(mildly) non-linear cosmological perturbations

* mildly non-linear scales as a window to beyond ΛCDM cosmologies
* beyond linear perturbation theory
* resummations and RG
* results: BAO's, neutrino mass, Non Gaussianity

M. Pietroni-INFN Padova

the future of precision cosmology: non-linear scales

matter density:

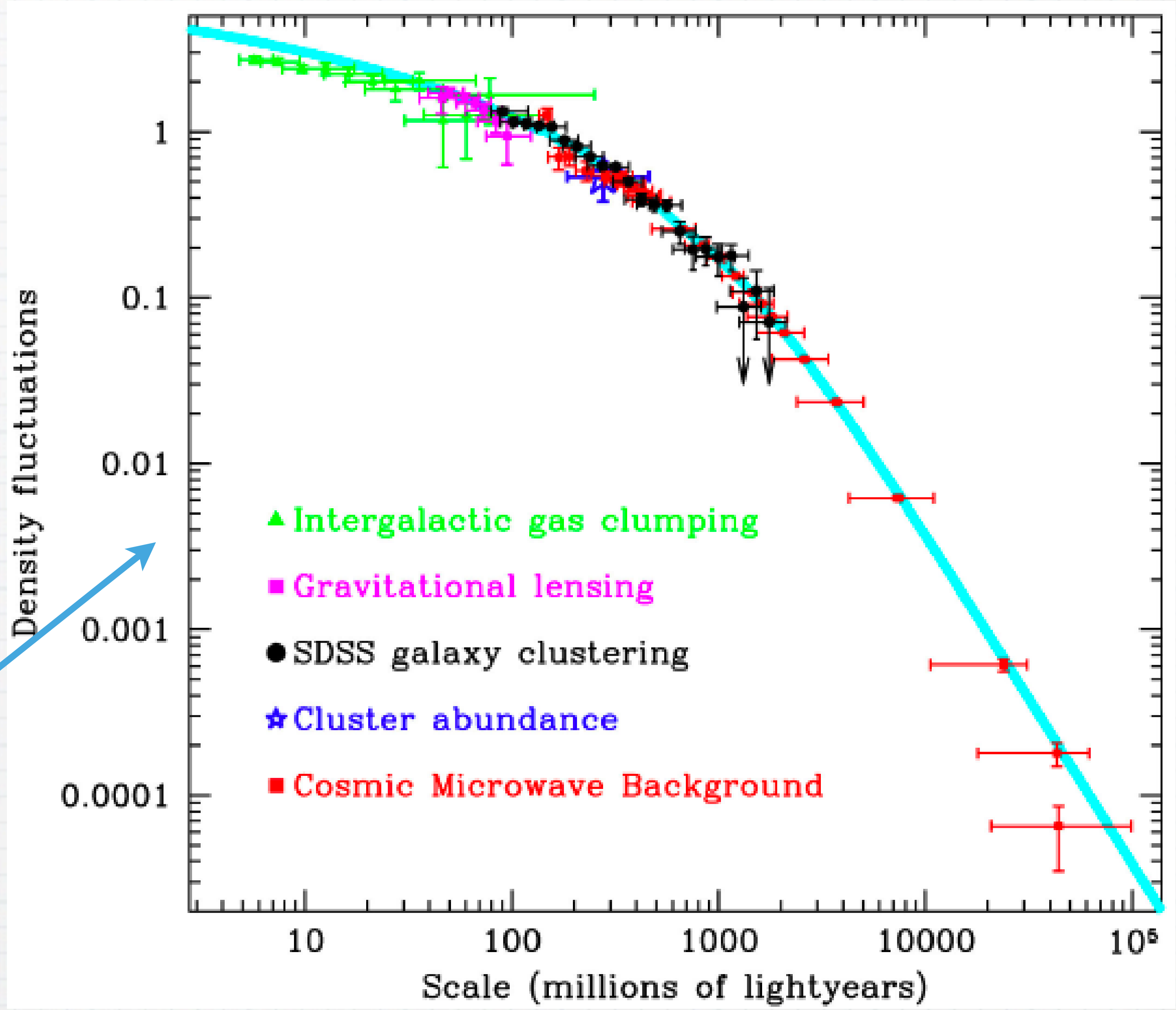
$$\rho(\mathbf{x},\,\tau) \equiv \bar{\rho}(\tau)[1+\delta(\mathbf{x},\,\tau)]$$

power spectrum:

$$\langle\delta(\mathbf{k},\,\tau)\,\delta(\mathbf{k}',\,\tau)\rangle = P(k,\,\tau)\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

`size' of the fluctuations at different scales/epochs:

$$4\pi k^3 P(k) \sim a^2\, k^{3+n}$$



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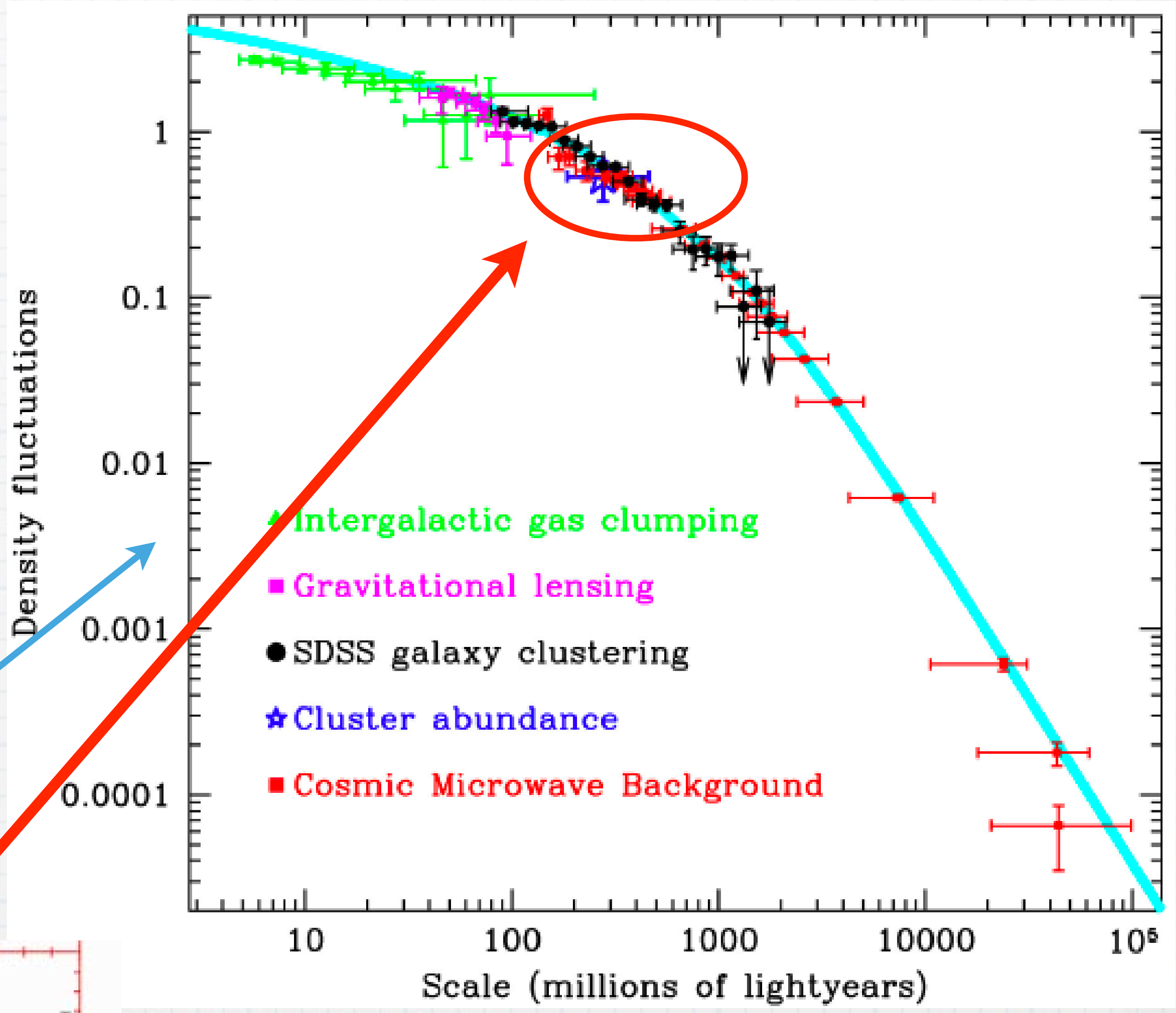
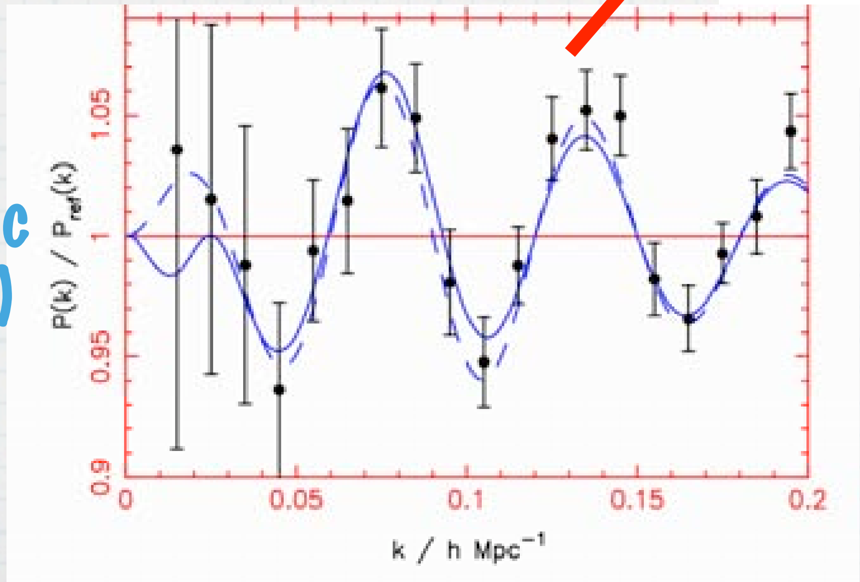


Baryonic Acoustic Oscillations (BAO)



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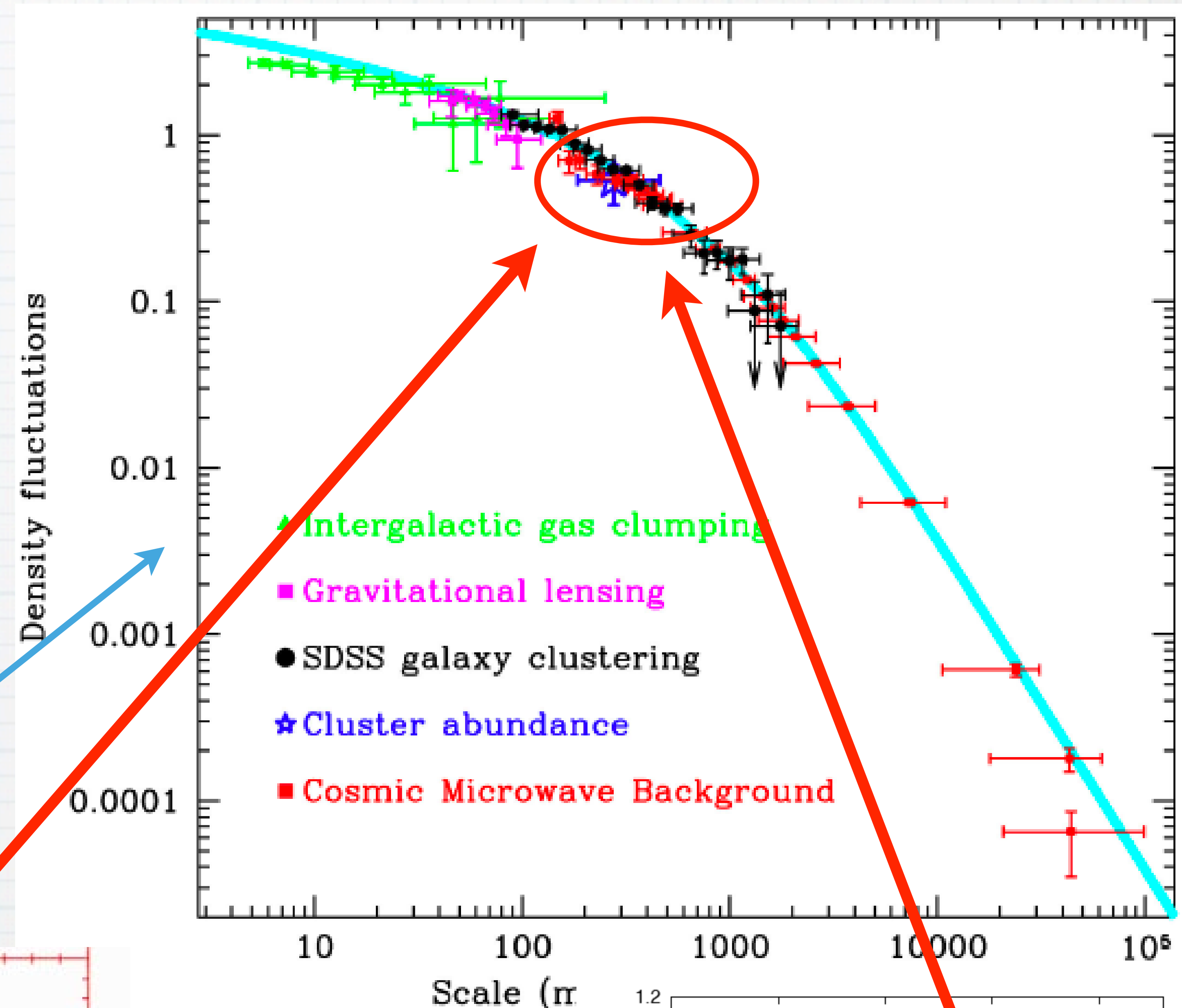

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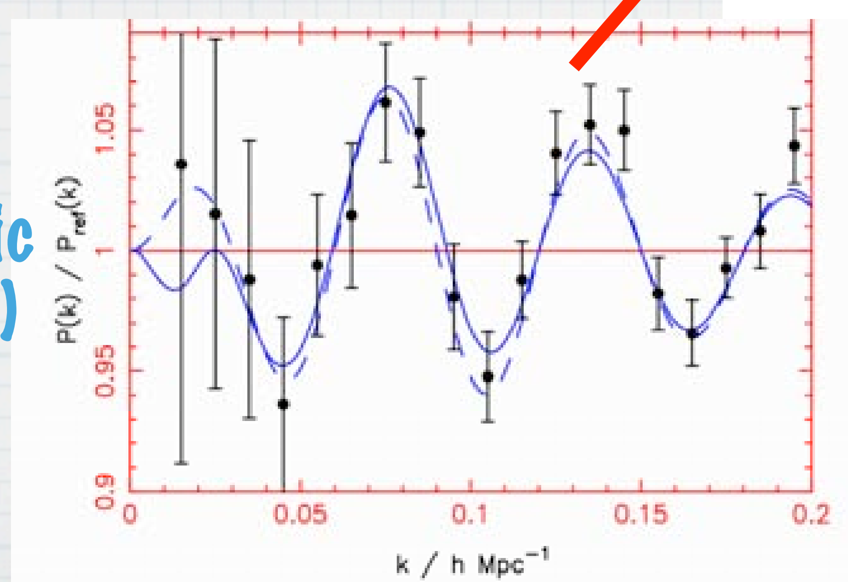
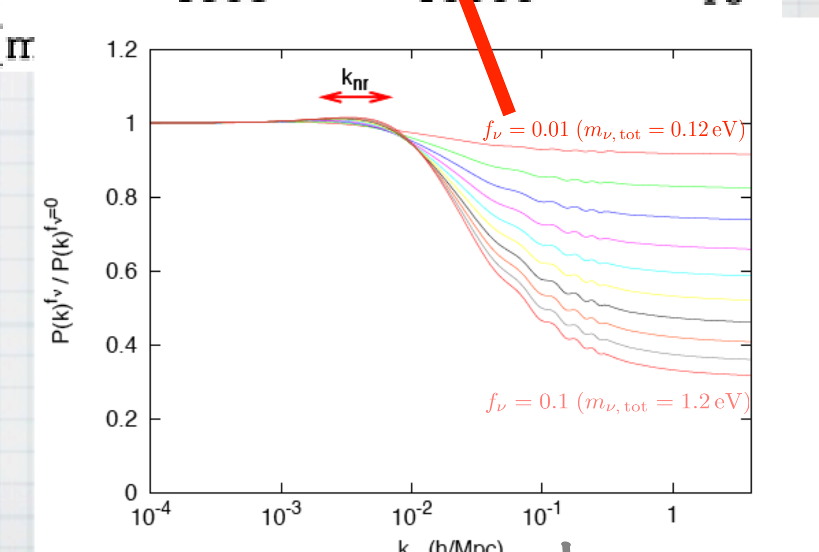
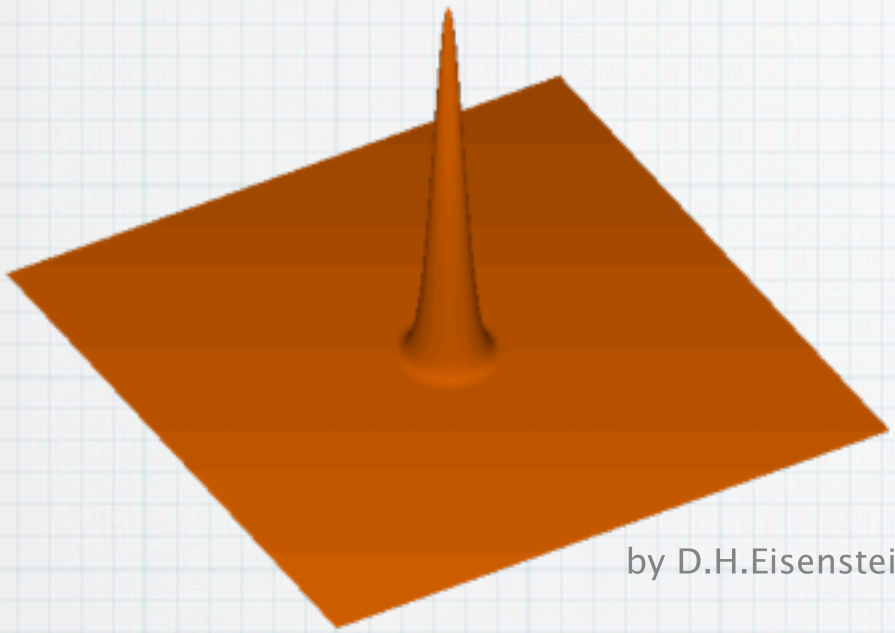


