FPCP 2010, Torino

Neutrino Physics: a theoretical review

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Neutrinos have mass!

add a neutrino mass term to the SM:

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{3} \bar{\nu}_{iL} U^*_{\alpha i} \gamma_\rho \ell_{\alpha L} + \text{h.c.}$$

$$\mathcal{L}_{\rm M} = -\frac{1}{2} \sum_{i=1}^{3} \nu^T_{iL} C^{-1} \nu_{iL} m^\nu_i - \sum_{\alpha=e,\mu,\tau} \bar{\ell}_{\alpha R} \ell_{\alpha L} m^\ell_\alpha + \text{h.c.}$$

“effective” mass term (violates gauge invariance)

neutrino mass requires physics beyond the SM

need new fields (right-handed neutrinos, new scalar reps.,...)

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The Pontecorvo-Maki-Nakagawa-Sakata lepton mixing matrix:

$$(U_{\alpha i}) \equiv U_{\rm PMNS}$$

U_{PMNS}

conventional parameterization: $U_{\mathrm{PMNS}} = V^{\mathrm{Dirac}} D^{\mathrm{Maj}}$

$$V^{\mathrm{Dirac}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- three angles

$$\tan\theta_{23} = \frac{U_{\mu 3}}{U_{\tau 3}} \qquad \sin\theta_{13} = |U_{e3}| \qquad \tan\theta_{12} = \frac{U_{e2}}{U_{e1}}$$

$$U_{\mathrm{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

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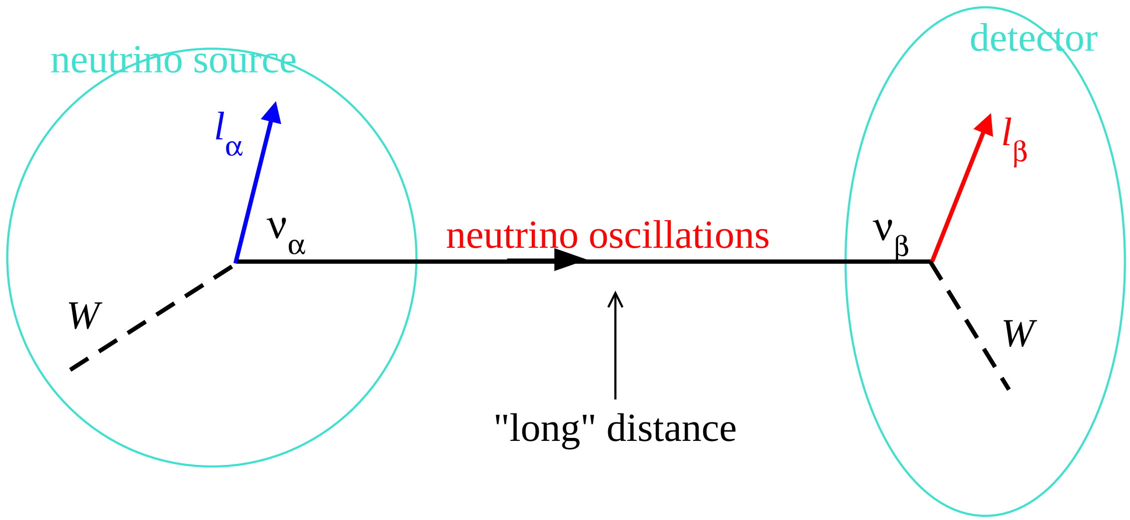
- three angles
- one Dirac phase $\delta \Rightarrow$ CP violation in oscillations
- and two physical Majorana phases in D^{Maj}
  (no effect in osc.)

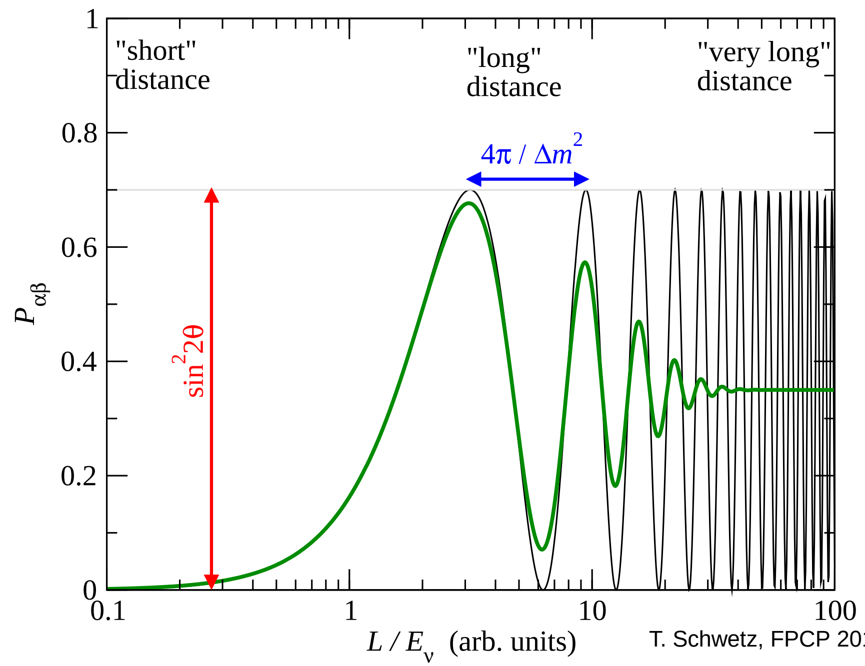
Neutrino oscillations

$|\nu_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle$ $\quad e^{-i(Et-p_i x)}$ $\quad |\nu_\beta\rangle = U^*_{\beta i}|\nu_i\rangle$



$P(\nu_\alpha \to \nu_\beta)$

(two-flavour limit)



3-flavour oscillation parameters

Δm^2_{31} Δm^2_{21}

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric+LBL Chooz solar+KamLAND

3-flavour effects are suppressed because

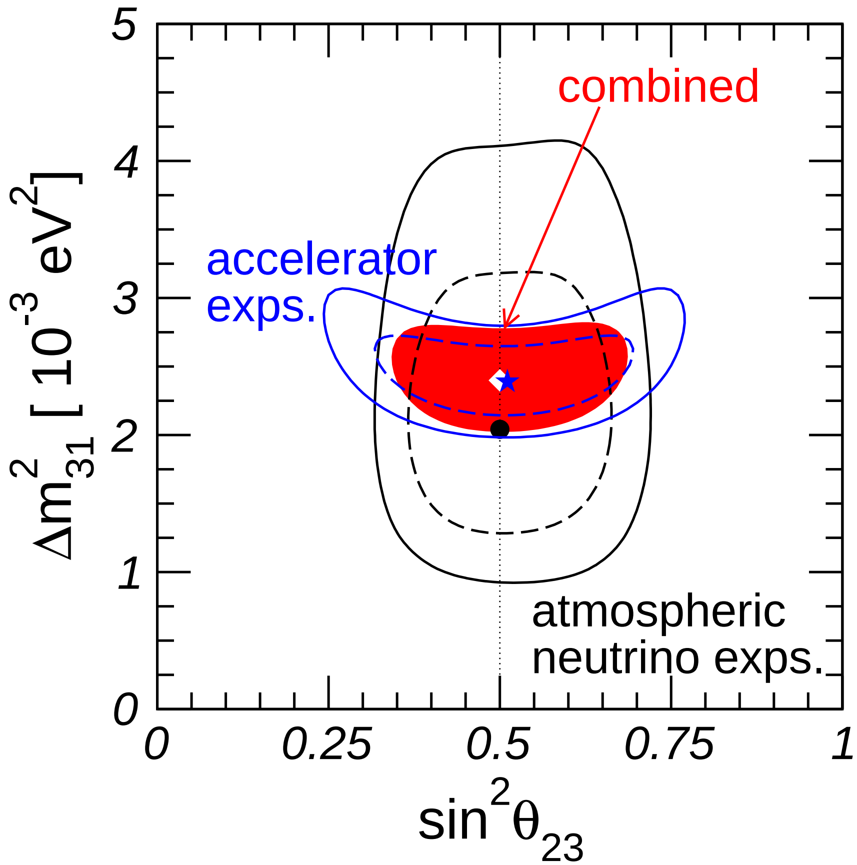
$\Delta m^2_{21} \ll |\Delta m^2_{31}|$ and $\theta_{13} \ll 1$

