -1/3; Q = -1/3)$ ;  $D^{7/6}(3, 2, 7/6; Q = 5/3, 2/3)$  Chakraverty, D. Choudhuri, Datta 2001; Biggio, Bordone 2014; Queiroz, Shepherd 2014; Coluccio Leskow, D'Ambrosio, Crivellin, Muller 2017

- **iii)** one massive **vector boson**: only possibility  $\rightarrow$  abelian gauge extensions –  $Z'$ , dark photon (Biggio, Bordone, Di Luzio, Ridolfi 2016; Davoudiasl, H.-S.Lee, Marciano 2014; Altmannshofer, C.-Y. Chen, Dev, Soni 2016; )
- **iv)** **ALPs** (ALP-photon photon + ALP Yukawa interactions with leptons)

$$\mathcal{L} = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} + i y_{a\psi} a \bar{\psi} \gamma_5 \psi$$



Chen, Davoudiasl,  
Marciano, Zhang 2016

One-loop  
contribution

Two-loop  
contributions

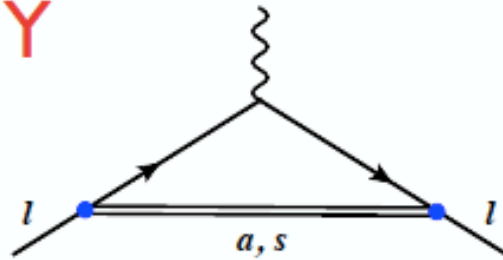
Marciano, AM, Paradisi, Passera 2016



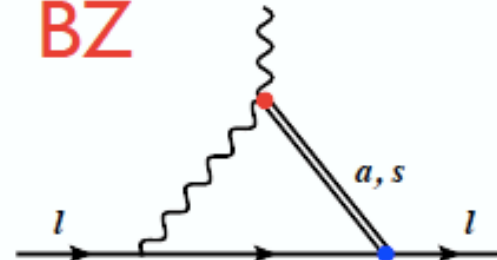
# ALPs contributions to the muon g-2?

