Lattice input to CKM measurements

Jack Laiho
University of Glasgow

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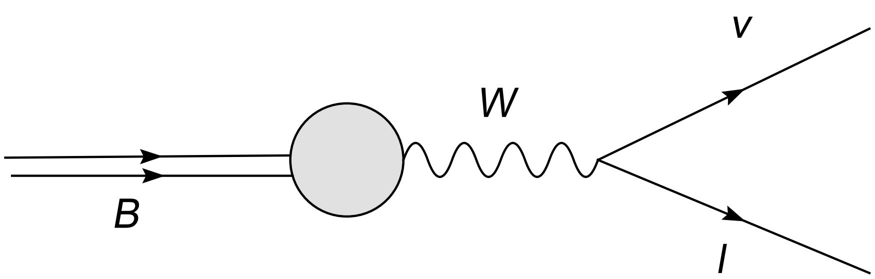
Introduction

Lattice now delivers unquenched (2+1 flavor) determinations for all quantities that enter the UT fit.

Several groups now have results for various quantities. Averages are necessary!

Nonperturbative input needed



$$\epsilon_K = (\text{known factor})\,(\text{CKM factor})(\text{QCD factor}) \quad (1)$$

$$\mathcal{B}(B \to \tau \overline{\nu}_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B \quad (2)$$

Lattice QCD

- Allows non-perturbative calculations from first principles
- Can be simulated on a computer using Monte Carlo methods
- Calculations require a finite-sized grid with lattice spacing a and size L
- Still very expensive! Large supercomputers are needed.

Types of Errors

Because QCD with physical quark masses is a nonlinear multiscale problem ($\Lambda_{QCD} \approx 100 - 200$ MeV, $m_{u,d} \approx 2 - 6$ MeV, $m_b \approx 4.3$ GeV), it is very expensive to simulate at the physical quark masses.

1.) Statistics and fitting

2.) Tuning lattice spacing, a, and quark masses

3.) Matching lattice gauge theory to continuum QCD

4.) Extrapolation to continuum

5.) Chiral extrapolation to physical up, down quark masses

6.) Quenching. Uncontrolled!

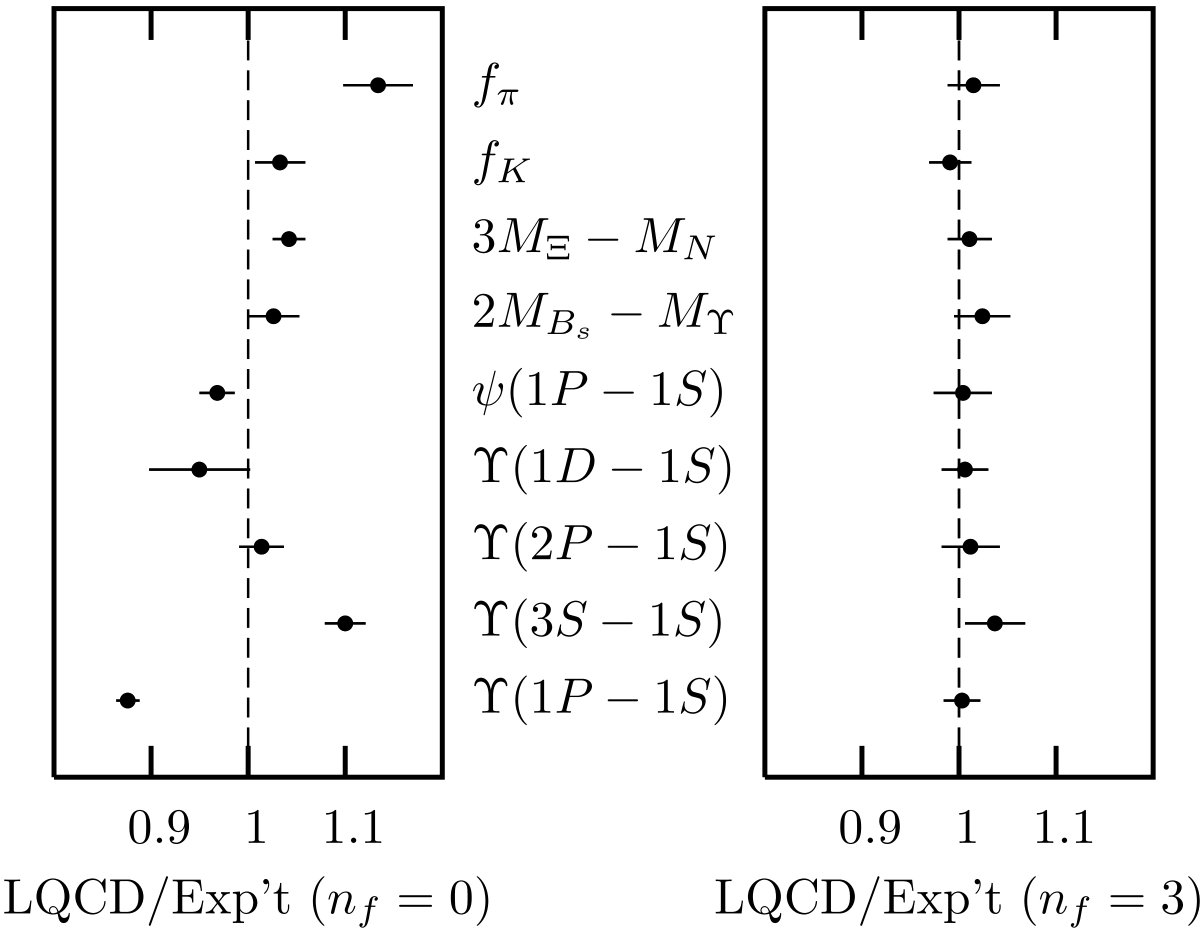
Quenched Approximation

Configurations are generated with a weighting given by the gauge field and fermion determinant. Including the fermion determinant in this weighting is the most computationally demanding step in lattice QCD.