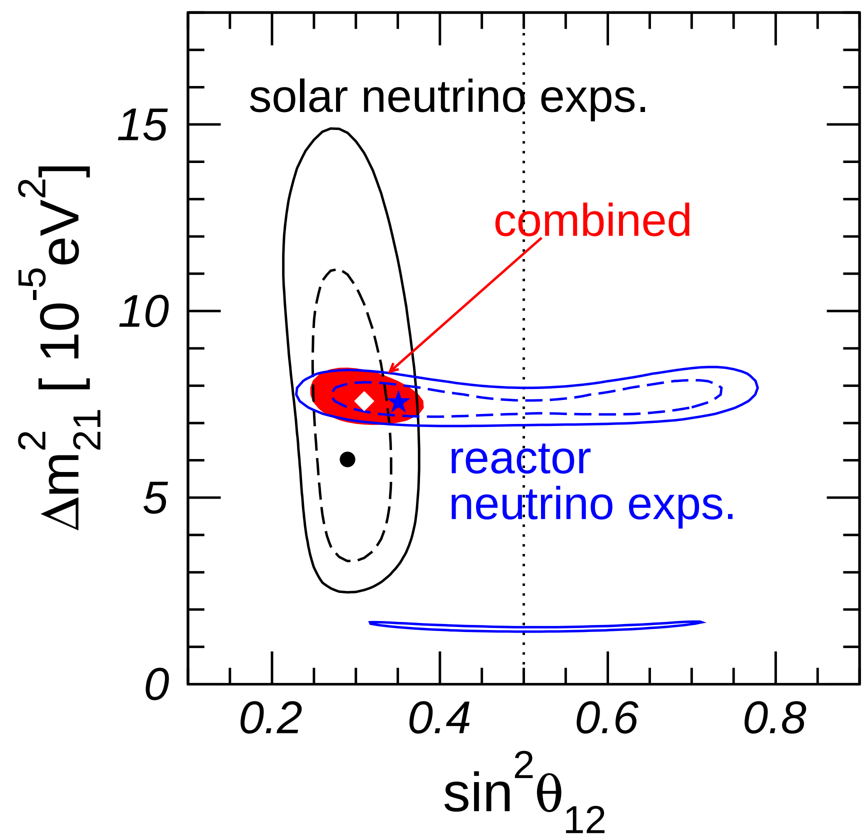
$\Rightarrow$ dominant oscillations are well described by effective two-flavour oscillations

Leading oscillation modes

$$|\Delta m^2_{31}| = (2.4 \pm 0.12)\, 10^{-3}\, \mathrm{eV}^2 \qquad \Delta m^2_{21} (7.6 \pm 0.2)\, 10^{-5}\, \mathrm{eV}^2$$

$$\sin^2\theta_{23} = 0.50 \pm 0.07 \qquad \sin^2\theta_{12} = 0.32 \pm 0.019$$

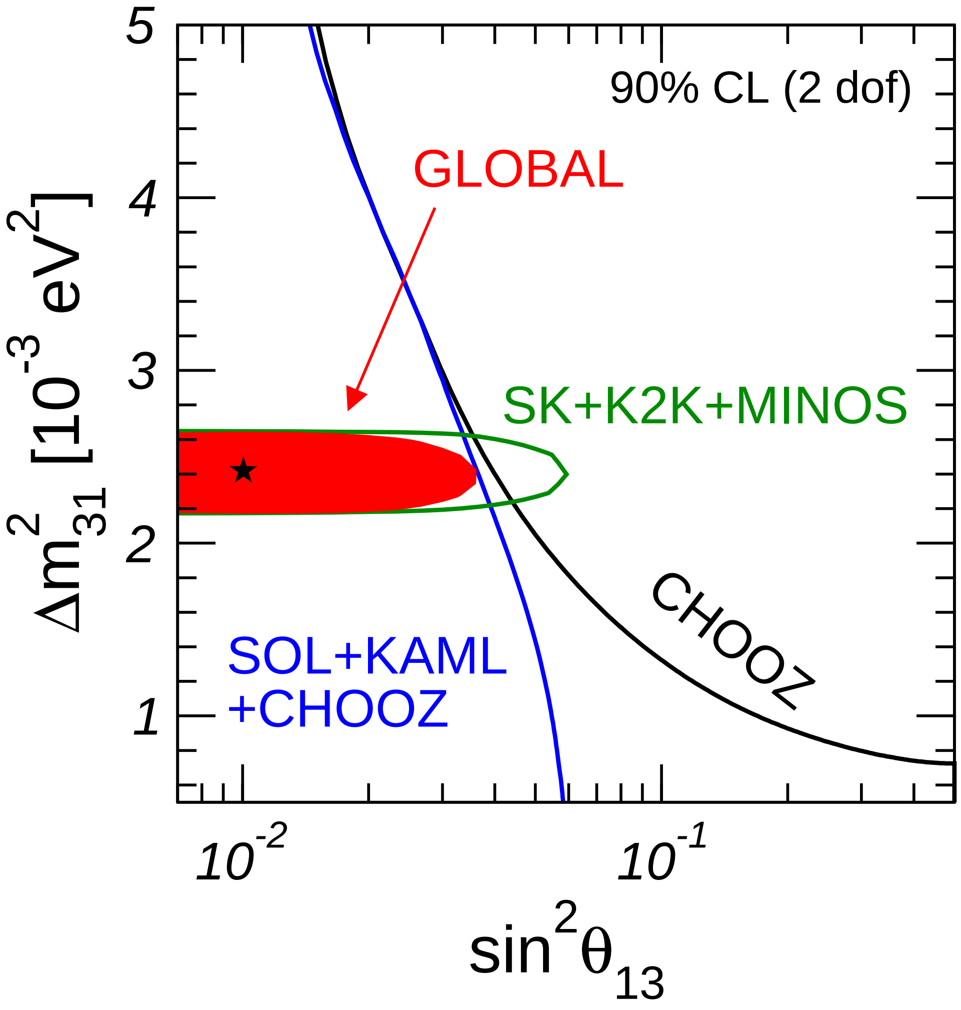




TS, Tortola, Valle, 0808.2016 (arXiv version updated)

The status of θ_{13}

global data: $\sin^2\theta_{13} < 0.031\,(0.047)$ at 90% CL (3σ)



