“Broken translation invariance for fluids in external magnetic fields”

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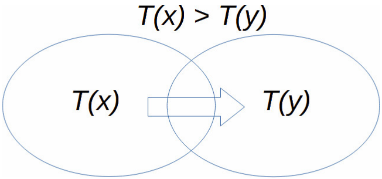
What is hydrodynamics?



- Describes the flow of charges between local equilibria - “short scattering time”.
- Hydrodynamics broadly consists of three elements:
  1. Spatially varying thermodynamic parameters e.g. $T(x)$, $u^\mu(x)$, $\ldots$,
  2. Conservation eqns e.g. $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu J^\mu = 0$, $\ldots$,
  3. Constitutive relations with transport coefficients e.g.

$$J^\mu = q u^\mu + \sigma^{\mu\nu} \partial_\mu \left(\frac{\mu}{T} \right) + \mathcal{O}(\partial^2) \, .$$

Linearised hydrodynamics, qnms and Green's functions

- Hydrodynamics $\to$ non-linear evolution equations.
- Simplification - linearise about equilibrium:

$$\delta\langle J^\mu\rangle \sim \int G_R^{\mu\nu}\,\delta A_\nu \ . \tag{1}$$

- Structure G_R^{hydro}:

$$G_R^{\text{hydro}}(\omega,\vec{k}) \sim \frac{\alpha_0(\vec{k})}{\omega-\omega_0(\vec{k})} + \alpha_1(\vec{k}) \ . \tag{2}$$

e.g. $\omega_0(\vec{k}) = -iD\vec{k}^2 + \mathcal{O}(\vec{k}^4)$.

- Hydrodynamics is a "good" description if
  1. $\omega(k) = \sum_n c_n k^n$ converges,
  2. and $g = G_R - G_R^{\text{hydro}}$ is small.

Linearised hydrodynamics, qnms and Green's functions

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$$\delta\langle J^{\mu}\rangle \sim \int G_R^{\mu\nu}\,\delta A_{\nu}\ . \tag{3}$$

- Structure G_R^{hydro}:

$$G_R^{\mathrm{hydro}}(\omega,\vec{k}) \sim \left(\alpha_1(\vec{k}) - \frac{\alpha_0(\vec{k})}{\omega_0(\vec{k})}\right) + \frac{\alpha_0(\vec{k})\omega}{\omega_0(\vec{k})^2} + \mathcal{O}(\omega^2)\ .$$

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- Hydrodynamics is a "good" description if
  1. $\omega_0(k) = \sum_n c_n k^n$ converges,
  2. and $g = G_R - G_R^{\mathrm{hydro}}$ is small about $\omega = 0$.

Quasihydrodynamics & magnetic fields

- A simple, relevant, example is a charged fluid in a magnetic field.
- Cousin of hydrodynamics where we allow conservation to "relax" e.g.

$$\partial_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu \neq 0 \,, \qquad \partial_\mu J^\mu = 0 \,. \tag{4}$$

- Regardless of the microscopic nature of such a fluid the DC conductivities take the form:
  1. $\sigma_{\rm L} = \alpha_{\rm L} = 0$,
  2. $\sigma_{\rm H} = n$ and $\alpha_{\rm H} = sT + \mu n$.
- Why?

Ward identities

- Let's define the canonical heat current - $Q^i = P^i - \mu J^i$.
- We will assume spatial rotation invariance and spatial parity invariance.
- There are three interesting observables: $\sigma^{ij}(\omega)$, $\alpha^{ij}(\omega)$ and $\kappa^{ij}(\omega)$.
- There is a tower structure in the Ward identities because of how translation invariance is broken:

$$
\begin{aligned}
\langle Q^i J^j \rangle &= -\left(\mu \delta^i{}_k - \frac{iB}{\omega} \epsilon^i{}_k \right) \langle J^k J^j \rangle - n\delta^{ij} \,, \\
\langle Q^i Q^j \rangle &= -\left(\mu \delta^i{}_k - \frac{iB}{\omega} \epsilon^i{}_k \right) \langle Q^k J^j \rangle \\
&\quad -(sT + \mu n)\delta^{ij} \,.
\end{aligned}
$$