$\mu$

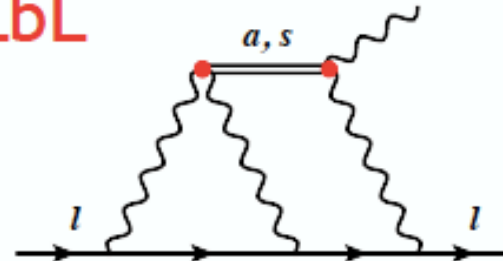
Y



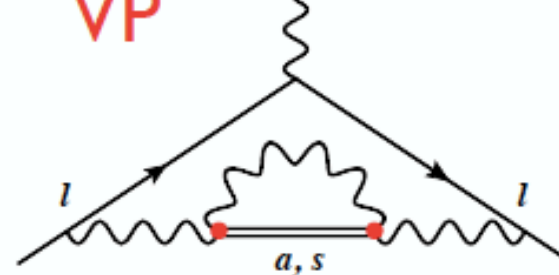
BZ



LbL



VP

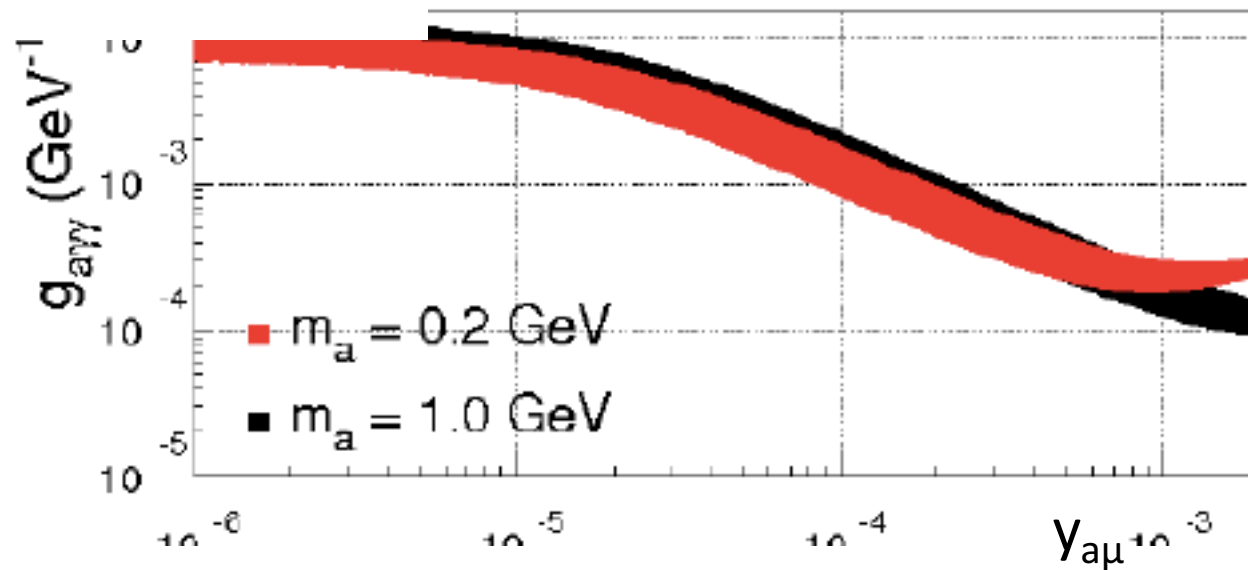


Marciano, AM, Paradisi, Passera 2016;  
Bauer, Neubert, Thamm 2017

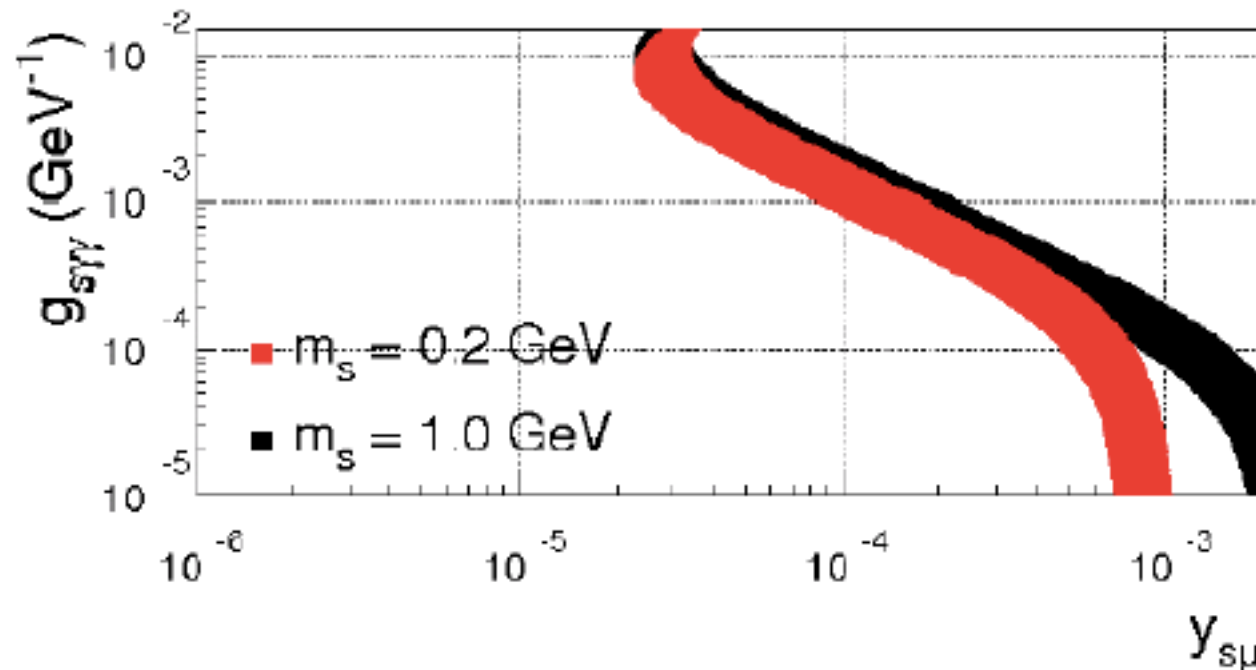
- Both scalar and pseudoscalar ALPs can solve  $\Delta a_\mu$  for masses  $\sim [100\text{MeV}-1\text{GeV}]$  and couplings allowed by current experimental constraints.
- They can be tested at present low-energy  $e^+e^-$  experiments, via dedicated  $e^+e^- \rightarrow e^+e^- + \text{ALP}$  &  $e^+e^- \rightarrow \gamma + \text{ALP}$  searches.

$$\mathcal{L} = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} + i y_{a\psi} a \bar{\psi} \gamma_5 \psi$$

$$g_{a\gamma\gamma} \equiv \frac{2\sqrt{2}\alpha}{\Lambda} c_{a\gamma\gamma}$$



Pseudoscalar 1σ solution bands to the g-2 muon anomaly taking  $\Lambda = 1$  TeV