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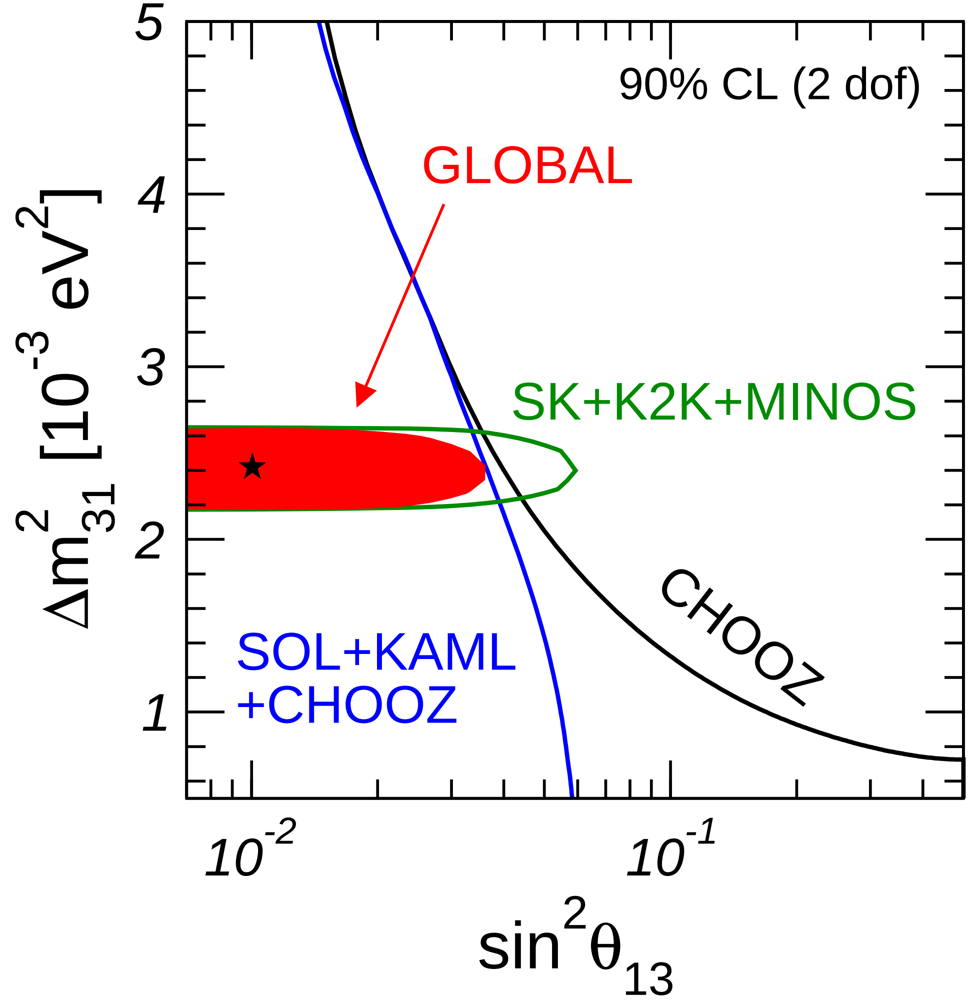

$$\sin\theta_{13} = |U_{e3}| < 0.217\,(3\sigma) \quad \leftrightarrow \quad |V_{us}| = 0.2257 \pm 0.0021$$

Hint for non-zero θ_{13}?

Fogli, Lisi, Marrone, Palazzo, Rotunno
"Hints of theta(13) > 0 from global neutrino data analysis." PRL08 [0806.2649]

- Hint from solar+KamLAND data ($\sim 1.5\sigma$)
  fragile, but agreement among groups
  depends somewhat on assumptions on solar metalicity

- Hint from atmospheric data
  controversial, not confirmed by SuperK Wendell et al., 1002.3471

- MINOS appearance data ($\nu_\mu \to \nu_e$)
  initial $\sim 1.5\sigma$ hint has recently decreased to $\sim 0.7\sigma$

Hint for non-zero θ_{13}?

reference	best-fit and 1σ errors	significance
Fogli et al. [84]	$\sin^2\theta_{13} = 0.02 \pm 0.01$	2σ
Gonzalez-Garcia et al. [18] (GS98)	$\sin^2\theta_{13} = 0.0095^{+0.013}_{-0.007}$	1.3σ
Gonzalez-Garcia et al. [18] (AGSS09)	$\sin^2\theta_{13} = 0.008^{+0.012}_{-0.007}$	1.1σ
Schwetz et al. [14] (GS98)	$\sin^2\theta_{13} = 0.013^{+0.013}_{-0.010}$	1.5σ
Schwetz et al. [14] (AGSS09)	$\sin^2\theta_{13} = 0.010^{+0.013}_{-0.008}$	1.3σ

[84] Fogli, Lisi, Marrone, Palazzo, Rotunno, arxiv:0905.3549 (MINOS 2010 not yet incl.)

[18] Gonzalez-Garcia, Maltoni, Salvado, 1001.4524

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[14] TS, Tortola, Valle, 0808.2016 (updated 2010)

good prospect for upcoming reactor (Double-Chooz, RENO, Daya Bay) and accelerator (T2K, NOνA) experiments

