# Beyond the shadow

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# Outline

1. **Motivations**
   - Black hole at the galactic center
2. **Spherical toy models**
   - Photosphere $\not\Rightarrow$ horizon
   - Shadow $\not\Rightarrow$ photosphere
3. **Nonspherical examples**

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   - Photosphere $\neq$ horizon
   - Shadow $\neq$ photosphere
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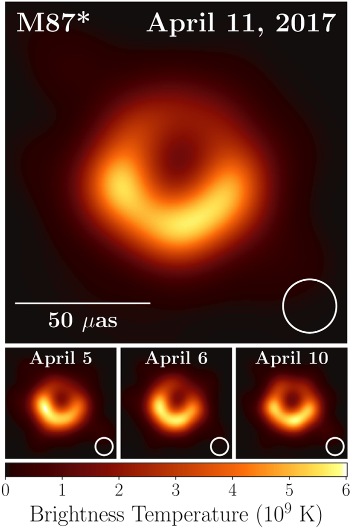
# The nature of the shadow




## The EHT Coll et al 2019 ApJL 875:L1

- images through VLBI (H Falcke et al, 2000 ApJL 528:L13 )
- library of solutions to GRMHD eqn (e.g. C F Gammie, J C McKinney and G Toth 2003 ApJ 589:444, O Porth et al 2019 ApJS 243:26)
- $T^{\mu\nu} = T^{\mu\nu}_{\rm fl} + T^{\mu\nu}_{\rm EM}$
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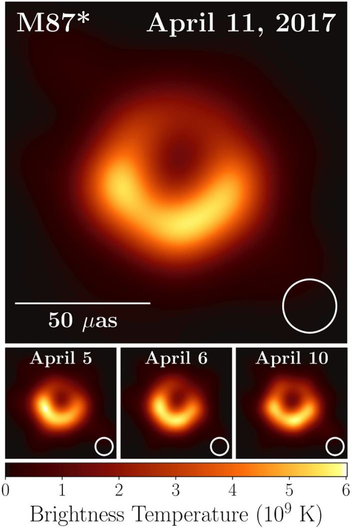


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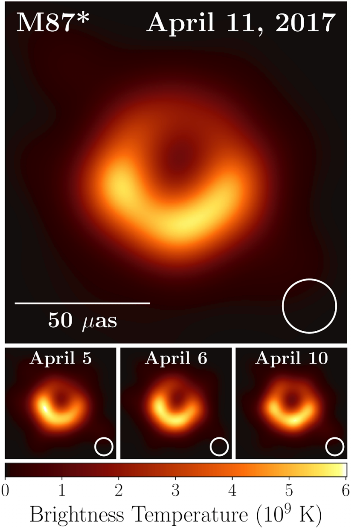


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## *Man–in–the–street* version

- a black hole surrounded by a event horizon
- (singularity $\Rightarrow$) horizon $\Rightarrow$ photon sphere (ring) $\Rightarrow$ shadow
- anticipated by nicely fitting computer simulations for supermassive BH (e.g. H Falcke et al, 2000 ApJ 528:L13; M Mościbrodzka, H Falcke and H Shiokawa, 2016 A&A 586:A38)

## Question

Are there alternatives to this scheme?

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At the same time, it is more difficult to rule out alternatives to black holes in GR, because a shadow can be produced by any compact object with a spacetime characterized by unstable circular photon orbits (Mizuno et al. 2018). Indeed, while the Kerr metric remains a solution in some alternative theories of gravity (Barausse & Sotiriou 2008; Psaltis et al. 2008), non-Kerr black hole solutions do exist in a variety of such modified theories (Berti et al. 2015). Furthermore, exotic alternatives to black holes, such as naked singularities (Shaikh et al. 2019), boson stars (Kaup 1968; Liebling & Palenzuela 2012), and gravastars (Mazur & Mottola 2004; Chirenti & Rezzolla 2007), are admissible solutions within GR and provide concrete, albeit contrived, models. Some of such exotic compact objects can already be shown to be incompatible with our observations given our maximum mass prior. For example, the shadows of naked singularities associated with Kerr spacetimes with $|a_*| > 1$ are substantially smaller and very asymmetric compared to those of Kerr black holes (Bambi & Freese 2009).

# Outline

# Photon sphere without event horizon

## Static models

$$g = -A(r)dt^2 + \frac{dr^2}{1-\frac{2m(r)}{r}} + r^2\left(\sin^2\theta d\varphi^2 + d\theta^2\right)$$

## P S Joshi, D Malafarina and R Narayan 2011 CQGra 28:235018

- vanishing radial pressure ($\Rightarrow$ matching with Schwarzschild exterior)
- $$g = -(1-M_0)\left(\frac{r}{r_b}\right)^{M_0/(1-M_0)} dt^2 + \frac{dr^2}{1-M_0} + r^2 d\Omega^2$$
- $\rho = M_0/r^2$
- $M_0 < 1 \Rightarrow$ no horizon

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# Photon sphere without event horizon

**Key feature**

regular matching with Schwarzschild exterior is always possible for spherical models when $p_r = 0$

**R Shaikh, P Kocherlakota, R Narayan and P S Joshi 2019 MNRAS 482:52**

- if $M_{tot}/r_b$ is sufficiently large $\Rightarrow \exists$ unstable photon orbits (due to Schwarzschild exterior)
- $M_0 \geq 2/3 \Rightarrow \exists$ photon sphere

**C Bambi 2013 PhRvD 87:107501**

Shadows and images of emission nearby region are Schwarzschild–like.