The quenched approximation ignores fermion-antifermion vacuum bubbles. This is an uncontrolled systematic error.

“Unquenched” calculations, where the fermion determinant is included, are now the norm.

Unquenching Lattice QCD



- Hadron spectroscopy – masses and decay constants (hep-lat/0304004)
- *Good agreement for simple quantities!*

Quenching the strange quark

- Strange threshold lies in the nonperturbative regime.
- Most quantities show no difference with 2+1 flavor results, with precision at the 3-5% level.
- Error is difficult to estimate. Could be as much as 5%, and only sure way to quantify it is to compare with 2+1 flavor results.
- Since the lattice world averages are approaching the level where quenching the strange is likely to be important, we do not include 2 flavor results in our averages.

Quenching the charm quark

- The charm threshold lies above the nonperturbative regime.

- Error due to neglecting charm can be estimated by Heavy Quark Effective Theory, $\alpha_s(\Lambda/m_c)^2$

- This is $\mathcal{O}(1\%)$. (Likely smaller for specially constructed ratios.)

  New lattice calculations from MILC and ETM Collaborations are being done with 2+1+1 flavors and will address this issue.

Rooted staggered quarks

- Cheapest fermions on the market at the present time
- Many 3 flavor calculations use staggered quarks
- The staggered action has extra unphysical species of fermions (called “tastes”) due to lattice artifacts which vanish in the continuum limit. There is no rigorous proof that staggered quarks recover QCD in the continuum limit. There has been much recent theoretical progress, and the recovery of the correct continuum limit appears plausible. (Sharpe, hep-lat/0610094; Kronfeld, arXiv:0711.0699.)
- Extra “tastes” complicate the analysis with staggered fermions, as compared to “chiral” fermions such as domain-wall or overlap, which are many times more expensive.
- Staggered chiral perturbation theory gives good control over staggered discretization effects (MILC, arXiv:hep-lat/0407028). If rooting is correct, the error is incorporated into the chiral extrapolation error.

Effective field theories

Effective field theories can be used to quantify systematic errors due to extrapolations in light quark masses (chiral perturbation theory), or the treatment of heavy-quarks like charm and bottom (Heavy-Quark Effective Theory).

Symanzik effective theory can be used to quantify systematic errors due to finite-lattice spacing.

Chiral perturbation theory

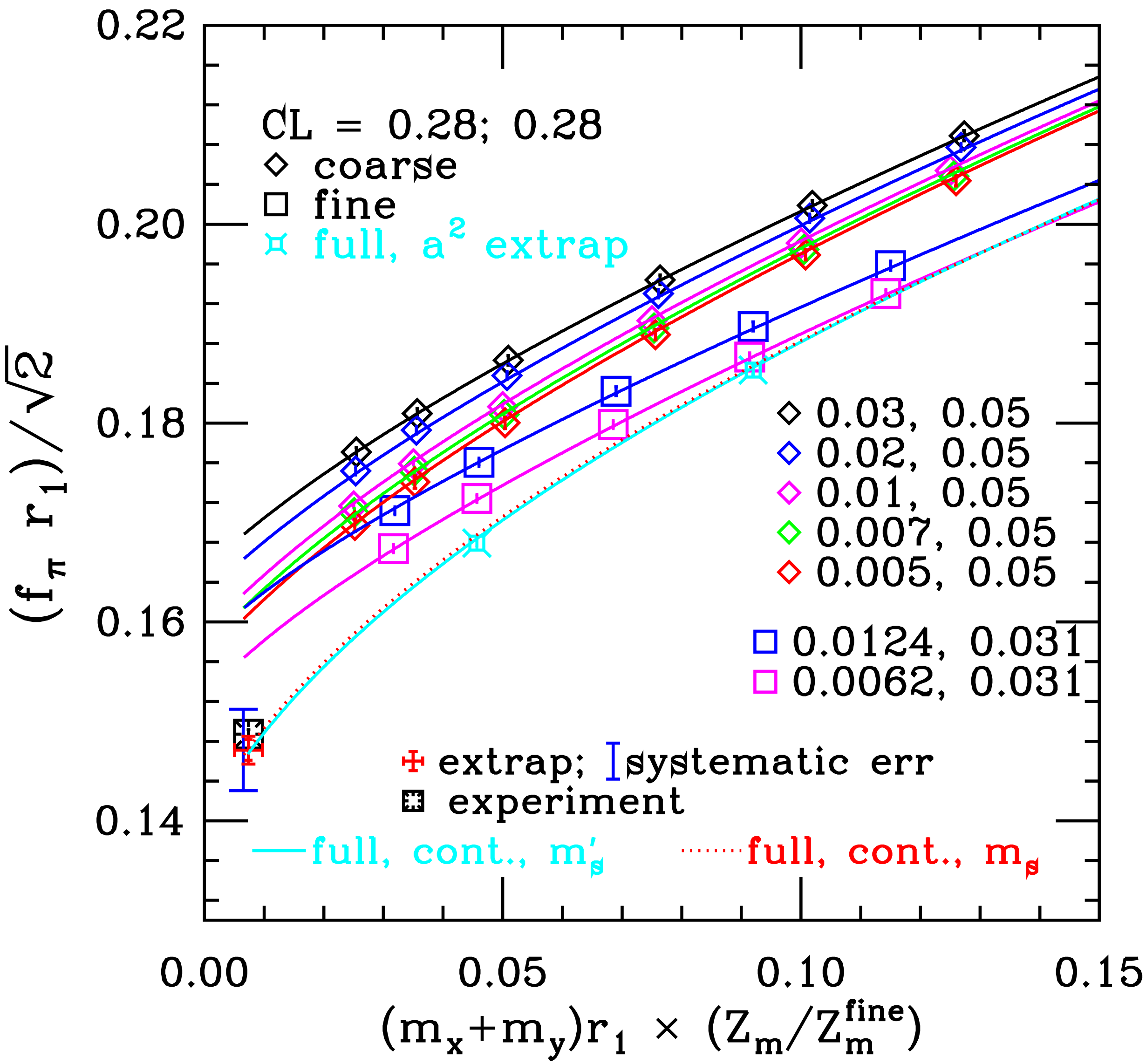
Chiral perturbation theory (ChPT) is an expansion about small quark masses and momenta.