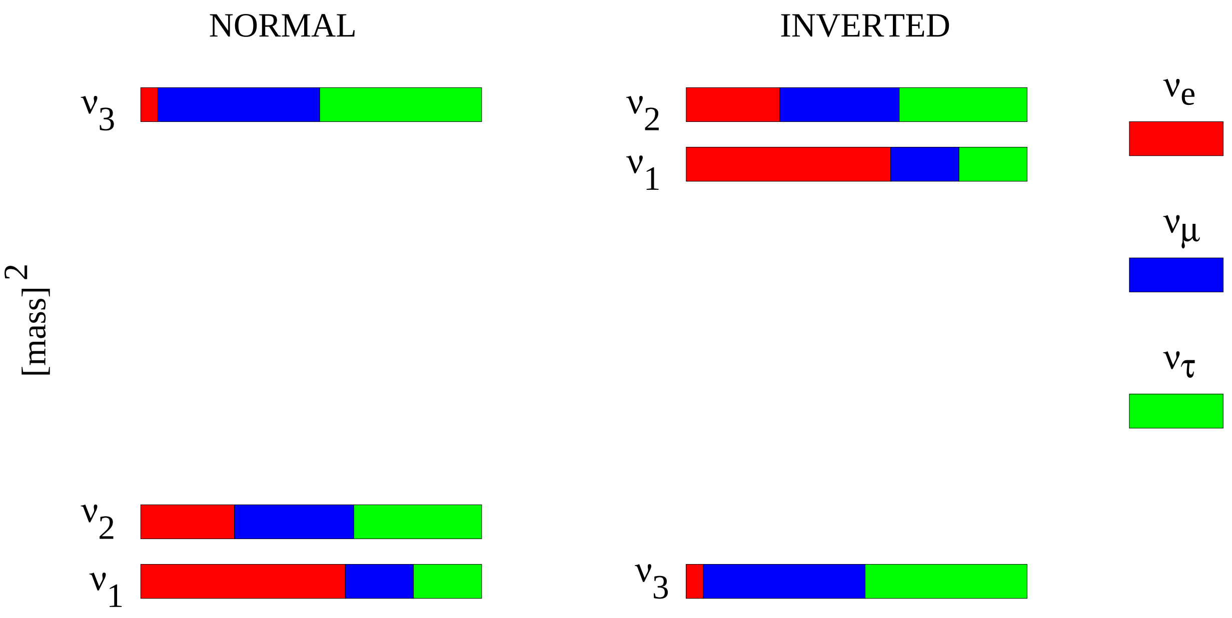
see talk by M. Mezzetto

3-flavour neutrino parameters

	bf $\pm 1\sigma$	acc. @ 3σ	
Δm^2_{21}	$(7.6 \pm 0.2)\, 10^{-5}\, \mathrm{eV}^2$	(8%)	KamLAND
$\lvert\Delta m^2_{31}\rvert$	$(2.4 \pm 0.12)\, 10^{-3}\, \mathrm{eV}^2$	(14%)	MINOS
$m_0 \lesssim 0.5\,\mathrm{eV}$			^{3}H, cosmo
$\sin^2\theta_{12}$	0.32 ± 0.019	(17%)	SNO
$\sin^2\theta_{23}$	0.50 ± 0.07	(30%)	SK atm
$\sin^2\theta_{13} < 0.047$ @ 3σ			CHOOZ
$0 \leq \delta < 2\pi$, $0 \leq \alpha, \beta < \pi$			

TS, Tortola, Valle
0808.2016v3
(updated 2010)

two possible mass spectra:



to-do list

oscillation experiments:

- search for CP violation
  compare oscillation probabilities for neutrinos and anti-neutrinos
- determine neutrino mass ordering sgn(Δm^2_{31})
  need to see the matter effect in Δm^2_{31} oscillations

under intense study (EUROν, NF-IDS), see talk by M. Mezzetto

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under intense study (EUROν, NF-IDS), see talk by M. Mezzetto

non-oscillation experiments:

- find out the absolute neutrino mass scale
- are neutrinos Dirac or Majorana particles?

What is the absolute neutrino mass?

non-oscillation experiments:

- neutrino-less double beta-decay

  several-σ claim $0.16\,\text{eV} \lesssim m_{\beta\beta} \lesssim 0.52\,\text{eV}$

  GERDA, Majorana, Cuoricino, NEMO3,...

  sensitivities $m_{\beta\beta} \sim 0.05\text{eV}$

  talk by F. Bellini

- Tritum beta-decay

  current bound $m_\beta \lesssim 2$ eV, KATRIN sens.: $m_\beta \sim 0.2\text{eV}$

- Cosmology (galaxy surveys, CMB), current bound:

$$\sum_i m_i \lesssim 0.5\text{eV}$$

Neutrinoless double beta decay

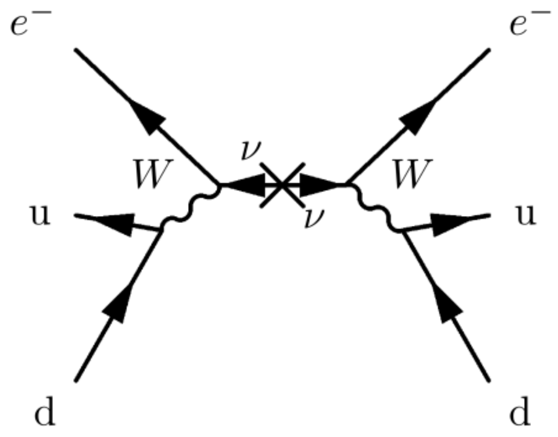
$$(A, Z) \longrightarrow (A, Z+2) + 2e^-$$

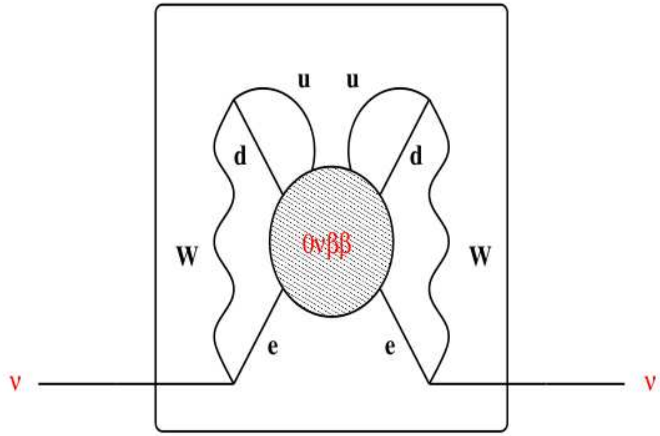
rate proportional to $|\sum U_{ei}^2 m_i|$

obs. of neutrinoless DBD implies violation of Lepton number

proves Majorana nature of neutrinos

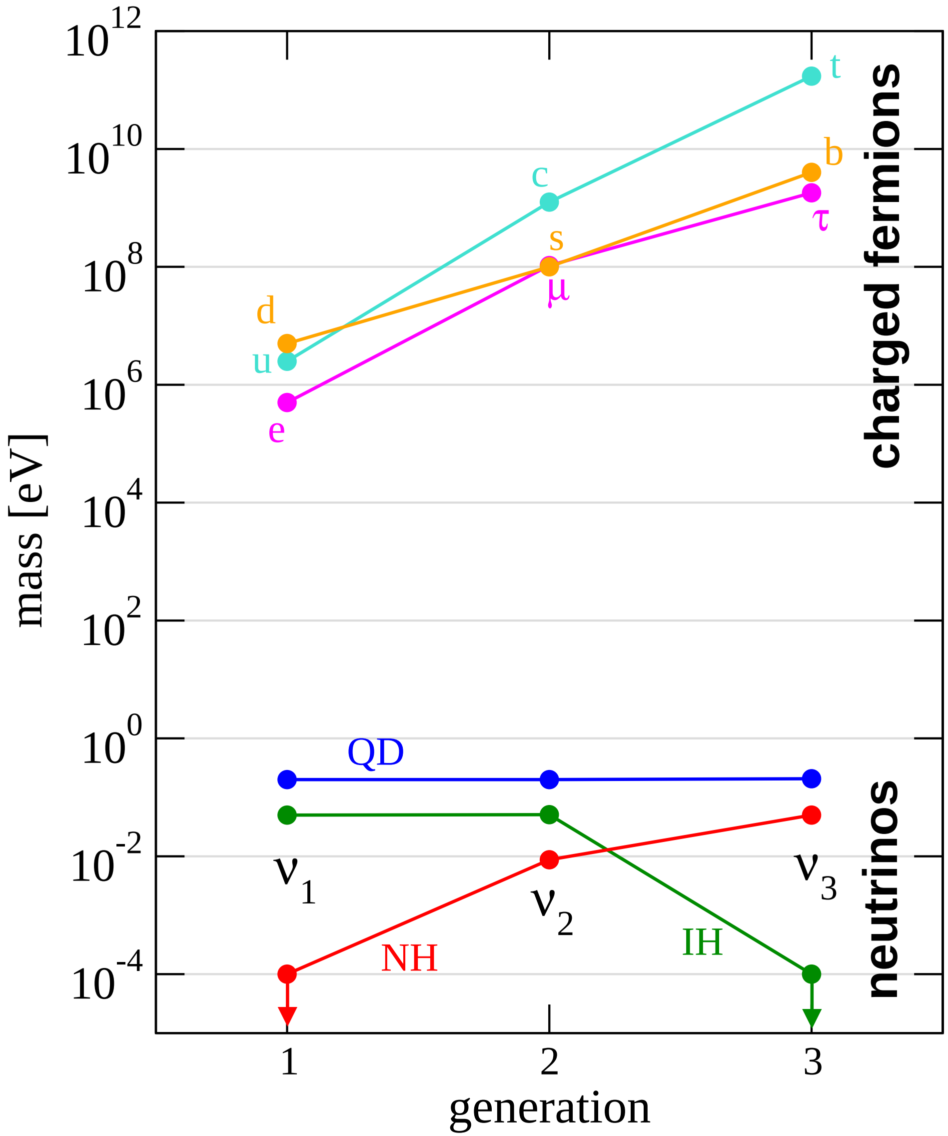
Schechter, Valle, 1982; Takasugi, 1984





Neutrinos and beyond SM physics

Why are neutrino masses so small?