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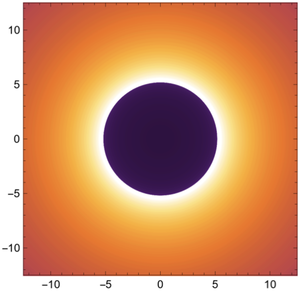
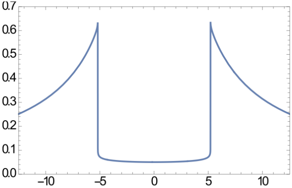



(b) $M_0 = 0.7$, JMN-1 naked singularity




(e) $M_0 = 0.7$, JMN-1 naked singularity

# Shadow without photon sphere

## Photon sphere existence

- Let us consider again

$$g = -(1 - M_0)\left(\frac{r}{r_b}\right)^{M_0/(1-M_0)} dt^2 + \frac{dr^2}{1 - M_0} + r^2 d\Omega^2$$

- $M_0 < 2/3 \Rightarrow$ no photon sphere (and no shadow)

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Build a singular model with shadow and without photon sphere

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## P S Joshi et al 2021 PhRvD 103:024015

- Start from $g = -A(r)dt^2 + \frac{dr^2}{1-\frac{2m(r)}{r}} + r^2\left(\sin^2\theta d\varphi^2 + d\theta^2\right)$
- Null geodesic equation is

$$\frac{A(r)}{\ell^2(1-2m(r)/r)}(u^r)^2 + \frac{A(r)}{r^2} = \frac{\epsilon^2}{\ell^2}$$

- the profile of $V_{eff}(r) := A(r)/r^2 \Rightarrow$ info on photosphere and shadow

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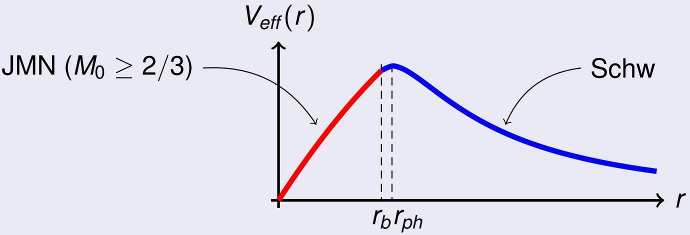

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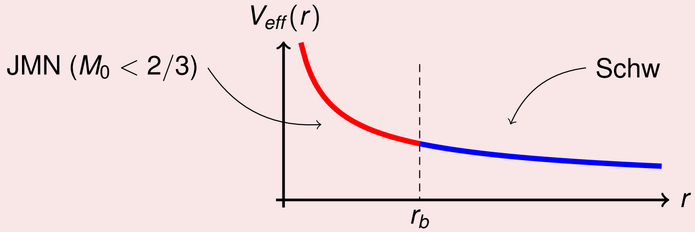

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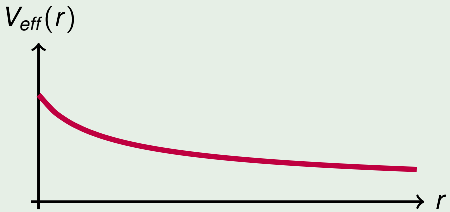
## $V_{eff}(r)$ bounded and decreasing



# Shadow without photon sphere

## An explicit example

- $$g = -\left(1+\frac{M}{r}\right)^{-2} dt^2 + \left(1+\frac{M}{r}\right)^{2} dt^2 + r^2 d\Omega^2$$
- Kerr-Schild metric (anisotropic generalization of deSitter $\rho + p_r = 0$, RG 2002 CQGra 19:4399) where $m(r) = \frac{r(M^2+2Mr)}{2(M+r)^2}$.
- Singular solution without horizon
- $V_{eff} = \frac{1}{(M+r)^2}$ ($\Rightarrow V_{eff}(0) \in \mathbb{R}$, $V'_{eff}(r) < 0$)
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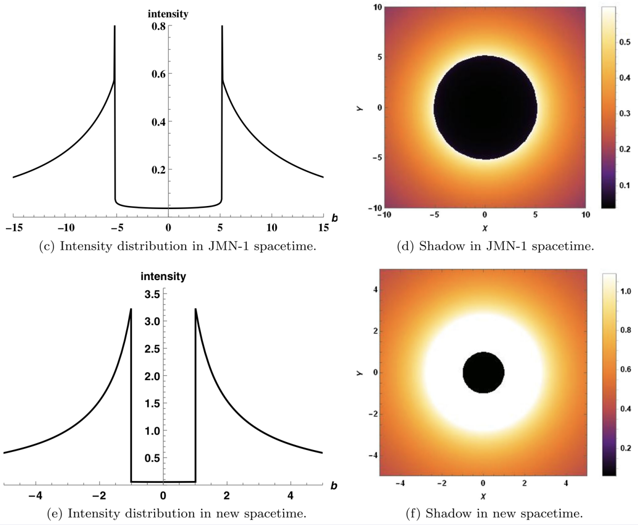
# Shadow without photon sphere

## P S Joshi et al 2020 PhRvD 102:024022




(c) Intensity distribution in JMN-1 spacetime.