At each order new terms must be introduced to cancel the renormalization scale dependence. These terms are not determined within ChPT.

When combined with lattice calculations, these constants can be determined.

It is possible to account for lattice artifacts in the ChPT by introducing the appropriate symmetry breaking terms in the chiral lagrangian (finite lattice spacing) or restricting the Feynman integrals to finite volume.

Chiral Extrapolation



(MILC, hep-lat/0407028)

Heavy quarks on the lattice

The lattice cut-off is smaller than the heavy quark masses for realistic lattices.
The solution(s): heavy quark effective theory(HQET) or nonrelativistic QCD

Fermilab Method:

Continuum QCD $\longrightarrow$ Lattice gauge theory
(using HQET)

nonrelativistic QCD method:

Continuum QCD $\longrightarrow$ Nonrelativistic QCD $\longrightarrow$ Lattice gauge theory

- Both methods require tuning parameters of the lattice action
- The currents and 4-quark operators must be matched as well. Typically this is done in lattice perturbation theory.

Other Approaches

The extrapolation method (Becirevic, et al, hep-lat/0002025; QCDSF, hep-lat/0701015):

In this case one simulates at masses around the charm quark (or heavier) and extrapolates to bottom with fit functions motivated by HQET.

The step-scaling method (Guazzini, Sommer, and Tantalo, hep-lat/0609065):

One starts with a small volume where the b quark can be computed directly, where the finite size effects can be eliminated through step scaling functions which give the change of the observables when L is changed to $2L$.

Matching Errors

One must estimate errors due to inexact matching of the lattice to the continuum.

In the Fermilab method, all errors associated with discretizing the action are combined. These errors are then estimated using knowledge of HQET power counting.

In the nonrelativistic QCD method, there are "relativistic errors" associated with using NRQCD [$O(\alpha_s \Lambda_{QCD}/m_Q)$, $O(\Lambda_{QCD}^2/m_Q^2)$], and "perturbation theory errors" associated with matching NRQCD to the lattice [$O(\alpha_s^2)$].

Treatment of correlations

We don't have a complete correlation matrix between various lattice calculations. We combine lattice errors with the following assumptions:

- Whenever a source of error is at all correlated between two lattice calculations, we assign the degree-of-correlation a value of 100%.
- This assumption is conservative, and will lead to an overestimate in the total error of the averages.
- It still takes better advantage of the available results then assigning the smallest systematic error of any of the individual lattice calculations appearing in the average.

For example, statistical errors of results derived from the same ensemble of configurations are treated as 100% correlated. A perturbative matching calculation between common schemes (for B_K) is treated as 100% correlated, since this leads to the same renormalization factor.

Treatment of correlations

$$
\begin{aligned}
A \pm \delta A_1 \pm \delta A_2 \pm \delta A_3 &= A \pm \delta A \\
B \pm \delta B_1 \pm \delta B_2 \pm \delta B_3 &= B \pm \delta B \\
C \pm \delta C_1 \pm \delta C_2 \pm \delta C_3 &= C \pm \delta C
\end{aligned} \tag{3}
$$

Suppose $(\delta A_2, \delta B_2)$ and $(\delta B_3, \delta C_3)$ are 100% correlated and $\delta X^2 = \sum(\delta X_i)^2$. Then the covariance matrix C_{ij} is