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\begin{aligned}
\alpha^{ij}(\omega) &= -\left(\mu\delta^i{}_k - \frac{iB}{\omega}\epsilon^i{}_k\right)\sigma^{kj}(\omega) - \frac{n}{\omega}\delta^{ij}\,, \\
\kappa^{ij}(\omega) &= -\left(\mu\delta^i{}_k - \frac{iB}{\omega}\epsilon^i{}_k\right)\alpha^{kj}(\omega) - \frac{(sT+\mu n)}{\omega}\delta^{ij}\,.
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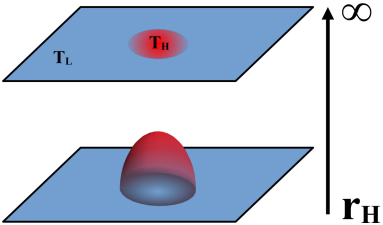
- There is a tower structure in the Ward identities because of how translation invariance is broken:

$$\begin{aligned}\sigma_{\mathrm{L}}(\omega) &= -\frac{i\chi_{\pi\pi}\omega}{B^2} + \frac{\kappa_{\mathrm{L}}(0)\omega^2}{B^2} + \dots\ , \\ \sigma_{\mathrm{H}}(\omega) &= n + \left(\kappa_{\mathrm{H}}(0) + \mu\left(2\chi_{\pi\pi} - \mu n\right)\right)\frac{\omega^2}{B^2} + \dots\ .\end{aligned}$$

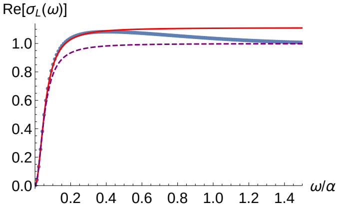
where $\chi_{\pi\pi} = sT + \mu n$.

Fluids dual to black holes



- It's still hard to compute G_R.
- A very useful set of toy models are those given by gauge-gravity.
- Important facts:
  1. gravitational fluctuations of black holes in AdS $\leftrightarrow$ fluid dynamics of some ("strongly coupled" field theory),
  2. solving Einstein's equations in AdS space allows us to compute retarded Greens functions for some field theory.

The dyonic black hole

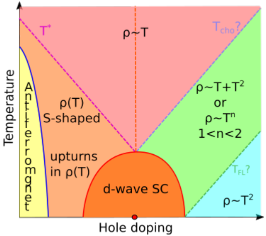


- The dyonic black hole is a special one to $(3+1)$-dimensions.
- It has a charge and magnetic field.
- In our paper (2005.09662) we used quasihydrodynamic framework to argue for new transport coefficients - the incoherent Hall conductivities.

Summary of introduction - key ideas

- Hydrodynamics describes late time behaviour of interacting systems.
- Quasi-hydrodynamics describes situations where hydrodynamic charges relax.
- Holographic models are useful toys for testing ideas.

Phase diagram of high $-T_c$ superconductors



A cartoon of a high $-T_c$ superconductor phase diagram.

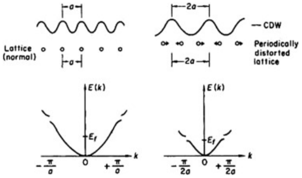
- **What are they?** Materials with $T_c \gg 30$ K.
- **What is the strange metal phase?**
  - Comparison of Fermi liquid and strange metal scaling

$$\mathrm{FL}: \quad \rho_{xx} \sim T^2 , \quad \cot(\Theta_{\mathrm{H}}) = \frac{\rho_{xx}}{\rho_{xy}} \sim T^2 ,$$

$$\mathrm{Strange}: \quad \rho_{xx} \sim T , \quad \cot(\Theta_{\mathrm{H}}) = \frac{\rho_{xx}}{\rho_{xy}} \sim T^2 .$$

  - QCP under dome?

Charge density wave order



1D CDW distortion and energy band gaps from Bhadeshia et al. (2014).