Neutrino mass bounds



a standard ruler: Baryonic Acoustic Oscillations



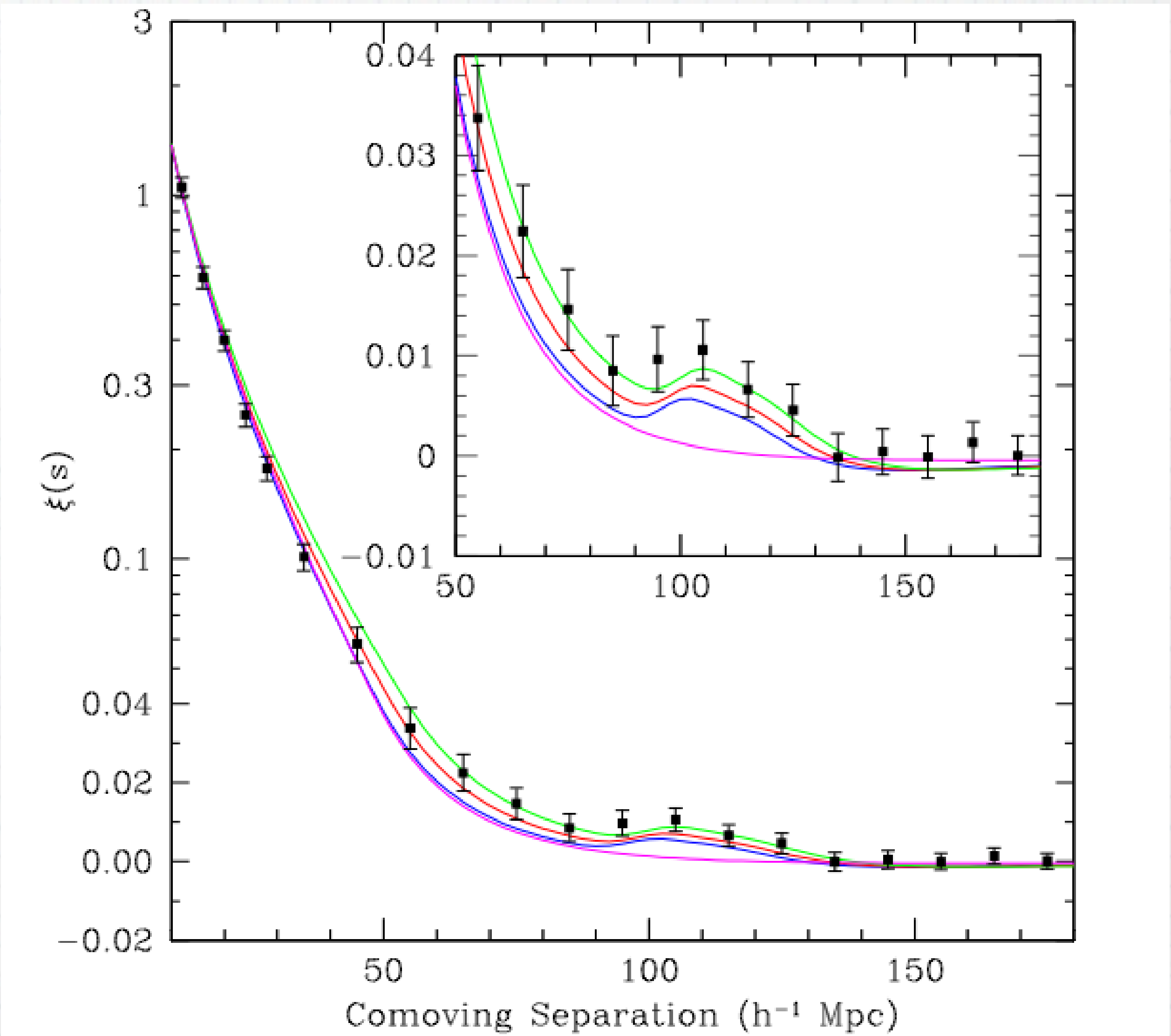
by D.H.Eisenstein

excess probability of finding galaxies 100 Mpc away from a given one

3.5 sigma evidence

Eisenstein et al. '05
Padmanabhan et al. 06
Blake et al. 06

The same scale `seen' in the CMB acoustic peaks,
but at a much later epoch



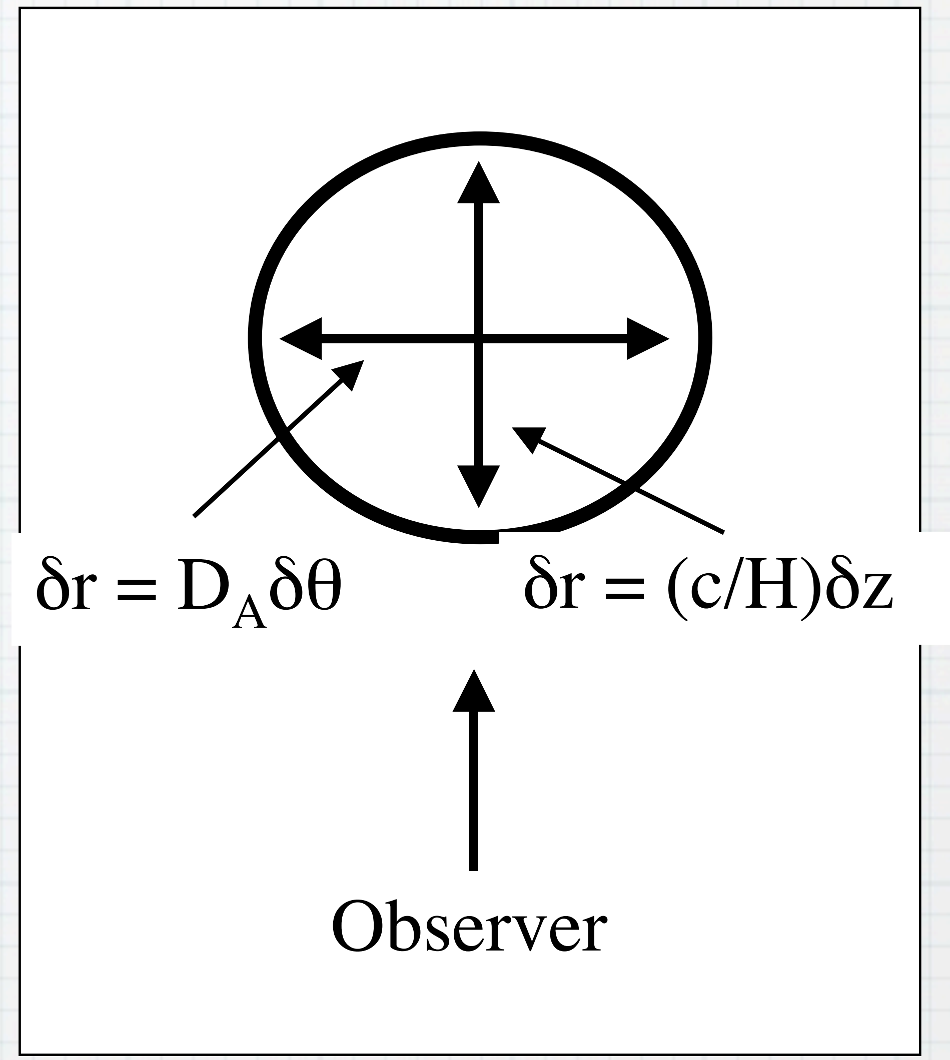
a standard ruler: Baryonic Acoustic Oscillations

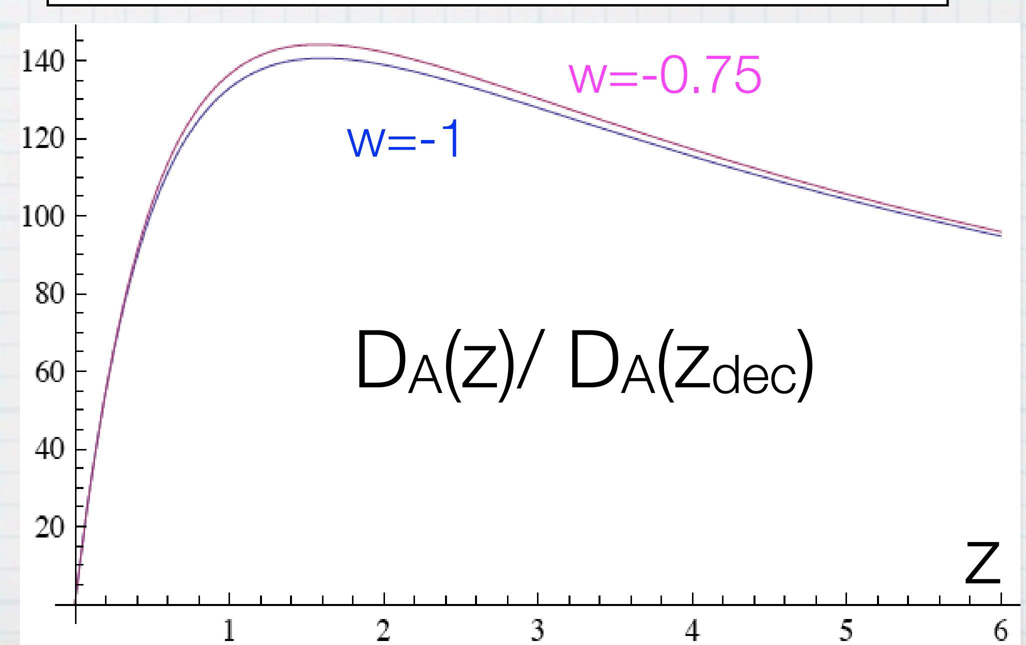
The same acoustic oscillation scale is imprinted on the CMB anisotropies

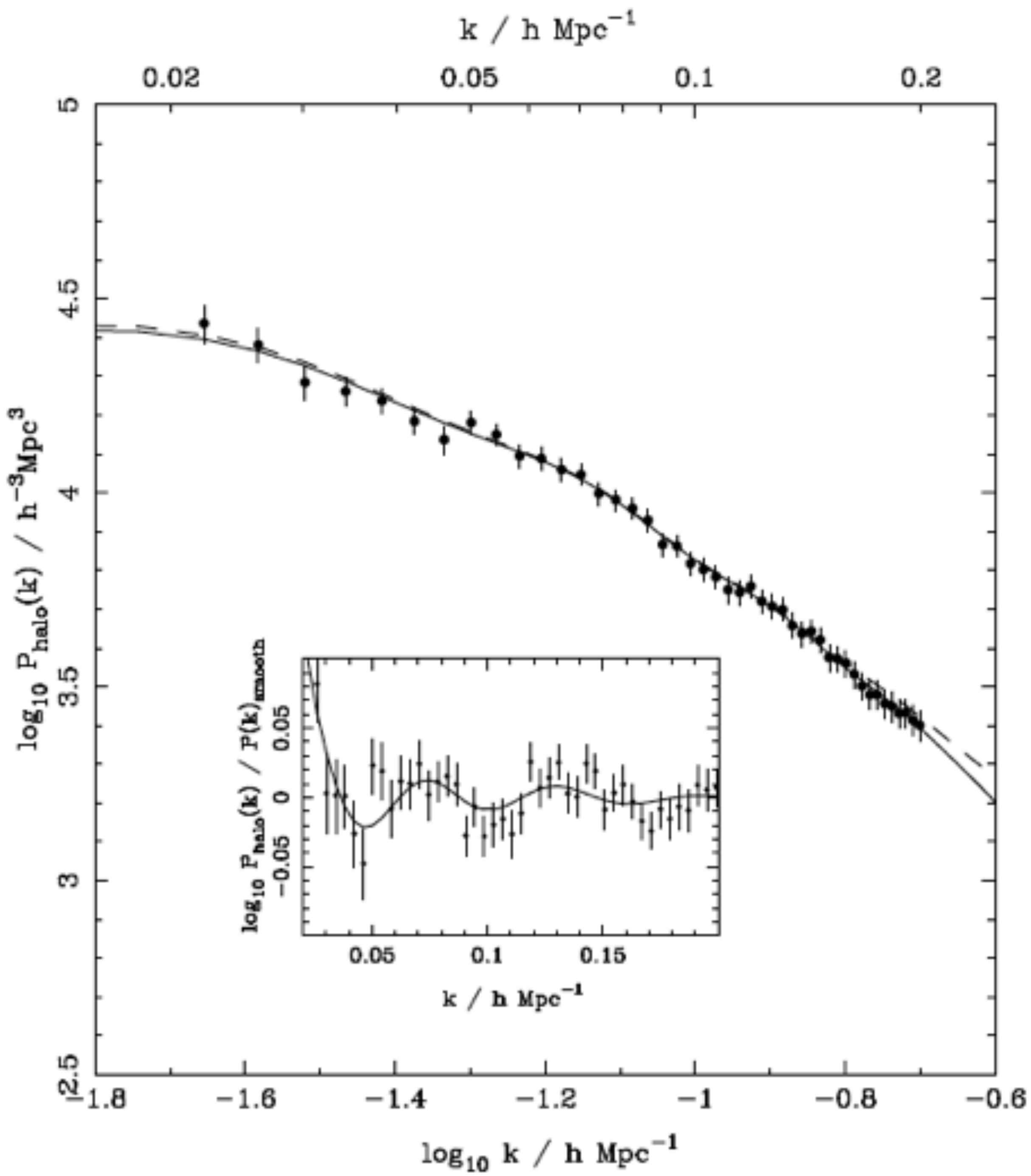
Redshift surveys of galaxies (e.g. Sloan) measure this scale both along and across the line of sight

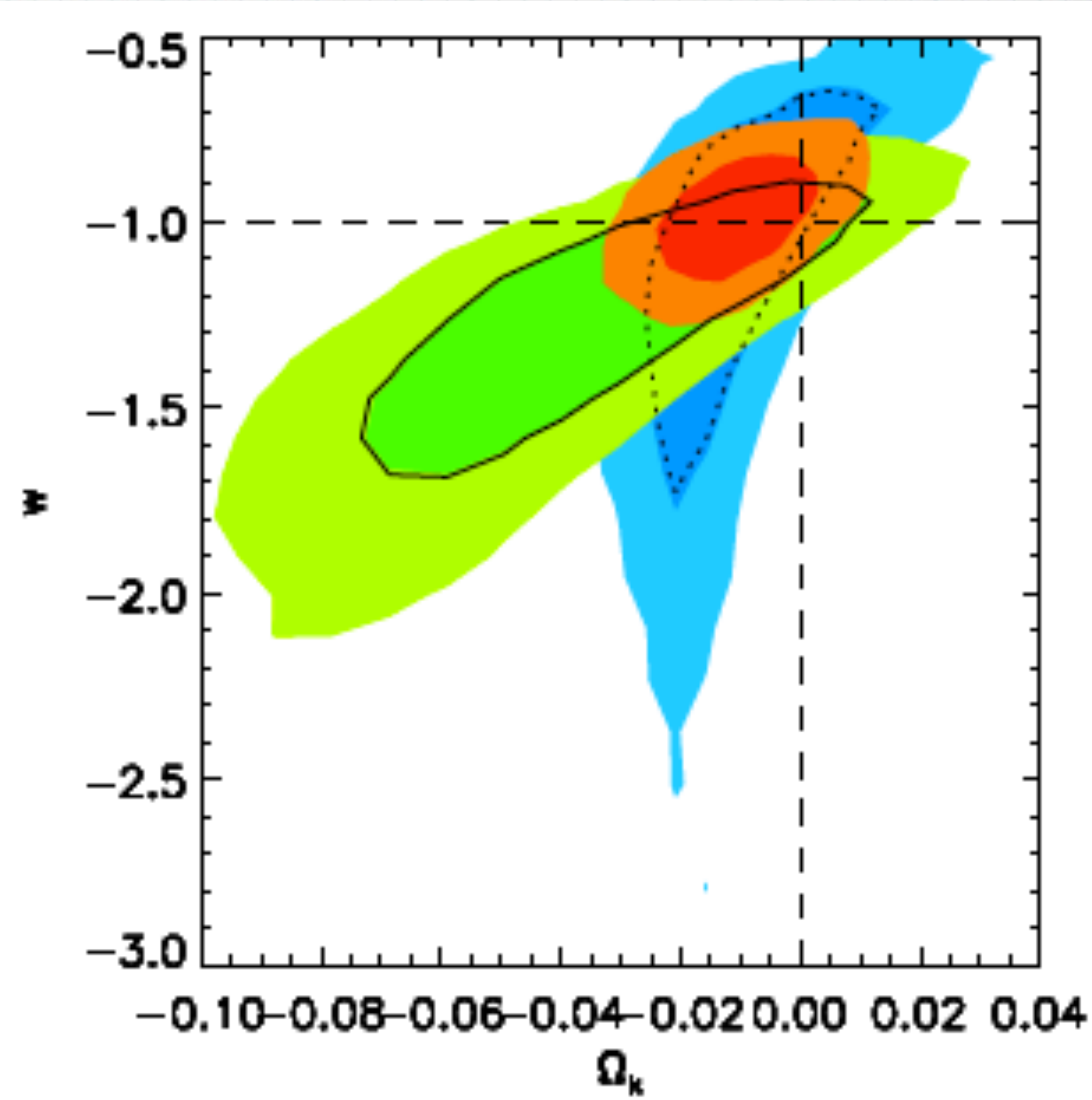
Reconstruct the expansion history of the Universe from $z_{dec} \simeq 1100$ to today!

for $z \lesssim$ 1-3, reconstruct the Dark Energy !