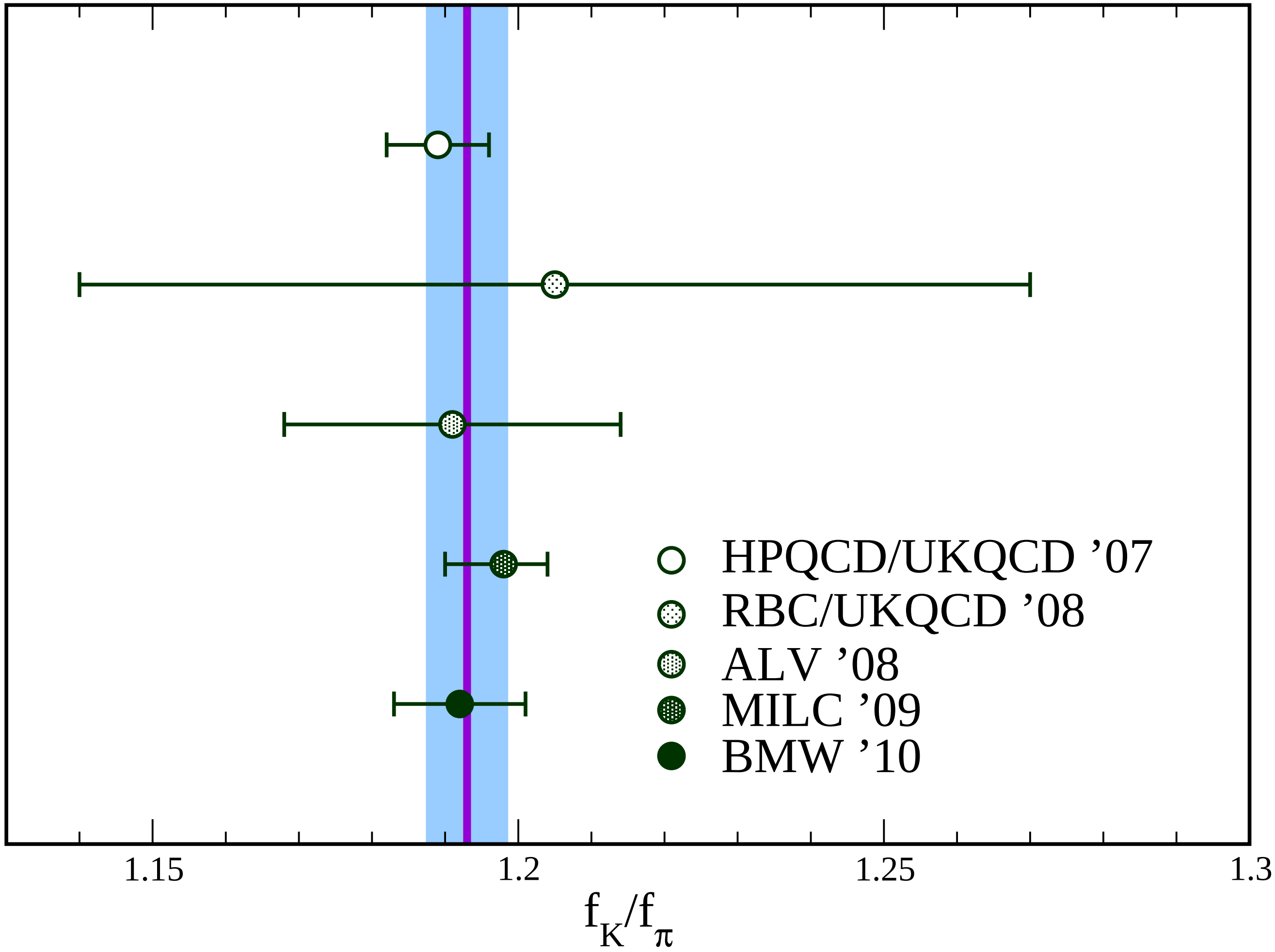
$$
\begin{pmatrix}
\delta A^2 & \delta A_2 \delta B_2 & 0 \\
\delta A_2 \delta B_2 & \delta B^2 & \delta B_3 \delta C_3 \\
0 & \delta B_3 \delta C_3 & \delta C^2
\end{pmatrix} \tag{4}
$$

Following Schmelling, Phys. Scripta 51, 676 (1995), we take

$$
x_{\rm avg} = \sum \omega_i x_i, \qquad \delta x_{\rm avg} = \sum \omega_i C_{ij} \omega_j, \tag{5}
$$

where the quantities being averaged are $x_i \pm \sigma_i$ and $\omega_i = \sigma_i^{-2} / \sum \sigma_j^{-2}$

