Sommerfeld enhancements in the MSSM

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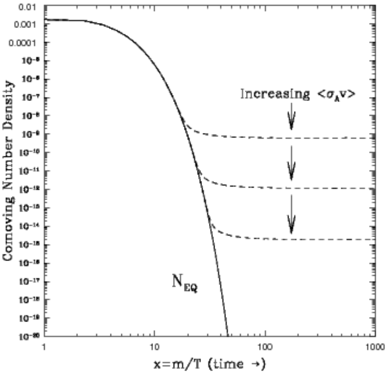
with R. Iengo and P. Ullio, in preparation

Introduction

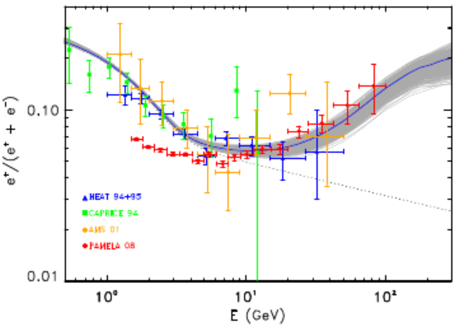


[Kolb, Turner]

Thermal relic density of dark matter particles is determined by the annihilation cross section at the about freeze-out temperature

$$\Omega h^2 \approx 0.1 \left(\frac{3 \cdot 10^{-26} cm^3 s^{-1}}{\langle \sigma_A \mathrm{v} \rangle_{T_{\mathrm{f.o.}}}} \right)$$

Introduction



Indirect detection depends on the same cross section, but for low velocity WIMPs in DM halos

$$\text{flux} \sim n^2 \langle \sigma_A v \rangle_{\text{DM halo}},$$

and to explain e^+/e^- puzzle we need

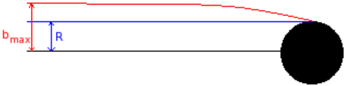
$$\langle \sigma_A v \rangle_{\text{DM halo}} \gg \langle \sigma_A v \rangle_{T_{\text{f.o.}}}$$

[Grasso et al. '09]

How one can solve this? $\Rightarrow$ Sommerfeld enhancement!

Sommerfeld effect - a classical analogy

[Arkani-Hamed et al. '09]



Without gravity the cross section: $\sigma_0 = \pi R^2$

With gravity it is enhanced: $\sigma = \pi b_{max}^2$ and

$$\sigma = \sigma_0 \left(1 + \frac{v_{esc}^2}{v^2}\right)$$

where $v_{esc} = \sqrt{2G_N M/R}$ is the escape velocity.

For $v \ll v_{esc}$ there is large enhancement!

Sommerfeld enhancement

Sommerfeld enhancement (effect) is a non-relativistic effect changing the cross section due to the wave function distortion by a long range potential.

Conditions for significant enhancement:

- slow incoming particles

$$\underbrace{m_\chi \mathrm{v}^2}_{\text{kinetic energy}} \lesssim \underbrace{\alpha^2 m_\chi}_{\text{Bohr energy}}$$

- long range force

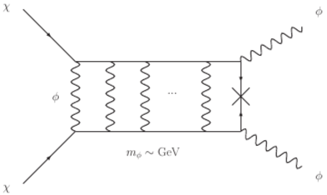
$$\underbrace{\frac{1}{m_\phi}}_{\text{force range}} \gtrsim \underbrace{\frac{1}{\alpha m_\chi}}_{\text{Bohr radious}}$$

m_χ - mass of DM particle, m_ϕ - mass of force carrier ϕ, v - CM velocity, $\alpha = g^2/(4\pi)$

Sommerfeld effect and dark matter

There has been plenty of work done to study this effect in context of

- indirect detection signals [Arkani-Hamed et al. '09, Chen et al. '09...]
- thermal relic density [Hisano et al. '06, Feng et al. '10,...]
- constraints from other data [Buckley et al. '09, Zavala et al. '10,...]

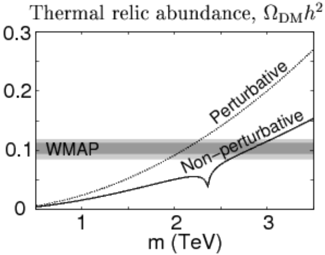


Common feature: new force in dark sector mediated by $m_\phi \sim 1\,\mathrm{GeV}$ particle
$\rightarrow$ to explain the cosmic flux anomalies (PAMELA, ATIC, etc.), but...

