D^0-hadronic decays related to the extraction of γ

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On behalf of the CLEO-c collaboration

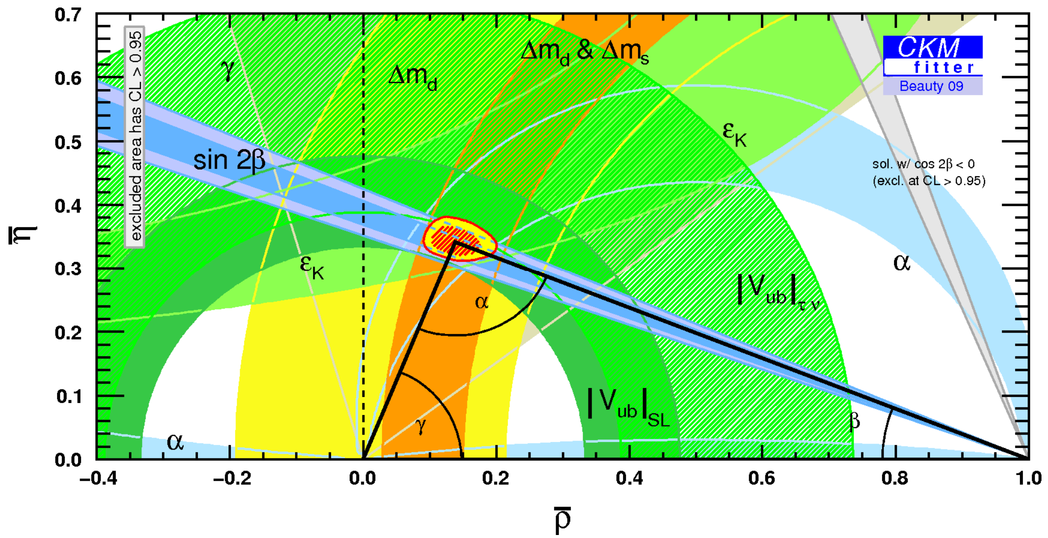
Outline

- Introduction
- Quantum-correlated decays at CLEO-c
- D^0-hadronic decays
  - Dalitz analysis
  - ADS + GLW method
  - Multibody ADS method
  - Influence on γ precision
- The future

Introduction

Introduction

- Currently γ is the least constrained of the CKM angles



$$\alpha = (89.0^{+4.4}_{-4.2})^\circ$$

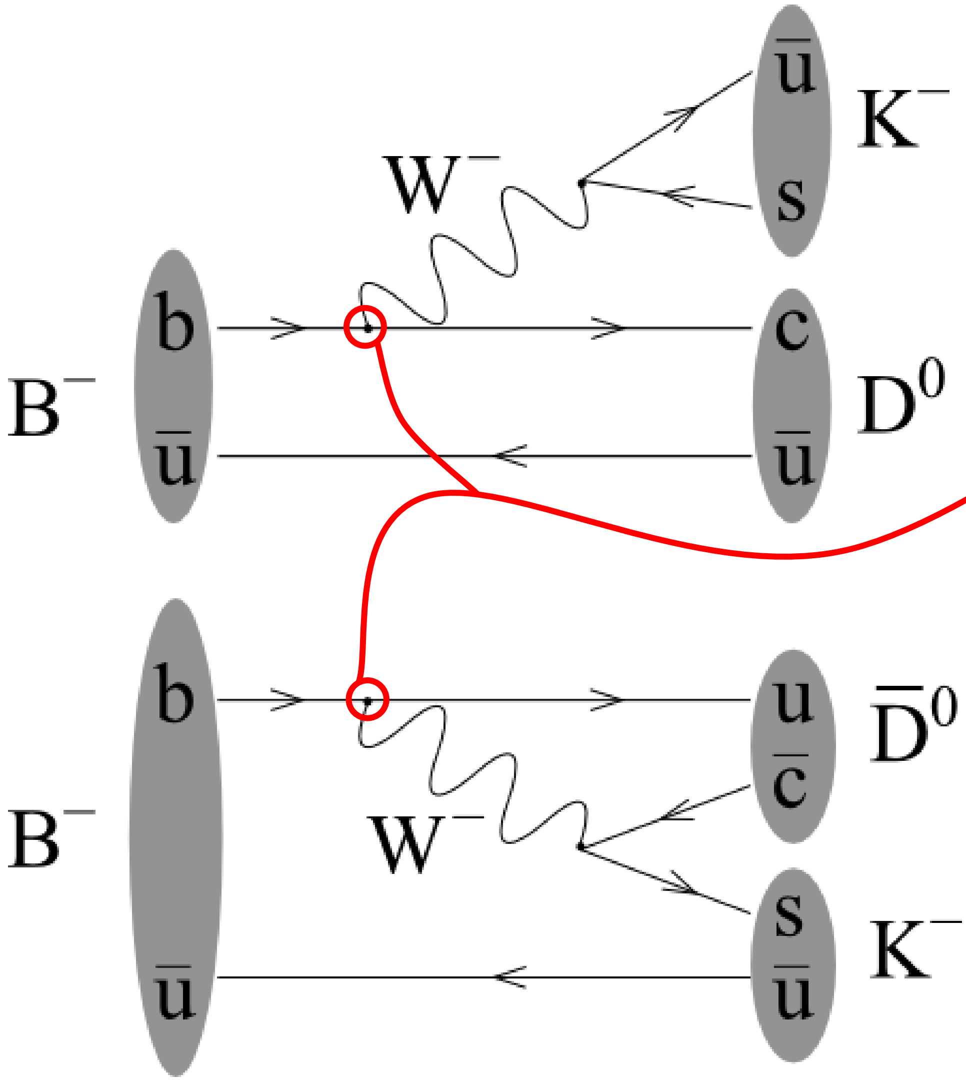
$$\beta = (21.15^{+0.90}_{-0.88})^\circ$$

$$\gamma = (73^{+22}_{-25})^\circ$$

CKMfitter 2009

- Analysis of $B^\pm \to (D^0/\overline{D}^0)\, K^\pm$ channels is a very promising method to determine γ
- Concept is outlined on next two slides; see talk by D. Derkach for more detail

Use of $B^{\pm} \to (D^0/\overline{D}^0)\, K^{\pm}$ to determine γ



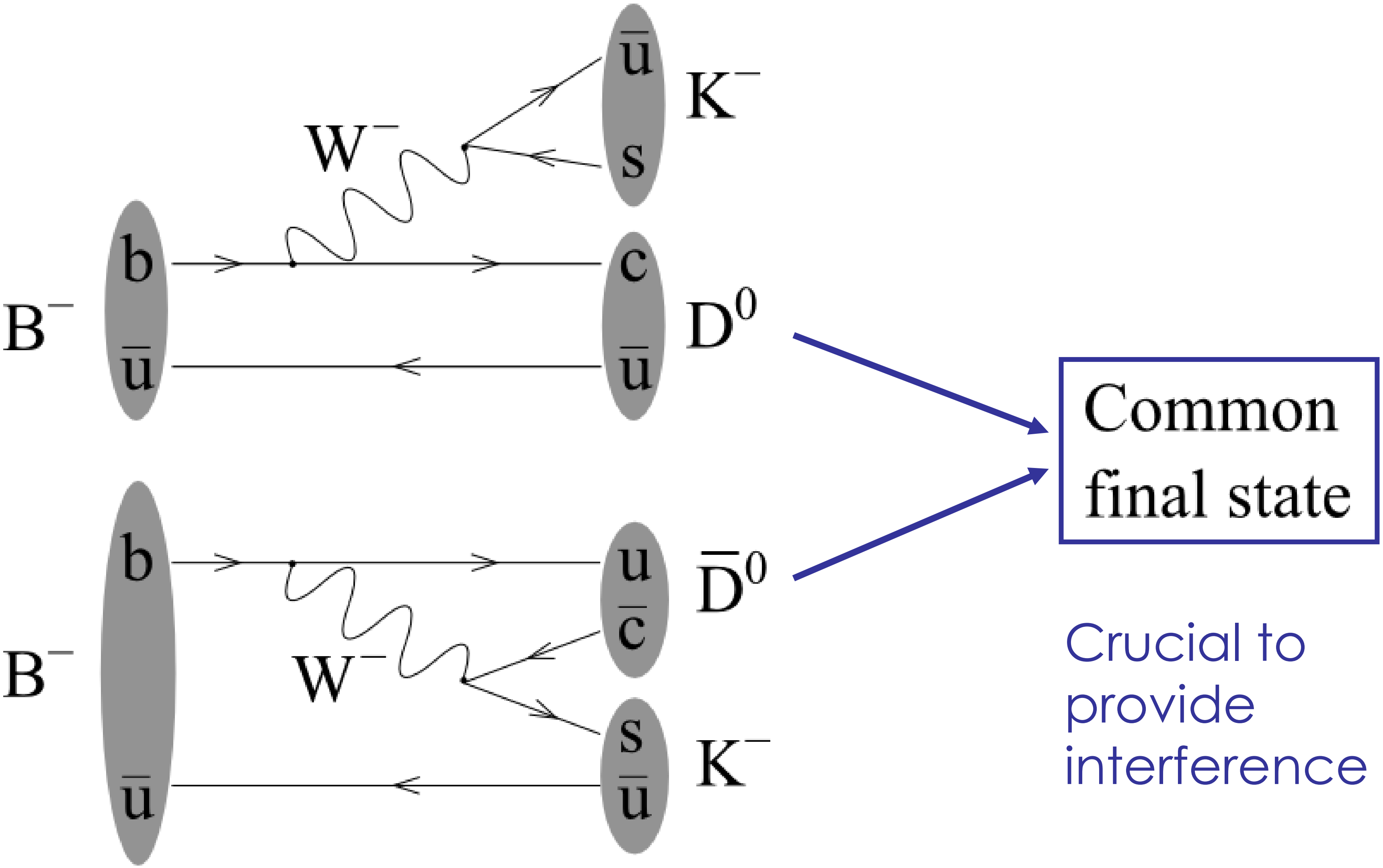
$$\gamma \equiv \arg\left(-\frac{V_{\mathrm{ud}}V_{\mathrm{ub}}^{*}}{V_{\mathrm{cd}}V_{\mathrm{cb}}^{*}}\right)$$

$$r_B e^{i(\delta_B - \gamma)} \equiv \frac{\mathcal{A}(B^- \to \overline{D^0} K^-)}{\mathcal{A}(B^+ \to D^0 K^-)}$$

DCS/CF magnitude ratio

B strong phase

Use of $B^{\pm} \rightarrow (D^0/\bar{D}^0)\, K^{\pm}$ to determine γ

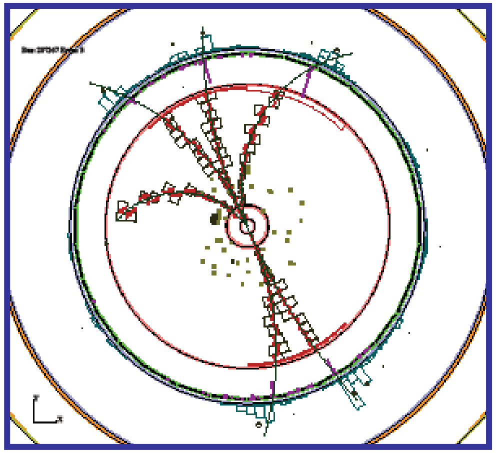


Crucial to provide interference

Quantum-correlated decays at CLEO-c

CLEO-c

- Located at Cornell Electron Storage Ring
- e^+e^- collisions
- Data collected at and above $c\bar{c}$ threshold
- $(818 \pm 8)\ \mathrm{pb}^{-1}$ of $e^+e^- \to \psi(3770)$ events
- $\psi(3770) \to D^0\overline{D}^0$ system is quantum-correlated with overall CP -1
- Very clean environment
  - Low backgrounds
  - Can reconstruct 'missing' particles (K_L, ν)