An example: f_K/f_π



$f_K/f_\pi = 1.1925 \pm 0.0056$

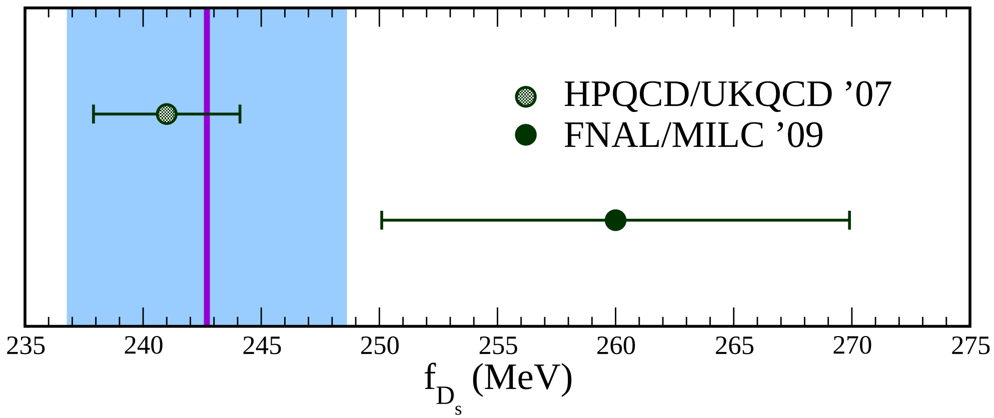
PDG prescription

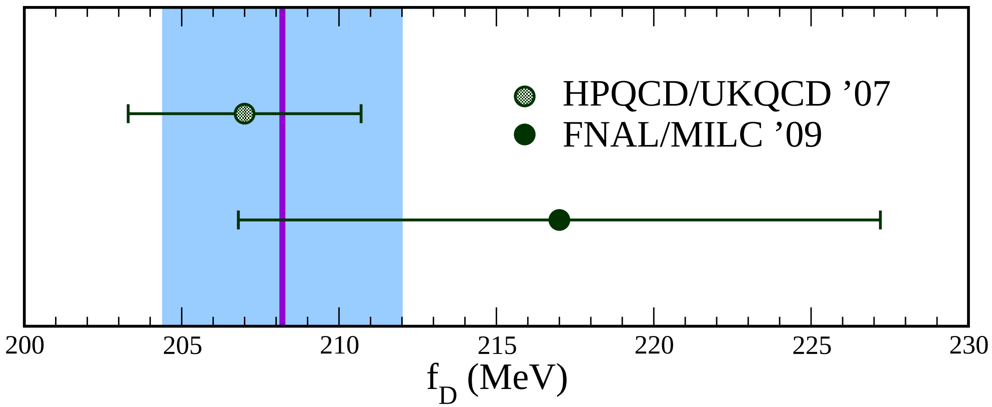
We adopt the PDG prescription to combine several measurements whose spread is wider than what is expected from the quoted errors.

The error on the average is rescaled by the square root of the minimum of the chi-square per degree of freedom:

$$\sqrt{\sum (x_i - x_{\mathrm{avg}})(C^{-1})_{ij}(x_j - x_{\mathrm{avg}})/(n-1)} \tag{6}$$

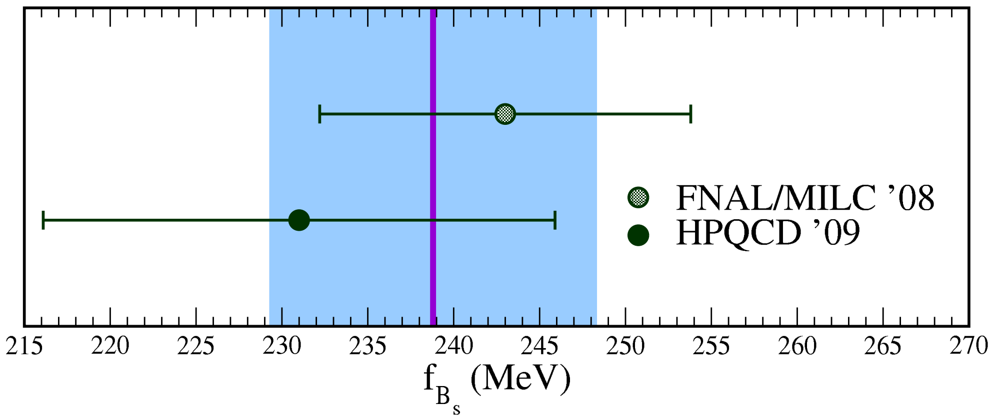
f_{D^+}, f_{D_s}

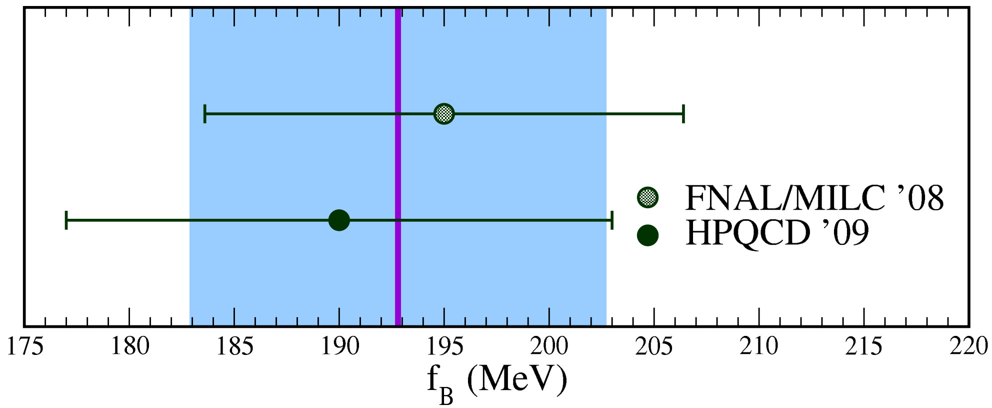




$f_{D_s} = 242.8 \pm 6.0$ MeV, $\quad f_D = 208.1 \pm 3.7$ MeV.