Scalar 1σ solution bands to the g-2 muon anomaly taking  $\Lambda = 1$  TeV

# Experimental tests at $e^+e^-$ colliders

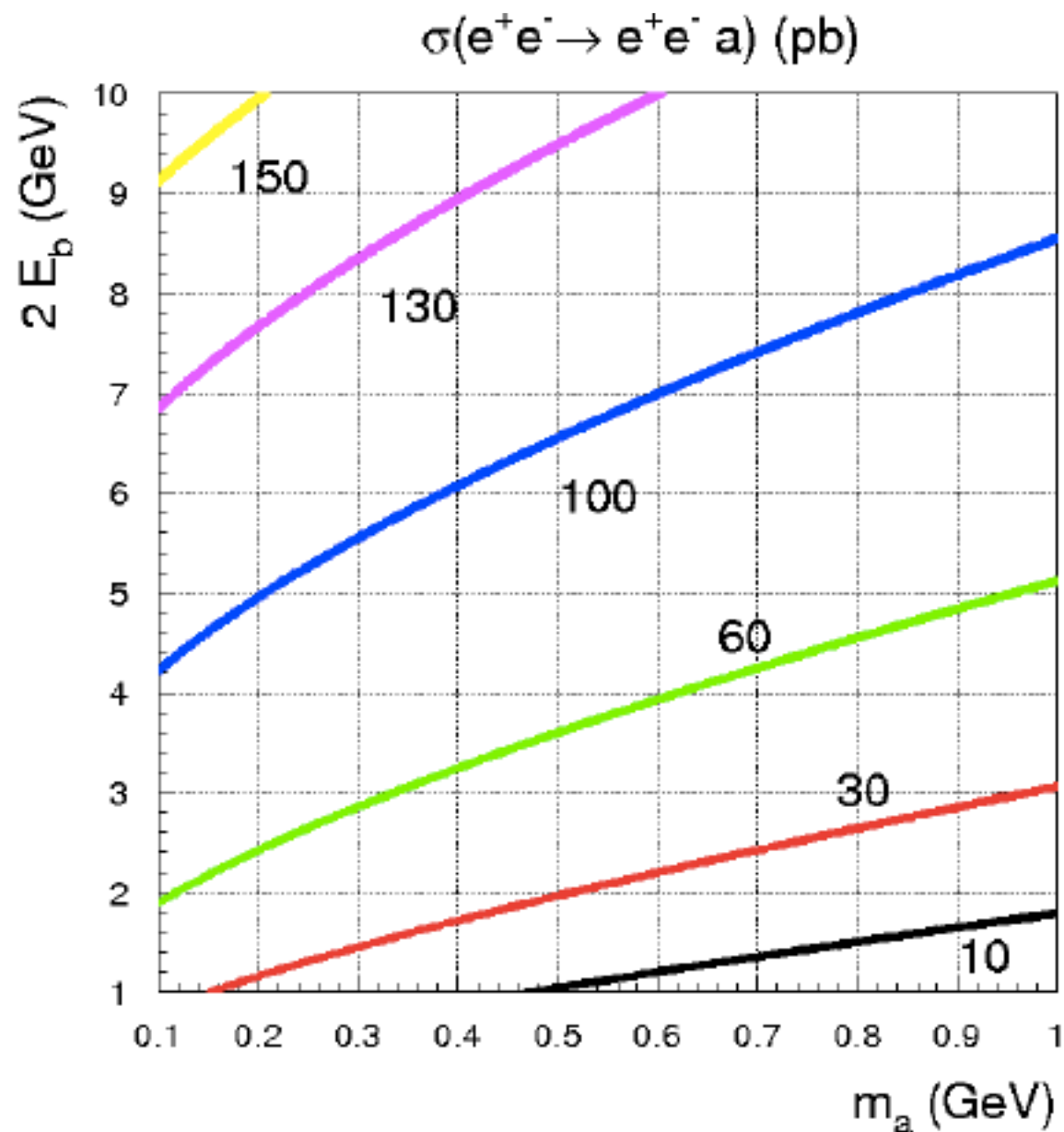
MMPP 2016

$$\begin{aligned} e^+e^- &\rightarrow e^+e^-\gamma^*\gamma^* \rightarrow e^+e^-a, \\ e^+e^- &\rightarrow \gamma^* \rightarrow \gamma a, \end{aligned}$$

$$\sigma_{eea} \simeq \frac{\alpha^2}{4\pi} g_{a\gamma\gamma}^2 \left( \ln \frac{E_b}{m_e} \right)^2 f\left(\frac{m_a}{2E_b}\right)$$
$$E_b \equiv \sqrt{s}/2$$

$$\sigma_{eea}(\sqrt{s} = 1 \text{ GeV}) \approx 31 \text{ pb} \left( \frac{g_{a\gamma\gamma}}{10^{-2} \text{ GeV}^{-1}} \right)^2,$$

$$\sigma_{\gamma a}(\sqrt{s} = 1 \text{ GeV}) \approx 9 \text{ pb} \left( \frac{g_{a\gamma\gamma}}{10^{-2} \text{ GeV}^{-1}} \right)^2,$$