Sommerfeld enhancement without dark force

... even before the recent boom there have been works on such an effect coming only from gauge bosons

→ for the pure wino or pure higgsino in MSSM [Hisano et al. '03, '05]



→ for the Minimal Dark Matter model [Strumia et al. '07]

Effect not so big as in models with dark force, but stil important and much less speculative! However, it was studied in a simple models. What about more realistic setups?

Sommerfeld effect in the MSSM

In the MSSM:

- Dark matter $\rightarrow$ lightest neutralino χ_1^0
- possible intermediate bosons: $\gamma, W^{\pm}, Z^0, h_1^0, h_2^0, \; H^+$

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It would seem that to have a seizable effect

$$\frac{1}{m_W} \gtrsim \frac{1}{\alpha m_\chi} \qquad \Rightarrow \qquad m_\chi \gtrsim 2.3\ \text{TeV}$$

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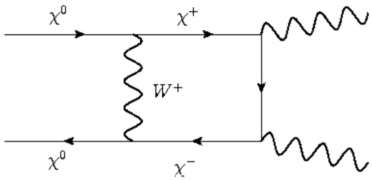
Moreover, if $\delta m = m_{\chi^+} - m_\chi$ is too large then the effect coming from

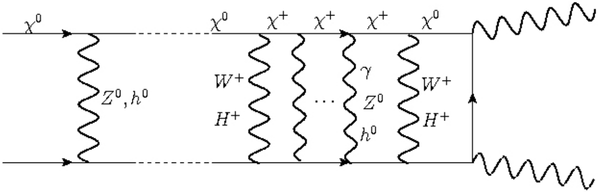


is very small $\rightarrow$ no enhancement at all...?

Sommerfeld effect in the MSSM

... but

- as soon as one can produce nearly on-shell χ^+, i.e. when $\mathcal{E} \approx 2\delta m$:

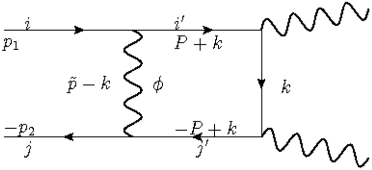


- for relic density also co-annihilations are important $\rightarrow$ one needs to compute Sommerfeld effect also for incoming $\chi^+\chi^-$, $\chi^+\chi_1^0$, ...

$\Rightarrow$ co-annihilation σ's can be strongly enhanced or suppressed

Our approach

The idea is to compute separately every possible interaction potential.



Then after some manipulations we arrive to set of coupled Schrödinger equations for 2-body wave functions $\varphi_{ij}(x)$ [Iengo '09]

$$\frac{d^2\varphi_{ij}(x)}{dx^2} + \frac{m^r_{ij}}{m^r_{ab}}\left[\left(1 - \frac{2\delta m_{ij}}{\mathcal{E}}\right)\varphi_{ij}(x) + \frac{1}{\mathcal{E}}\sum_{i'j'} V^{\phi}_{ij,i'j'}(x)\varphi_{i'j'}(x)\right] = 0,$$

$$V^{\phi}_{ij,i'j'}(x) = p\frac{c_{ij,i'j'}(\phi)}{4\pi}\frac{e^{-\frac{m_\phi}{p}x}}{x}$$

$\mathcal{E} = \vec{p}^2/2m_r^{ab}$, m^r_{ij} - reduced mass of ij state,
m^r_{ab} - reduced mass of initial state,
$2\delta m_{ij} = m_i + m_j - (m_a + m_b)$,
$x = p \cdot r$, p - CM momentum

Note that

$\bar\Psi^i(z)\gamma_5\Psi^j(w)|0>$ – singlet spin state, $\bar\Psi^i(z)\vec\gamma\Psi^j(w)|0>$ – triplet spin state,

The full potential with $c_{ij,i'j'}(\phi)$ can be computed from transition $ij \to i'j'$.

Using general notation γ for both cases and $\Gamma = 1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5$

$$\frac{1}{2!}\int [dx]\,[dy] \underbrace{\sum_{i,i'} \left(\bar\Psi^i(x)\Gamma\Psi^{i'}(x)+h.c.\right)\left(\bar\Psi^i(y)\Gamma\Psi^{i'}(y)+h.c.\right)}_{interactions} G_\phi(|\vec x-\vec y|)\cdot$$

$$\cdot\int [dz]\,[dw]\underbrace{N_{ij}\bar\Psi^i(z)\gamma\Psi^j(w)|0>}_{initial\ \ state}\Phi^{ij}(z,w)=$$

$$=\int [dx]\,[dy]\underbrace{N_{i'j'}\bar\Psi^{i'}(x)\gamma\Psi^{j'}(y)|0>}_{final\ \ state}V_\phi^{ij,i'j'}(|\vec x-\vec y|)\Phi^{ij}(x,y),$$

where:
N_{ij} - normalization of ij state,
$G_\phi(|\vec x-\vec y|)$ - ϕ boson propagator,
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Summary of our approach

Remarks:

- force can be attractive or repulsive,
- coefficients $c_{ij,i'j'}(\phi)$ depend on ϕ but also on the incoming and outgoing states (Dirac or Majorana, identical or not)
- the effect is different for the incoming particles being in the singlet or triplet spin state.