Exploiting quantum correlations

- Overall CP of the system is known (-1)
- Determine quantum numbers of one D^0 that has decayed to a known CP state
  - e.g. K^+K^- which has CP +1
- Quantum correlation enables deduction of quantum numbers of the other D^0 – which has decayed to the final state of interest
- Known as tagging
- Currently, unique results at CLEO-c
  - BES-III measurements have begun

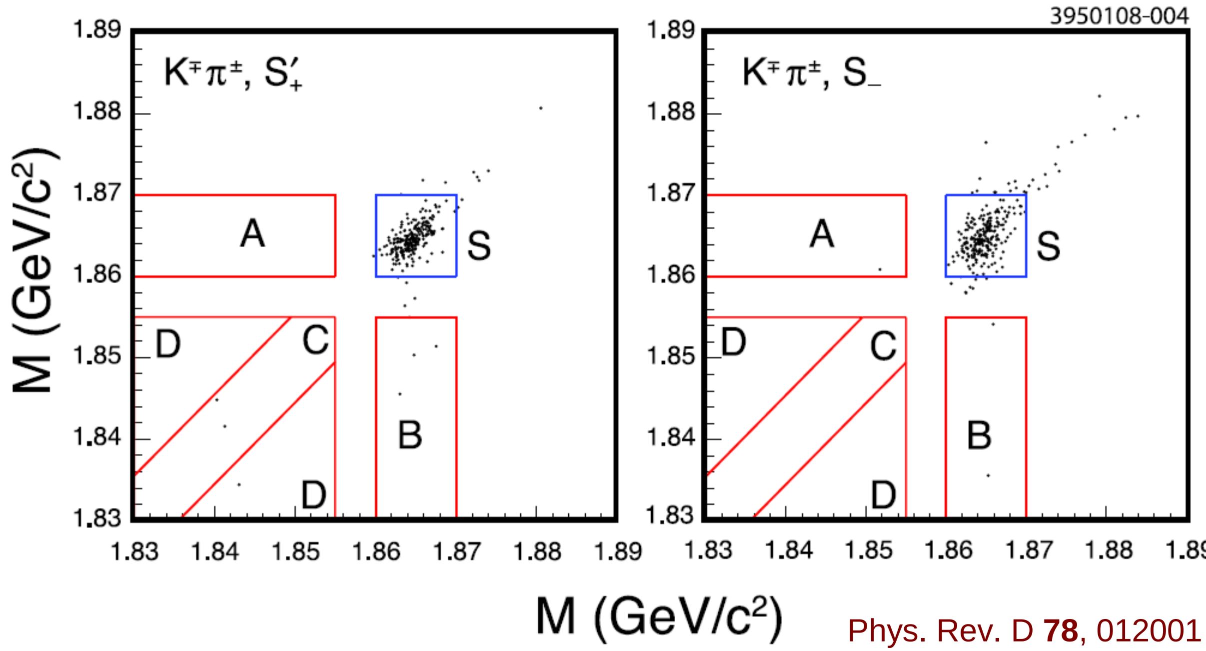
Selections at CLEO-c

- Plane of D^0 and $\overline{D}^0$ $m_{BC} \equiv \sqrt{E_{beam}^2 - \mathbf{p}_D^2}$

Data

Signal region, centred on nominal D^0 mass

Sidebands



Phys. Rev. D **78**, 012001 (2008)

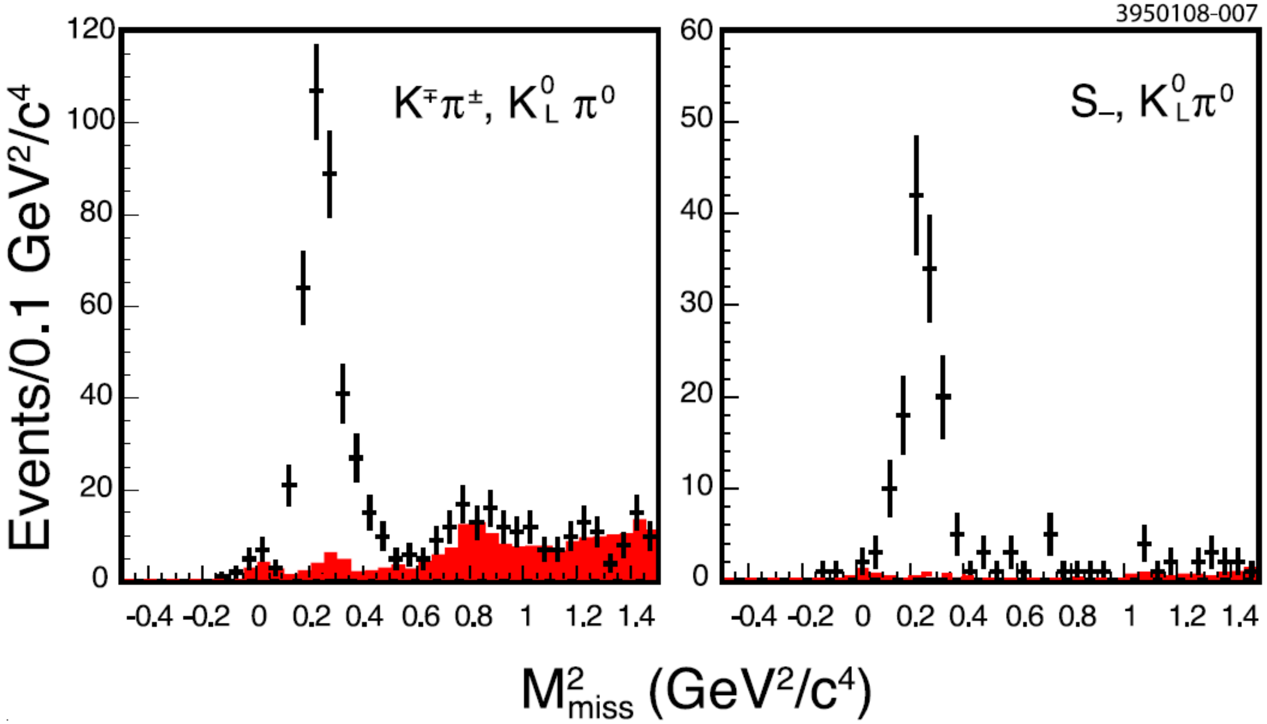
- Flat background estimated using sidebands
- Peaking background estimated using MC

Selections at CLEO-c

- For tags containing K_L, reconstruct missing momentum and hence determine (missing mass)^2
  - Expected to be centred on K_L mass^2 (0.25 GeV^2/c^4)

Data

Estimated background



BES-III

- Located at BEPCII e^+e^- collider, Beijing
- Design luminosity 10^{33} cm^{-2} s^{-1}
- This year collected ~700 pb^{-1} at $\psi(3770)$ – aiming for total of 1 fb^{-1}
- 2011: narrow resonance (J/ψ, ψ')
- 2012: $\psi(3770)$
  - Estimated total integrated luminosity $\gtrsim$ 4 fb^{-1}
- Hadronic charm physics program
  - Quantum correlations, γ input, …
- For much more information see talk by H. Li

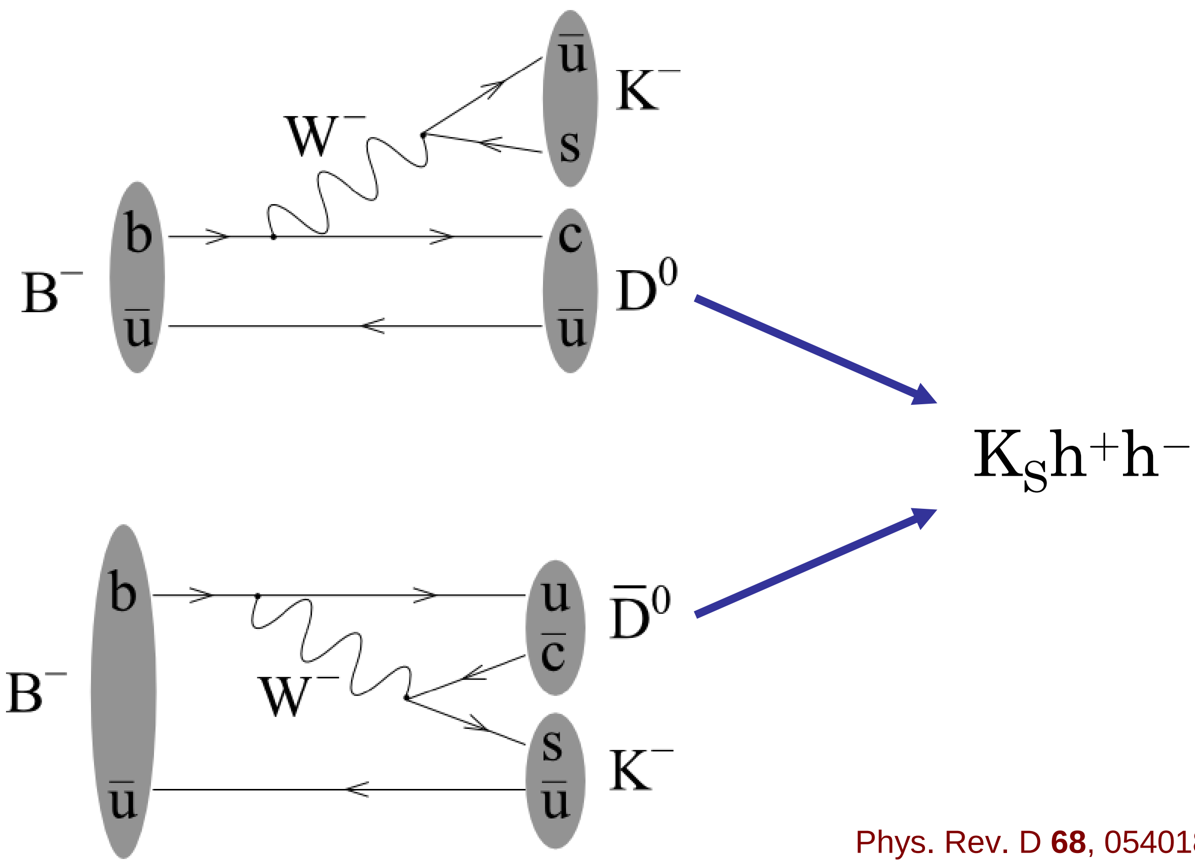
D^0-hadronic decays

Introduction

- D^0 decays to final states containing kaons ($K^{\pm}$, K_S) and pions ($\pi^{\pm}$, π^0)
- For a successful determination of γ from $B^{\pm} \rightarrow DK^{\pm}$, the D-decay parameters must be well-known for each final state
  - CLEO-c input very useful
- When D^0 and $\overline{D}^0$ are considered together they are referred to as D