- **BR( $\ell_i \rightarrow \ell_j \gamma$ ) vs.  $(g - 2)_\mu$**

$$\text{BR}(\mu \rightarrow e \gamma) \approx 3 \times 10^{-13} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{e\mu}}{10^{-5}} \right)^2$$

$$\text{BR}(\tau \rightarrow \mu \gamma) \approx 4 \times 10^{-8} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{\mu\tau}}{10^{-2}} \right)^2$$

- **EDMs vs.  $(g - 2)_\mu$**

$$d_e \simeq \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 10^{-28} \left( \frac{\phi_e^{CPV}}{10^{-4}} \right) e \text{ cm},$$

$$d_\mu \simeq \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22} \phi_\mu^{CPV} e \text{ cm}.$$

- **Main messages:**

- ▶  $\Delta a_\mu \approx (3 \pm 1) \times 10^{-9}$  requires a nearly flavor and CP conserving NP
- ▶ Large effects in the muon EDM  $d_\mu \sim 10^{-22} e \text{ cm}$  are still allowed!

[Giudice, P.P., & Passera, '12]

# DM and g-2 as windows to New Physics

- **Minimal extensions of the SM to account for the DM:** one additional field that being neutral and stable might have been in thermal equilibrium interacting with ordinary matter and today have the correct density to account for the DM
- **Minimal extensions of the SM to account for the g-2 anomaly:** one single additional field (leptoquark or additional Higgs doublet or ALPs) coupling sizeably to leptons and/or photons
- Is it possible to have just one single additional field to account for both the DM **and** the g-2 anomaly? No, the DM fields in these minimal SM extensions decay too quickly to ordinary matter particles. **One needs at least two new fields** (for instance one additional fermion and one additional scalar)

Calibbi, Ziegler, Zupan 2018

# Muon-electron scattering: The MUonE Project

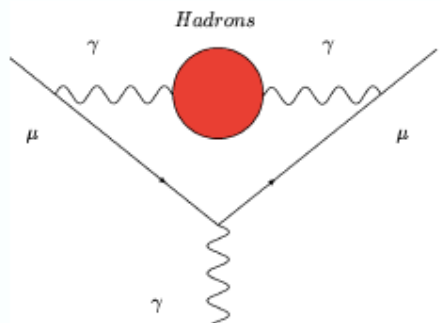
Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,  
Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni  
EPJC 2017 - arXiv:1609.08987



A new approach to  $a_\mu^{\text{HLO}}$

C. Carloni Calame, MP, L. Trentadue, G. Venanzoni  
PLB 2015 - arXiv:1504.02228

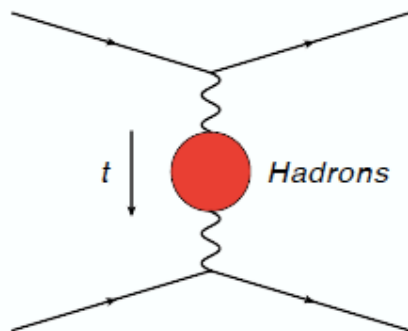
- At present, the leading hadronic contribution  $a_\mu^{\text{HLO}}$  is computed via the **timelike** formula:



$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds K(s) \sigma_{\text{had}}^0(s)$$

$$K(s) = \int_0^1 dx \frac{x^2 (1-x)}{x^2 + (1-x)(s/m_\mu^2)}$$

- Alternatively, exchanging the  $x$  and  $s$  integrations in  $a_\mu^{\text{HLO}}$



$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{\text{had}}(t)$  is the hadronic contribution to the running of  $\alpha$  in the **spacelike** region:  $a_\mu^{\text{HLO}}$  can be extracted from scattering data!