- **What are charge density waves?**
  - Peierls (1955) suggested periodic distortion of 1D lattice can lower total energy.
  - Start with first Brillouin zone $k = \pm\pi/a$ half filled.
  - CDW distortion $\rightarrow$ new superlattice of spacing $2a$. New first Brillouin zone band gap at $k = \pm\pi/2a$.
  - Gain in creating energy gaps can overcome loss of lattice distortion.
- Incommensurate CDW $\rightarrow$ broken translation invariance.

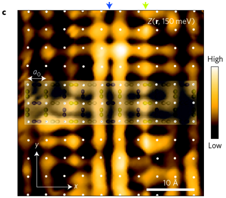
An interlude: broken translation and Goldstone bosons

- Traditional Goldstone theorem - compact group e.g. $SU(2)$, unbroken spatial translation invariance.
- Naïve application of Goldstone theorem needs Fourier transforms/spatial translation invariance.
- Idea: break a double copy $ISO(d) \times ISO(d) \to ISO(d)$.
- Lagrangian for Goldstone bosons will look like

$$\begin{aligned} L &= \partial_\mu \phi^I \partial_\mu \phi_I + \text{higher derivative terms}, \\ \langle O_I \rangle &= x_i + \delta\phi_I(x) \; . \end{aligned}$$

- Shift symmetry of "internal" $ISO(d)$: $\phi_I \to \phi_I - \alpha_I$.
- Spatial translation of $ISO(d)$: $x_i \to x_i + a_i$.
- Diagonal $ISO(d)$ symmetry remains if $a_i = \alpha_I$ - Goldstone theorem follows.

An intriguing observation...

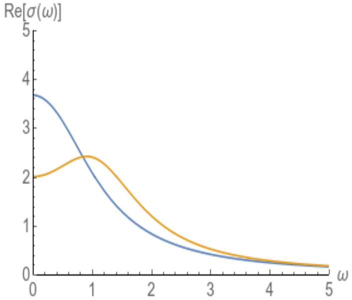


Scan of electronic structure in $Bi_2Sr_2CaCu_2O_8$. Image taken from Hamidian et al. (2016).

- Charge-density wave (CDW) order appears to be a ubiquitous feature of cuprate superconductors.
- Our material, $Bi_2Sr_2CuO_6$:
  - 2D CDW confirmed (by X-ray diffraction) to extend to optimal and over-doped region,[1]
  - low critical temperature ($T_c \sim 10 - 33$ K).

[1] Peng et al., "Re-entrant charge order in overdoped $(Bi,Pb)_{2.12}Sr_{1.88}CuO_{6+\delta}$ outside the pseudogap regime".

CDWs and pinning



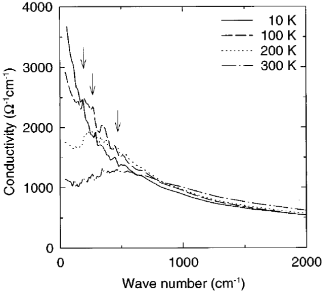


Image from Tsvetkov et al. (1997).

- A non-zero mass for Goldstone of broken translation (i.e. "a pinning frequency") can lead to off-axis peaks.
- Off-axis peak if there is a pinning frequency

$$\sigma(\omega) \quad = \quad \sigma_{(\mathrm{L})} + \frac{n^2}{\chi_{\pi\pi}} \frac{\Omega_{(\mathrm{L})} - i\omega}{\left(\Omega_{(\mathrm{L})} - i\omega\right)\left(\Gamma_{(\mathrm{L})} - i\omega\right) + \omega_0^2} \,.$$

Aims

- **What did we want to do?**
  1. Experiment suggests CDW order important for all thermoelectric transport.[2]
  2. Try to understand properties of strange metal phase as a combination of QCP and CDW order.
  3. Match against results from a single material.
- **Problem:** no quasi-particle description, strong coupling.
- **Answer:** use hydrodynamics.
  - No need for specific microscopic mechanism.[3]
  - Right thing to use when no quasiparticles, only long-lived modes are conserved currents.

[2] Cyr-Choinière et al., "Enhancement of the Nernst effect by stripe order in a high-Tc superconductor".