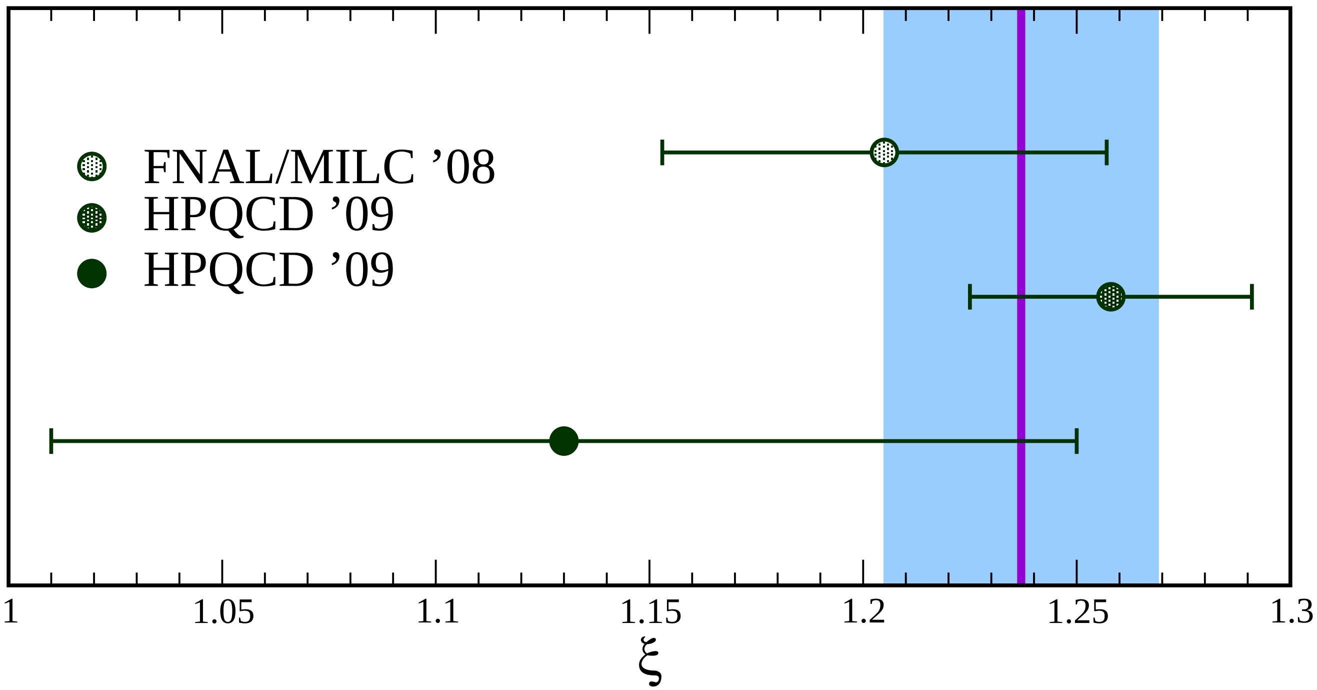
f_B, f_{B_s}





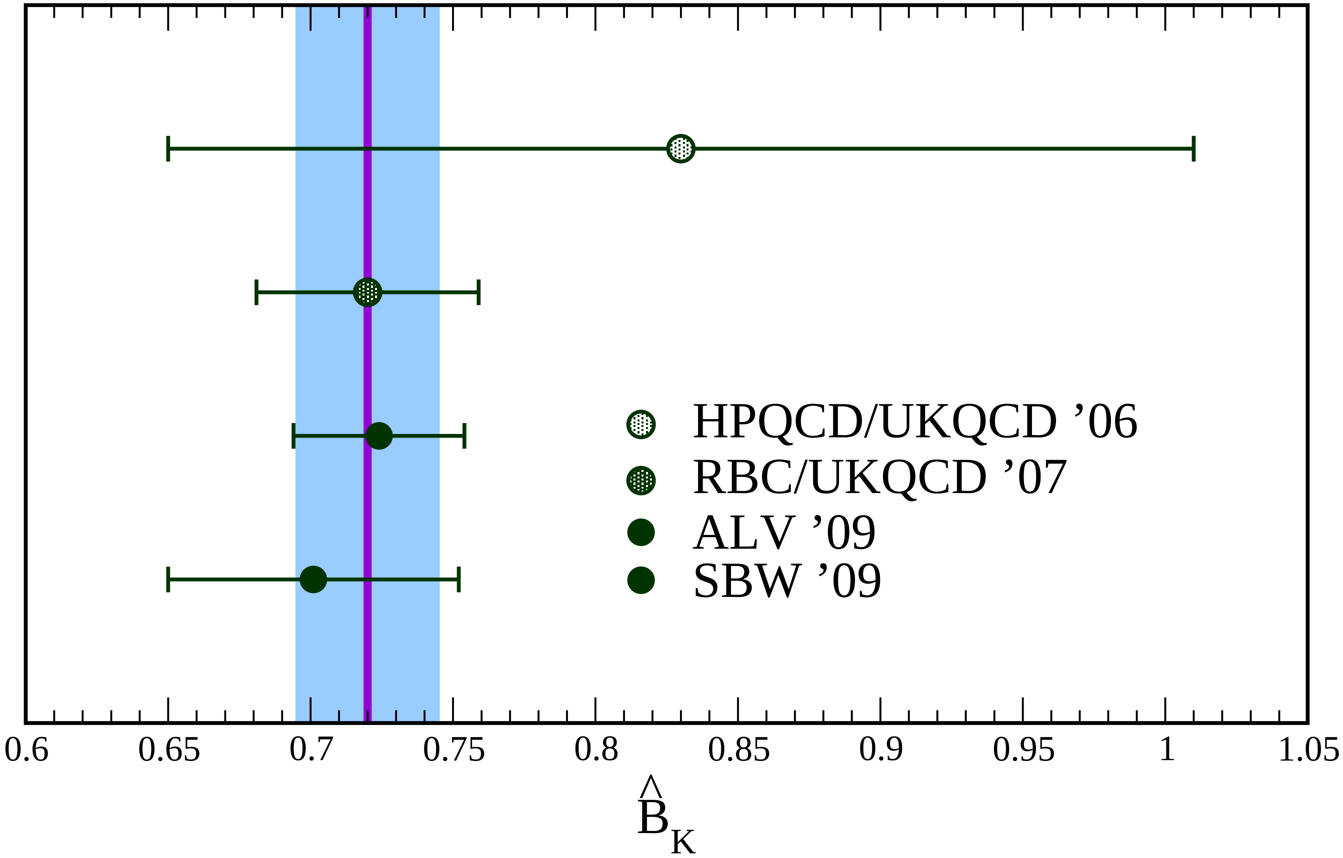
$f_{B_s} = 238.8 \pm 9.5,\quad f_B = 192.8 \pm 9.9$ MeV.