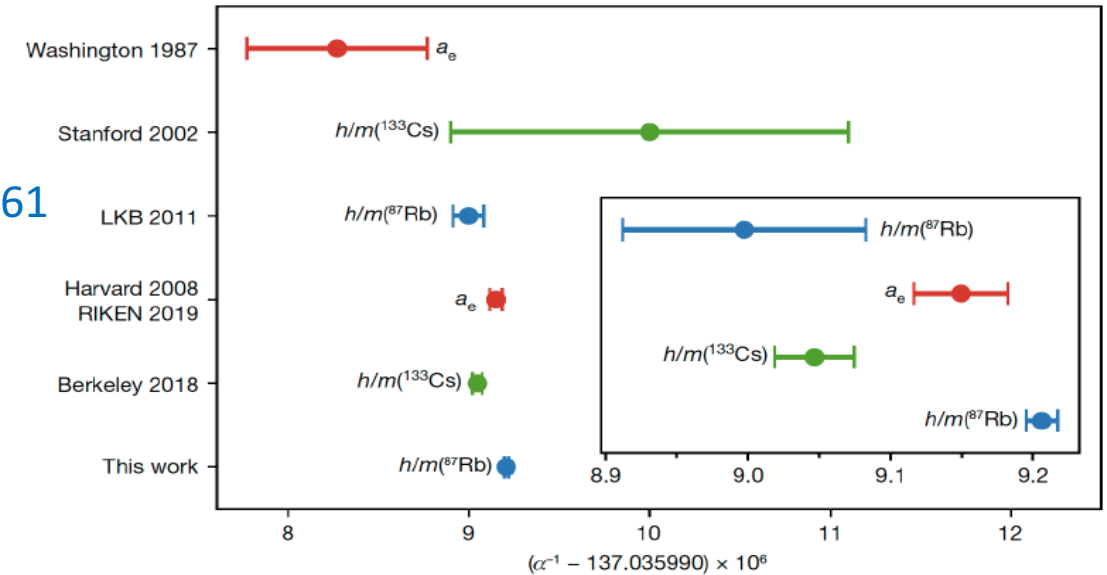


# A new life for an old protagonist: the g-2 of the ELECTRON

$$a_e^{\text{EXP}} = 115\,965\,218\,07.3(2.8) \times 10^{-13} \quad \text{Hanneke et al, PRL 2008}$$

The g-2 of the electron **no longer** provides the best determination of  $\alpha$

Morel et al.,  
Nature 588 (2020) 61



$$\begin{aligned} \Delta a_e &= a_e^{\text{EXP}} - a_e^{\text{SM}} = -8.9(3.6) \times 10^{-13} [2.5\sigma] \text{ [Cs18]} \\ &= +4.7(3.0) \times 10^{-13} [1.6\sigma] \text{ [Rb20]} \end{aligned}$$

The  $(g-2)_e$  exp. error may soon become  $< 10^{-13} \rightarrow$   
 $(g-2)_e$  can soon play a pivotal role in probing NP in the leptonic sector

Giudice, Paradisi, Passera 2012

- In a broad class of BSM theories, contributions to  $a_\ell$  scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left( \frac{m_{\ell_i}}{m_{\ell_j}} \right)^2 \quad \text{This Naive Scaling leads to:}$$

$$\Delta a_e = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}; \quad \Delta a_\tau = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.8 \times 10^{-6}$$



- The sensitivity in  $\Delta a_e$  may soon drop below  $10^{-13}$ ! This will bring  $a_e$  to play a pivotal role in probing new physics in the leptonic sector.
- NP scenarios exist which **violate Naive Scaling**. They can lead to larger effects in  $\Delta a_e$  and contributions to EDMs, LFV or lepton universality breaking observables.

Giudice, Paradisi & MP, JHEP 2012

Crivellin, Hoferichter, Schmidt-Wellenburg, PRD 2018

- One real scalar with a mass of  $\sim 250-1000$  MeV could explain the deviations in  $a_\mu$  and  $a_e$ , through one- and two-loop processes, respectively.

Davoudiasl & Marciano, PRD 2018

## ...some final thoughts on the leptonic $g-2$

- $(g-2)_\mu$  certainly represents the most longstanding possible hint (constantly at the 3 – 4  $\sigma$  significance level) of NP BSM (now, thanks to this last exp. result, growing to 4.2  $\sigma$ )
- the recent **BMW remarkable ab initio lattice computation** of  $(g-2)_\mu$  is nicely quite close to the experimental value of  $(g-2)_\mu$ , however its discrepancy w.r.t. the traditional dispersive relation **cannot** be accounted for by  $\Delta\sigma(s)$  shifts **above  $\sim 1\text{GeV}$**  (otherwise the global EW fit is ruined) and by shifts **below  $\sim 1\text{GeV}$**  to avoid conflicts with the quoted exp. error of  $\sigma(s)$
- An important  $(g-2)$  leptonic synergy:  **$(g-2)_\mu$  and  $(g-2)_e$**
- The NP accounting for  $(g-2)_\mu$  can lead to potentially relevant enhancements in **leptonic EDMs and LFV physics**
- **MUonE** can soon provide an **independent determination** of the leading hadronic contributions to  $a_\mu$  alternative to both the dispersive and lattice methods