[3] Delacrétaz et al., "Theory of hydrodynamic transport in fluctuating electronic charge density wave states"; Delacrétaz et al., "Theory of collective magnetophonon resonance and melting of a field-induced Wigner solid".

Conservation laws and hydrodynamic flows

- Hydrodynamic model: conservation equations

$$\partial_t n + \partial_i J^i = 0 \,, \qquad \partial_t s + \partial_i \left(\frac{Q^i}{T} \right) = 0 \,, \tag{5}$$

$$\partial_t P^i + \partial_j T^{ij} = -\Gamma^{ij} P_j + F^{ij} J_j - \partial^j \Phi^i O_j \,, \tag{6}$$

- a Josephson relation

$$\partial_t O^i + \Omega^{ij} O_j = -v^i + \text{higher order in } \partial \,, \tag{7}$$

- and the constitutive relations e.g.

$$\begin{aligned} J^i &= n v^i + \sigma^{ij} \left(E_j - B v_j - \partial_j \mu \right) \\ &\quad + \alpha^{ij} \partial_j T + \gamma^{ij} \partial_j s \,, \end{aligned} \tag{8}$$

where e.g. $\sigma^{ij} = \sigma_{(\mathrm{L})} \delta^{ij} + \sigma_{(\mathrm{H})} F^{ij}$.

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with spatial parity + $B \sim \mathcal{O}(\partial)$ + relativistic + assumption.

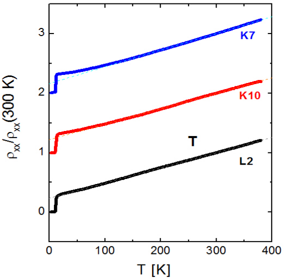
Results

- **Result:** Taking the DC transport coefficients to lowest order in B:
  - Charge resistivity: $\rho_{xx} = \frac{1}{\sigma_{(\mathrm{L})}+\tilde{\sigma}} + \mathcal{O}(B^2)$.
  - Magnetoresistance: $\frac{\Delta\rho}{\rho} = B^2 \frac{\sigma_{(\mathrm{L})}^3\ \tilde{\sigma}}{n^2} \frac{1}{(\sigma_{(\mathrm{L})}+\tilde{\sigma})^2} + \mathcal{O}(B^4)$.
  - Thermal Hall conductivity: $\kappa_{xy} = -BT\frac{\tilde{\sigma}^2 s}{n^4}\left(ns + 2\frac{\alpha_{(\mathrm{L})}n^2}{\tilde{\sigma}}\right) + \mathcal{O}(B^3)$.
  - Hall angle: $\cot\Theta_H = \frac{n}{B\tilde{\sigma}}\frac{1+\frac{\sigma_{(\mathrm{L})}}{\tilde{\sigma}}}{1+2\frac{\sigma_{(\mathrm{L})}}{\tilde{\sigma}}} + \mathcal{O}(B)$.
  - Nernst coefficient: $N = \frac{B\ \sigma_{(\mathrm{L})}\ \tilde{\sigma}}{n^2(\sigma_{(\mathrm{L})}+\tilde{\sigma})^2}(s\sigma_{(\mathrm{L})} - n\alpha_{(\mathrm{L})}) + \mathcal{O}(B^3)$.
- DC electric longitudinal conductivity

$$\sigma_{\mathrm{DC}} = \sigma_{(\mathrm{L})} + \tilde{\sigma} \qquad \text{with} \qquad \tilde{\sigma} = \frac{n^2}{\chi_{\pi\pi}}\frac{\Omega_{(\mathrm{L})}}{\Omega_{(\mathrm{L})}\Gamma_{(\mathrm{L})} + \omega_0^2}\ .$$

- Only four variables $\sigma_{(\mathrm{L})}$, $\tilde{\sigma}$, n and s. But we will measure five observables - system overconstrained.

Matching data to experiment



Data:

- $\underline{\rho_{xx} \sim (T, B^0)}$,
- $\Delta\rho/\rho \sim (T^{-4}, B^2)$,
- $\cot(\Theta_H) \sim (T^{\frac{3}{2}}, B^{-1})$
- and $\kappa_{xy} \sim (T^{-3}, B)$.