Dalitz analysis

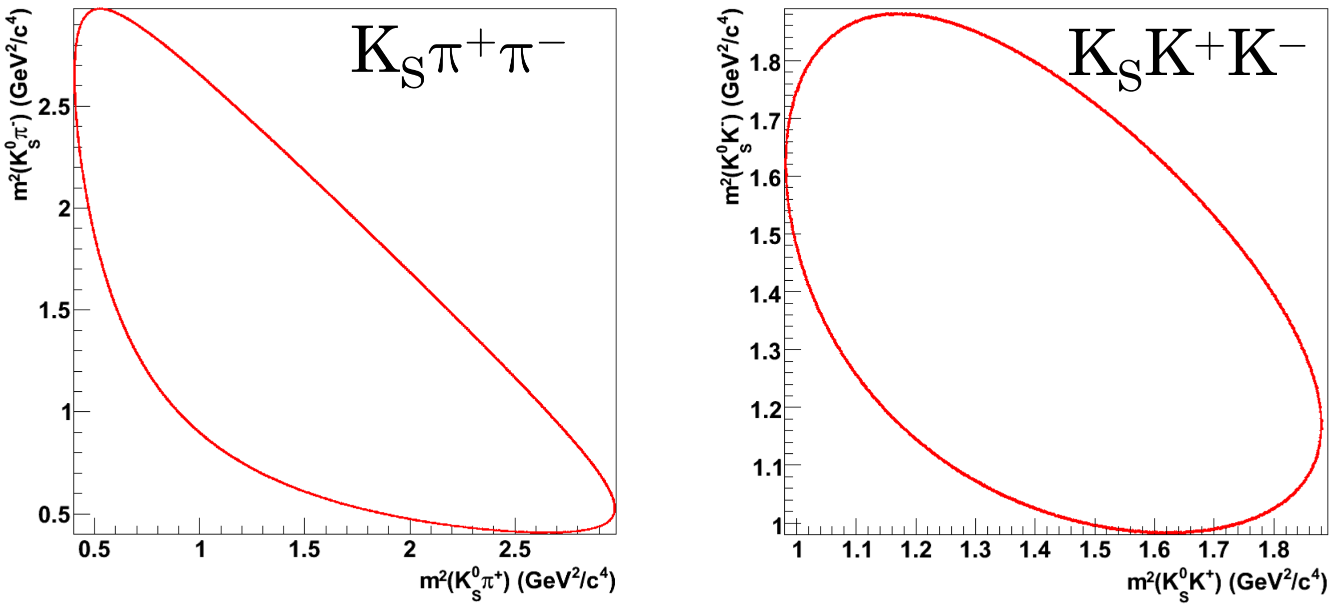
- D decays to $K_S h^+ h^-$ (h = K, π)



Phys. Rev. D **68**, 054018 (2003)

Dalitz analysis

- Three-body final state is a natural candidate for a Dalitz analysis



$$\boxed{x \equiv m^2_{K^0_S h^+} \quad y \equiv m^2_{K^0_S h^-}}$$

Dalitz analysis

- D^0 decay amplitude:

$$f_{D^0}(x,y) \equiv A_{xy} e^{i\delta_{xy}} = \mathcal{A}(\mathrm{D}^0 \to \mathrm{K}^0_\mathrm{S}\mathrm{h}^+\mathrm{h}^-)(x,y)$$

- In absence of D-mixing and CPV:

$$f_{D^0}(x,y) = f_{\overline{D^0}}(y,x)$$

- B^- decay amplitude:

$$\mathcal{A}(\mathrm{B}^- \to (\mathrm{K}^0_\mathrm{S}\mathrm{h}^+\mathrm{h}^-)_\mathrm{D}\mathrm{K}^-) \propto f_{D^0}(x,y) + r_B e^{i(\delta_B-\gamma)} f_{D^0}(y,x)$$

- B^- decay rate:

$$\Gamma(\mathrm{B}^- \to (\mathrm{K}^0_\mathrm{S}\mathrm{h}^+\mathrm{h}^-)_\mathrm{D}\mathrm{K}^-) \propto A^2_{xy} + r^2_B A^2_{yx} + 2r_B A_{xy} A_{yx}[\cos(\delta_B-\gamma)\cdot\cos(\Delta\delta_D) - \sin(\delta_B-\gamma)\cdot\sin(\Delta\delta_D)]$$

Dalitz analysis

Sensitivity to γ

$$\Gamma(\mathrm{B}^- \to (\mathrm{K}^0_\mathrm{S}\mathrm{h}^+\mathrm{h}^-)_\mathrm{D}\mathrm{K}^-) \propto A_{xy}^2 + r_B^2 A_{yx}^2 + 2 r_B A_{xy} A_{yx} [\cos(\delta_B - \gamma) \cdot \cos(\Delta\delta_D) - \sin(\delta_B - \gamma) \cdot \sin(\Delta\delta_D)]$$

B-decay parameters

$$\boxed{\Delta\delta_D \equiv \delta_{xy} - \delta_{yx}}$$

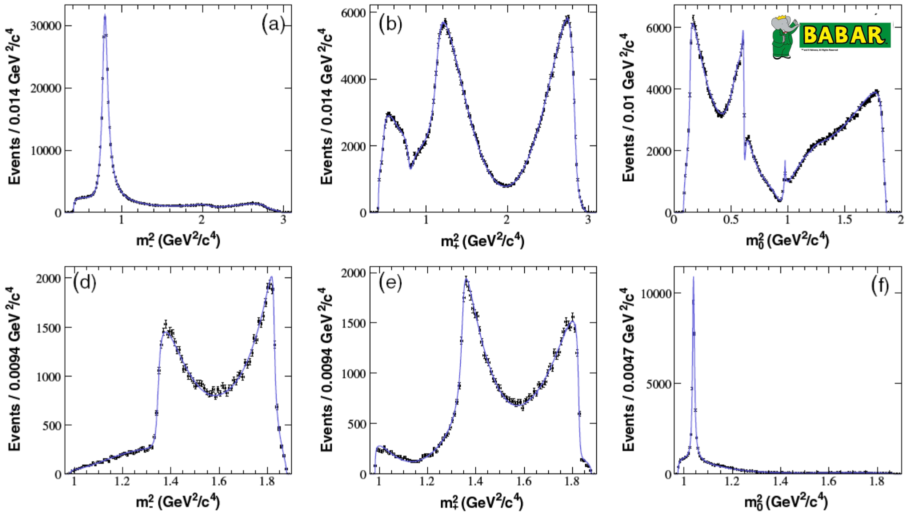
- Options:
  - Measure all parameters with $\mathrm{B}^\pm$ decays
  - Use an additional flavour-tagged D decay sample to determine D model parameters
  - Measurements of binned D-Dalitz plot to determine D model parameters

Flavour-tagged D decays

- Model $D^0 \rightarrow K_S h^+ h^-$ decays using tagged $D^{*\pm} \rightarrow D^0\pi^\pm$ events:

$K_S\pi^+\pi^-$

$K_SK^+K^-$



$m^2(K_S h^-)$ $m^2(K_S h^+)$ $m^2(h^+h^-)$

BaBar Dalitz projections

Phys. Rev D **78**, 034023 (2008)

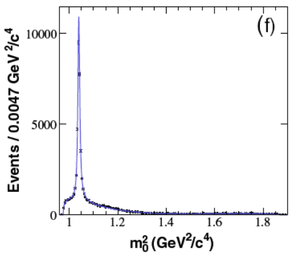
- Must use decay model to make assumptions about strong phase values – uncertainty

Flavour-tagged D decays

- BaBar $K_S K^+ K^-$ model (others in backups):

Component	a_r	ϕ_r (deg)	Fraction (%)
$K_S^0 a_0(980)^0$	1	0	55.8
$K_S^0 \phi(1020)$	0.227 ± 0.005	-56.2 ± 1.0	44.9
$K_S^0 f_0(1370)$	0.04 ± 0.06	-2 ± 80	0.1
$K_S^0 f_2(1270)$	0.261 ± 0.020	-9 ± 6	0.3
$K_S^0 a_0(1450)^0$	0.65 ± 0.09	-95 ± 10	12.6
$K^- a_0(980)^+$	0.562 ± 0.015	179 ± 3	16.0
$K^- a_0(1450)^+$	0.84 ± 0.04	97 ± 4	21.8
$K^+ a_0(980)^-$	0.118 ± 0.015	138 ± 7	0.7

Phys. Rev D **78**, 034023 (2008)



- Latest BaBar result (468M $B\bar{B}$):

arXiv:hep-ex/1005.1096

$$\gamma = 68^\circ \pm 14^\circ(\text{stat}) \pm 4^\circ(\text{syst}) \pm 3^\circ(\text{model})$$

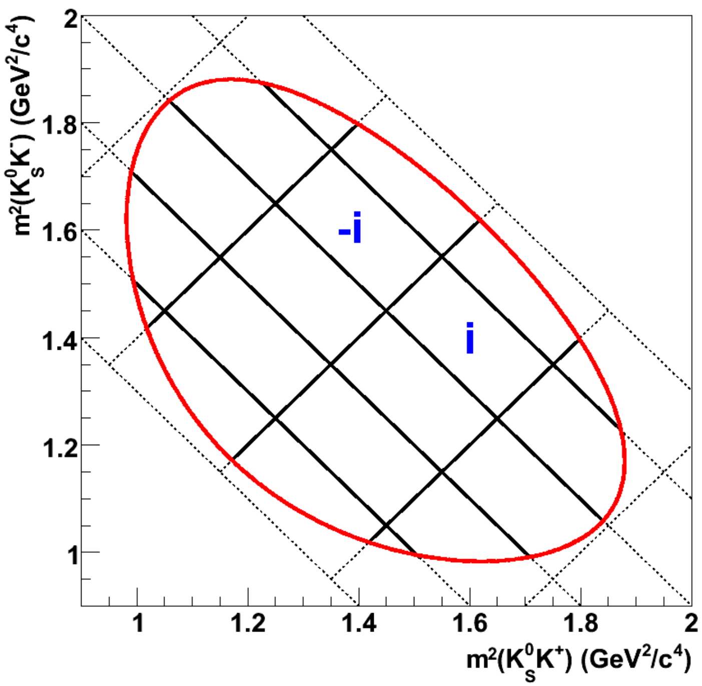
- Latest Belle result (657M $B\bar{B}$):

arXiv:hep-ex/1003.3360

$$\gamma = 78.4^{\circ}{}^{+10.8^\circ}_{-11.6^\circ}(\text{stat}) \pm 3.6^\circ(\text{syst}) \pm 8.9^\circ(\text{model})$$

Binned Dalitz analysis

- Divide the Dalitz plot into bins and measure yields in each bin
- Symmetric division of bins about $y = x$