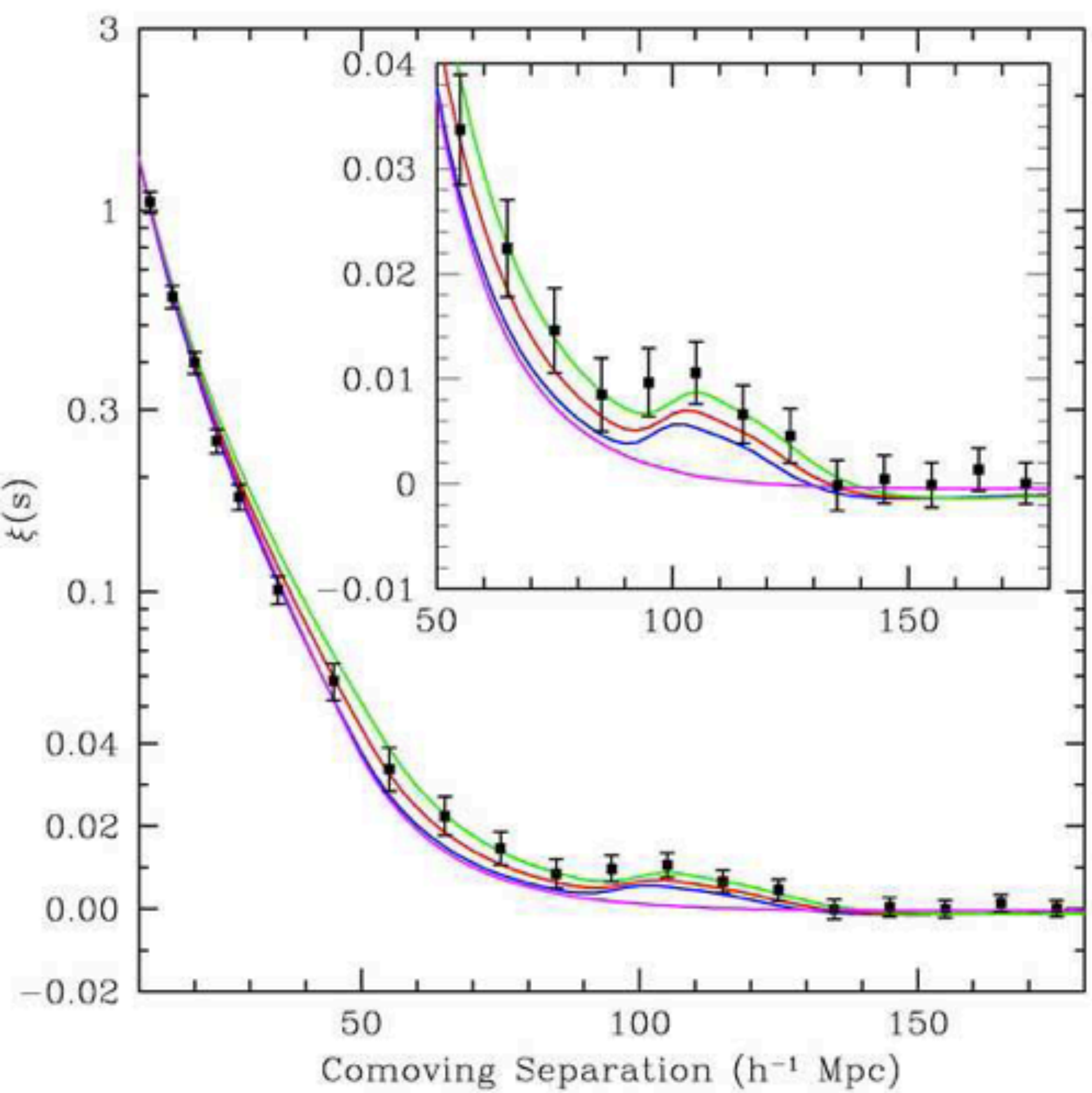


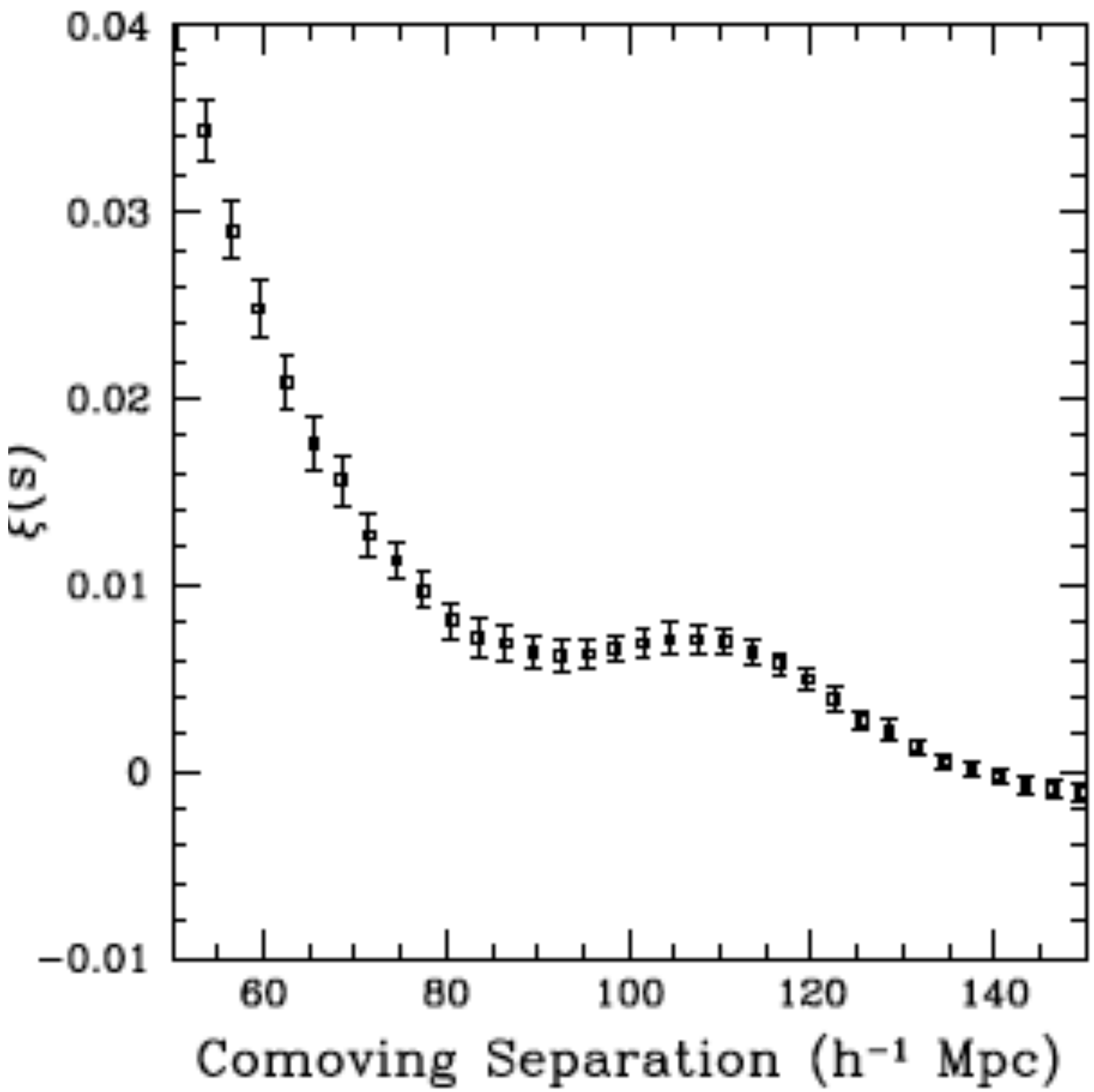
the current situation...

... and constraints on cosomological parameters

from SDSS, Reid et al. ('09)

future surveys



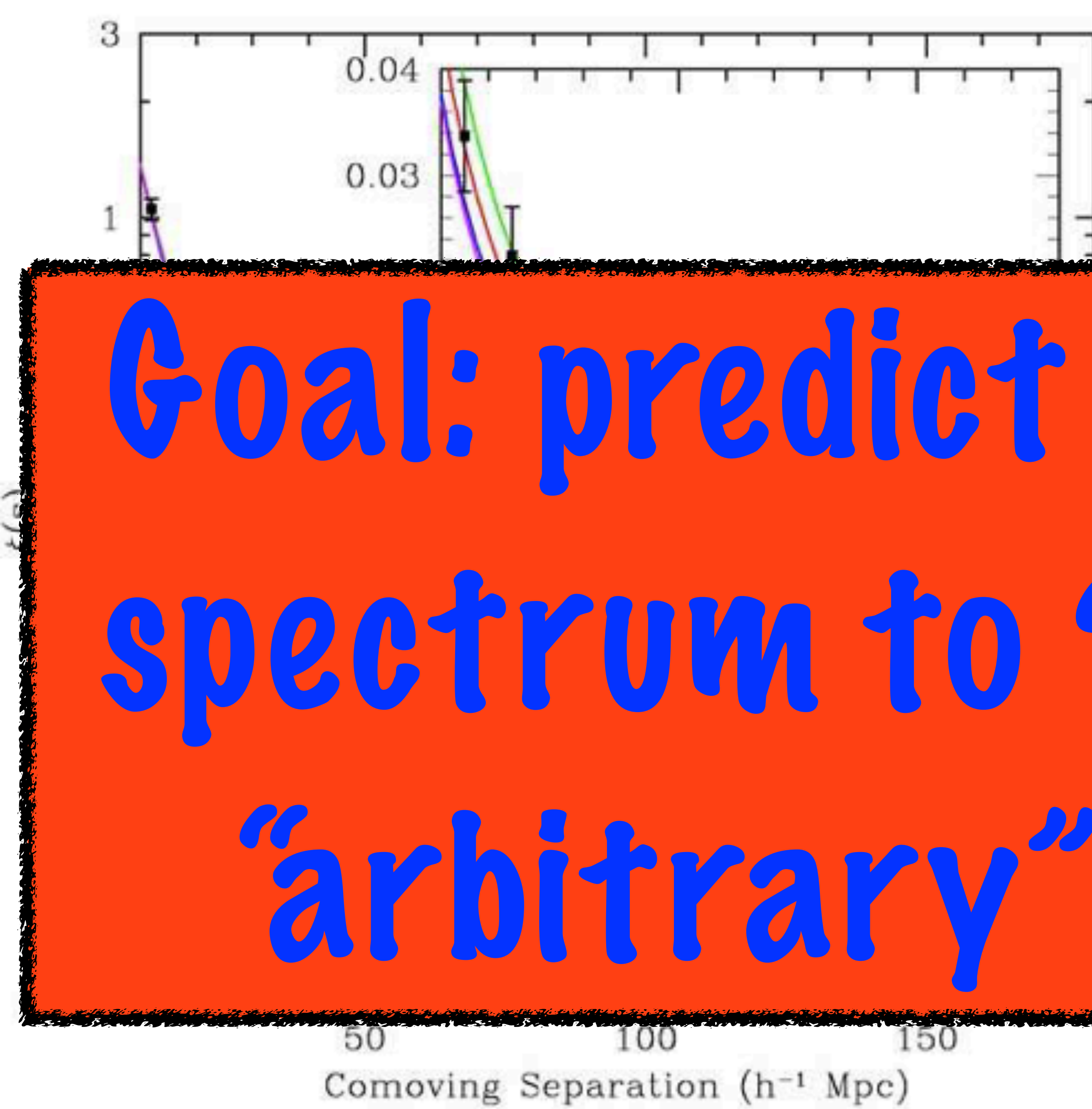


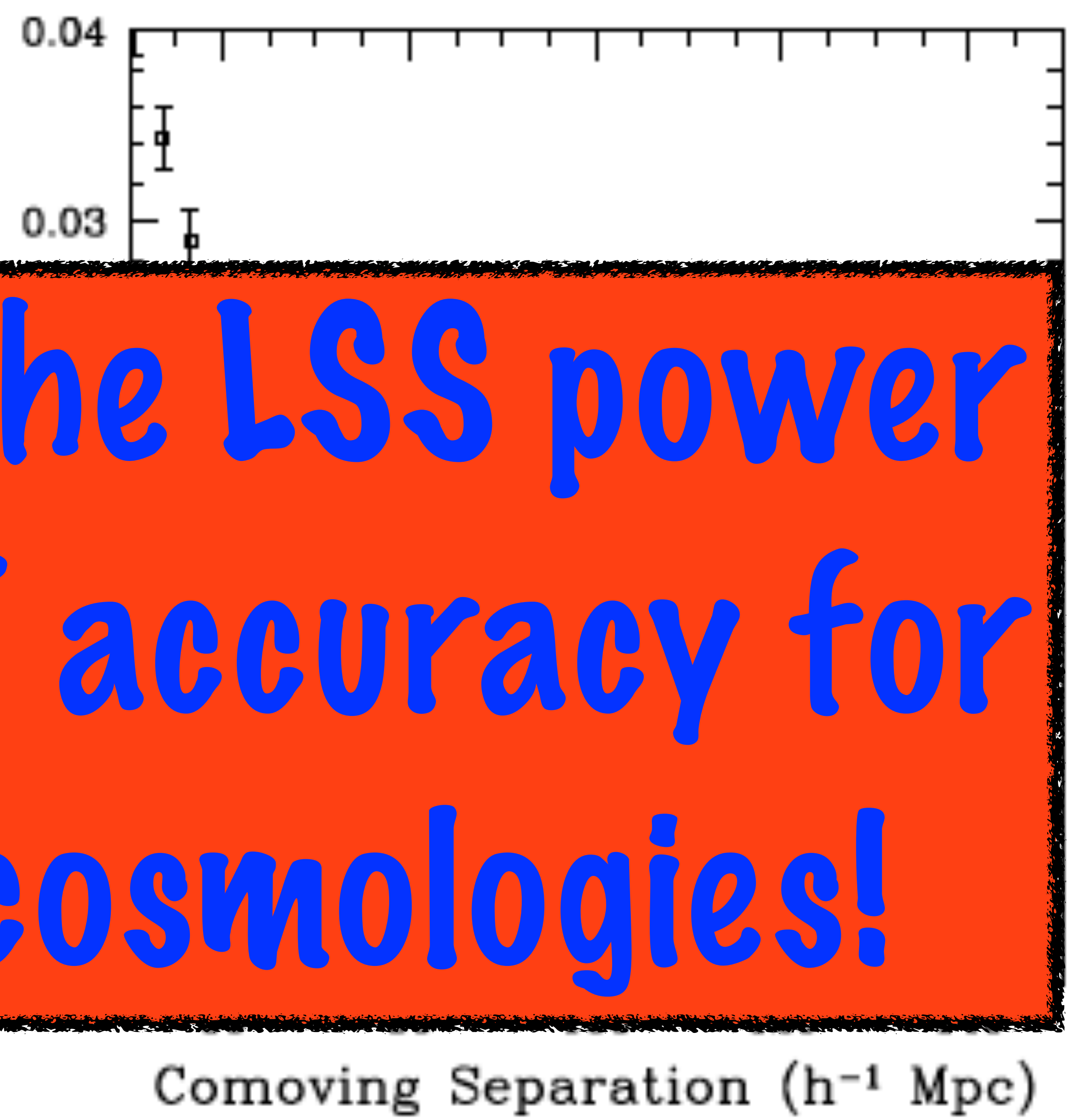
Now

Boss

1.5M of luminous
red galaxies up to z < 0.8

future surveys





Goal: predict the LSS power spectrum to % accuracy for "arbitrary" cosmologies!

Now

Boss

1.5M of luminous
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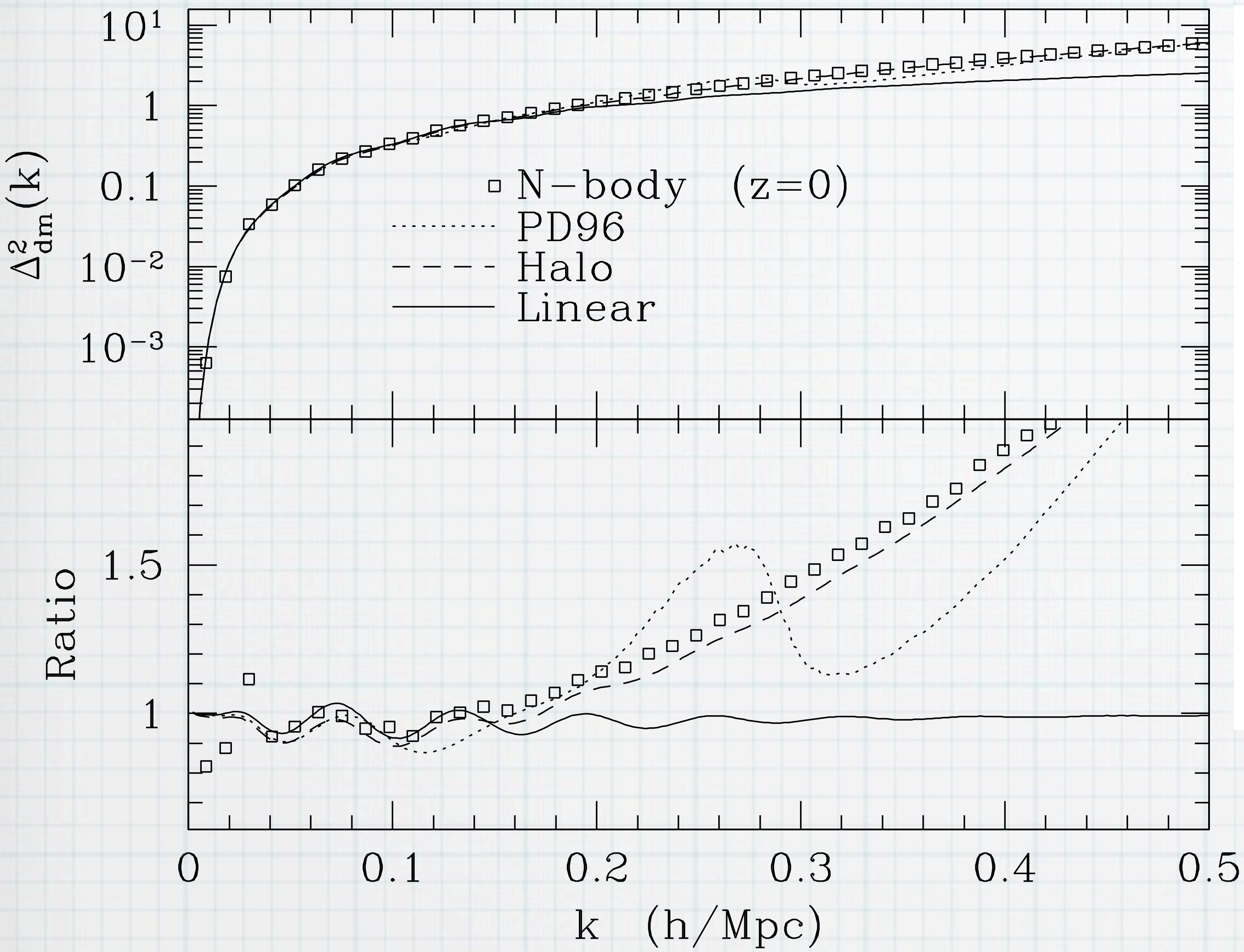
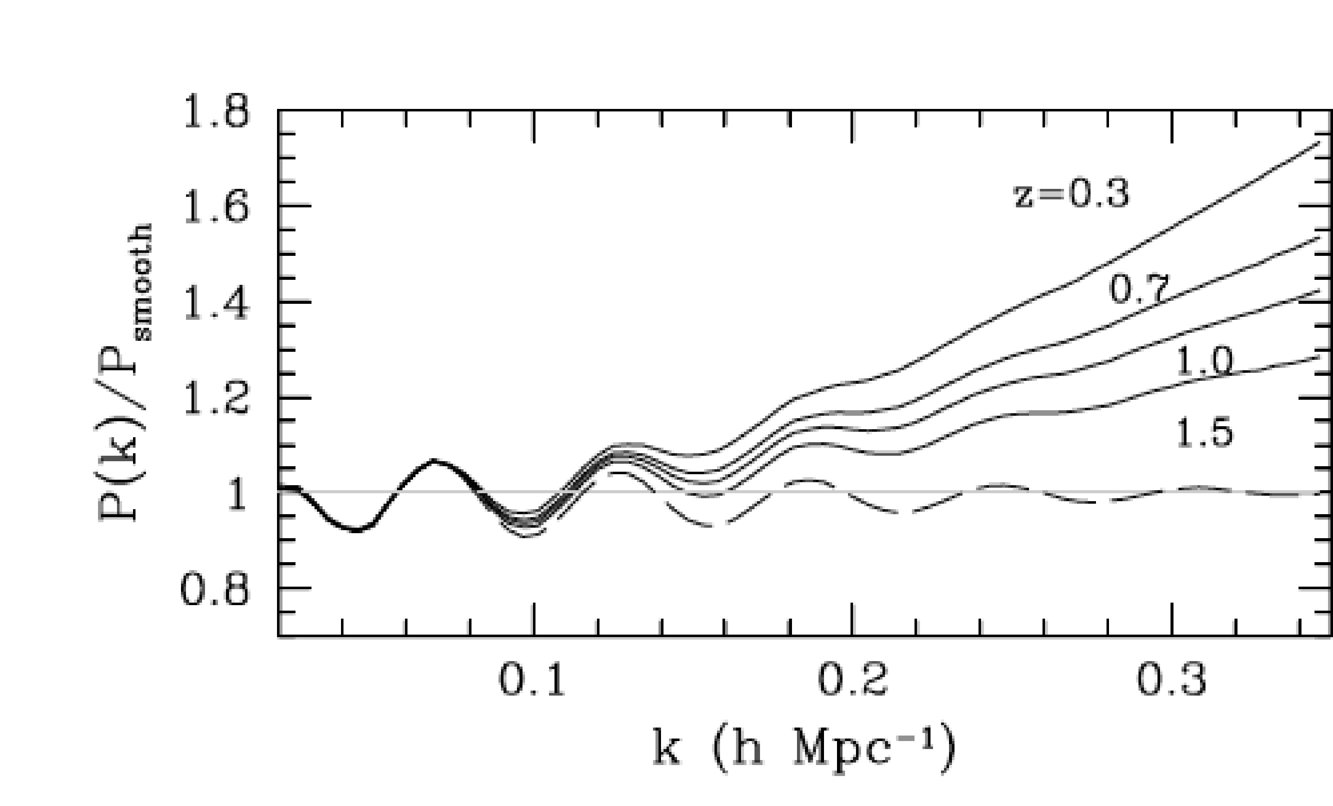
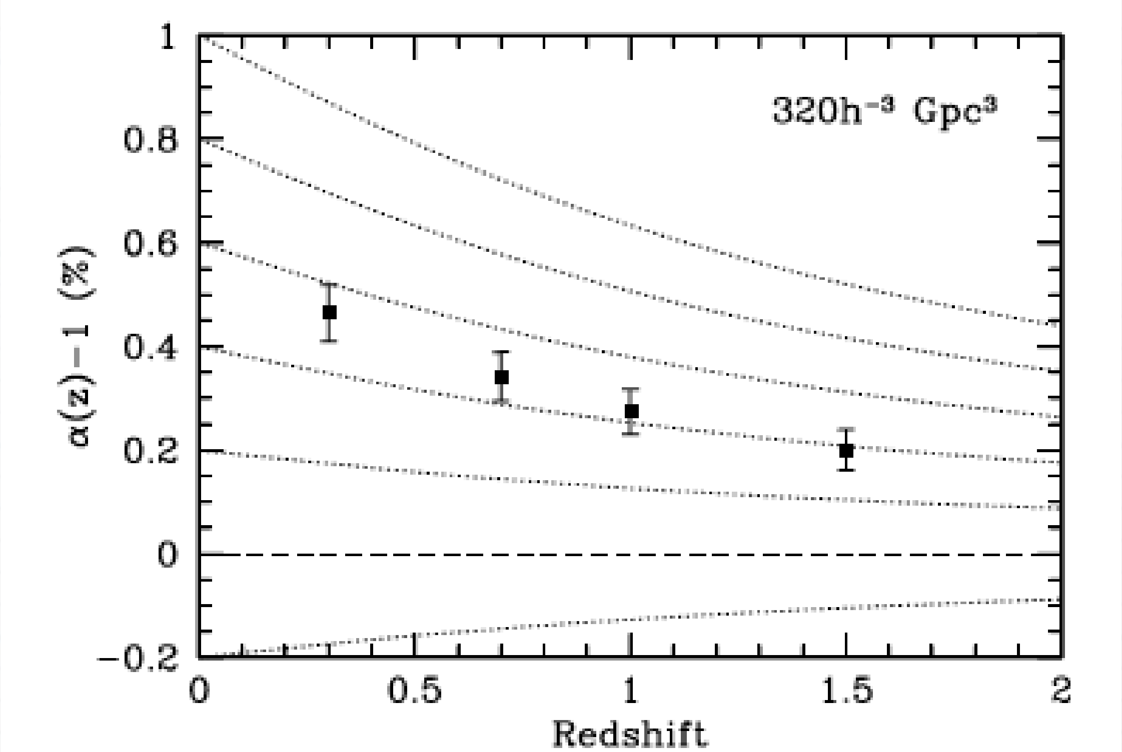
Standard Approach: N-body simulations+fitting functions