at least

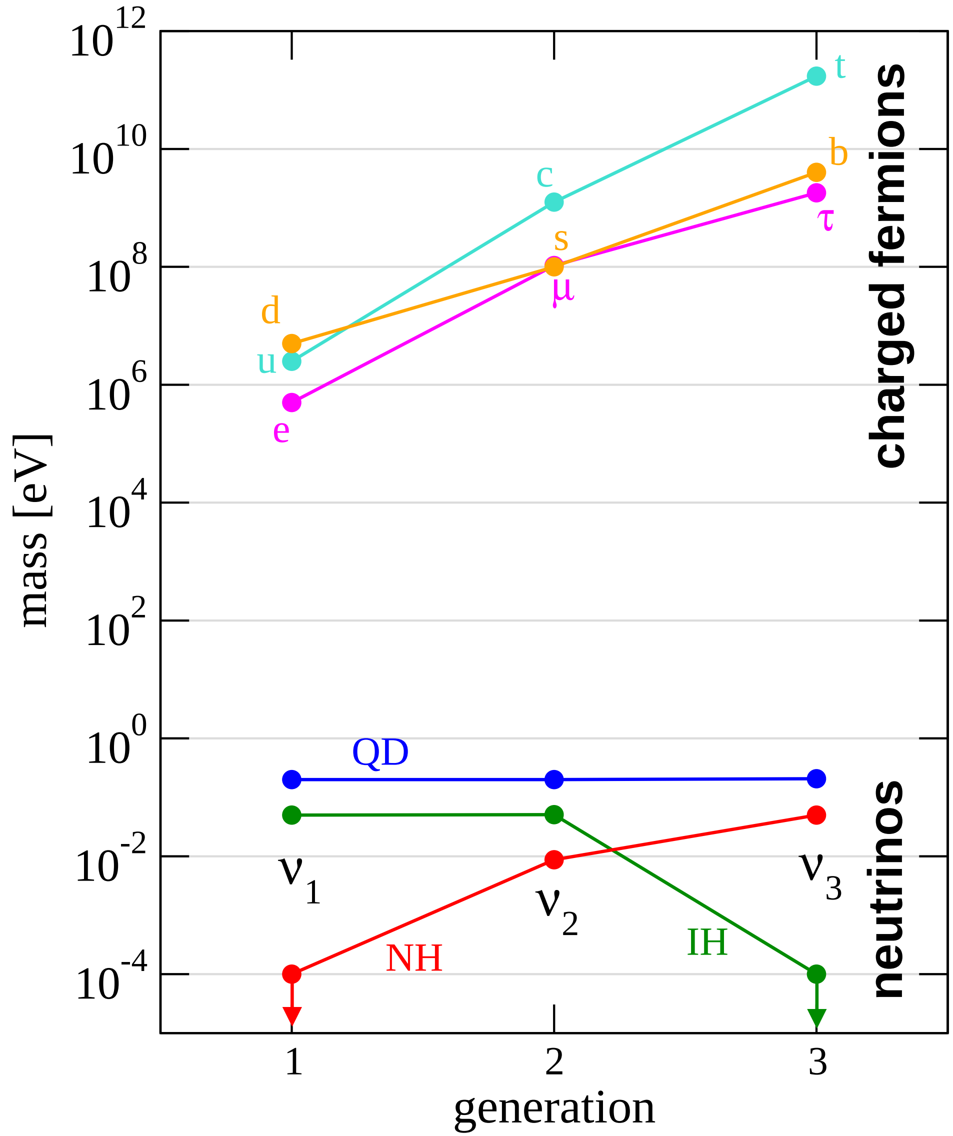
6 orders of magnitude

hierarchy

within one generation!

"ν_1" := state with dominant mixing with electron

Why are neutrino masses so small?



Is the smallness of m_ν related to a high scale Λ (GUT scale?) via the seesaw mechanism?

or

is new physics at the TeV scale responsible for neutrino mass generation?

Seesaw

Weinberg 1979: there is one dim-5 operator in the SM, which will lead to a Majorana mass term for neutrinos after EWSB:

$$Y^2 \frac{L^T \tilde{\phi}^* \, \tilde{\phi}^\dagger L}{\Lambda} \quad \longrightarrow \quad m_\nu \sim Y^2 \frac{v^2}{\Lambda}$$

for $v \approx 246 \text{ GeV}$ and $Y \sim 1$ this implies that the physics responsible for neutrino masses lives at the very high scale

$$\Lambda \sim 10^{14} \, \text{GeV} \, .$$

- impossible to probe at any imaginable collider experiment
- somewhat close to the GUT scale $\Lambda \simeq 10^{16} \text{ GeV}$

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3 tree-level realizations of the Weinberg operator:

- Type I: fermionic singlet (right-handed neutrinos)
- Type II: scalar triplet
- Type III: fermionic triplet

Neutrino masses from GUT physics?

- SM fermions of one generation + RH neutrino:

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, u_R, u_R, d_R, d_R, d_R, \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}, \ell_R, N_R$$

fit into 16-dim. spinorial representation of SO(10).

- SO(10) GUTs usually include Higgs representations containing SU(2) triplets

$\Rightarrow$ SO(10) provides a very natural framework for type-I and/or type-II seesaw

An SO(10) example

fermion masses from 10 and $\overline{126}$ Higgses: Aulakh, Mohapatra, 83

$$
\begin{aligned}
M_d &= v_d^{10} Y_{10} + v_d^{126} Y_{126} \\
M_u &= v_u^{10} Y_{10} + v_u^{126} Y_{126} \\
M_\ell &= v_d^{10} Y_{10} - 3 v_d^{126} Y_{126} \\
M_D &= v_u^{10} Y_{10} - 3 v_u^{126} Y_{126} \\
M_L &= v_L Y_{126} \\
M_R &= v_R Y_{126}
\end{aligned}
$$

light neutrino masses:

$$M_\nu = M_L - M_D M_R^{-1} M_D$$

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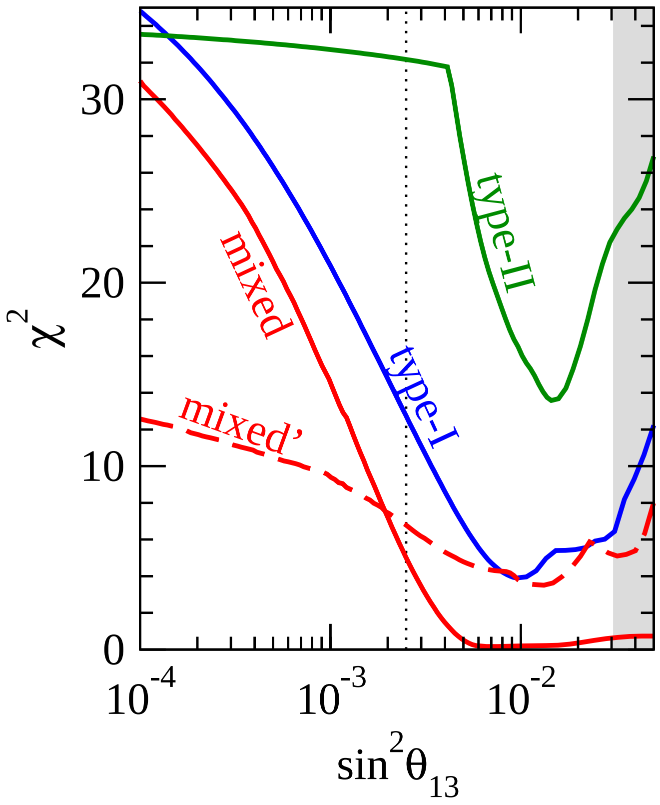


Bertolini, Schwetz, Malinsky, 06

light neutrino masses:

$$M_\nu = M_L - M_D M_R^{-1} M_D$$

very constrained ⇒ definite predictions

An SO(10) example

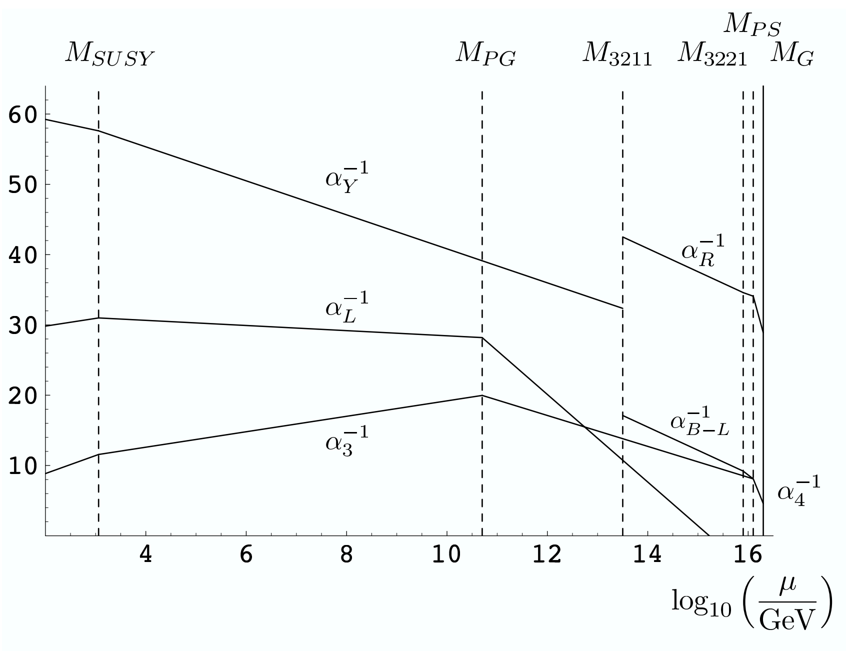
“minimal” renormalizable SUSY SO(10) GUT:

Matter: 16_m, Higgses: $10_H, \overline{126}_H, 126_H, 210_H$

Aulakh, Bajc, Bertonlini, Fukuyama, Malinski, Melfo, Mohapatra, Okada, Senjanovic, Vissani, . . .

Higgs potential + Yukawa $\Rightarrow$ very constrained (26 parameters)

$\Rightarrow$ failure of gauge coupling unification due to m_ν-scale



pseudo-Goldstone bosons at

$$M_{PG} \sim \frac{M_{\mathrm{seesaw}}^2}{M_{\mathrm{GUT}}} \sim 10^{10}\,\mathrm{GeV}$$

due to the breaking of accidental global symmetries

Bertolini, Schwetz, Malinsky, 06

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one possible alternative: e.g., Albright, Barr

give up renormalizability and allow for non-ren. operators

- get rid of large representations like 126 or 210
- more freedom for fermion masses
- lose predictivity and conceptional beauty

Neutrino masses from the TeV scale?

Maybe the BSM physics expected around TeV can also be responsible for neutrino masses:

$\Rightarrow$ TeV scale neutrino mass models

Generically in such models seesaw suppression is not sufficient, one needs additional means to obtain small neutrino masses:

- putting small numbers by hand
- cancellations between large terms
- radiative neutrino masses
- ...

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at LHC $\Rightarrow$ dilepton (or multi-lepton) events, e.g.:

lepton number violating: $\ell^\pm \ell^\pm$ + jets or

lepton flavour violating: $\ell_\alpha^\pm \ell_\beta^\mp$ + jets

Type I seesaw at LHC

$$m^\nu_{\alpha\beta} = v^2 \sum_i \frac{Y_{\alpha i} Y_{\beta i}}{M_i}$$

If heavy neutrino masses M_i are not so heavy there are two possibilities to obtain small neutrino masses:

1. small Yukawas $Y_{\alpha i} \sim 10^{-6}$ (electron Yukawa)
2. cancellations in the sum over N_i

   Buchmüller, Wyler, 1990; Pilaftsis, 1992

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add 1: since N_i are SM singlets they interact only via Yukawas $\Rightarrow$ tiny Yukawas imply negligible production rate at LHC.

Type I seesaw at LHC

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add 2: cancellations could be motivated by symmetries, but decouple LHC signatures from light neutrino mass matrix Kersten, Smirnov, 07

Seesaw at LHC with tiny Yukawas

Way out: give N_i some interaction for prod. @ LHC, e.g.:

- Low scale Left-Right symmetry
  $q\bar{q} \to W_R \to N\ell \to \ell\ell$ + jets e.g., Keung, Senjanovic, 1983
- Type II seesaw: $N \to \Delta$ scalar triplet
  $q\bar{q} \to Z^0(\gamma) \to \Delta^{--}\Delta^{++} \to \ell^-\ell^-\ell^+\ell^+$
  Hektor et al., 07; Han, Mukhopadhyaya, Si, Wang, 07; Garayoa, Schwetz, 07;
  Akeroyd, Aoki, Sugiyama, 07; Kadastik, Raidal, Rebane, 07; Fileviez Perez et al., 08
  Br$(\Delta \to \ell_\alpha \ell_\beta) \propto (m_\nu)_{\alpha\beta}$
- Type III seesaw: $N \to T$ fermionic triplet
  $q\bar{q} \to W^- \to T^- T^0 \to \ell^-\ell^-$ + jets
  Foot et al., 89; Ma, 98; Bajc, Senjanovic, 07; Franceschini, Hambye, Strumia, 08; . . .

R-parity violating SUSY

Allow for "tiny" R-parity violation
(be sure of not violating proton decay bounds!)

$\Rightarrow$ neutrino mass generation is related to lepton number violating terms in superpotential

can study neutrino properties by observing R-parity violating decays of the LSP (neutralino) at LHC

e.g.: Diaz, Dedes, Eboli, Hirsch, Porod, Restrepo, Romao, Senjanovic, Valle, . . .