Variables:

- n,
- s,
- $\sigma_{(L)}$,
- and $\tilde{\sigma}$.

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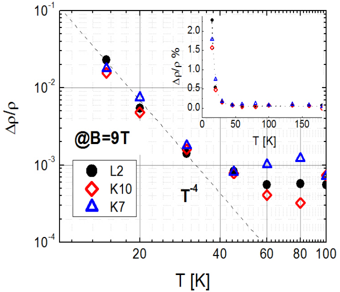

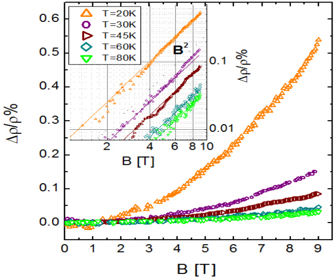

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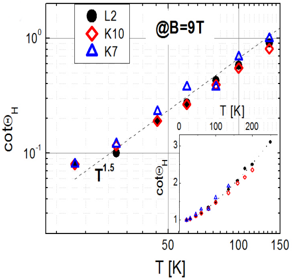

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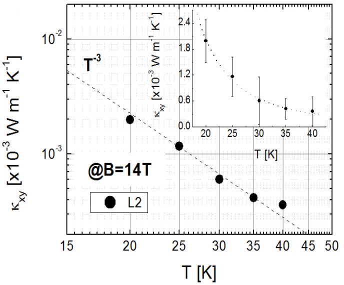

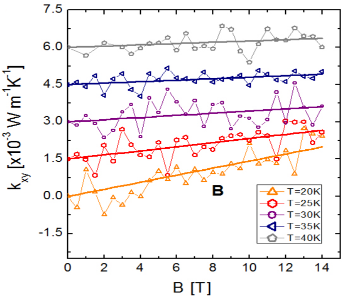

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Variables:

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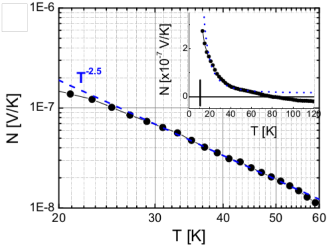
Recovering the Nernst behaviour



- The Nernst coefficient behaves as

$$N \sim \frac{\mu B \tilde{\sigma}}{nT} \sim \frac{\mu}{T \cot \Theta_H} \sim T^{-2.5} \,. \tag{13}$$

- We have complete consistency with experiment!
- N dominated by CDW relaxation, anomalously large Nernst signal can be explained.[4]

[4] Cyr-Choinière et al., "Enhancement of the Nernst effect by stripe order in a high-Tc superconductor".

Summary of our results

- Theory results:
  1. developed a hydrodynamic theory of CDW order with magnetic field,
  2. determined the heat current (previously not found in literature),
  3. and computed the DC transport coefficients in low B limit.
- Experimental results:
  1. rare case of measuring five transport coefficients in the same material,
  2. determined the low temperature scaling of the transport coefficients,
  3. and matched them to our theoretical model getting perfect agreement.
- Still need AC correlators if we are to claim perfect match.

AC response

- Subsequently Martina went on to study the non-zero frequency responses.
- Everything agreed with our model.
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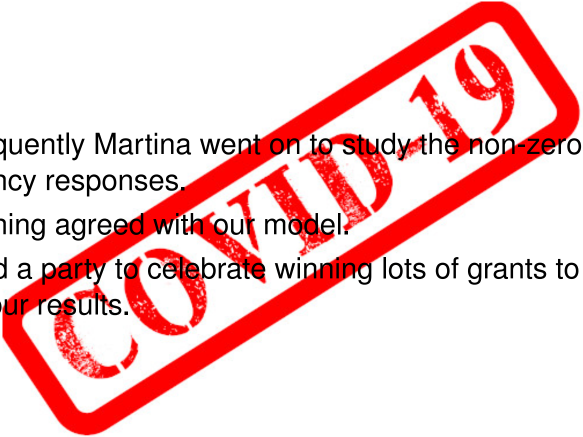