Huff et al, '06



Seo et al, '08



5-10% discrepancies between fitting functions and simulations

peak displacement by nonlinearities

Cold dust cosmology: equations of motion

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$

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In Fourier space, (defining $\theta(\mathbf{x},\tau) \equiv \nabla\cdot\mathbf{v}(\mathbf{x},\tau)$),

$$\frac{\partial\,\delta(\mathbf{k},\tau)}{\partial\,\tau} + \theta(\mathbf{k},\tau) + \int d^3\mathbf{k_1} d^3\mathbf{k_2}\,\delta_D(\mathbf{k}-\mathbf{k_1}-\mathbf{k_2})\alpha(\mathbf{k_1},\mathbf{k_2})\theta(\mathbf{k_1},\tau)\delta(\mathbf{k_2},\tau) = 0$$

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mode-mode coupling controlled by:

$$\alpha(\mathbf{k_1},\mathbf{k_2}) \equiv \frac{(\mathbf{k_1}+\mathbf{k_2})\cdot\mathbf{k_1}}{k_1^2}$$

$$\beta(\mathbf{k_1},\mathbf{k_2}) \equiv \frac{|\mathbf{k_1}+\mathbf{k_2}|^2(\mathbf{k_1}\cdot\mathbf{k_2})}{2k_1^2k_2^2}$$

linear approximation: $\alpha(\mathbf{k_1},\, \mathbf{k_2}) = \beta(\mathbf{k_1},\, \mathbf{k_2}) = 0$

no mode-mode coupling

$$\frac{\partial\, \delta(\mathbf{k}, \tau)}{\partial\, \tau} + \theta(\mathbf{k}, \tau) = 0$$

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$$\Omega_M = 1 \to \mathcal{H} \sim a^{-1/2}$$

$$\downarrow$$

$$\begin{aligned} \delta(\mathbf{k},\, \tau) &= \delta(\mathbf{k},\, \tau_i) \left(\frac{a(\tau)}{a(\tau_i)}\right)^m \\ -\frac{\theta(\mathbf{k},\, \tau)}{\mathcal{H}} &= m\, \delta(\mathbf{k},\, \tau) \end{aligned}$$

$$m = \begin{cases} 1 & \text{growing mode} \\ -\frac{3}{2} & \text{decaying mode} \end{cases}$$

Compact Perturbation Theory

Crocce, Scoccimarro '05

Consider again the (non-linear) continuity and Euler equations

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

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define $\left(\begin{array}{c}\varphi_1(\eta,\mathbf{k})\\ \varphi_2(\eta,\mathbf{k})\end{array}\right) \equiv e^{-\eta}\left(\begin{array}{c}\delta(\eta\,,\mathbf{k})\\ -\theta(\eta\,,\mathbf{k})/\mathcal{H}\end{array}\right)$ with $\eta = \log\dfrac{a(\tau)}{a(\tau_i)}$ $\quad \mathbf{\Omega} = \left(\begin{array}{cc}1 & -1\\ -3/2 & 3/2\end{array}\right)$

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then we can write (we assume an EdS model):

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta,\mathbf{k}) = e^{\eta}\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

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and the only non-zero components of the mode-mode coupling are

$$\gamma_{121}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \gamma_{112}(\mathbf{k_1},\,\mathbf{k_3},\,\mathbf{k_2}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\frac{\alpha(\mathbf{k_2},\,\mathbf{k_3})}{2}$$

$$\gamma_{222}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\beta(\mathbf{k_2},\,\mathbf{k_3})$$

An action principle

Matarrese, M.P., '07

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The field equation can be derived by varying the **action**

$$S = \int d\eta_a d\eta_b \, \chi_a \, g^{-1}_{ab} \, \varphi_b - \int d\eta \, e^{\eta} \, \gamma_{abc} \, \chi_a \, \varphi_b \, \varphi_c$$

w.r.t. the auxiliary field $\chi_a(\eta, \mathbf{k})$

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$g_{ab}(\eta_1, \eta_2)$ is the retarded propagator: $(\delta_{ab}\partial_\eta + \Omega_{ab}) \, g_{bc}(\eta, \eta') = \delta_{ac} \, \delta_D(\eta - \eta')$

so that $\varphi^0_a(\eta, \mathbf{k}) = g_{ab}(\eta, \eta')\varphi^0_b(\eta', \mathbf{k})$ is the solution of the ***linear*** equation

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Explicitly, one finds: $\mathbf{g}(\eta_1, \eta_2) = \begin{cases} \mathbf{B} + \mathbf{A}\, e^{-5/2(\eta_1 - \eta_2)} & \eta_1 > \eta_2 \\ 0 & \eta_1 < \eta_2 \end{cases}$

$$\mathbf{B} = \frac{1}{5}\begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix}$$

$$\mathbf{A} = \frac{1}{5}\begin{pmatrix} 2 & -2 \\ -3 & 3 \end{pmatrix}$$

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growing mode

Initial conditions: $\varphi^0_b(\eta', \mathbf{k}) \propto u_b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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growing mode

decaying mode

Initial conditions: $\varphi^0_b(\eta', \mathbf{k}) \propto u_b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\qquad \begin{pmatrix} 1 \\ -3/2 \end{pmatrix}$

A generating functional

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$$
\begin{aligned}
Z[\mathbf{J},\,\mathbf{\Lambda}] \quad &= \quad \int \mathcal{D}\varphi\,\mathcal{D}\chi\,\exp\left\{\int d\eta_1 d\eta_2 \left[-\frac{1}{2}\,\chi\,\mathbf{g}^{-1}\mathbf{P^L}\mathbf{g^T}^{-1}\chi + i\,\chi\,\mathbf{g}^{-1}\,\varphi\right]\right. \\
&\qquad\qquad \left. -i\int d\eta\,[\mathbf{e}^{\eta}\gamma\,\chi\varphi\varphi - \mathbf{J}\varphi - \mathbf{\Lambda}\chi]\right\}
\end{aligned}
$$

the initial conditions are encoded
in the linear power spectrum:

A generating functional

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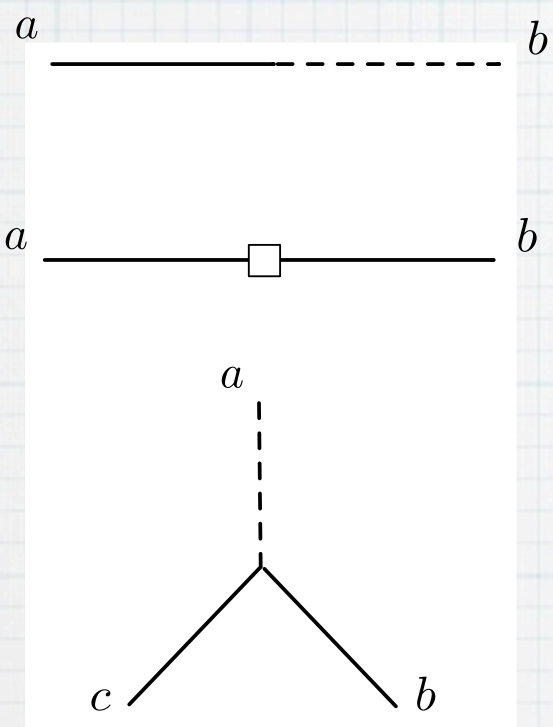
$$\left. -i\int d\eta\,[\mathbf{e}^{\eta}\gamma\,\chi\varphi\varphi - \mathbf{J}\varphi - \mathbf{\Lambda}\chi]\right\}$$

the initial conditions are encoded in the linear power spectrum:

$$P^L_{ab}(\eta,\,\eta';\,\mathbf{k}) \equiv \left(\mathbf{g}(\eta)\mathbf{P^0}(\mathbf{k})\mathbf{g}^T(\eta')\right)_{ab}$$

Derivatives of Z w.r.t. the sources J and Λ give all the n-point correlation functions (power spectrum, bispectrum, ...) and the full non-linear propagator

Perturbation Theory: Feynman Rules



propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$

interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

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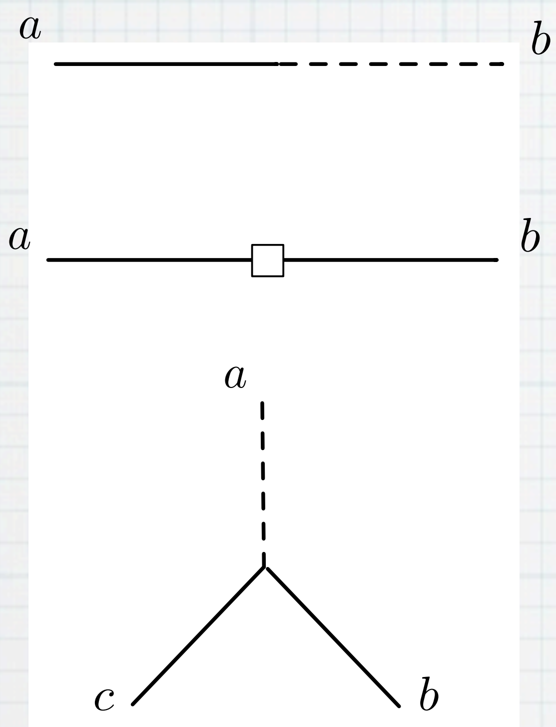

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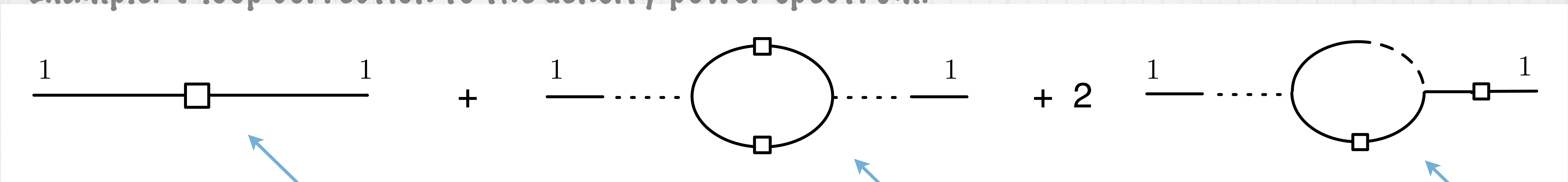
Example: 1-loop correction to the density power spectrum:

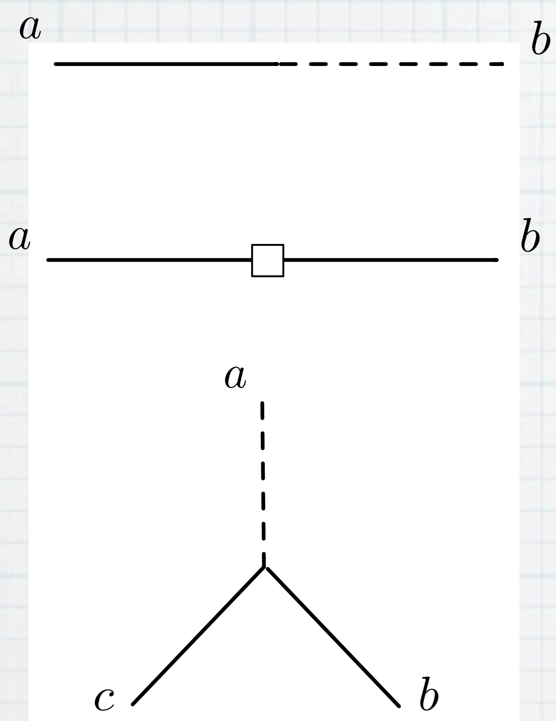


Linear Power spectrum

"P22"

"P13"

Perturbation Theory: Feynman Rules

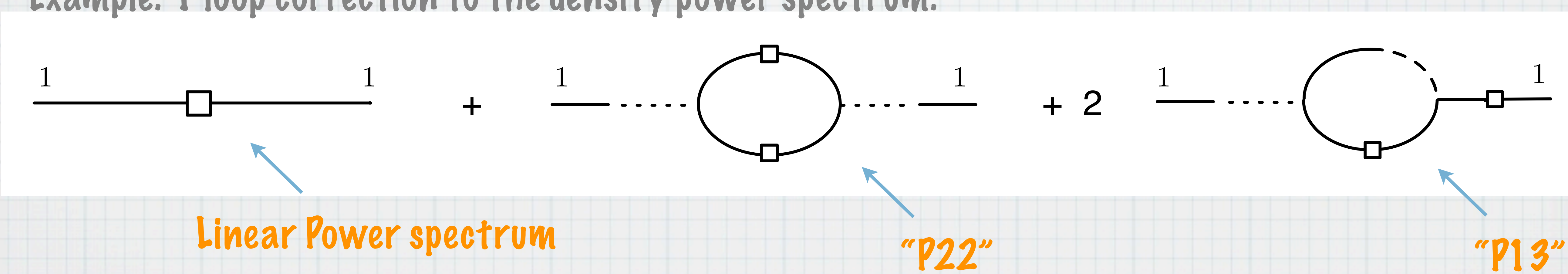


propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$

interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

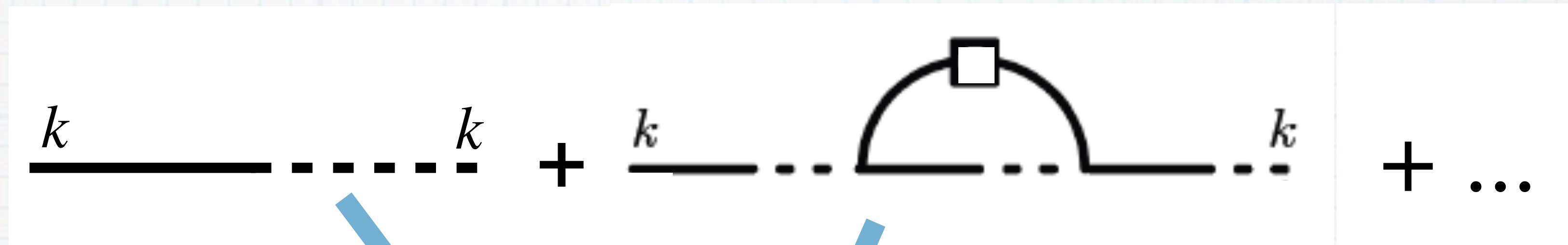
Example: 1-loop correction to the density power spectrum:



All known results in cosmological perturbation theory are expressible in terms of diagrams in which only a trilinear fundamental interaction appears

PT in the BAO range

1-loop propagator
@ large k:



$$G_{ab}(k;\eta_a,\,\eta_b) = g_{ab}(\eta_a,\,\eta_b)\left[1 - k^2\sigma^2\,\frac{(e^{\eta_a} - e^{\eta_b})^2}{2}\right] + O(k^4\sigma^4)$$

$$\left(\sigma^2 \equiv \frac{1}{3}\int d^3q\,\frac{P^0(q)}{q^2}\right)$$

$(\sigma\, e^{\eta_a})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$

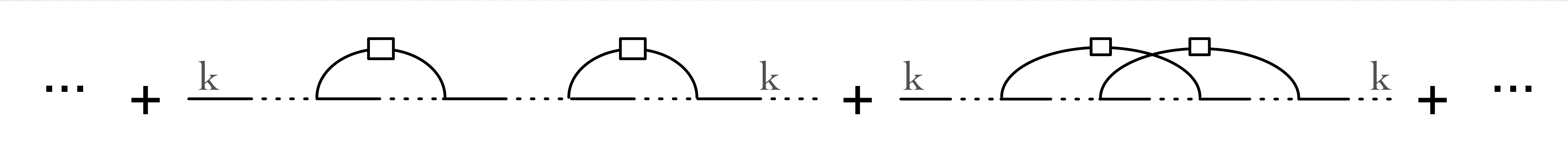
in the BAO range!

2-loop

the PT series blows up in the BAO range

But it can be resummed!!

(Crocce-Scoccimarro '06)



$$\boxed{G \sim e^{-\frac{k^2\sigma^2}{2} e^{2\eta}}}$$

physically, it represents the effect of multiple interactions of the k-mode with the surrounding modes

`coherence momentum' $k_{ch} = (\sigma\, e^{\eta})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$

damping in the BAO range!

Partial (!) list of contributors to the field

* "traditional" P.T.: see Bernardeau et al, Phys. Rep. 367, 1, (2002), and refs. therein; Jeong-Komatsu; Saito et al; Sefusatti;...
* resummation methods: Valageas; Crocce-Scoccimarro; McDonald; Matarrese-M.P.; Matsubara; M.P.; Taruya-Hiratamatsu; Bernardeau-Valageas; Bernardeau-Crocce-Scoccimarro;...

Time-RG

(also for cosmologies with $D^{\pm} = D^{\pm}(k,\, z)$)

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta,\mathbf{k}) = e^\eta \gamma_{abc}(\mathbf{k},\ -\mathbf{k_1},\ -\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

$$\partial_\eta \varphi = -\Omega\,\varphi + e^\eta \gamma\,\varphi\,\varphi$$

$$\partial_\eta \langle \varphi\,\varphi \rangle = -\sum \Omega\,\langle \varphi\,\varphi \rangle + \sum e^\eta \gamma\,\langle \varphi\,\varphi\,\varphi \rangle$$

$$\partial_\eta \langle \varphi\,\varphi\,\varphi \rangle = -\sum \Omega\,\langle \varphi\,\varphi\,\varphi \rangle + \sum e^\eta \gamma\,\langle \varphi\,\varphi\,\varphi\,\varphi \rangle$$

$$\cdots$$

infinite tower of equations

Advantages

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Works also for cosmologies with $\Omega_{ab} = \Omega_{ab}(k,\, \eta)$

not only for $\Omega_{ab} = \begin{pmatrix} 1 & -1 \\ -3/2 & 3/2 \end{pmatrix}$

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Power spectrum ($\langle \varphi\, \varphi \rangle$) and bispectrum ($\langle \varphi\, \varphi\, \varphi \rangle$) from a single run!