Binned Dalitz analysis

- Number of B^+ events in bin i has dependence on weighted average cosine and sine of $\Delta\delta_D$ in bin i

$$N_i(\mathrm{B}^\pm \to (\mathrm{K}^0_\mathrm{S}\mathrm{h}^+\mathrm{h}^-)_\mathrm{D}\mathrm{K}^\pm) \propto T_i + r_B^2 T_{-i} + 2r_B\sqrt{T_i T_{-i}}[\cos(\delta_B \pm \gamma)c_i + \sin(\delta_B \pm \gamma)s_i]$$

$$T_i \equiv \int_i |f_{D^0}(x,y)|^2 \,\mathrm{d}x\,\mathrm{d}y$$

$$c_i \equiv \frac{1}{\sqrt{T_i T_{-i}}} \int_i |f_{D^0}(x,y)||f_{D^0}(y,x)| \cos(\Delta\delta_D(x,y))\,\mathrm{d}x\,\mathrm{d}y$$

$$s_i \equiv \frac{1}{\sqrt{T_i T_{-i}}} \int_i |f_{D^0}(x,y)||f_{D^0}(y,x)| \sin(\Delta\delta_D(x,y))\,\mathrm{d}x\,\mathrm{d}y$$

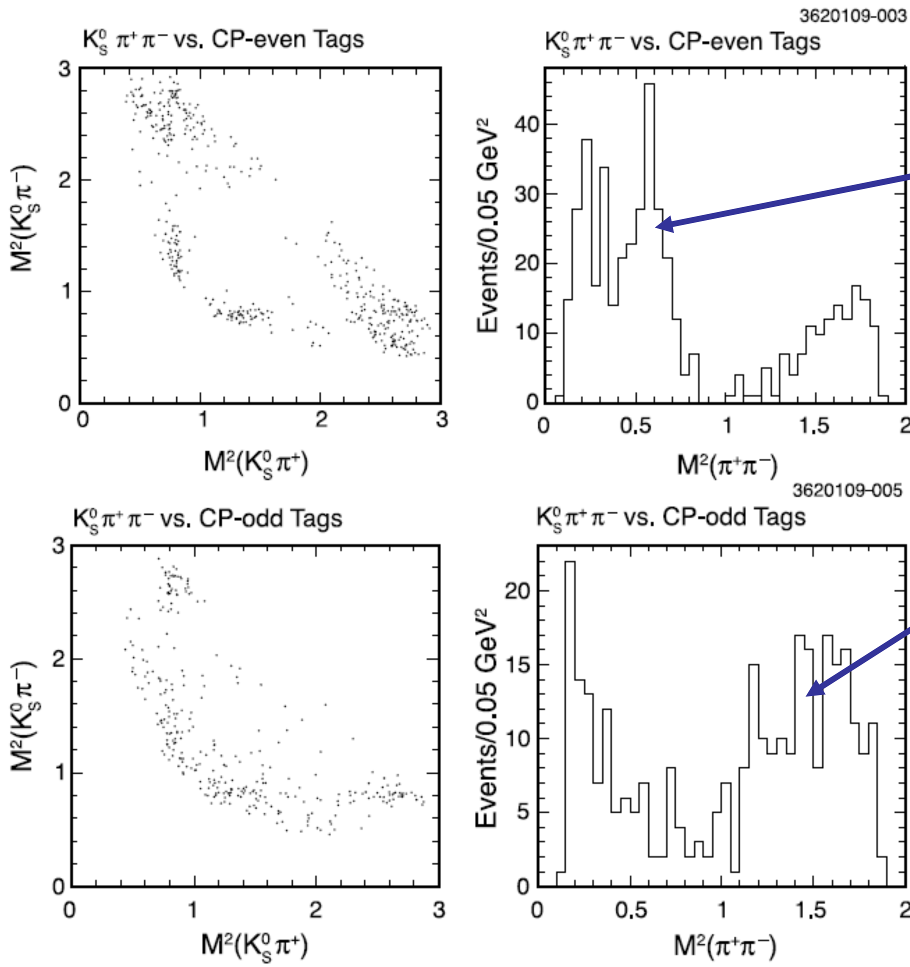
- No model error (but increased stat. error)

Determining c_i and s_i

- c_i and s_i can be measured with quantum-correlated $K_{S,L}h^+h^-$ events at CLEO-c
- $K_{S,L}h^+h^-$ tagged with CP eigenstates can be used to constrain c_i
  - To first order, $K_Sh^+h^-$ vs CP$\pm$ $\equiv$ $K_Lh^+h^-$ vs CP$\mp$
- $K_{S,L}h^+h^-$ tagged with $K_{S,L}h^+h^-$ can be used to constrain c_i and s_i
- Log likelihood used to extract c_i and s_i
- Detailed method and yields in backups

CLEO-c $K_S\pi^+\pi^-$ Dalitz plots

- Clearly see resonant substructures

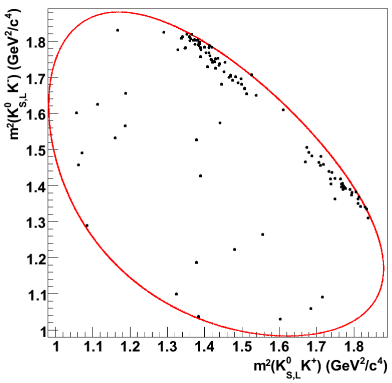


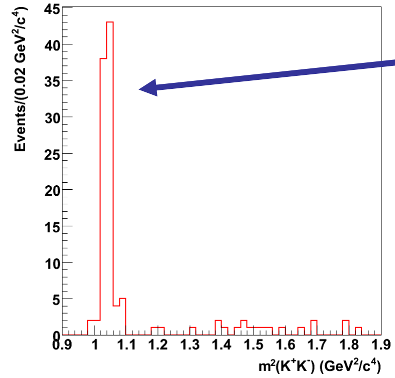
Phys. Rev. D **80**, 032002 (2009)

CLEO-c $K_S K^+ K^-$ Dalitz plots

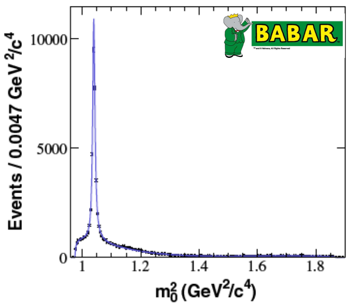
CLEO-c preliminary

$K_S K^+ K^-$ against CP+, $K_L K^+ K^-$ against CP-

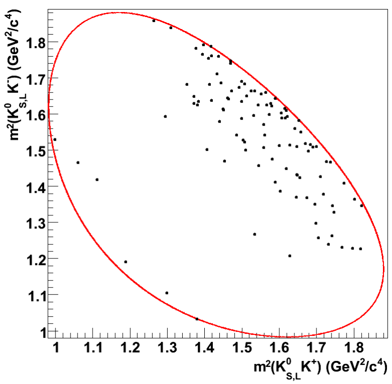


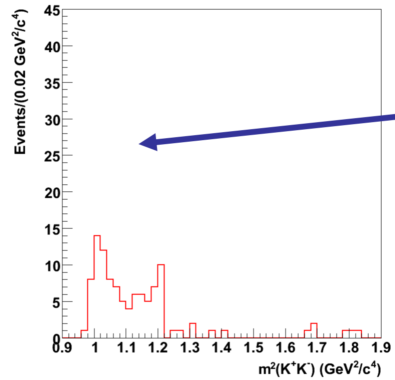


$K_S\ \varphi$



$K_S K^+ K^-$ against CP-, $K_L K^+ K^-$ against CP+





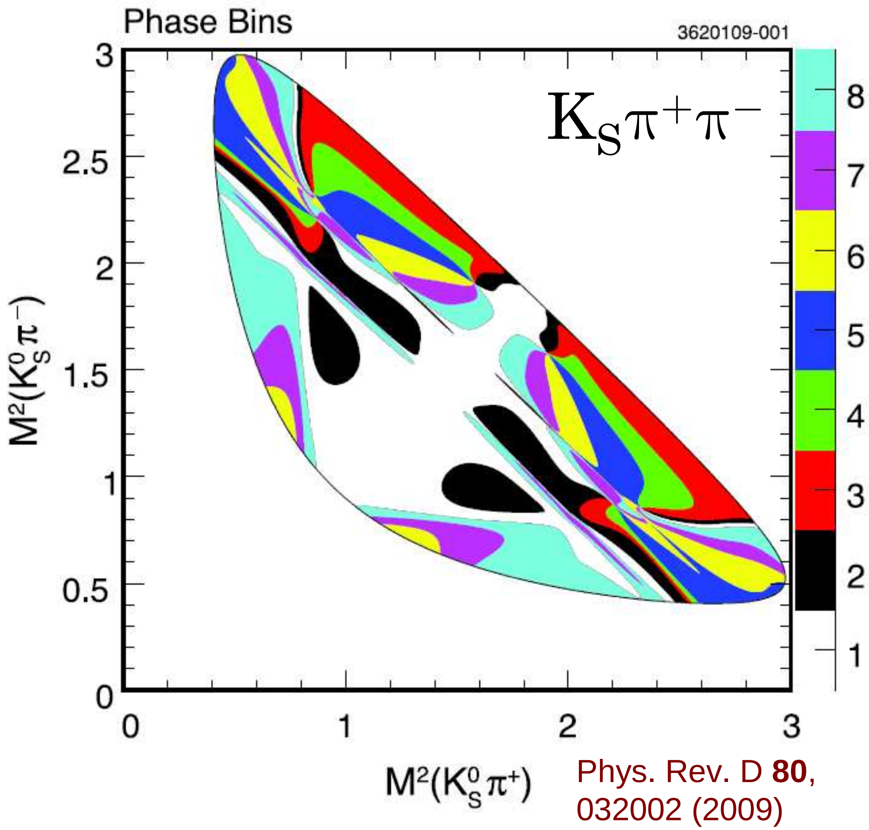
No $K_S\ \varphi$

- Quantum correlations in action!

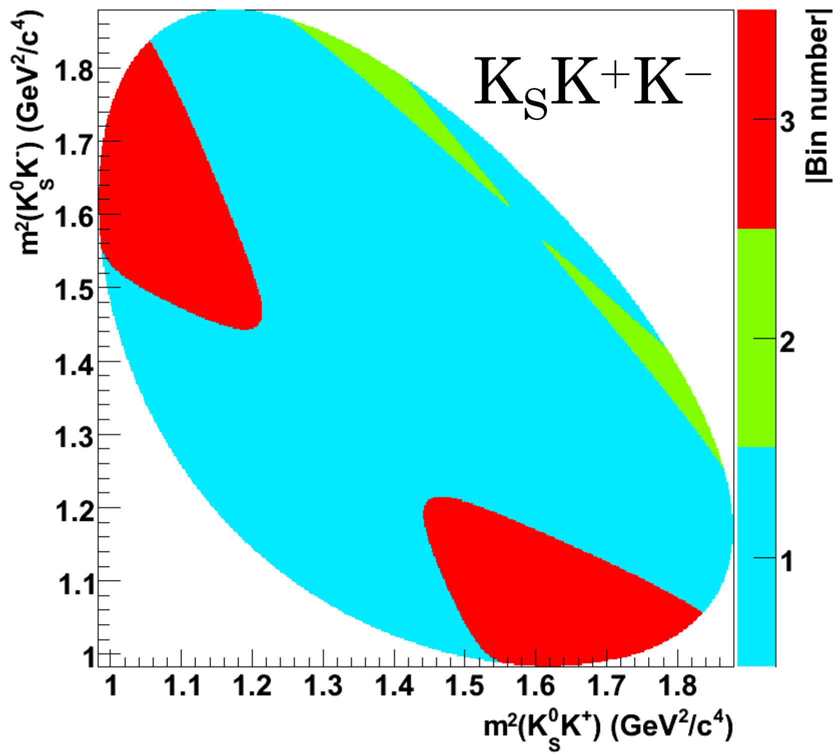
Dalitz plane binning

- Statistically-sensitive binning is in regions of similar $\Delta\delta_D$

Eur. Phys. J. C **47**, 347 (2006)