Ward identities and broken translation invariance

- Suppose our system satisfies relativistic conservation equations

$$\partial_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu - \left(\partial^\nu \Phi^i\right) O_i \,, \qquad \partial_\mu J^\mu = 0 \,. \quad (14)$$

- Again we assume rotation invariance and spatial parity invariance.
- The Ward identities are generally modified by presence of O_i

$$\alpha^{ij}(\omega) = -\left(\mu \delta^i{}_k - \frac{1}{i\omega} F^i{}_k\right) \sigma^{kj}(\omega) + \frac{\omega_0^2 \chi_{\pi\pi}}{(i\omega)^2} \langle J^i O^j \rangle(\omega) - \frac{n}{i\omega} \delta^{ij} \,. \quad (15)$$

- Again these have consequences for the AC conductivities.

Ward identities at low frequencies

- Spontaneous breaking

$$\sigma_{(\mathrm{L})}(\omega) = -\frac{i}{B^2}\left(\alpha_{(\mathrm{H})}(0) + \mu n\right)\omega + \frac{\kappa_{(\mathrm{L})}(0)}{B^2}\omega^2 + \mathcal{O}(\omega^3)\,, \quad (16)$$

$$\sigma_{(\mathrm{H})}(\omega) = \sigma_{(\mathrm{H})}(0) + \frac{\kappa_{\mathrm{H}}(0) + \mu\left(2\chi_{\pi\pi} - \mu n\right)}{B^2}\omega^2 + \mathcal{O}(\omega^3) \quad (17)$$

- Pseudo-spontaneous breaking

$$\sigma_{(\mathrm{L})}(\omega) = -\frac{\omega_0^4\chi_{\pi\pi}^2\zeta_{(\mathrm{L})}(0)}{B^2} + \mathcal{O}(\omega)\,, \quad (18)$$

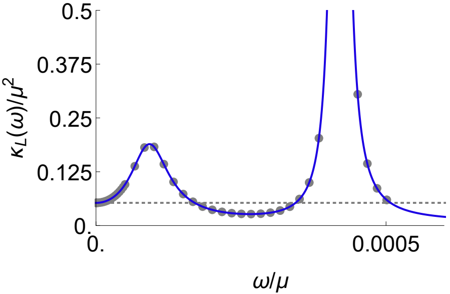
$$\zeta^{ij}(\omega) = \frac{1}{i\omega}\left(\langle O^j O^i\rangle - \delta^{ji}\right)\,. \quad (19)$$

Now use hydrodynamics

- Hydrodynamics also gives expressions for the DC conductivities.
- For $\Gamma = 0$, turns out that there is a one-to-one relationship between diffusive hydrodynamic transport coeffs. and DC.
- **What we learn:**
  1. We have analytic expressions for all the hydrodynamic transport coefficients in terms of $\sigma^{ij}_{\mathrm{DC}}$, $\alpha^{ij}_{\mathrm{DC}}$ and $\kappa^{ij}_{\mathrm{DC}}$.
  2. $\sigma^{ij} \neq \sigma_{(\mathrm{L})}\delta^{ij}$ but $\sigma^{ij} = \sigma_{(\mathrm{L})}\delta^{ij} + \sigma_{(\mathrm{H})}F^{ij}$.
  3. One can have off and on-axis Drude peaks from phase relaxation alone (no need for a Γ).
  4. Freezing out the "phonon"

$$\Gamma^i_j \equiv \omega_0^2 \left(\Omega^{-1}\right)^i_j \ . \tag{20}$$

Let me show you that this works...



The AC longitudinal thermal conductivity at $B/\mu^2 \approx 3 \times 10^{-4}$ and $T/\mu = 10^{-2}$ in the pseudo-spontaneous regime. Notice the appearance of a "phonon" peak displaced from $\omega = 0$.

Ideas for future work

- Return to our experimental paper and check results in light of new observations.
- By carefully using Ward identities we can analyse how different mechanisms contribute to the DC e.g.
  - a non-zero Γ,
  - charged lattices,
  - helical phases,
  - anyons,
  - anomalous transport...
- I have a new powerful procedure - I'm hungry for condensed matter systems to apply it to.

Thanks for listening!