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Power spectrum ($\langle \varphi\, \varphi \rangle$) and bispectrum ($\langle \varphi\, \varphi\, \varphi \rangle$) from a single run!

Systematic approximation scheme straightforward

Approximation

$$\langle\varphi_a(\mathbf{k},\ \eta)\varphi_b(\mathbf{q},\ \eta)\rangle \equiv \delta_D(\mathbf{k}+\mathbf{q})P_{ab}(\mathbf{k}\,,\eta)\ ,$$

$$\langle\varphi_a(\mathbf{k},\ \eta)\varphi_b(\mathbf{q},\ \eta)\varphi_c(\mathbf{p},\ \eta)\rangle \equiv \delta_D(\mathbf{k}+\mathbf{q}+\mathbf{p})B_{abc}(\mathbf{k},\ \mathbf{q},\ \mathbf{p};\ \eta)\ ,$$

$$\begin{aligned}\langle\varphi_a(\mathbf{k},\ \eta)\varphi_b(\mathbf{q},\ \eta)\varphi_c(\mathbf{p},\ \eta)\varphi_d(\mathbf{r},\ \eta)\rangle \equiv&\\ &[\delta_D(\mathbf{k}+\mathbf{q})\,\delta_D(\mathbf{p}+\mathbf{r})P_{ab}(\mathbf{k}\,,\eta)P_{cd}(\mathbf{p}\,,\eta)\\ &+\delta_D(\mathbf{k}+\mathbf{p})\,\delta_D(\mathbf{q}+\mathbf{r})P_{ac}(\mathbf{k}\,,\eta)P_{bd}(\mathbf{q}\,,\eta)\\ &+\delta_D(\mathbf{k}+\mathbf{r})\,\delta_D(\mathbf{q}+\mathbf{p})P_{ad}(\mathbf{k}\,,\eta)P_{bc}(\mathbf{q}\,,\eta)\\ &+\delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q}+\mathbf{r})\,T_{abcd}(\mathbf{k}\,,\mathbf{q}\,,\mathbf{p}\,,\mathbf{r}\,,\eta)]\ ,\end{aligned}$$

Only approximation: $T_{abcd} = 0$

Equations to solve:

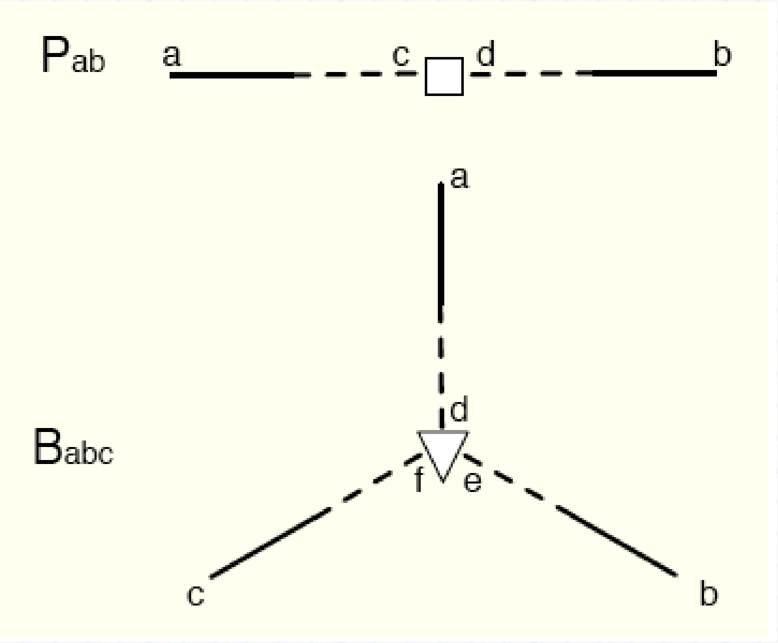
$$
\begin{aligned}
\partial_\eta P_{ab}(\mathbf{k}\,,\eta) = & -\Omega_{ac}(\mathbf{k}\,,\eta)P_{cb}(\mathbf{k}\,,\eta) - \Omega_{bc}(\mathbf{k}\,,\eta)P_{ac}(\mathbf{k}\,,\eta) \\
& + e^\eta \int d^3q\, \left[\gamma_{acd}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k})\, B_{bcd}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k};\, \eta)\right. \\
& \left. + B_{acd}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k};\, \eta)\, \gamma_{bcd}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k})\right]\,,
\end{aligned}
$$

$$
\begin{aligned}
\partial_\eta B_{abc}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k};\, \eta) = & -\Omega_{ad}(\mathbf{k}\,,\eta)B_{dbc}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k};\, \eta) \\
& - \Omega_{bd}(-\mathbf{q}\,,\eta)B_{adc}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k};\, \eta) \\
& - \Omega_{cd}(\mathbf{q}-\mathbf{k}\,,\eta)B_{abd}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k};\, \eta) \\
& + 2e^\eta \left[\gamma_{ade}(\mathbf{k},\, -\mathbf{q},\, \mathbf{q}-\mathbf{k})P_{db}(\mathbf{q}\,,\eta)P_{ec}(\mathbf{k}-\mathbf{q}\,,\eta)\right. \\
& + \gamma_{bde}(-\mathbf{q},\, \mathbf{q}-\mathbf{k},\, \mathbf{k})P_{dc}(\mathbf{k}-\mathbf{q}\,,\eta)P_{ea}(\mathbf{k}\,,\eta) \\
& \left. + \gamma_{cde}(\mathbf{q}-\mathbf{k},\, \mathbf{k},\, -\mathbf{q})P_{da}(\mathbf{k}\,,\eta)P_{eb}(\mathbf{q}\,,\eta)\right]\,.
\end{aligned}
$$

initial conditions given at $\eta = 0$, corresponding to $z = z_{in}$

Iterative solution: step 1

$O(\gamma^0)$: linear PT



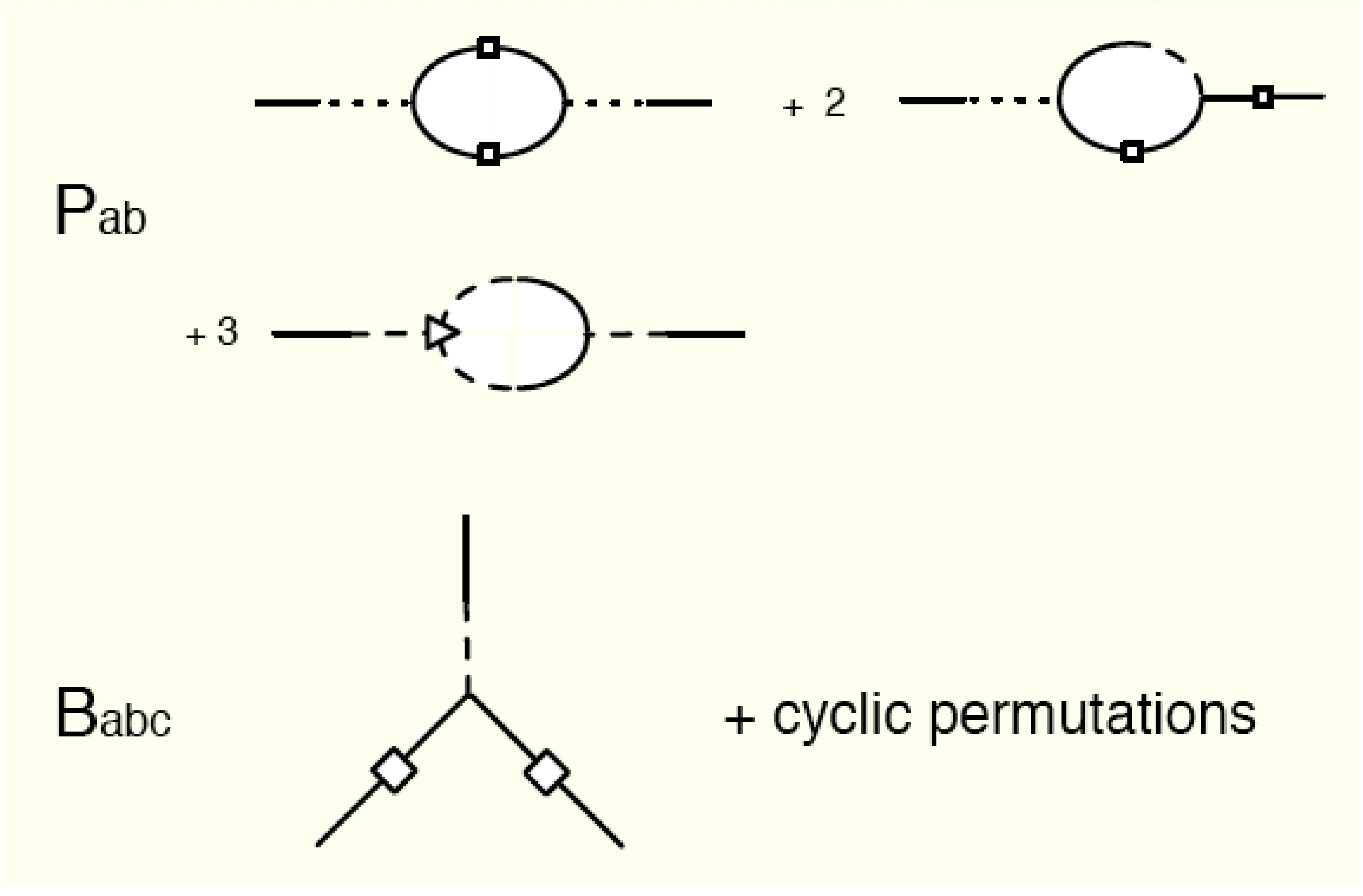
$$P^L_{ab}(\mathbf{k}\,,\eta) = g_{ac}(\mathbf{k}\,,\eta,0)\,g_{bd}(\mathbf{k}\,,\eta,0)P_{cd}(\mathbf{k}\,,\eta=0)\,,$$

$$B^L_{abc}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k};\,\eta) =$$
$$g_{ad}(\mathbf{k}\,,\eta,0)g_{be}(-\mathbf{q}\,,\eta,0)g_{cf}(\mathbf{q}-\mathbf{k}\,,\eta,0)B_{def}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k};\,\eta=0)$$

Iterative solution: step 2

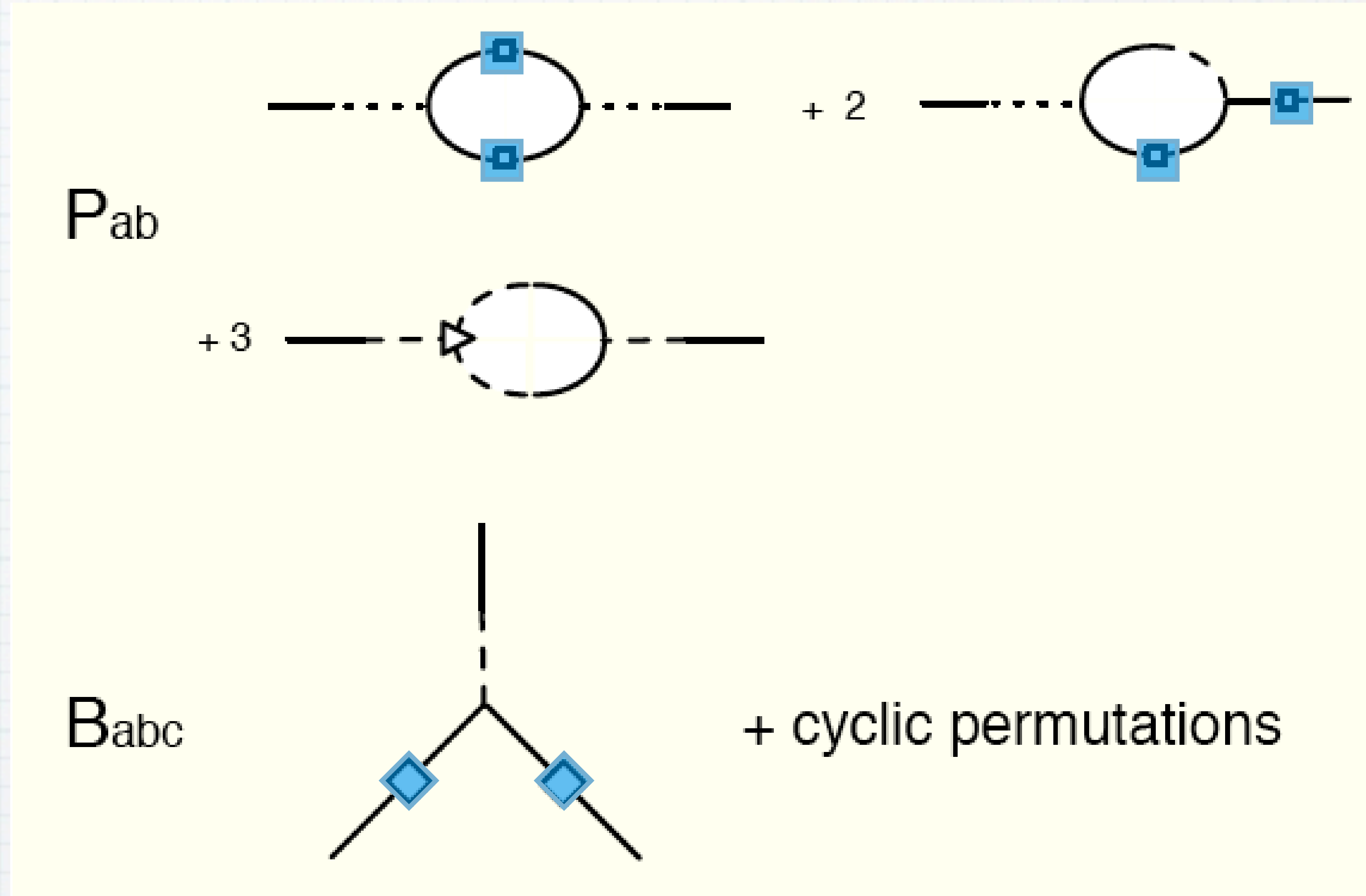
1-loop corrections for P_{ab} :

$O(\gamma)$ corrections for B_{abc} :