Short summary of our approach:

1. We took every possible initial state ij
2. Computed coefficients $c_{ij,i'j'}(\phi)$ and potential $V^{\phi}_{ij,i'j'}(x)$
3. Wrote a code to solve the set of coupled Schrödinger equations numerically
4. Implemented it into `DarkSUSY` to use the full cross section σ_{eff} for computing the relic density

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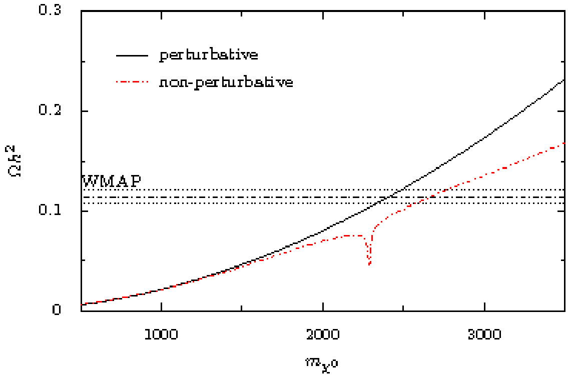
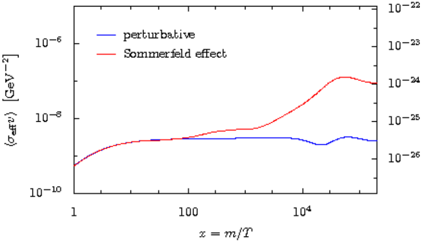
Results - χ_1^0 pure wino

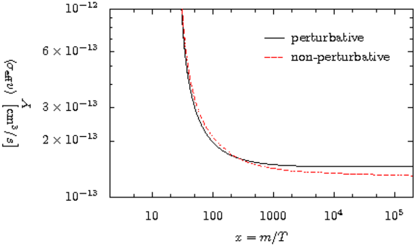


Similar results as obtained by Hisano et al. '06 (and also Strumia et al. '07).
We obtained smaller effect (apart from resonance) due to different sign in $\chi^+\chi_1^0$ potential

Results - χ_1^0 pure wino

Away from resonance:





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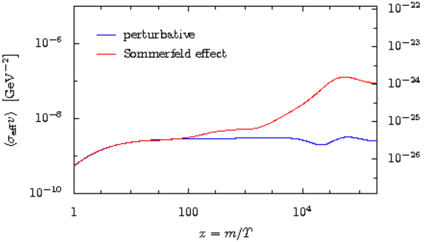
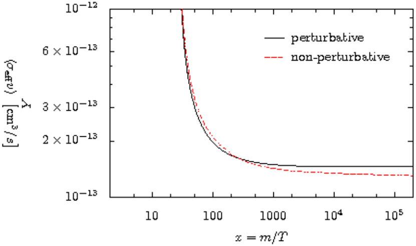
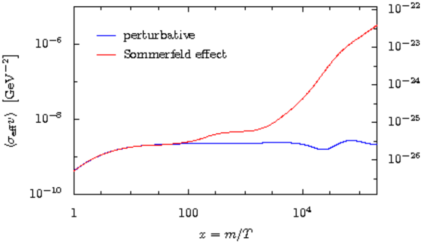
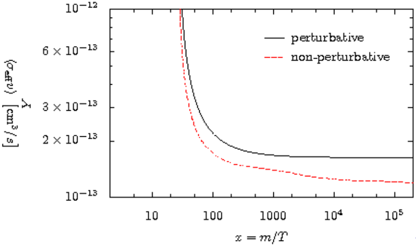


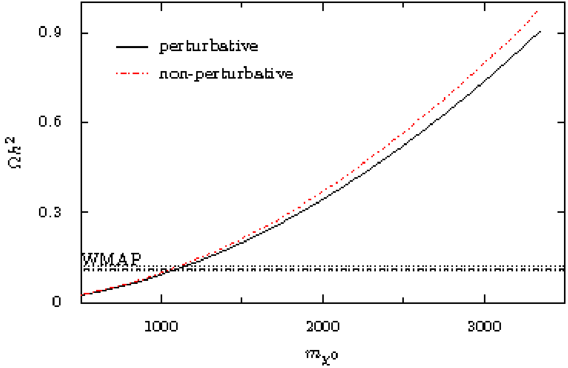


Near resonance:





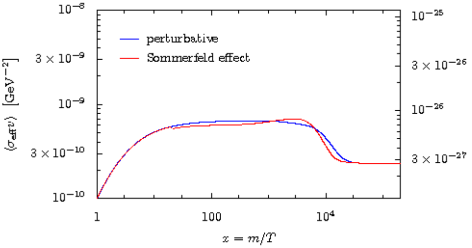
Results - χ_1^0 pure higgsino

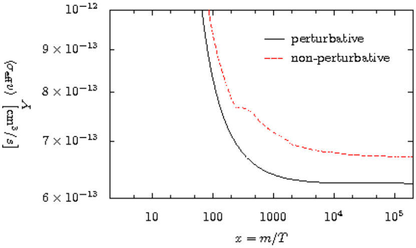


We included χ_2^0 and more interactions $\Rightarrow$ bigger repulsive effect than obtained by Hisano et al. '06.

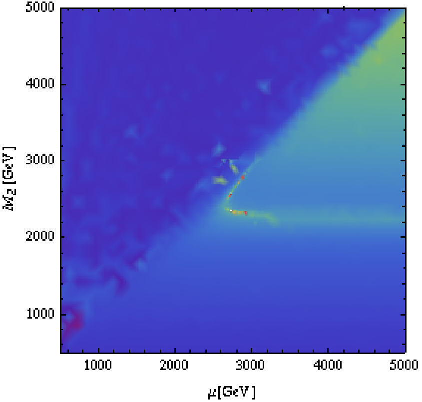
Results - χ_1^0 pure higgsino

No resonance in this case:





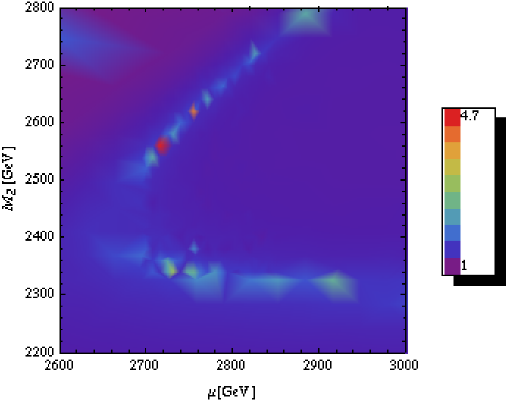
Results - relic density effect



The ratio of relic densities without and with Sommerfeld effect,

$$\epsilon = \frac{(\Omega h^2)_0}{(\Omega h^2)_{SE}}$$

Results - relic density effect



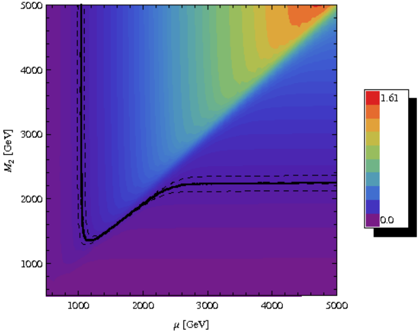
Zoom on the resonace effect on the relic densitiy.
The resonance occurs for $m_\chi \approx 2.3$ TeV, i.e. for the value satisfying

$$\frac{1}{m_W} \approx \frac{1}{\alpha m_\chi}$$

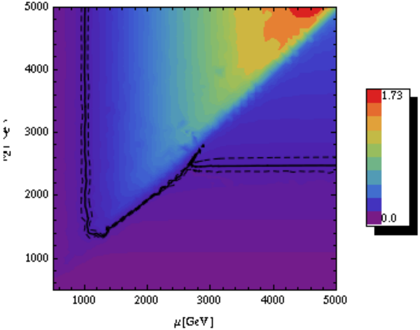
$\rightarrow$ resonance in relic densitiy is due to possibility of creating a bound state

Results - relic density

perturbative



Sommerfeld effect



The black bands are **WMAP7** regions,

$$\Omega h^2 = 0.1123 \pm 0.0035 \qquad \text{(68\% CL uncerinities)}$$

Conclusions & Prospects

- We have computed the Sommerfeld effect coming from gauge bosons and Higgs in the MSSM and then the relic density using `DarkSUSY`
- The results show that this effect is important for non-neglible parameter regions, also away from the resonance
- We wrote a code to solve the Schrödinger equations numerically and implemented this into `DarkSUSY` $\Rightarrow$ tool to investigate Sommerfeld effect for general MSSM parameters

Future work:

- Search for other regions of MSSM where Sommerfeld effect can be relevant
- Look at indirect detection signals with the Sommerfeld effect included, both for the relic density and annihilation cross section