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LSP is not stable

- neutralino no viable DM candidate
- gravitino DM (decay suppressed by R-parity and M_{Pl})

  Takayama, Yamaguchi, 00; Buchmuller, Covi, Hamaguchi, Ibarra, Yanagida, 07

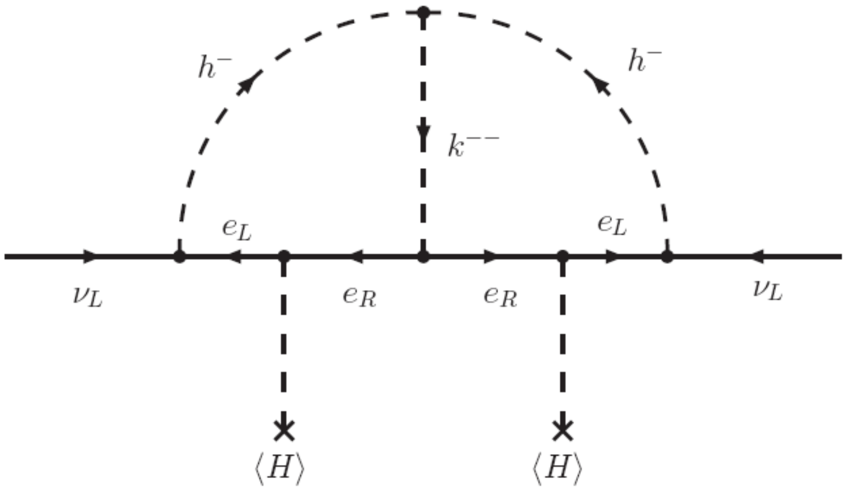
Radiative neutrino mass generation

Example: Zee-Babu model Zee, 85, 86; Babu 88

add two SU(2) singlet scalars: h^+, k^{++}

$$\mathcal{L}_\nu = f_{\alpha\beta} L_\alpha^T C i\sigma_2 L_\beta h^+ + g_{\alpha\beta} \overline{e^c_{R\alpha}} e_{R\beta} k^{++} + \mu h^- h^- k^{++} + \text{h.c.}$$

$$m_\nu \approx \frac{\mu}{48\pi^2 m_k^2} f m_\ell g^* m_\ell f^T$$



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$$m_\nu \approx \frac{\mu}{48\pi^2 m_k^2} f m_\ell g^* m_\ell f^T$$

good prospects to see doubly-charged scalar at LHC $\rightarrow$ like-sign lepton events; BR$(k^{\pm\pm} \rightarrow \ell_\alpha^\pm \ell_\beta^\pm) \propto |g_{\alpha\beta}|^2$

if k^{++} is within reach for LHC the model is tightly constrained by perturbativity requirements and bounds from LFV

Babu, Macesanu, 02; Aristizabal, Hirsch, 06; Nebot et al., 07, Ohlsson, TS, Zhang, 09

Lepton mixing and the problem of flavour

Why is lepton mixing large?

Lepton mixing:

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \epsilon \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

Quark mixing:

$$U_{CKM} = \begin{pmatrix} 1 & \epsilon & {\scriptstyle \epsilon} \\ \epsilon & 1 & {\scriptstyle \epsilon} \\ {\scriptstyle \epsilon} & {\scriptstyle \epsilon} & 1 \end{pmatrix}$$

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consistent with random (“anarchical”) neutrino mass matrix

in this case θ_{13} has to be close to the bound and θ_{23} should not be very close to 45°

e.g., deGouvea, Murayama, hep-ph/0301050

Is there a special pattern in lepton mixing?

example: Tri-bimaximal mixing

Harrison, Perkins, Scott, PLB 2002, hep-ph/0202074

$$\sin^2\theta_{12} = 1/3\,, \quad \sin^2\theta_{23} = 1/2\,, \quad \sin^2\theta_{13} = 0 \qquad \Rightarrow$$

$$U_{\rm TMB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

TBM mass matrix

TBM implies a special pattern for the neutrino mass matrix

$$
\begin{aligned}
m_{\mathrm{TBM}} &= U_{\mathrm{TBM}}\,\mathrm{diag}(m_1, m_2, m_3)\,U_{\mathrm{TBM}}^T \\
&= \begin{pmatrix} a & b & b \\ \% & \frac{1}{2}(a+b+c) & \frac{1}{2}(a+b-c) \\ \% & \% & \frac{1}{2}(a+b+c) \end{pmatrix}
\end{aligned}
$$

with

$$
a = \frac{1}{3}(3m_1 + m_2) \qquad b = \frac{1}{3}(-m_1 + m_2) \qquad c = m_3
$$

Flavor symmetry?

maybe the special values of the mixing angles indicate a flavour symmetry (μ-τ exchange sym., $S_2, S_3, A_4, D_4, D_5, D_8, \Delta(27), ...$)

see recent review Altarelli, Feruglio, 1002.0211

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example A_4:

- symmetry group of the regular tetrahedron
- representations $1, 1', 1'', 3$
- smallest group with 3-dimensional irrep.
- $m_{\rm TBM}$ is invariant under one of the generators

A_4 models

Table 1: Particle assignments of A_4 models in the literature. Lepton doublets, charged lepton singlets and right-handed neutrinos are denoted by L_i, ℓ_i^c and ν_i^c, respectively. Δ denotes the Higgs triplets in the type II seesaw mechanism. Models that also study the quark sector have the superscript #, those that embed A_4 into a GUT group have the superscript *.

Type	L_i	ℓ_i^c	ν_i^c	Δ	References
A1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	-	[11–20] [21]#
A2				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[22, 23]
B1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	-	[14, 24–27]# [28, 29]* [30–41]
B2				$\underline{1}, \underline{3}$	[42]#
C1	$\underline{3}$	$\underline{3}$	-	-	[12]
C2				$\underline{1}$	[43, 44] [45]#
C3				$\underline{1}, \underline{3}$	[46]
C4				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[47]
D1	$\underline{3}$	$\underline{3}$	$\underline{3}$	-	[48, 49]* [50, 51]
D2				$\underline{1}$	[52] [53]*
D3				$\underline{1}'$	[54]*
D4				$\underline{1}', \underline{3}$	[55]*
E	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	[56, 57]
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$	[58]
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$	-	[59]
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	-	-	[60]
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$	-	[61]*
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$	-	[62, 63]

Barry, Rodejohann, 1003.2385

Flavor symmetries - models

some general (maybe personal) remarks

- full models get often quite invovled
- “strong” breaking of the symmetry required (charged leptons)
- need many fields with complicated charge assignments
- VEV alignment
- difficult to extend flavour symmetry to quarks (very few examples)