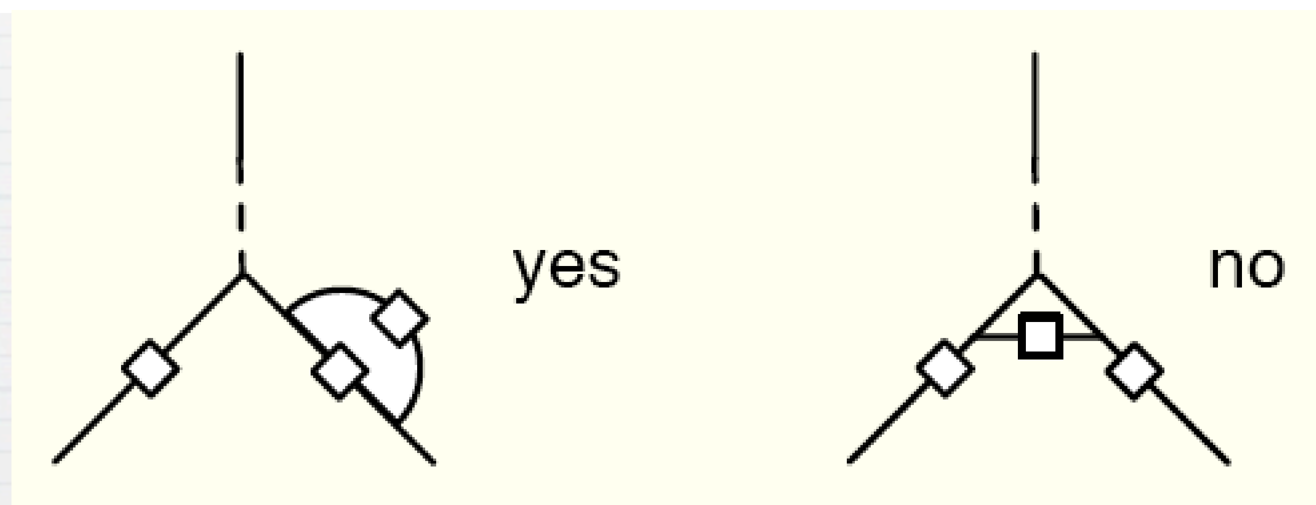
Iterative solution: step n>2

n-1-loop corrections for P_{ab}, <u>but not all!!</u>

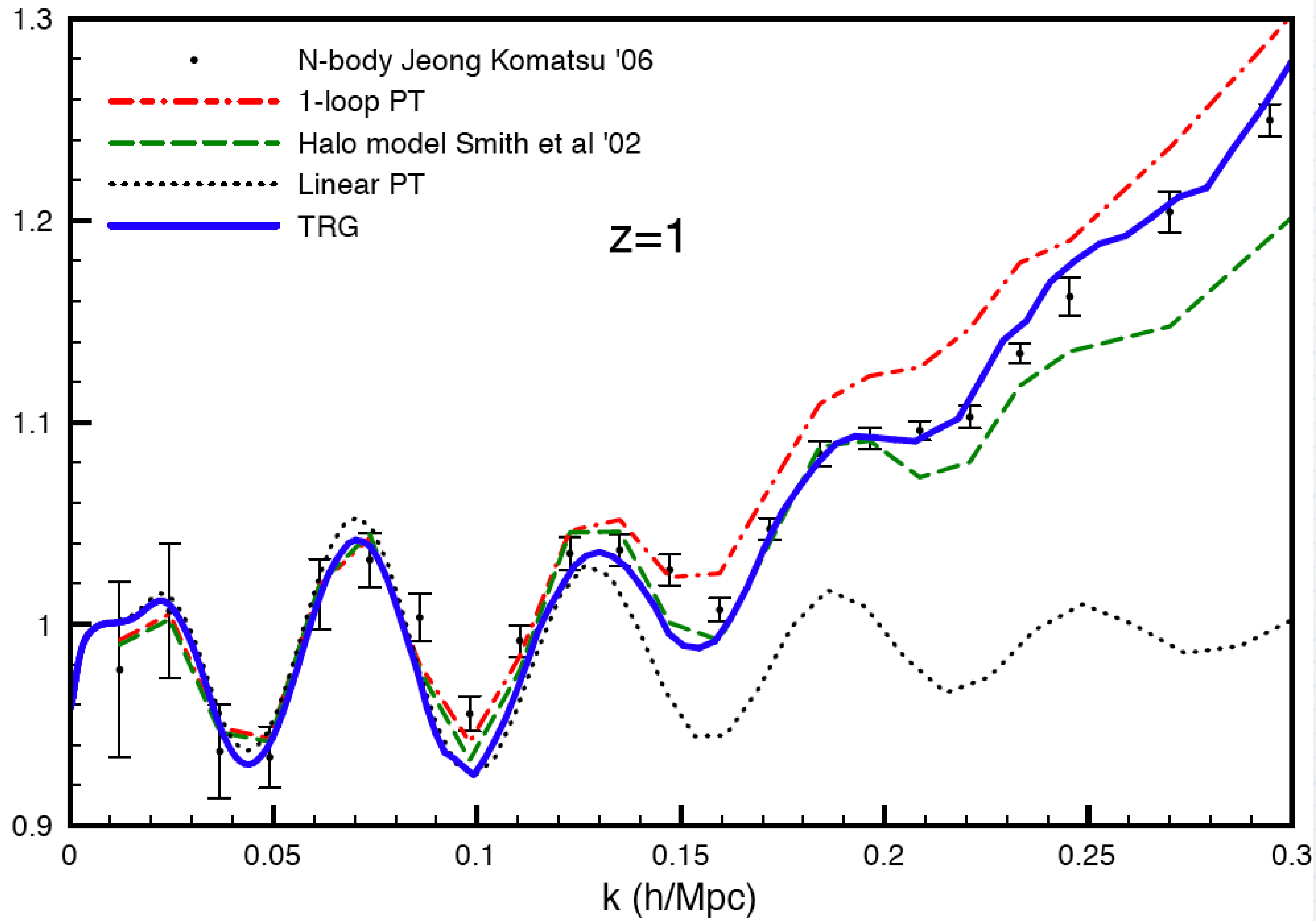


$O(\gamma^{2n-3})$ corrections for B_{abc} <u>but not all!</u>

e.g.: for n=3



Full equation: numerical results



M. Pietroni 0806.0971 (JCAP)

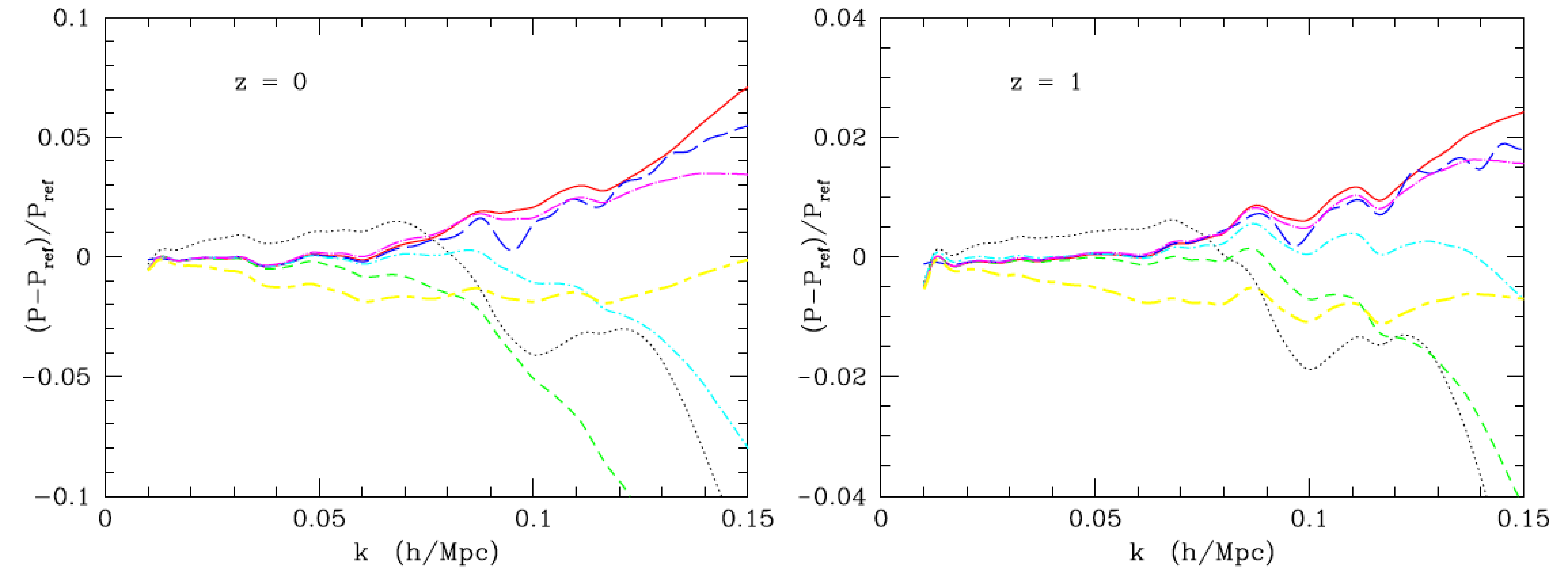
initial conditions:

$$P_{ab}(\mathbf{k}, 0) = P_{\rm Lin}(\mathbf{k}, z_{in}) u_a u_b$$

$$B_{abc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}, 0) = 0$$

Comparison with other methods

Carlson, White,
Padmanabhan, '09



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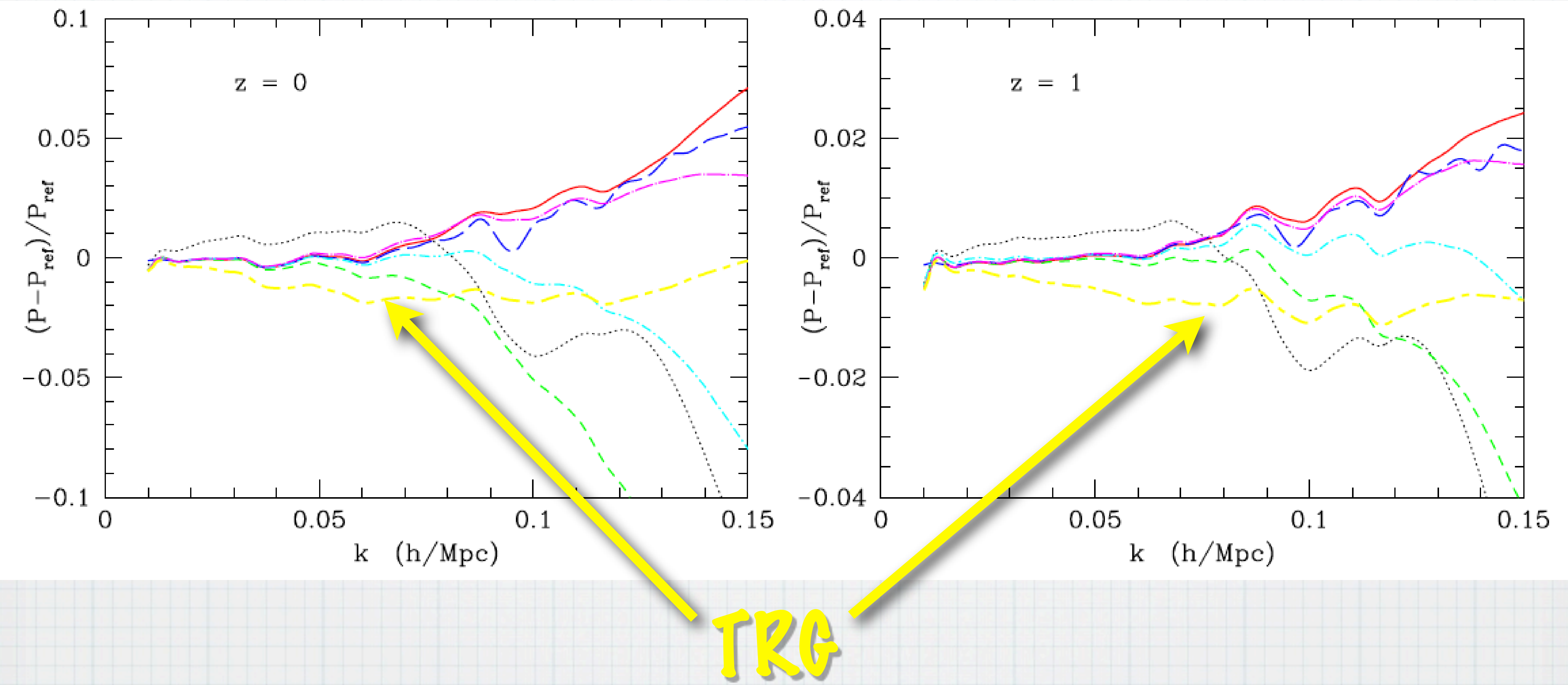

Fractional difference w.r.t. high resolution N-body below 2% in the BAO range down to z=0!

More General Cosmologies

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,,$$

$$\frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}(1+A(\vec{x},\,\tau))\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

$$\nabla^2\phi = 4\pi G\,(1+B(\vec{x},\,\tau))\,\rho\,a^2\,\delta$$

More General Cosmologies

deviation from geodesic
(e.g. DM-scalar field interaction)

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$$\left(\delta_{ab}\partial_\eta + \Omega_{ab}(\eta,\,\mathbf{k})\right)\varphi_b(\eta,\mathbf{k}) = e^\eta\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

$$\boldsymbol{\Omega}_{ab} = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_M(1+B(\eta,\,\mathbf{k})) & 2+\frac{\mathcal{H}'}{\mathcal{H}}+A(\eta,\,\mathbf{k}) \end{pmatrix} \qquad (\eta = \log a)$$

Ex: Scalar-Tensor: $\mathcal{A} = \alpha\,d\varphi/d\log a$ $\qquad \mathcal{B} = 2\alpha^2$ $\qquad \alpha^2 = 1/(2\omega+3)$

see Saracco, MP, Tetradis, Pettorino, Robbers '09

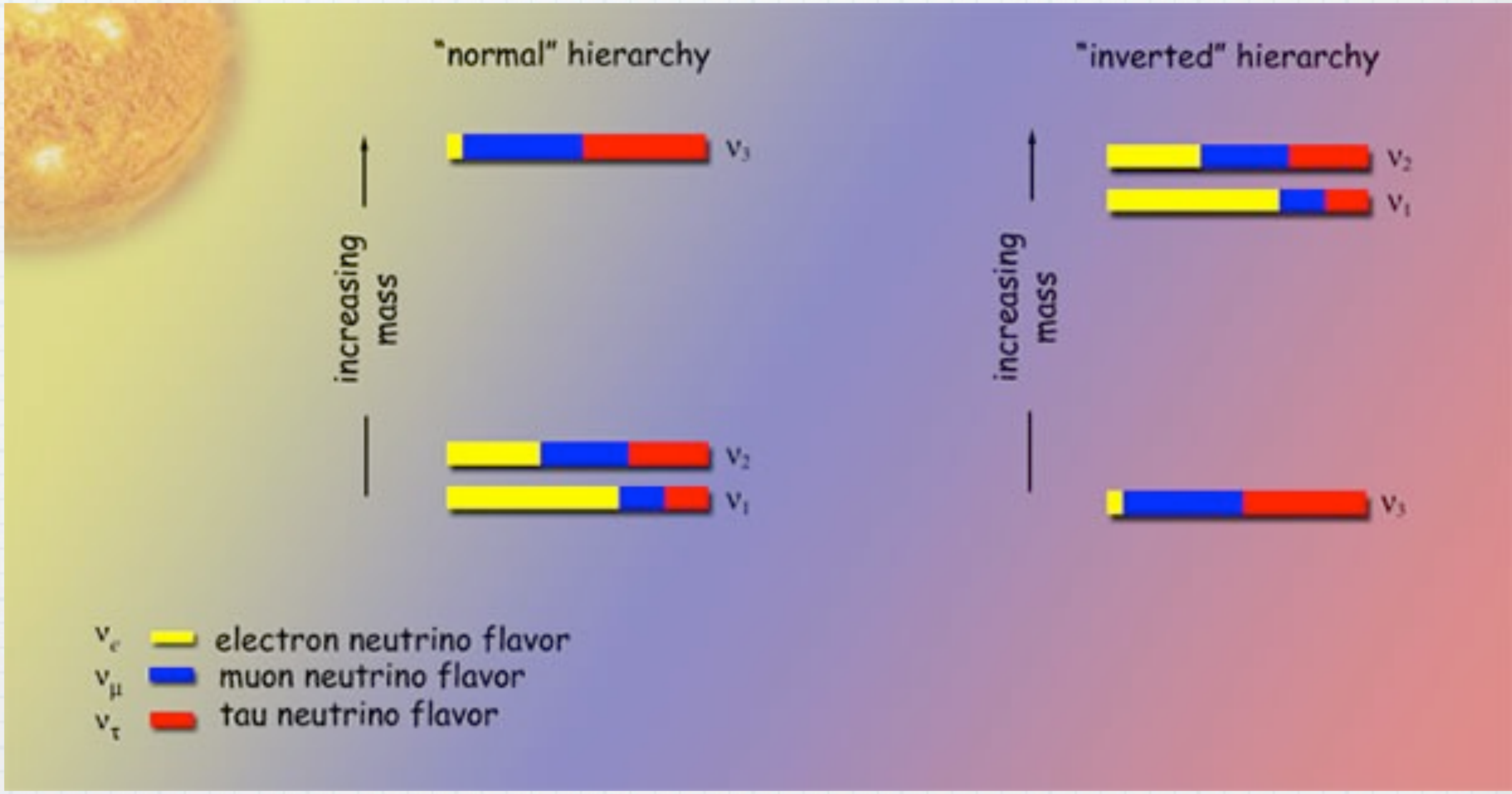
Neutrino Masses

neutrino mass exps:

$$|\Delta m^2_{12}| = (5.4 - 9.5) \times 10^{-5}\,\mathrm{eV}^2 \qquad \text{solar}$$

$$|\Delta m^2_{23}| = (1.2 - 4.8) \times 10^{-3}\,\mathrm{eV}^2 \qquad \text{atmosferic}$$

$$m_{\nu_e} < 2.5\,\mathrm{eV} \qquad {}^3H\ \beta\,\text{decay}$$



$$m_\nu^{\rm tot} < 7.5\,\mathrm{eV} \ll \mathrm{MeV}$$

all three SM neutrinos decouple when they are relativistic: $\left.\dfrac{n_{\nu i}}{n_\gamma}\right|_{T_{\rm dec}} = \dfrac{2\times 3/4}{2} = \dfrac{3}{4}$ **Hot!**

Neutrinos as Hot Dark Matter

neutrino free-streaming erases all structures up to mass scales of order:

$$M_{FS} = 3 \cdot 10^{18} \frac{M_\odot}{m_\nu^2(\mathrm{eV})} \gg M_{\mathrm{Gal}} = O(10^{12}\ M_\odot)$$

large structures would form first, at odds with observations (galaxies already at z ~6, clusters only at z<1)

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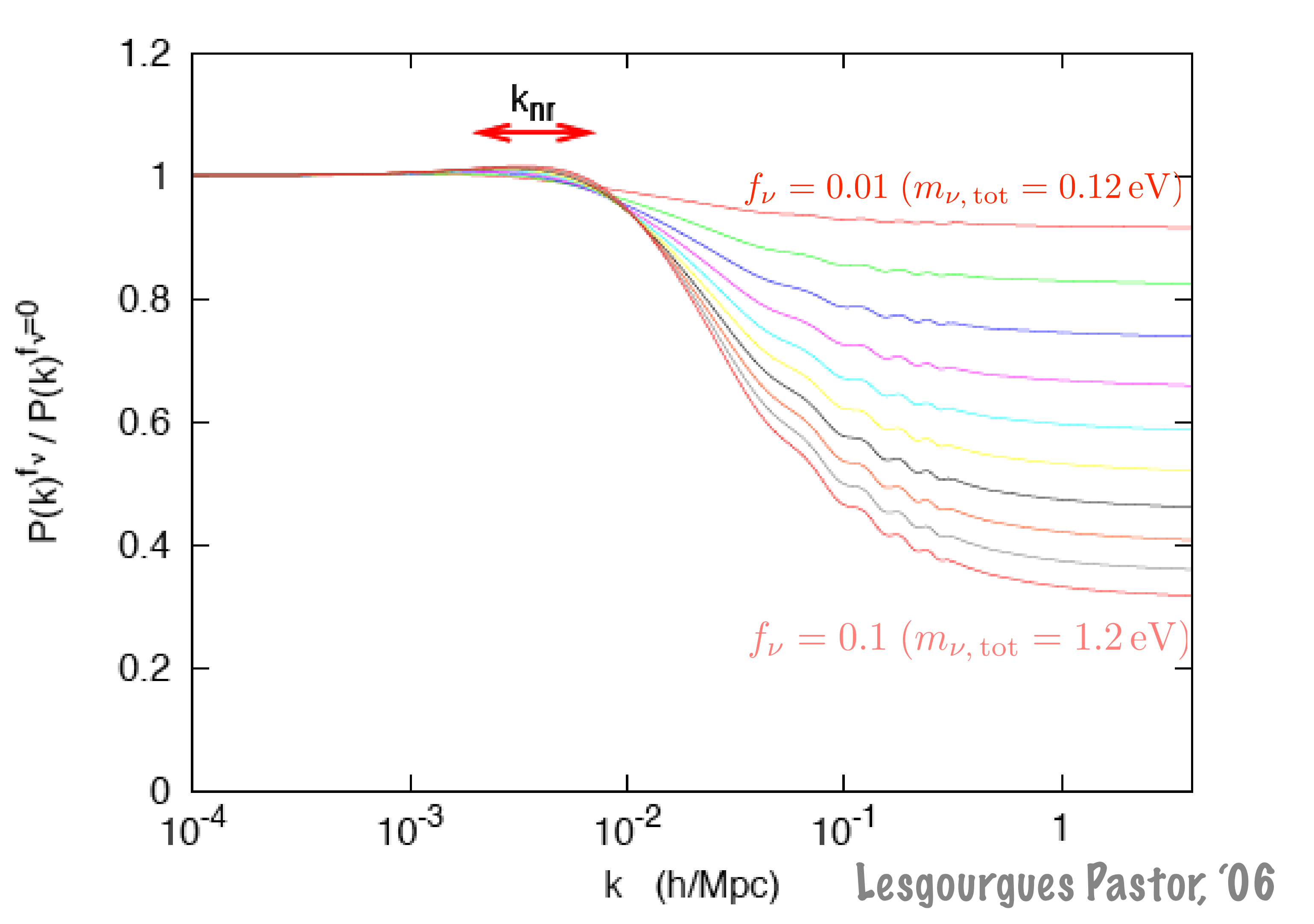
neutrinos cannot be all the DM. We need cold DM

relic neutrino energy density: $$\Omega_{\nu_i} \equiv \frac{m_{\nu_i} n_{\nu_i}}{\rho_c} = \frac{m_{\nu_i}}{94\, eV\ h^2} \equiv f_\nu\, \Omega_m$$

Massive Neutrinos and the Power Spectrum



$$f_\nu = \frac{\Omega_\nu}{\Omega_m} = \frac{m_{\nu,\,\mathrm{tot}}}{94.1\,\Omega_m\,h^2\,\mathrm{eV}}$$

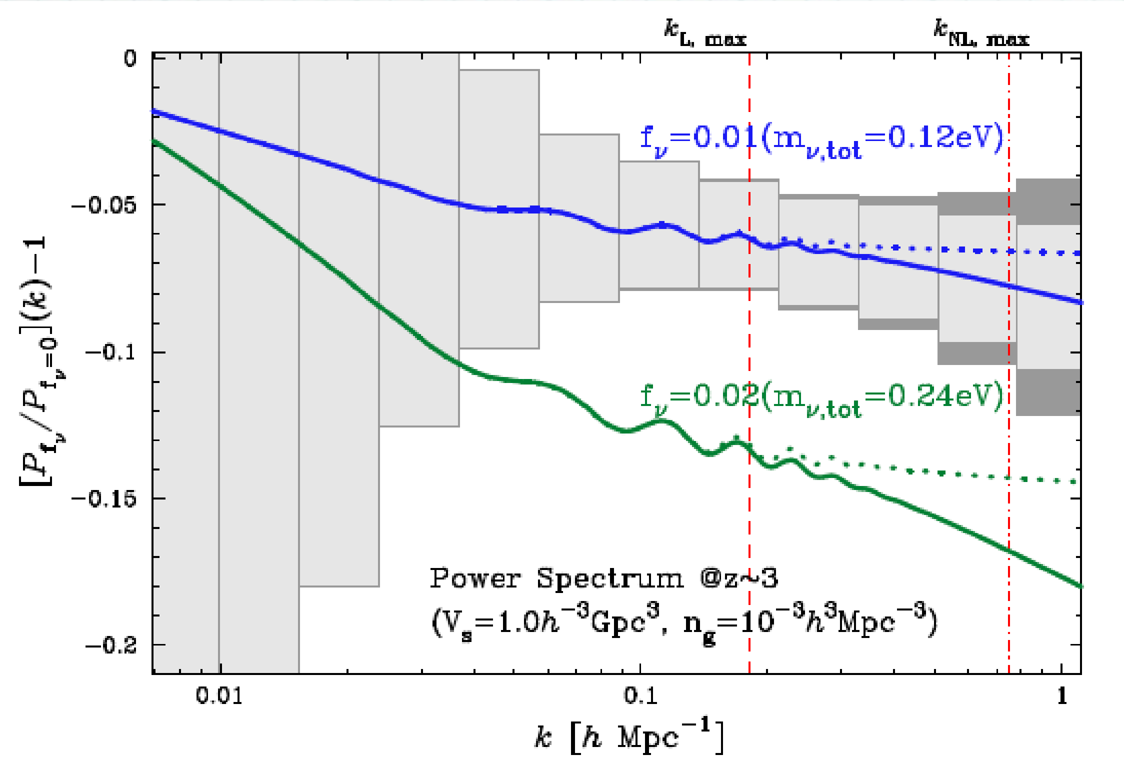
$$k_{nr} \simeq 0.018\,\Omega_m^{1/2}\left(\frac{m_\nu}{1\mathrm{eV}}\right)\,h\,\mathrm{Mpc}^{-1}$$

$$\frac{\Delta P}{P^{f_\nu=0}} \sim -8 f_\nu$$

The linear growth factor is scale dependent: $\delta_m(k \ll k_{nr}) \sim a\ , \quad \delta_m(k \gg k_{nr}) \sim a^{1-3/5 f_\nu}$