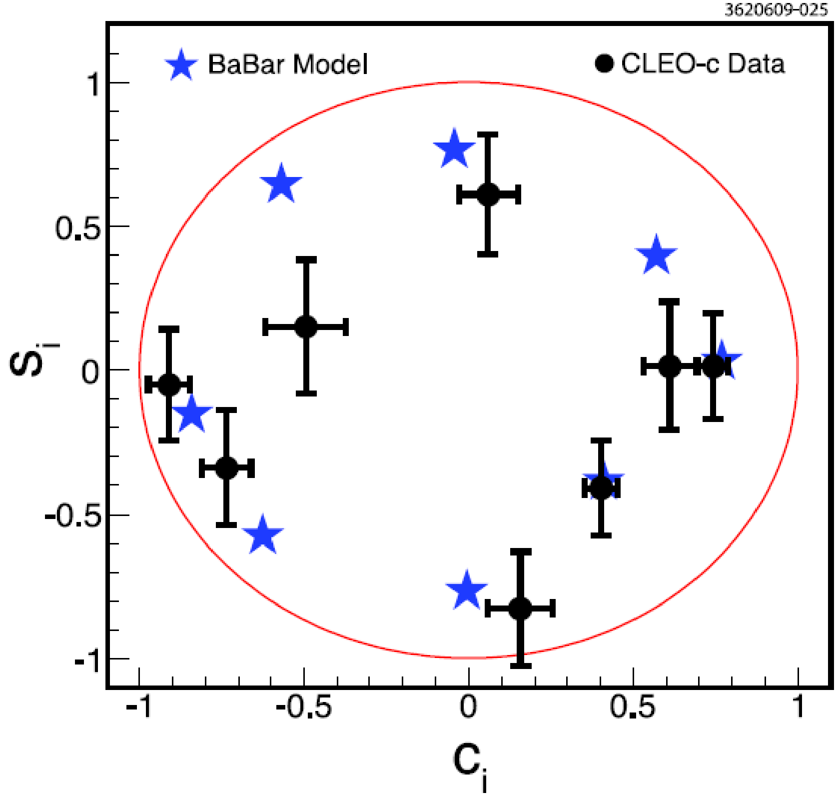


Phys. Rev. D **80**, 032002 (2009)

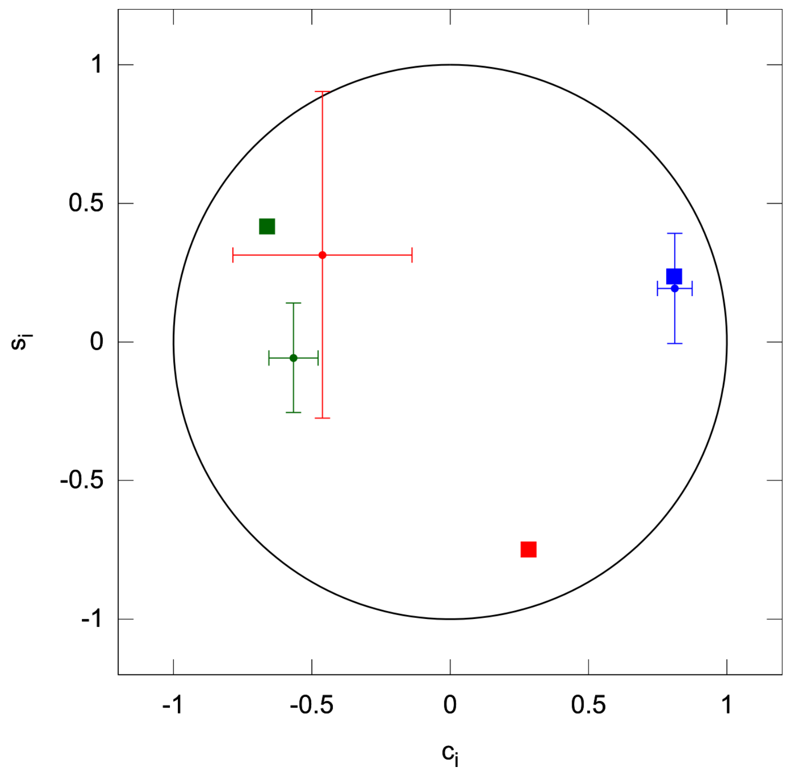


(c_i, s_i) results

$K_S\pi^+\pi^-$



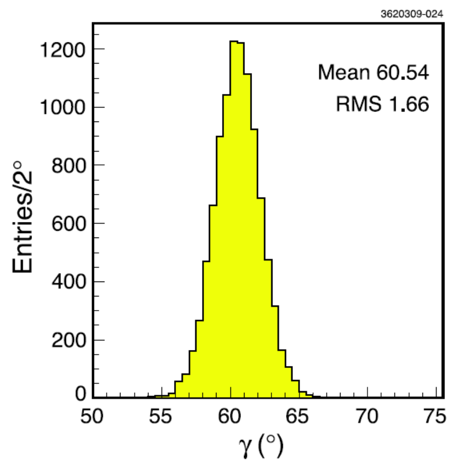
$K_SK^+K^-$



CLEO-c preliminary

Projected impact on γ

- Toy MC study of $B^{\pm} \to DK^{\pm}$
  - Large number of B events
  - Use fully correlated c_i and s_i
  - Float r_B, δ_B, γ (initially 0.1, 130°, 60°)
  - Smear & extract distributions



- CLEO-c statistical error (replaces model error)
  - $K_S\pi^+\pi^-$ ~1.7°
  - $K_SK^+K^-$ ~3.7°

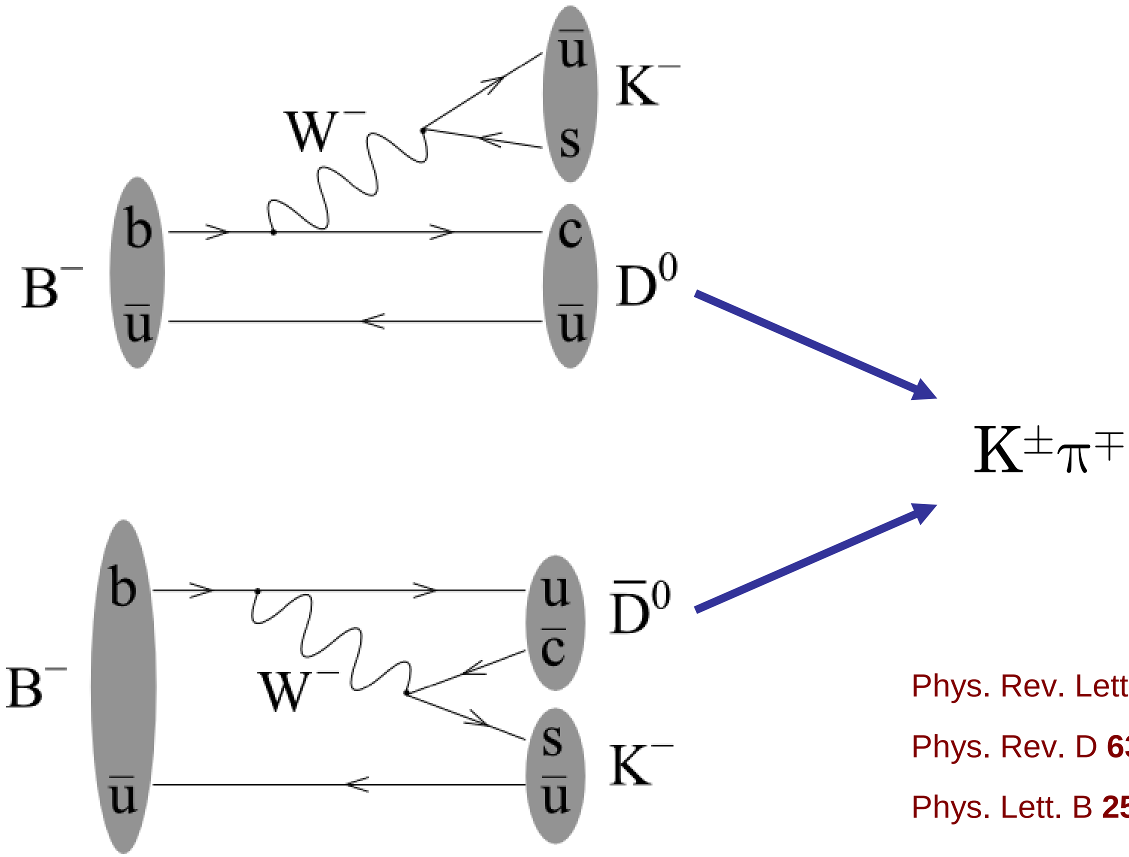
BaBar $K_S\pi^+\pi^-$ & $K_SK^+K^-$: 3°

Belle $K_S\pi^+\pi^-$: 8.9°

ADS + GLW method

Atwood, Dunietz, Soni   Gronau, London, Wyler

- D decays to two-body non-CP eigenstate



Phys. Rev. Lett. **78**, 3257 (1997)

Phys. Rev. D **63**, 036005 (2001)

Phys. Lett. B **253**, 483 (1991)

Phys. Lett. B **265**, 172 (1991)

ADS + GLW method

- Four permutations of $B^{\pm}$ and $K^{\pm}\pi^{\mp}$:

$$\Gamma(B^+ \to (K^+\pi^-)_D K^+) \propto 1 + (r_B r_D^{K\pi})^2 + 2 r_B r_D^{K\pi} \cdot \cos(\delta_B + \gamma - \delta_D^{K\pi})$$
$$\Gamma(B^+ \to (K^-\pi^+)_D K^+) \propto r_B^2 + (r_D^{K\pi})^2 + 2 r_B r_D^{K\pi} \cdot \cos(\delta_B + \gamma + \delta_D^{K\pi})$$
$$\Gamma(B^- \to (K^+\pi^-)_D K^-) \propto r_B^2 + (r_D^{K\pi})^2 + 2 r_B r_D^{K\pi} \cdot \cos(\delta_B - \gamma + \delta_D^{K\pi})$$
$$\Gamma(B^- \to (K^-\pi^+)_D K^-) \propto 1 + (r_B r_D^{K\pi})^2 + 2 r_B r_D^{K\pi} \cdot \cos(\delta_B - \gamma - \delta_D^{K\pi})$$

DCS/CF magnitude ratio

Strong phase

$$r_D^{K\pi} e^{i\delta_D^{K\pi}} \equiv \frac{\mathcal{A}(D^0 \to K^+\pi^-)}{\mathcal{A}(D^0 \to K^-\pi^+)}$$

- Strong phase world average (HFAG): $\boxed{\delta_D^{K\pi} = (26.4^{+9.6}_{-9.9})^\circ}$