Altarelli, Feruglio, 1002.0211

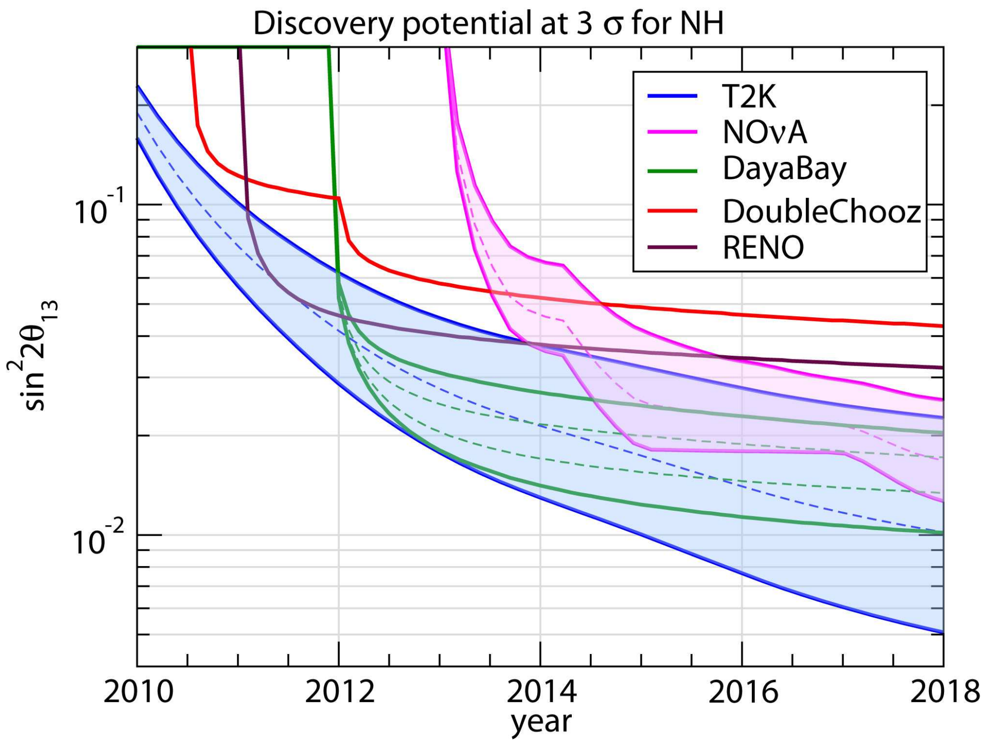
Conclusion

Maybe the surprising properties of neutrinos tell us something important about the physics beyond the Standard Model and/or about the origin of flavour.

Thank you for your attention!

Additional slides

Prospects for θ_{13}



Mezzetto, TS, "θ_{13}: phenomenology, status and prospect", 1003.5800;
Huber, Lindner, TS, Winter, 0907.1896

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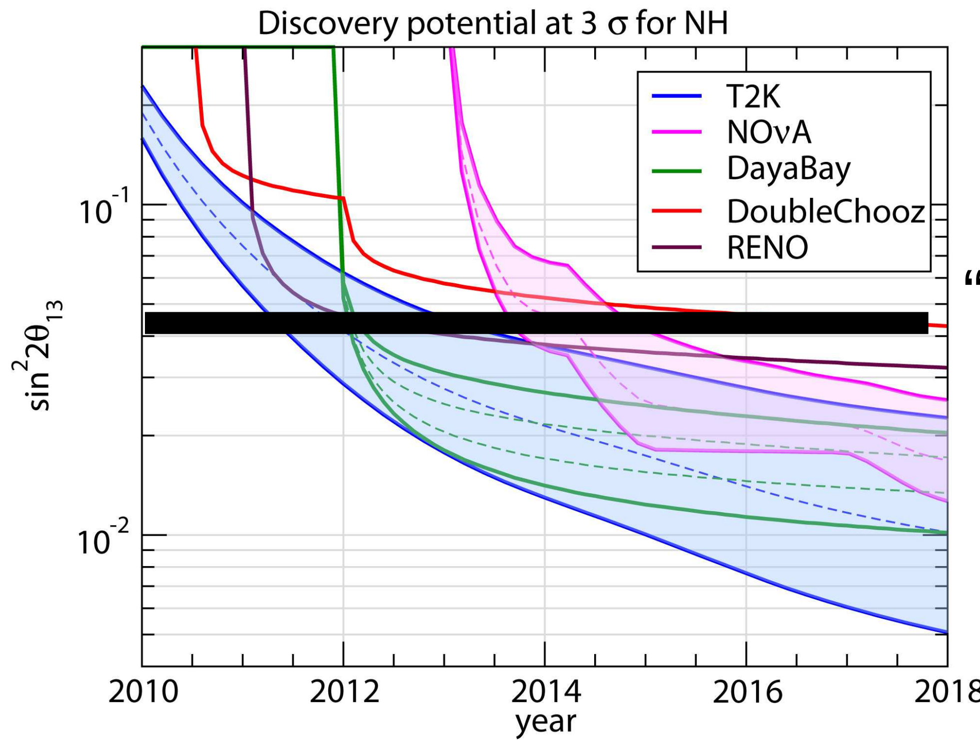

"hint"

$\sin^2\theta_{13} = 0.01$

Mezzetto, TS, "θ_{13}: phenomenology, status and prospect", 1003.5800;
Huber, Lindner, TS, Winter, 0907.1896

CP violation - in theory

neutrino oscillations in vacuum:

$$P_{\nu_\alpha \to \nu_\beta} = \sum_{jk} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \, e^{-i\Delta m^2_{jk} L/2E}$$

$$P_{\bar\nu_\alpha \to \bar\nu_\beta} = \sum_{jk} U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \, e^{-i\Delta m^2_{jk} L/2E}$$

“weak phase”: Arg$(U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k})$

“strong phase”: $\Delta m^2_{jk} L/2E$

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$$\Delta P^{CP} \equiv P_{\nu_\alpha \to \nu_\beta} - P_{\bar\nu_\alpha \to \bar\nu_\beta} \propto \mathrm{Im}(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k}) \propto \sin\delta \sin\theta_{13}$$

$\Rightarrow$ need $\alpha \neq \beta$, $i \neq k$, $\sin\theta_{13} > 0$

only the appearance channel shows explicit CPV

CP violation - in practice

Real exps are performed in matter. . .

$$H_{\text{eff}}^{\nu} = U \text{diag}\left(0, \frac{\Delta m_{21}^2}{2E_\nu}, \frac{\Delta m_{31}^2}{2E_\nu}\right) U^\dagger + \text{diag}(\sqrt{2} G_F N_e, 0, 0)$$

$$H_{\text{eff}}^{\bar\nu} = U^* \text{diag}\left(0, \frac{\Delta m_{21}^2}{2E_\nu}, \frac{\Delta m_{31}^2}{2E_\nu}\right) U^T - \text{diag}(\sqrt{2} G_F N_e, 0, 0)$$

matter potential changes sign for ν and $\bar\nu$

- for experiments in matter $P_{\nu_\alpha \to \nu_\beta} \neq P_{\bar\nu_\alpha \to \bar\nu_\beta}$ even for real U

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- cross section and fluxes are different for ν and $\bar\nu$

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$\Rightarrow$ Assume standard 3-flavour oscillations
perform a parametric fit to δ

Determination of the mass hierarchy

the vacuum oscillation probability is invariant under

$$\Delta m^2_{31} \to -\Delta m^2_{31} \qquad \delta_{\rm CP} \to \pi - \delta_{\rm CP}$$

key to resolve the mass hierarchy is the matter effect

resonance condition for $\nu_\mu \to \nu_e$ oscillations:

$$\pm \frac{2EV}{\Delta m^2_{31}} = \cos 2\theta_{13} \approx 1$$ can be fulfilled for

neutrinos if $\Delta m^2_{31} > 0$ (normal hierarchy)

anti-neutrinos if $\Delta m^2_{31} < 0$ (inverted hierarchy)

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need $L \gtrsim 1000$ km and $E_\nu \gtrsim 3$ GeV in order to reach the regime of sizable matter effect