Non-linear effects on neutrino mass bounds

(Saito, Takada, Taruya,'08)



$$P_m = (1-f_\nu)^2 \underbrace{P_{cc}}_{\text{1-loop}} + 2f_\nu(1-f_\nu)\underbrace{P^L_{c\nu}}_{\text{linear}} + f_\nu^2 \underbrace{P^L_{\nu\nu}}_{\text{linear}}$$

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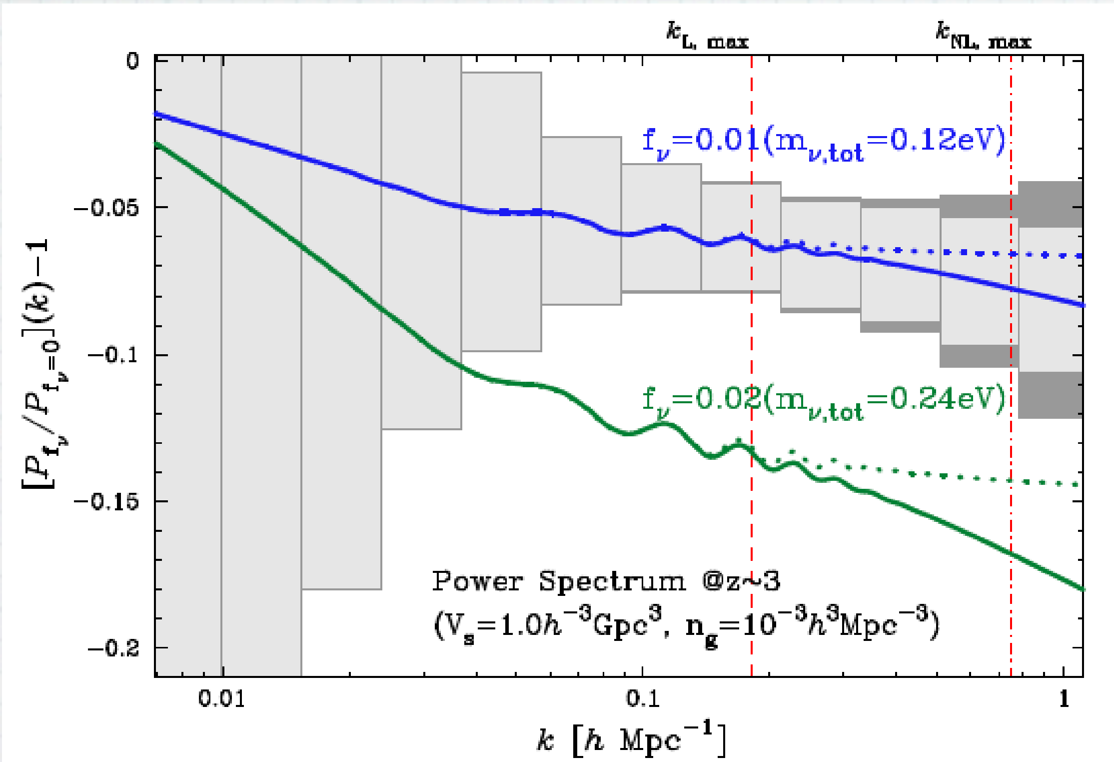
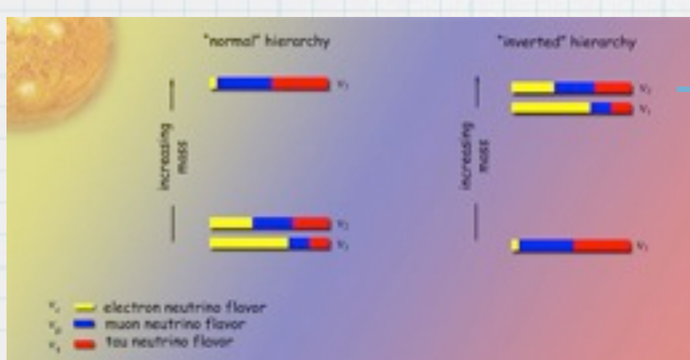


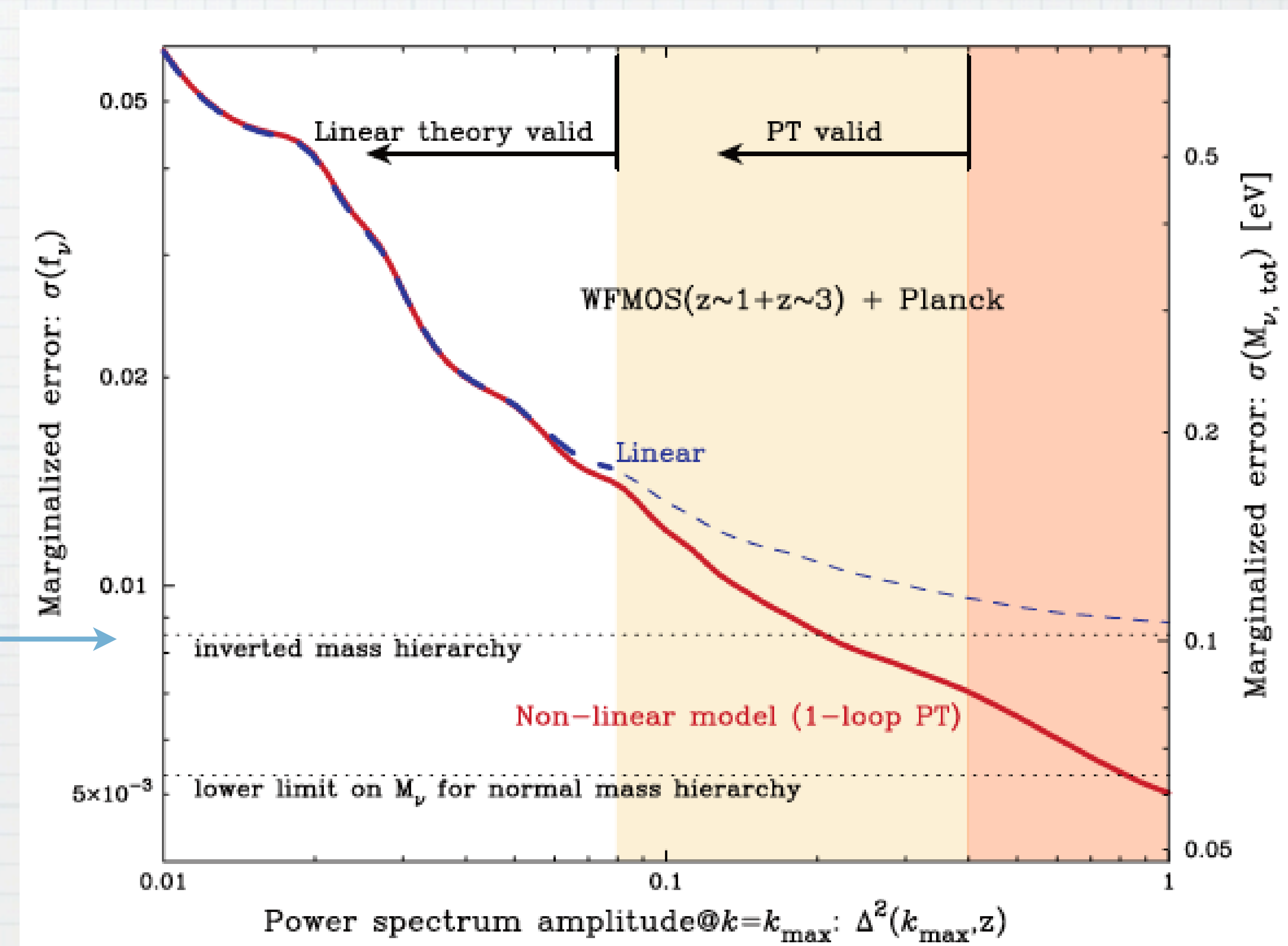
$$P_m = (1-f_\nu)^2 P_{cc} + 2f_\nu(1-f_\nu)\,P^L_{c\nu} + f_\nu^2\,P^L_{\nu\nu}$$

1-loop

linear

the neutrino mass bound could go down to ~ 0.09 eV !





Poisson equation for Cold DM + Neutrinos

$$\nabla^2\phi = 4\pi G\, a^2\, (\rho_c\, \delta_c + \rho_\nu\, \delta_\nu) = 4\pi G\, a^2\, \left(1 + \frac{\rho_\nu\, \delta_\nu}{\rho_c\, \delta_c}\right) \rho_c\, \delta_c$$

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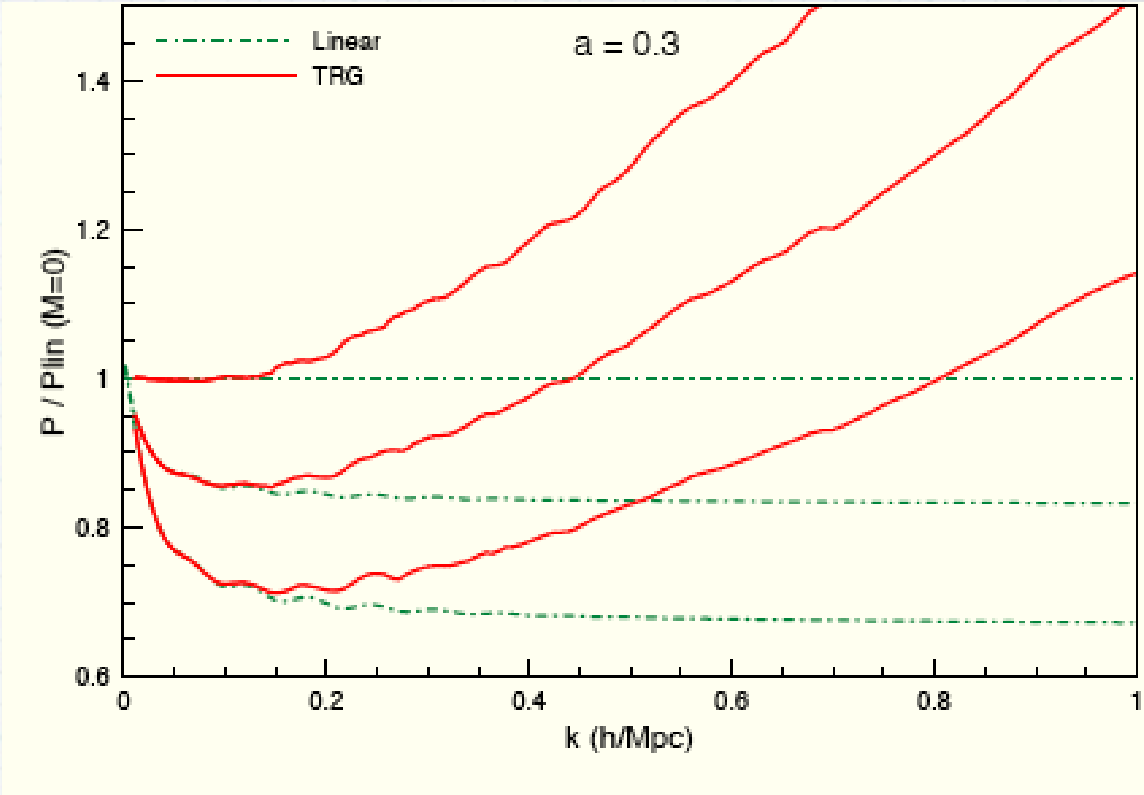
$$\mathcal{B}(k,\, \tau) \simeq \left.\frac{\rho_\nu\, \delta_\nu}{\rho_c\, \delta_c}\right|_{\mathrm{linear\,th.}}$$

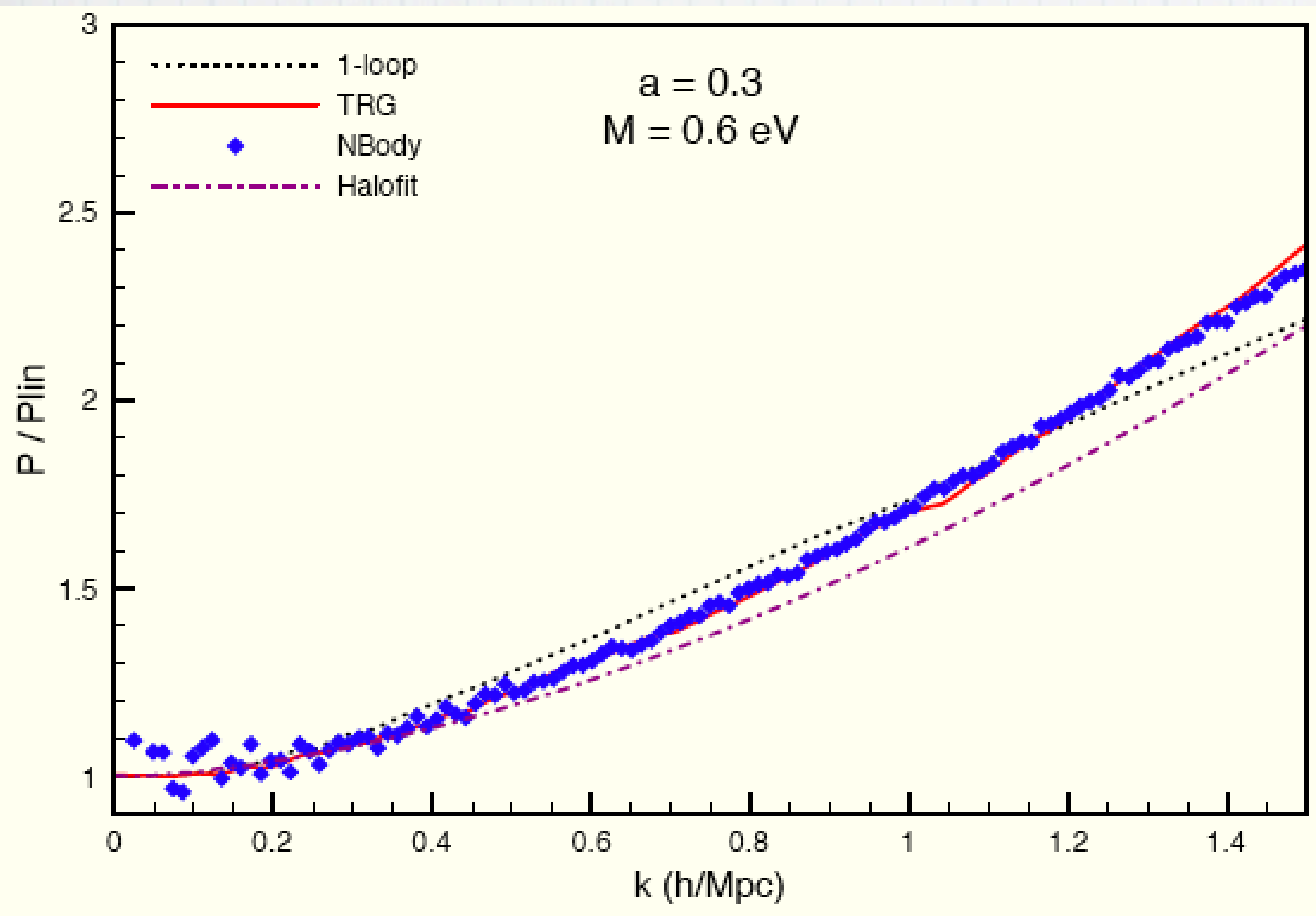
Non-Linear effects on νmass bounds: RG vs. 1-loop approximation

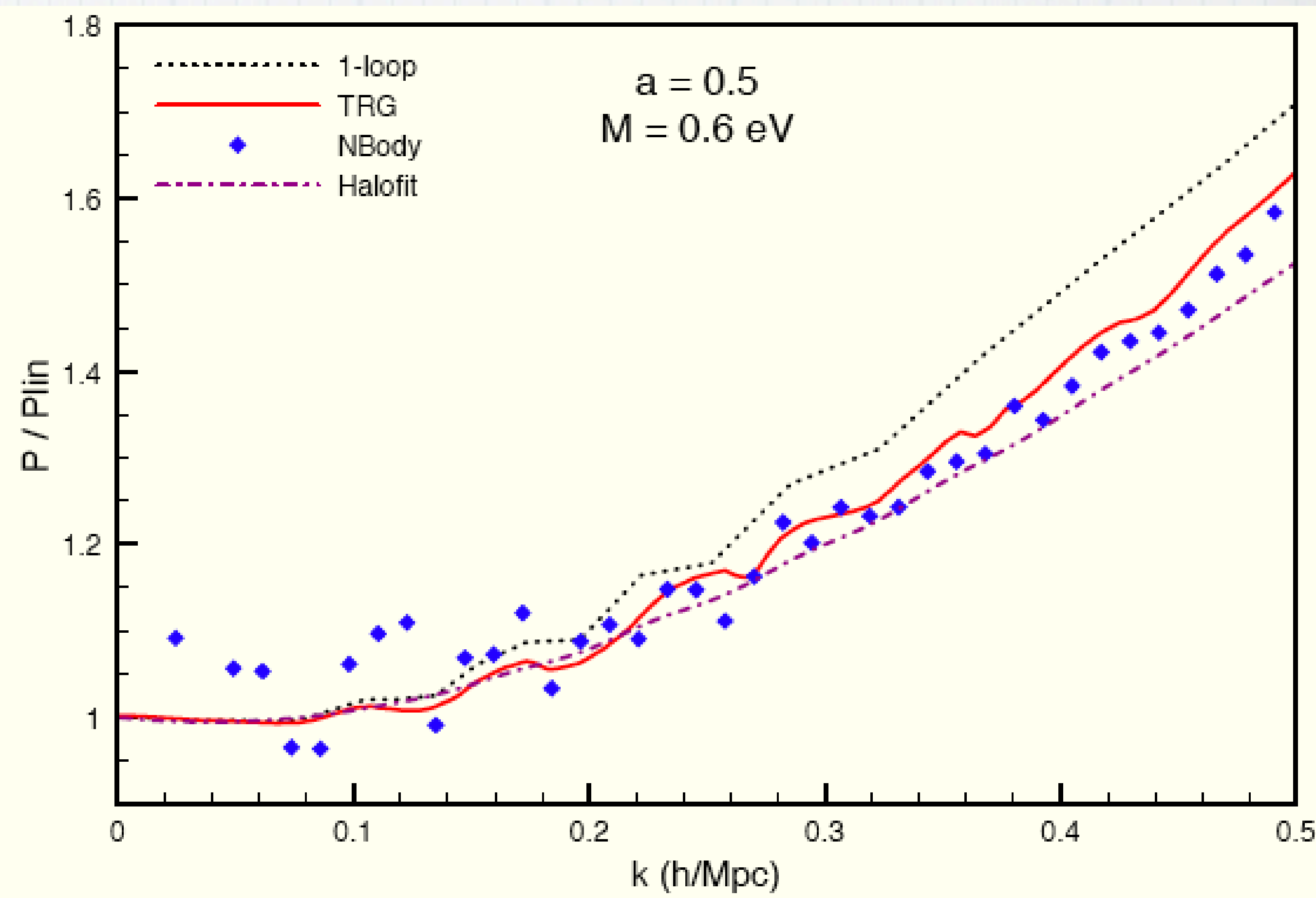
Lesgourgues, Matarrese, M.P., Riotto, '09

Cosmological Neutrino mass bounds should take non-linear effects into account!