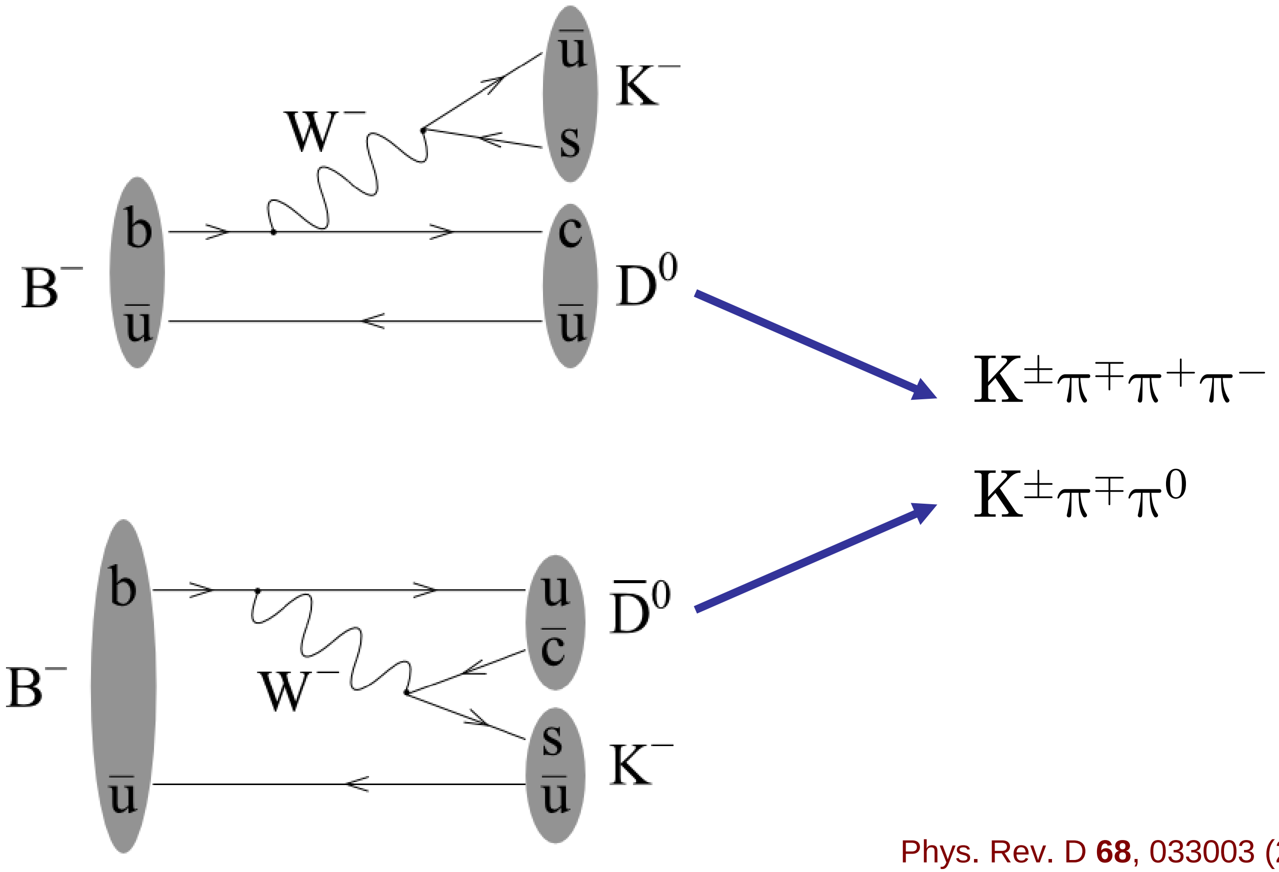
http://www.slac.stanford.edu/xorg/hfag

- Input from B-factory mixing measurements & CLEO-c ($281 pb^{-1}$): CP- and flavour-tagged $K^{\pm}\pi^{\mp}$

Phys. Rev. D **78**, 012001 (2008)

Multibody ADS method

- D decays to three or more bodies



Phys. Rev. D **68**, 033003 (2003)

Multibody ADS method

- Multibody decays can typically occur via intermediate resonances
  - e.g. $K^-\pi^+\pi^+\pi^-$ can be produced via $K(1270)^-\pi^+$
- Unlike the case of two bodies, the D^0 decay amplitude varies across phase space
- Each intermediate resonance has a different amplitude and phase
- Does one resonance dominate, or are there several with ~equal contributions?

Coherence factor

- Coherence factor $R_{K3\pi}$ can classify intermediate resonances:

No single resonance dominates

A single resonance dominates

$$0 \leq R_{K3\pi} \leq 1$$

Intermediate resonances are out of phase

Intermediate resonances are in phase

Coherence factor

- Rate equations:

$$\Gamma(\mathrm{B}^+ \to (\mathrm{K}^+\pi^-\pi^+\pi^-)_\mathrm{D}\mathrm{K}^+) \propto 1 \;+ (r_B r_D^{K3\pi})^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos(\delta_B + \gamma - \delta_D^{K3\pi})$$
$$\Gamma(\mathrm{B}^+ \to (\mathrm{K}^-\pi^+\pi^+\pi^-)_\mathrm{D}\mathrm{K}^+) \propto r_B^2 + (r_D^{K3\pi})^2 \;+ 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos(\delta_B + \gamma + \delta_D^{K3\pi})$$
$$\Gamma(\mathrm{B}^- \to (\mathrm{K}^+\pi^-\pi^+\pi^-)_\mathrm{D}\mathrm{K}^-) \propto r_B^2 + (r_D^{K3\pi})^2 \;+ 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos(\delta_B - \gamma + \delta_D^{K3\pi})$$
$$\Gamma(\mathrm{B}^- \to (\mathrm{K}^-\pi^+\pi^+\pi^-)_\mathrm{D}\mathrm{K}^-) \propto 1 \;+ (r_B r_D^{K3\pi})^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos(\delta_B - \gamma - \delta_D^{K3\pi})$$

- $R_{K3\pi}$ ~ 1: sensitive to γ
- $R_{K3\pi}$ ~ 0: lose sensitivity to γ, but obtain good constraint on r_B
- Similarly for $R_{K\pi\pi^0}$ etc.

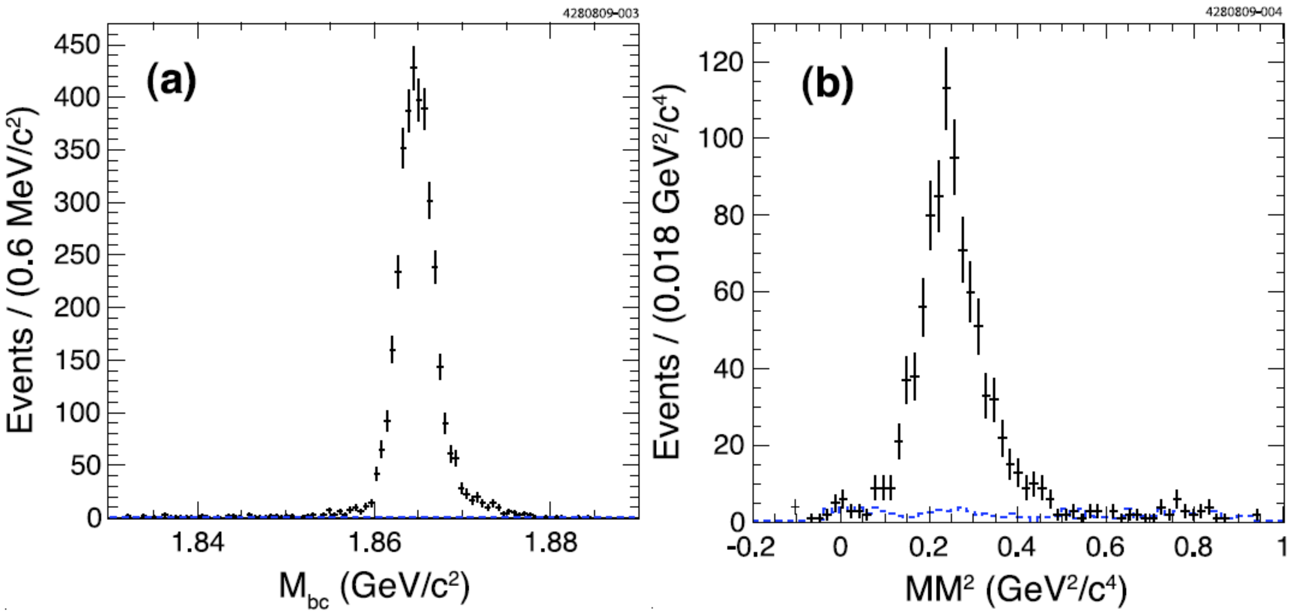
Measurement at CLEO-c

- As with two-body ADS, external constraints on D^0-decay parameters can be provided by quantum-correlated CLEO-c data Phys. Rev. D **80**, 031105(R) (2009)

$K\pi\pi^0$ against $K_S\pi^0$ $K3\pi$ against $K_L\pi^0$

Data

Estimated background



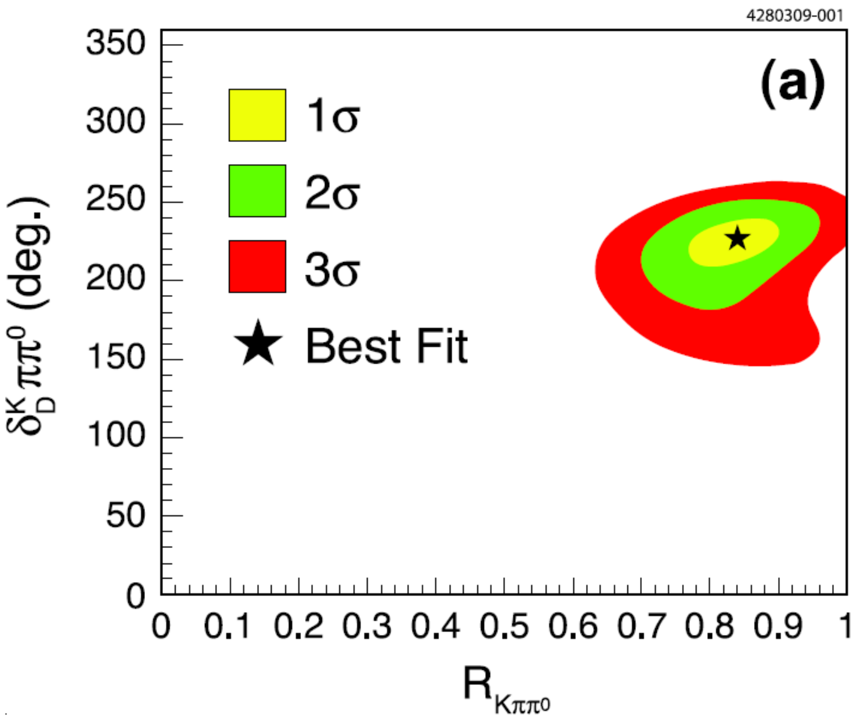
Measurement at CLEO-c

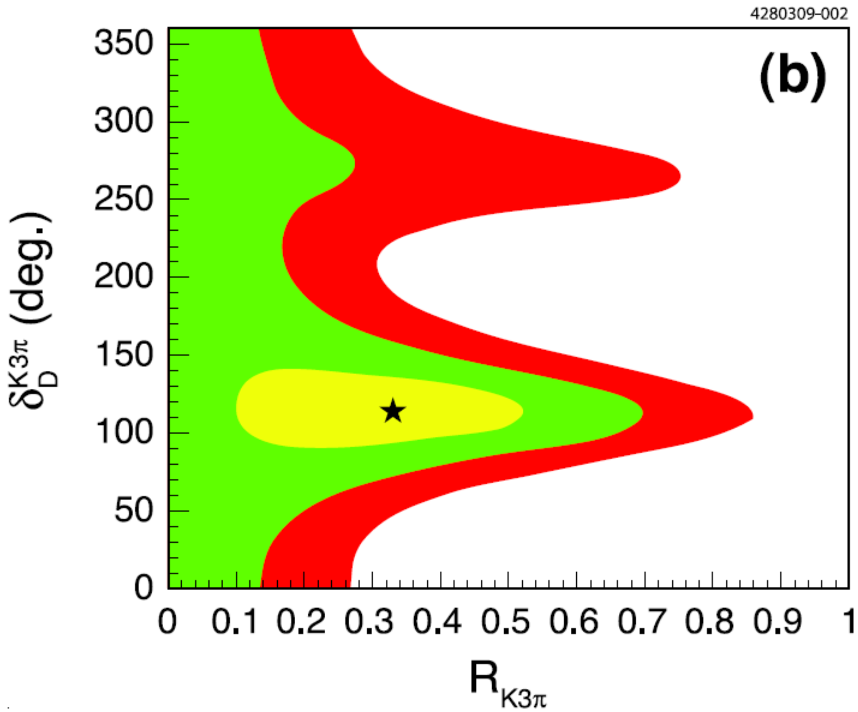
- Double-tagged yields are sensitive to various permutations of coherence factors and strong phases:

Double-tag yield	Sensitive to
$K^{\pm}\pi^{\mp}\pi^{+}\pi^{-}$ vs $K^{\pm}\pi^{\mp}\pi^{+}\pi^{-}$	${R_{K3\pi}}^2$
$K^{\pm}\pi^{\mp}\pi^{+}\pi^{-}$ vs $K^{\pm}\pi^{\mp}$	$R_{K3\pi}\cos(\delta^{K3\pi} - \delta^{K\pi})$
$K^{\pm}\pi^{\mp}\pi^{+}\pi^{-}$ vs $K^{\pm}\pi^{\mp}\pi^{0}$	$R_{K3\pi} R_{K\pi\pi^0} \cos(\delta^{K3\pi} - \delta^{K\pi\pi^0})$
$K^{\pm}\pi^{\mp}\pi^{+}\pi^{-}$ vs CP	$R_{K3\pi}\cos(\delta^{K3\pi})$

- 818 pb^{-1} data used; yields in backups

CLEO-c results





$$R_{K\pi\pi^0} = 0.84 \pm 0.07$$

$$\delta_D^{K\pi\pi^0} = (227^{+14}_{-17})^\circ$$

Highly coherent

$$R_{K3\pi} = 0.33^{+0.20}_{-0.23}$$

$$\delta_D^{K3\pi} = (114^{+26}_{-23})^\circ$$

Low coherence

Projected impact on γ

- Invaluable additional input for LHCb γ extraction
- Coherence factors, and $\delta_D^{K\pi}$ constraint, provide additional sensitivity equivalent to doubling the $\mathrm{B\overline{B}}$ dataset for 2 fb^{-1} LHCb-2008-031 public note

The future

- At CLEO-c:
  - $K^{\pm}\pi^{\mp}$ quantum-correlated analysis to be updated with full statistics (results expected autumn)
  - $K_S K^+ K^-$ analysis to be finalised
  - Update of $K_S\pi^+\pi^-$ analysis with newer BaBar amplitude model, Belle model, and statistically optimised binnings
  - Coherence factor analysis of $K_S K^{\pm}\pi^{\mp}$
- BES-III $\psi(3770)$ physics program
- D^0 results can be used in LHCb, etc.

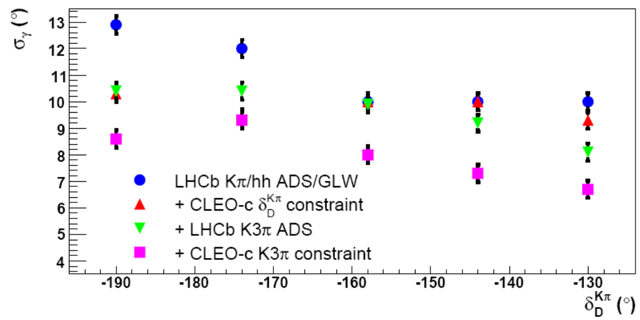
Conclusions

- D^0-hadronic decays play an important role in the precise measurement of γ
- Quantum correlations at CLEO-c enable precise measurement of D decay parameters
- BES-III taking data at $\psi(3770)$
- Dalitz analyses of $K_S\pi^+\pi^-$ and $K_SK^+K^-$
- ADS methods $K^\pm\pi^\mp$, $K^\pm\pi^\mp\pi^+\pi^-$, $K^\pm\pi^\mp\pi^0$
- Critical input for precise determination of γ at LHCb and future e^+e^- machines
- Combination of measurements, 10 fb^{-1} LHCb data: $\sigma_\gamma \sim (1.9 - 2.7)°$ LHCb-2008-031 public note

Backups

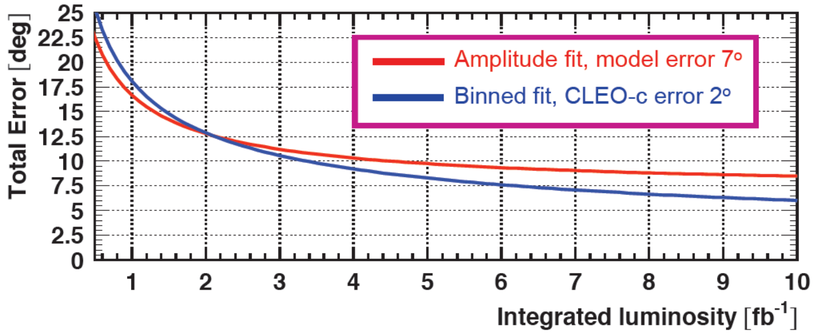
Influence on γ precision

- ADS method



LHCb-2008-031 public note

- $K_S\pi^+\pi^-$



BaBar $K_S\pi^+\pi^-$ model

TABLE II. CA, DCS, and CP eigenstates complex amplitudes $a_r e^{i\phi_r}$, $\pi\pi$ S-wave P-vector parameters, $K\pi$ S-wave parameters, and fit fractions, as obtained from the fit of the $D^0 \to K_S^0\pi^+\pi^-$ Dalitz plot distribution from $D^{*+} \to D^0\pi^+$. P-vector parameters f_{1v}^{prod}, for $v \neq 1$, are defined as $f_{1v}^{\text{prod}}/f_{11}^{\text{prod}}$. Errors for amplitudes are statistical only, while for fit fractions include statistical and systematic uncertainties, largely dominated by the latter. Upper limits on fit fractions are quoted at 95% confidence level.

Component	a_r		ϕ_r (deg)	Fraction (%)
$K^*(892)^-$	1.740 ± 0.010		139.0 ± 0.3	55.7 ± 2.8
$K_0^*(1430)^-$	8.2 ± 0.7		153 ± 8	10.2 ± 1.5
$K_2^*(1430)^-$	1.410 ± 0.022		138.4 ± 1.0	2.2 ± 1.6
$K^*(1680)^-$	1.46 ± 0.10		-174 ± 4	0.7 ± 1.9
$K^*(892)^+$	0.158 ± 0.003		-42.7 ± 1.2	0.46 ± 0.23
$K_0^*(1430)^+$	0.32 ± 0.06		143 ± 11	<0.05
$K_2^*(1430)^+$	0.091 ± 0.016		85 ± 11	<0.12
$\rho(770)^0$	1		0	21.0 ± 1.6
$\omega(782)$	0.0527 ± 0.0007		126.5 ± 0.9	0.9 ± 1.0
$f_2(1270)$	0.606 ± 0.026		157.4 ± 2.2	0.6 ± 0.7
β_1	9.3 ± 0.4		-78.7 ± 1.6	
β_2	10.89 ± 0.26		-159.1 ± 2.6	
β_3	24.2 ± 2.0		168 ± 4	
β_4	9.16 ± 0.24		90.5 ± 2.6	
f_{11}^{prod}	7.94 ± 0.26		73.9 ± 1.1	
f_{12}^{prod}	2.0 ± 0.3		-18 ± 9	
f_{13}^{prod}	5.1 ± 0.3		33 ± 3	
f_{14}^{prod}	3.23 ± 0.18		4.8 ± 2.5	
s_0^{prod}		-0.07 ± 0.03		
$\pi\pi$ S wave				11.9 ± 2.6
M (GeV/c^2)		1.463 ± 0.002		
Γ (GeV/c^2)		0.233 ± 0.005		
F		0.80 ± 0.09		
ϕ_F		2.33 ± 0.13		
R		1		
ϕ_R		-5.31 ± 0.04		
a		1.07 ± 0.11		
r		-1.8 ± 0.3		

Phys. Rev D **78**, 034023 (2008)

Belle $K_S\pi^+\pi^-$ model

TABLE I: Fit results for $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay. Errors are statistical only.

Intermediate state	Amplitude	Phase (°)	Fit fraction (%)
$K_S\sigma_1$	1.56 ± 0.06	214 ± 3	11.0 ± 0.7
$K_S\rho^0$	1.0 (fixed)	0 (fixed)	21.2 ± 0.5
$K_S\omega$	0.0343 ± 0.0008	112.0 ± 1.3	0.526 ± 0.014
$K_Sf_0(980)$	0.385 ± 0.006	207.3 ± 2.3	4.72 ± 0.05
$K_S\sigma_2$	0.20 ± 0.02	212 ± 12	0.54 ± 0.10
$K_Sf_2(1270)$	1.44 ± 0.04	342.9 ± 1.7	1.82 ± 0.05
$K_Sf_0(1370)$	1.56 ± 0.12	110 ± 4	1.9 ± 0.3
$K_S\rho^0(1450)$	0.49 ± 0.08	64 ± 11	0.11 ± 0.04
$K^*(892)^+\pi^-$	1.638 ± 0.010	133.2 ± 0.4	62.9 ± 0.8
$K^*(892)^-\pi^+$	0.149 ± 0.004	325.4 ± 1.3	0.526 ± 0.016
$K^*(1410)^+\pi^-$	0.65 ± 0.05	120 ± 4	0.49 ± 0.07
$K^*(1410)^-\pi^+$	0.42 ± 0.04	253 ± 5	0.21 ± 0.03
$K_0^*(1430)^+\pi^-$	2.21 ± 0.04	358.9 ± 1.1	7.93 ± 0.09
$K_0^*(1430)^-\pi^+$	0.36 ± 0.03	87 ± 4	0.22 ± 0.04
$K_2^*(1430)^+\pi^-$	0.89 ± 0.03	314.8 ± 1.1	1.40 ± 0.06
$K_2^*(1430)^-\pi^+$	0.23 ± 0.02	275 ± 6	0.093 ± 0.014
$K^*(1680)^+\pi^-$	0.88 ± 0.27	82 ± 17	0.06 ± 0.04
$K^*(1680)^-\pi^+$	2.1 ± 0.2	130 ± 6	0.30 ± 0.07
non-resonant	2.7 ± 0.3	160 ± 5	5.0 ± 1.0

arXiv:hep-ex/1003.3360

Determining c_i and s_i

- c_i and s_i can be measured with quantum-correlated $K_{S,L}h^+h^-$ events at CLEO-c
- CP-tagged data can be used to constrain c_i