ξ



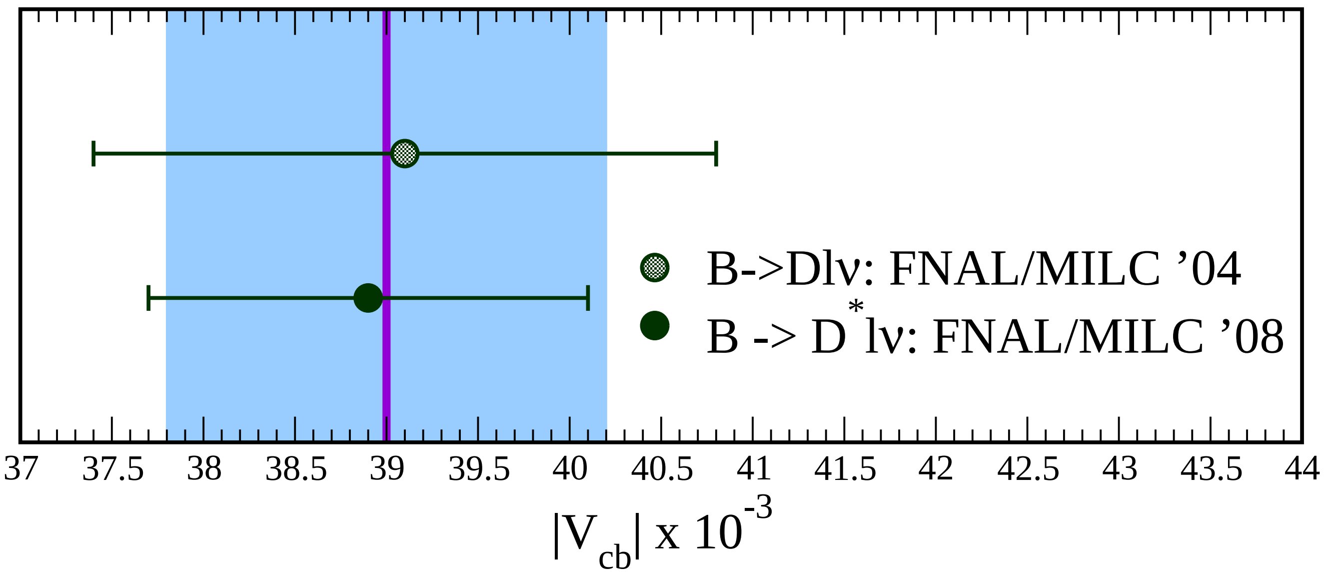
$$\xi = 1.237 \pm 0.032.$$

B_K



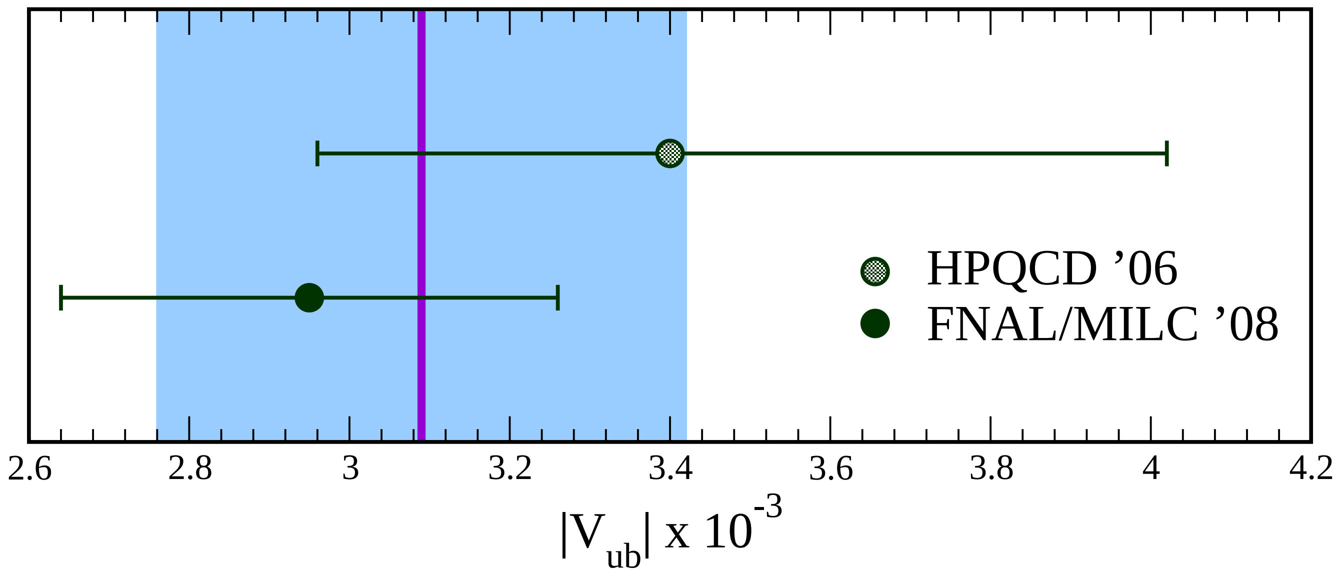
$$\widehat{B}_K = 0.720 \pm 0.025$$

$|V_{cb}|$



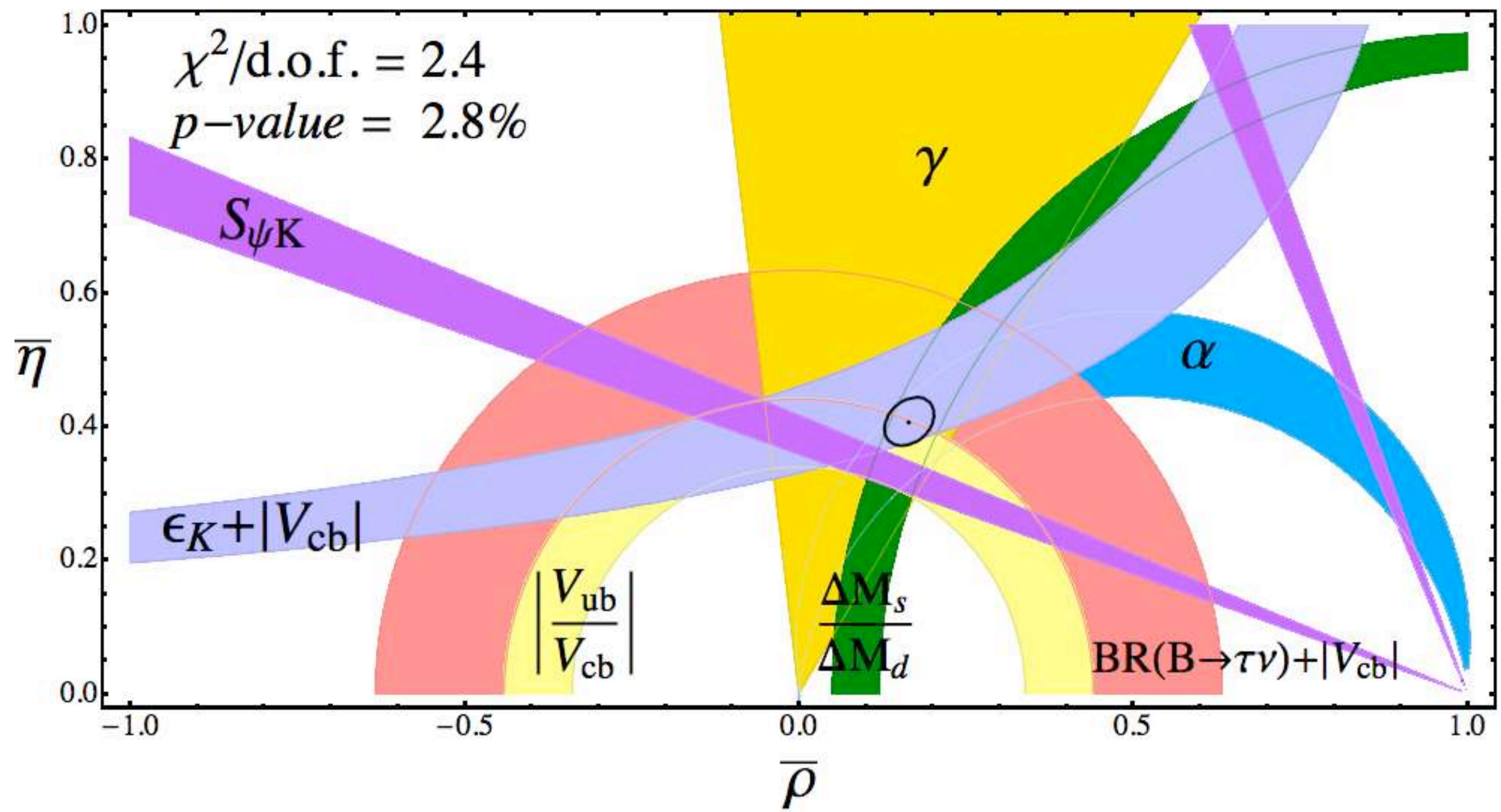
$$|V_{cb}| = (39.0 \pm 1.2) \times 10^{-3}$$

$|V_{ub}|$



$$|V_{ub}| = (3.09 \pm 0.33) \times 10^{-3}$$

Conclusion



Tension in fit to UT triangle. See Enrico's talk later in this session...

For latest averages, see www.latticeaverages.org