N-Body simulations including massive neutrinos still in their infancy ($\Delta P/P \approx -9.8/-10.5\ f_\nu$)







Non-Gaussianity

$$\Phi = \phi + f_{NL}(\phi^2 - \langle\phi^2\rangle)$$

CMB bispectrum: $-4 < f_{NL} < 80 = \ (95\% \ \ C.L.)$ K.M. Smith et al. '09

Galaxy bispectrum at large scales: complementary and competitive to CMB provided other sources of non-linearities under control

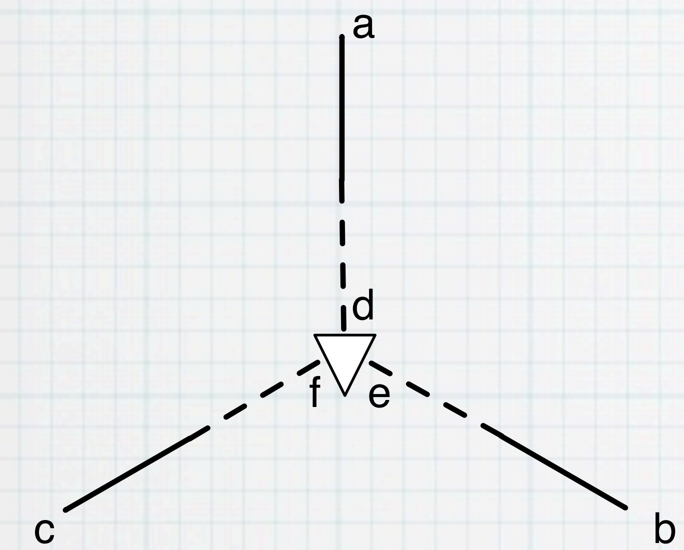
Fry & Scherrer, '94
Chodorowski & Bouchet '96
Scoccimarro '00
Verde et al. '00
...

DM (halos) power spectrum: large scales, rare objects → $\Delta f_{NL} \leq 1$

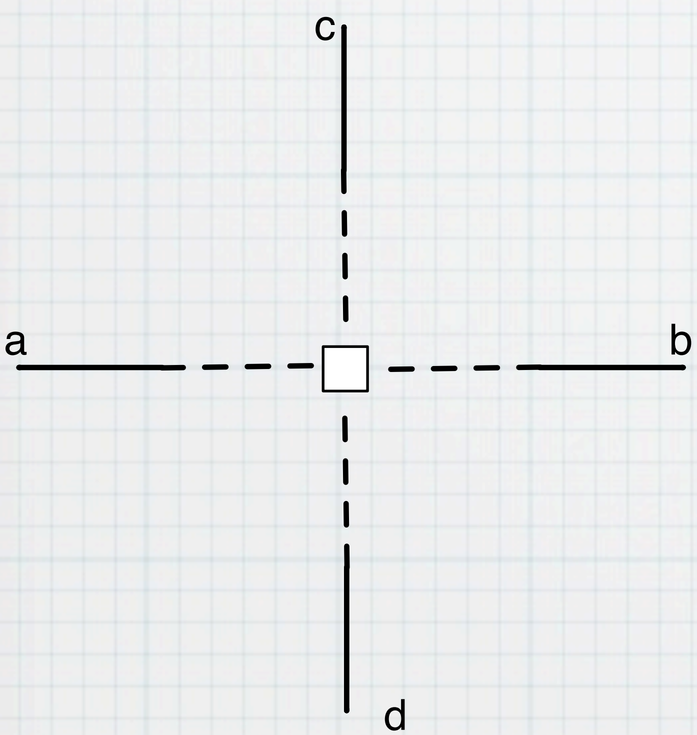
$\Delta P(f_{NL})/P \simeq 2 - 3\%$ on BAO scales

Dalal et al. '08,
Matarrese & Verde '08, Lam & Sheth '08,
Slosar et al '08, Mc Donald & Seljak '08,
Carbone et al, '08, Pillepich et al. '09, ...

Seeds from primordial NG



$$g_{ad}(\eta)\ g_{be}(\eta)\ g_{cf}(\eta)\ B_{def}(k_1, k_2, k_3; \eta = 0)$$



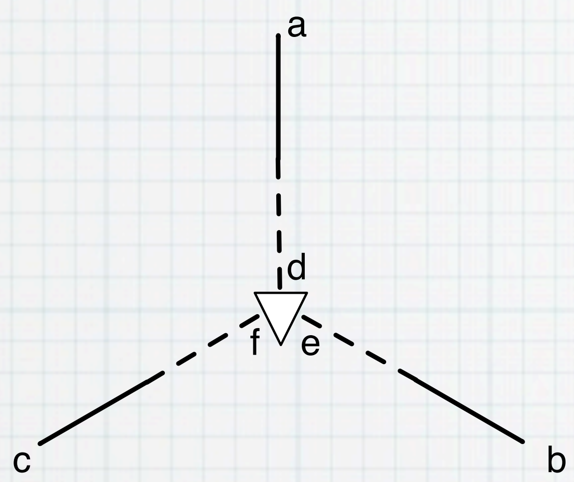
$$g_{ae}\ g_{bf}\ g_{cg}\ g_{dh}\ T_{efgh}(\eta = 0)$$

...

...

new "vertices" in the Feynman rules

Ex: late-time bispectrum



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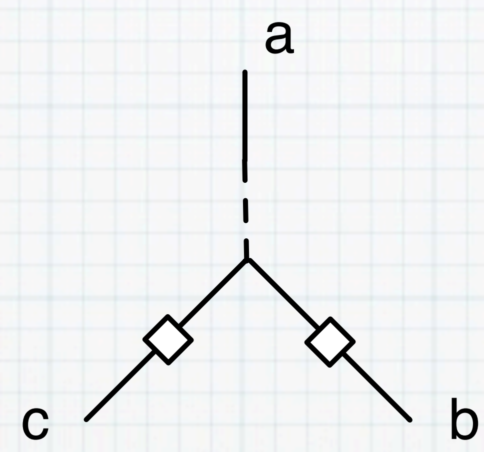
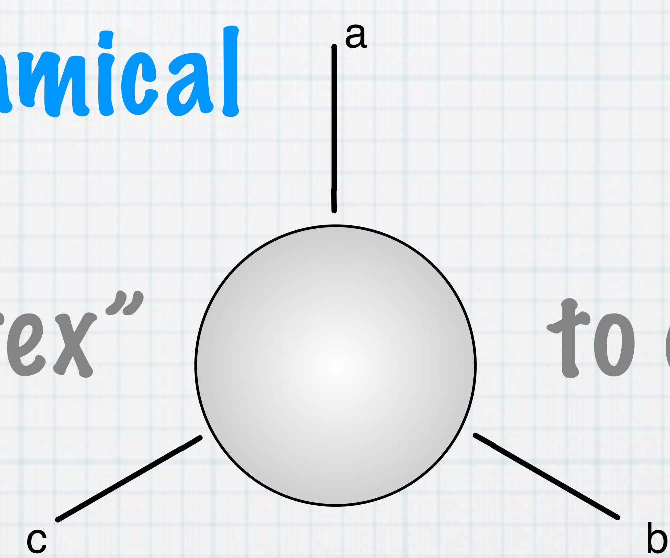


+ cyclic permutations

primordial

dynamical

Goal: probe the "effective vertex"

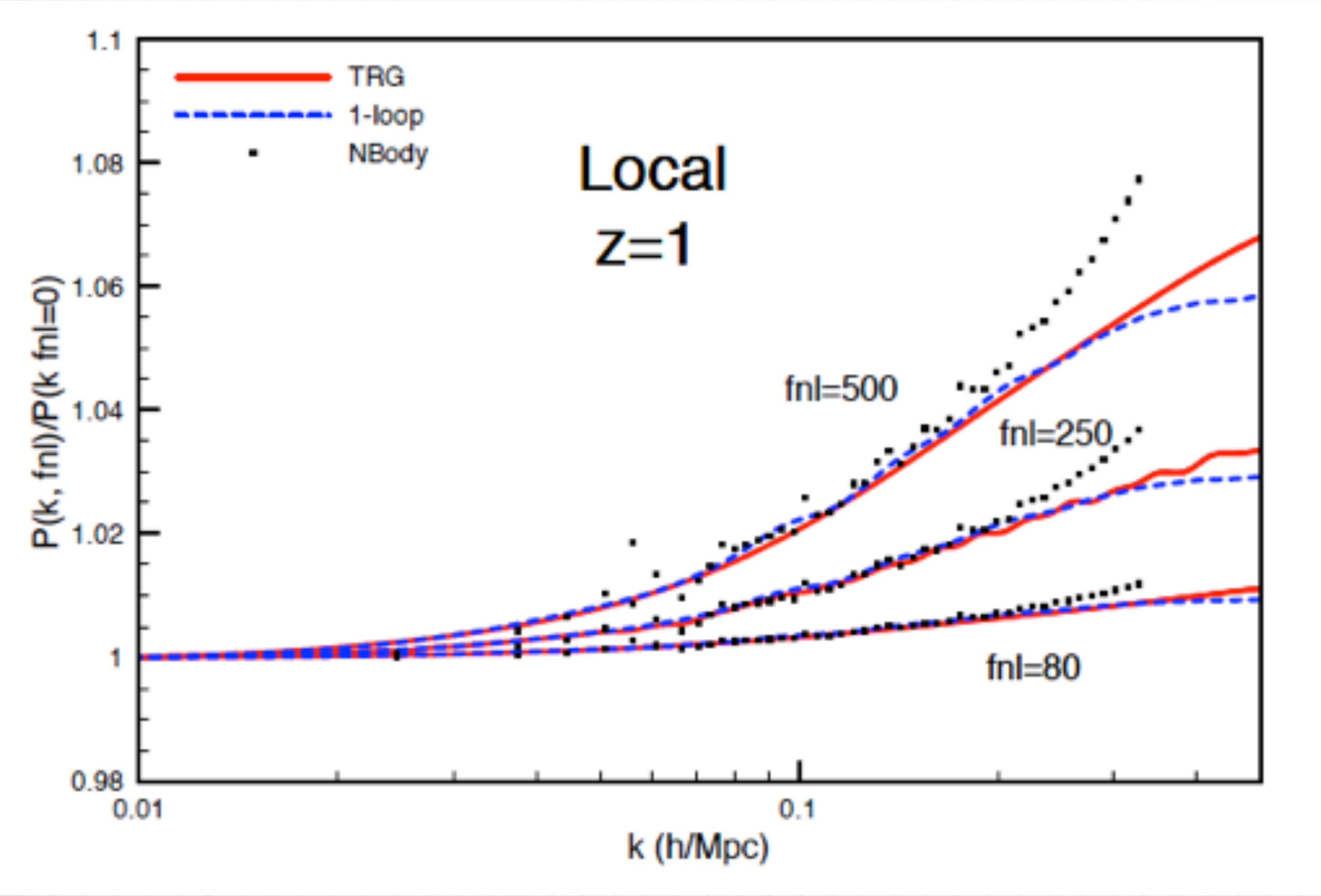


to extract $B(\eta = 0)$

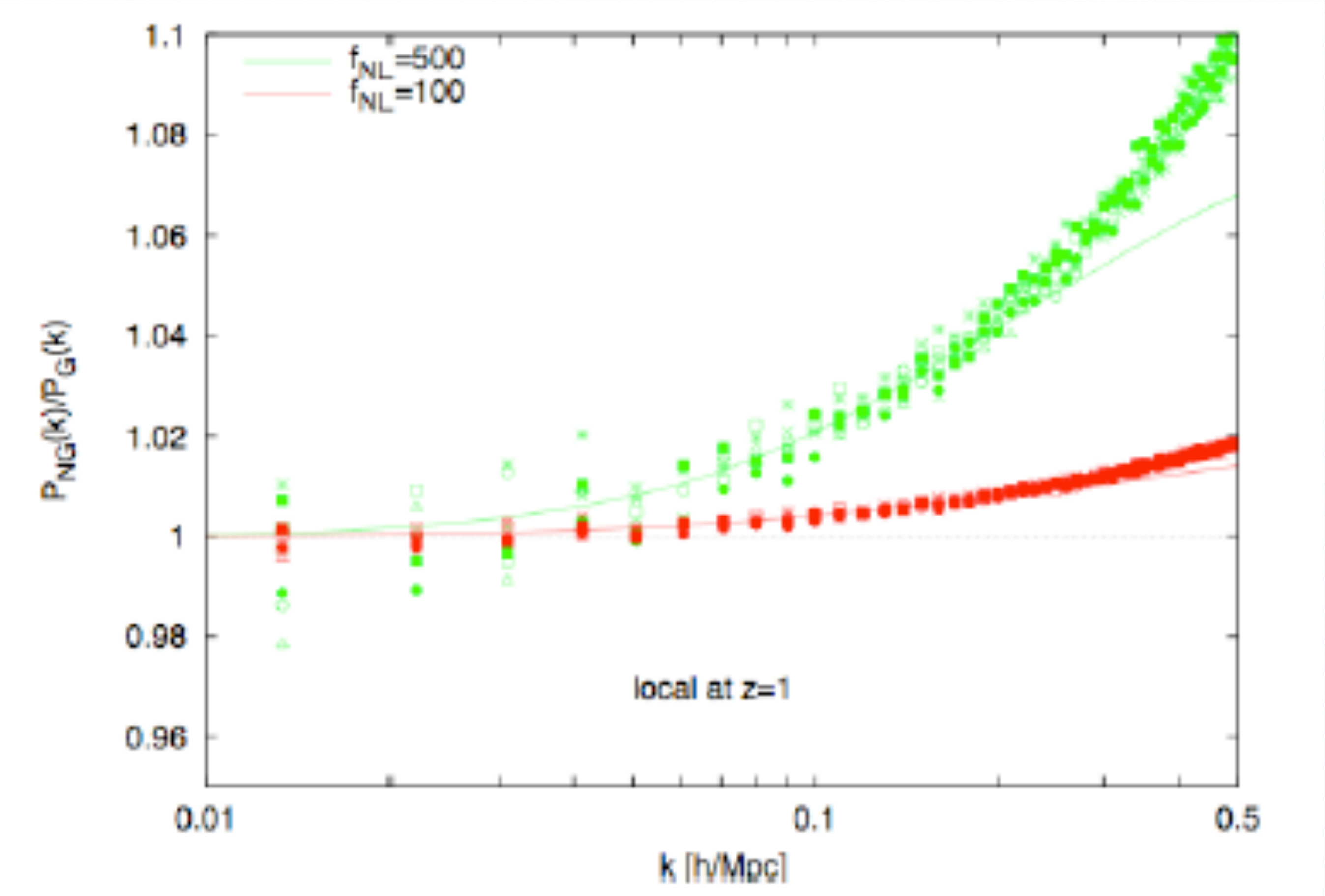
Need control on non-linear gravity dynamics!

TRG Results

Bartolo, Beltran-Almeida, Matarrese, M.P., Riotto, 0912.4276



(N-body from Pillepich et al. '09)



(N-body from Verde et al. '10-preliminary)

PS ratios in good agreement with 1-loop PT and (both) N-body simulations up to k <0.2 h/Mpc

Results also for "non-local" models for NG , difficult to simulate with NBody,

Conclusions

* Mildly non-linear scales are an unique opportunity to look for deviations from "vanilla" ΛCDM ($w \neq -1$, massive ν's, NonGaussianity, DE-DM interactions, exotic DM,...)
* Semi-analytic methods are needed to go beyond linear PT in a more transparent, flexible, and fast (!) way than NBody's
* Further improvements: bias, redshift space, effect of velocity dispersion...

A generating functional

A generating functional

The probability of the final configuration $\varphi_a(\eta_f)$, given the initial condition $\varphi_a(\eta_i)$, is

$$P[\varphi(\eta_f);\varphi(\eta_i)] = \delta\left[\varphi(\eta_f) - \overline{\varphi}[\eta_f;\varphi(\eta_i)]\right]$$

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fixed extrema

$$S[\varphi,\,\chi] = \int_{\eta_i}^{\eta_f} d\eta\, \chi_a\left[(\delta_{ab}\partial_\eta + \Omega_{ab})\varphi_b - e^{\eta}\gamma_{abc}\varphi_b\varphi_c\right]$$

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The generating functional ***at fixed initial conditions*** is

$$Z[J,\Lambda;\varphi(\eta_i)] = \int \mathcal{D}\varphi(\eta_f)\,\mathcal{D}''\varphi\,\mathcal{D}\chi\; e^{iS[\varphi,\,\chi] + i\int_{\eta_i}^{\eta_f} d\eta(J_a\varphi_a + \Lambda_b\chi_b)}$$

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Putting all together...

$$\begin{aligned}Z[\mathbf{J},\,\mathbf{\Lambda}] \quad = \quad &\int\mathcal{D}\varphi\,\mathcal{D}\chi\,\exp\left\{\int d\eta_1 d\eta_2\left[-\frac{1}{2}\,\chi\,\mathbf{g}^{-1}\mathbf{P^L}\mathbf{g^T}^{-1}\chi+i\,\chi\,\mathbf{g}^{-1}\,\varphi\right]\right.\\ &\left.-i\int d\eta\,[\mathbf{e}^{\eta}\gamma\,\chi\varphi\varphi-\mathbf{J}\varphi-\mathbf{\Lambda}\chi]\right\}\end{aligned}$$

where the initial conditions are encoded in the linear power spectrum: $P^L_{ab}(\eta,\,\eta';\,\mathbf{k})\equiv\left(\mathbf{g}(\eta)\mathbf{P^0}(\mathbf{k})\mathbf{g}^T(\eta')\right)_{ab}$

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Derivatives of Z w.r.t. the sources J and Λ give all the n-point correlation functions (power spectrum, bispectrum, ...) and the full non-linear propagator

Dark Matter Hydrodynamics

The DM particle distribution function, $f(\mathbf{x}, \mathbf{p}, \tau)$, obeys the Vlasov equation:

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{am} \cdot \nabla f - am \nabla \phi \cdot \nabla_{\mathbf{p}} f = 0$$

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Taking moments, i.e.,

$$\int d^3\mathbf{p}\, f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau) \equiv \overline{\rho}\,[1+\delta(\mathbf{x},\tau)]$$

$$\int d^3\mathbf{p}\, \frac{p_i}{am} f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau)v_i(\mathbf{x},\tau)$$

$$\int d^3\mathbf{p}\, \frac{p_i\,p_j}{a^2m^2} f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau)v_i(\mathbf{x},\tau)v_j(\mathbf{x},\tau) + \sigma_{ij}(\mathbf{x},\tau)$$

$$\cdots$$

and neglecting σ_{ij} and higher moments (single stream approximation), one gets...