Double CP tag yield

Single CP tag yield

$$\frac{M_i^{\pm}}{S_{\mp}} = \frac{K_i \pm 2c_i\sqrt{K_i K_{-i}} + K_{-i}}{2S_f}$$

Double flavour tag yield

Single flavour tag yield

Determining c_i and s_i

- $K_{S,L}h^+h^-$ tagged with $K_{S,L}h^+h^-$ can be used to constrain c_i and s_i

Double $K_{S,L}$hh tag yield

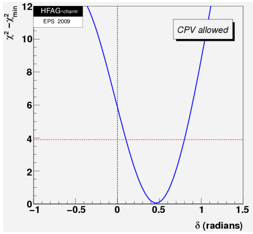
$$M_{ij} = \frac{N_{D,\overline{D}}}{2S_f^2}\left(K_i K_{-j} + K_{-i} K_j - 2\sqrt{K_i K_{-j} K_{-i} K_j}(c_i c_j + s_i s_j)\right)$$

Single flavour tag yield

Double flavour tag yield

- Log likelihood used to extract c_i and s_i

ADS: input from charm physics

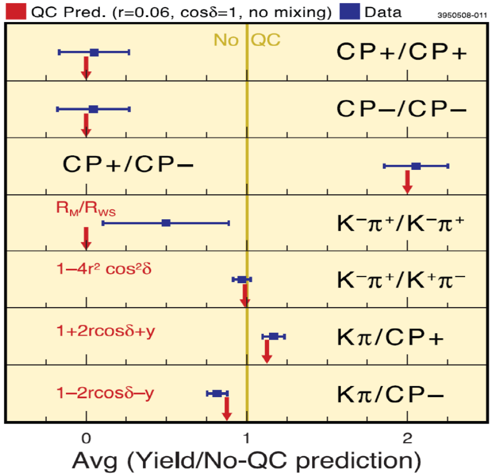


- Use external input of $\delta^{K\pi}$ to guide fit at B-physics experiments
- Current world average (HFAG): $\boxed{\delta_D^{K\pi} = (26.4^{+9.6}_{-9.9})^\circ}$

arXiv:hep-ex/0808.1297

http://www.slac.stanford.edu/xorg/hfag

  - B-factory mixing measurements
  - Input from CLEO-c (TQCA)
- TQCA: Phys. Rev. D **78**, 012001 (2008)
  - 281 pb^{-1} data used (to be updated)
  - $K^\pm\pi^\mp$ tagged with CP & flavour tags
  - Rate ~ $\mathcal{B}_{CP}\mathcal{B}_{K\pi}(1+2r^{K\pi}\delta^{K\pi}+\ldots)$
  - Quantum correlation observed!

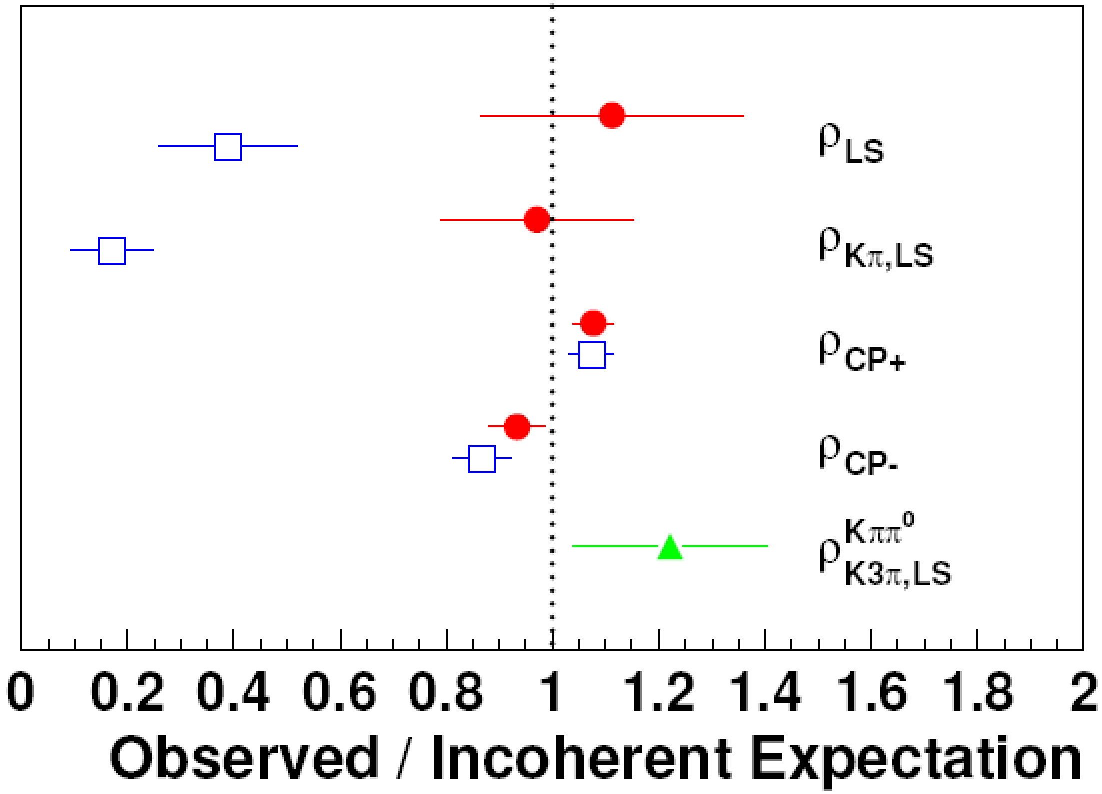


Coherence factors

- Coherence factor results normalised to incoherent expectation

$K3\pi$: low coherence

$K\pi\pi^0$: high coherence



Yields in Dalitz plot analyses

Tag group	$K_S\pi^+\pi^-$	$K_SK^+K^-$	$K_L\pi^+\pi^-$	$K_LK^+K^-$
CP+	471	50	517	59
CP-	786	46	322	64
Flavour	7829	745	12098	1174
$K_S\pi^+\pi^-$	475	56	867	126
$K_L\pi^+\pi^-$		140		
$K_SK^+K^-$		4		9

Yields in TQCA

Mode	Yield	Efficiency (%)
$K^-\pi^+, K^-\pi^+$	$2.0 \pm 1.4 \pm 0$	36.1 ± 3.4
$K^-\pi^+, K^+\pi^-$	$600 \pm 25 \pm 5$	41.1 ± 0.2
$K^-\pi^+, K^+K^-$	$71 \pm 8 \pm 1$	35.5 ± 0.6
$K^-\pi^+, \pi^+\pi^-$	$24 \pm 5 \pm 1$	44.4 ± 1.1
$K^-\pi^+, K^0_S\pi^0\pi^0$	$32 \pm 6 \pm 1$	8.0 ± 0.3
$K^-\pi^+, K^0_S\pi^0$	$88 \pm 9 \pm 1$	18.4 ± 0.3
$K^-\pi^+, K^0_S\eta$	$8.0 \pm 2.8 \pm 0.0$	6.0 ± 0.3
$K^-\pi^+, K^0_S\omega$	$29 \pm 5 \pm 0$	8.7 ± 0.2
$K^+\pi^-, K^+\pi^-$	$2.0 \pm 1.4 \pm 0.0$	44.1 ± 3.3
$K^+\pi^-, K^+K^-$	$54 \pm 7 \pm 0$	36.1 ± 0.6
$K^+\pi^-, \pi^+\pi^-$	$25 \pm 5 \pm 1$	48.1 ± 1.1
$K^+\pi^-, K^0_S\pi^0\pi^0$	$33 \pm 6 \pm 0$	8.0 ± 0.3
$K^+\pi^-, K^0_S\pi^0$	$76 \pm 9 \pm 0$	18.6 ± 0.3
$K^+\pi^-, K^0_S\eta$	$9 \pm 3 \pm 0$	6.1 ± 0.3
$K^+\pi^-, K^0_S\omega$	$33 \pm 6 \pm 1$	8.0 ± 0.2
$K^+K^-, K^0_S\pi^0$	$39 \pm 6 \pm 1$	17.1 ± 0.6
$K^+K^-, K^0_S\eta$	$7.0 \pm 2.7 \pm 0.0$	7.1 ± 0.8
$K^+K^-, K^0_S\omega$	$20 \pm 4 \pm 0$	6.8 ± 0.4
$\pi^+\pi^-, K^0_S\pi^0$	$13 \pm 4 \pm 0$	19.2 ± 1.2
$\pi^+\pi^-, K^0_S\eta$	$2.0 \pm 1.4 \pm 0.0$	8.1 ± 1.4
$\pi^+\pi^-, K^0_S\omega$	$7.0 \pm 2.7 \pm 0.0$	9.9 ± 0.9
$K^0_S\pi^0\pi^0, K^0_S\pi^0$	$14 \pm 4 \pm 0$	3.5 ± 0.2
$K^0_S\pi^0\pi^0, K^0_S\eta$	$4.0 \pm 2.0 \pm 0.0$	1.2 ± 0.2
$K^0_S\pi^0\pi^0, K^0_S\omega$	$4.0 \pm 2.0 \pm 0.0$	1.4 ± 0.2

Yields in coherence factor analyses

Mode	$K^\pm\pi^\mp\pi^\mp\pi^\pm$	$K^\pm\pi^\mp\pi^0$	$K^\pm\pi^\mp$
$K^\mp\pi^\pm\pi^\pm\pi^\mp$	$4,044 \pm 64$	–	–
$K^\pm\pi^\mp\pi^\mp\pi^\pm$	29.1 ± 5.9	–	–
$K^\mp\pi^\pm\pi^0$	$9,594 \pm 99$	$7,342 \pm 87$	–
$K^\pm\pi^\mp\pi^0$	63.6 ± 8.8	12.5 ± 4.1	–
$K^\mp\pi^\pm$	$5,206 \pm 72$	$7,155 \pm 85$	–
$K^\pm\pi^\mp$	35.6 ± 6.2	7.3 ± 3.3	–
K^+K^-	536 ± 23	764 ± 28	–
$\pi^+\pi^-$	246 ± 16	336 ± 18	–
$K^0_S\pi^0\pi^0$	283 ± 18	406 ± 21	221 ± 15
$K^0_L\pi^0$	827 ± 30	$1,236 \pm 38$	689 ± 28
$K^0_L\omega$	296 ± 18	449 ± 22	251 ± 17
$K^0_S\pi^0$	705 ± 27	891 ± 30	473 ± 22
$K^0_S\omega$	319 ± 19	389 ± 21	183 ± 14
$K^0_S\phi$	53.0 ± 7.5	90.9 ± 9.9	42.8 ± 6.9
$K^0_S\eta(\gamma\gamma)$	128 ± 12	116 ± 11	65.5 ± 8.3
$K^0_S\eta(\pi^+\pi^-\pi^0)$	35.9 ± 6.5	36.3 ± 7.2	27.2 ± 5.4
$K^0_S\eta'$	35.7 ± 6.0	60.6 ± 7.8	30.0 ± 5.5

Phys. Rev. D **80**, 031105(R) (2009)