(d) Shadow in JMN-1 spacetime.

(e) Intensity distribution in new spacetime.

(f) Shadow in new spacetime.

# Outline

# Shadow of double Schwarzschild

## The Bach-Weyl model (1921, repr. GReGr (2012) 44:817)

- Weyl metric $g_W = -e^{2U}dt^2 + e^{-2U}\left(e^{2K}[d\rho^2 + dz^2] + \rho^2 d\varphi^2\right)$
- $\Delta_{\mathbb{E}^3} U(\rho, z) = 0 \quad (ds_{\mathbb{E}^3} = d\rho^2 + \rho^2 d\varphi^2 + dz^2)$
  $K_\rho = \rho(U_\rho^2 - U_z^2),\ K_z = 2\rho U_\rho U_z$
- $e^{2U} = \gamma_1 \gamma_2^{-1} \gamma_3 \gamma_4^{-1}$
  $\gamma_k = \sqrt{\rho^2 + (z - a_k)^2} + a_k - z,\ \ a_1 < a_2 \le a_3 < a_4$
- The spacetime has a conical singularity between the two horizons

## P Cunha et al 2018 PhRvD 97:084020

- shadow representation techniques from A Bohn et al 2015 CQGra 32:065002
- shadow is insensitive to change in event horizon geometry

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  $\gamma_k = \sqrt{\rho^2 + (z - a_k)^2} + a_k - z,\ a_1 < a_2 \leq a_3 < a_4$
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## P Cunha et al 2018 PhRvD 97:084020

- shadow representation techniques from A Bohn et al 2015 CQGra 32:065002
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# Shadow of double Schwarzschild

## The Bach-Weyl model (1921, repr. GReGr (2012) 44:817)

- Weyl metric $g_W = -e^{2U}dt^2 + e^{-2U}\left(e^{2K}[d\rho^2 + dz^2] + \rho^2 d\varphi^2\right)$
- $\Delta_{\mathbb{E}^3} U(\rho, z) = 0 \quad (ds_{\mathbb{E}^3} = d\rho^2 + \rho^2 d\varphi^2 + dz^2)$
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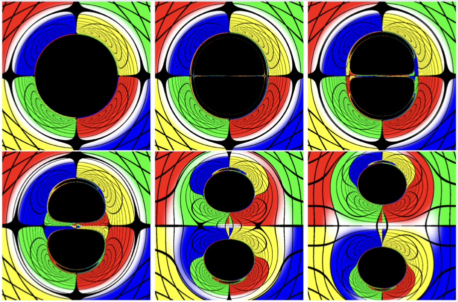

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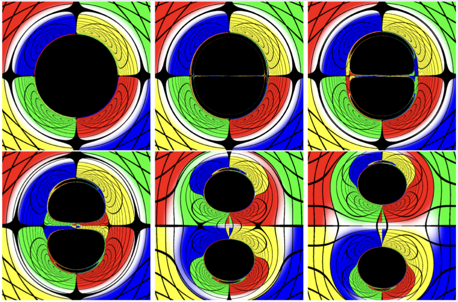

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# Shadow of the q-metric

## H Quevedo 2011 IJMPD 20:1779

- Apply $U \to (1+q)U$, $K \to (1+q)^2 K$ to the Weyl metric when $g_W = g_{Schw}$
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- $\forall q > -1 \Rightarrow$ singularity at $r = 0$ (naked if $q \neq 0$)

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- shadow exists though the singularity is naked
- correlation between shadow and repulsive gravity regions (O Luongo and H Quevedo 2014 PhRvD 90:084032, RG, O Luongo and H Quevedo 2020 PDU 30:100721)

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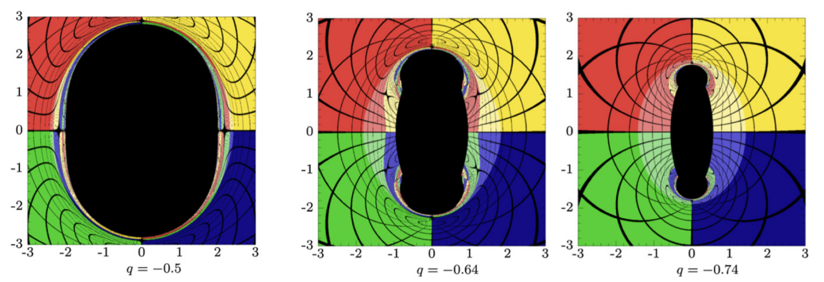


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**Does geodesical incompleteness...**

The formation of trapped surface implies BH existence, in the sense that $M \setminus J^{-}(\mathcal{I}^{+}) \neq \emptyset$.

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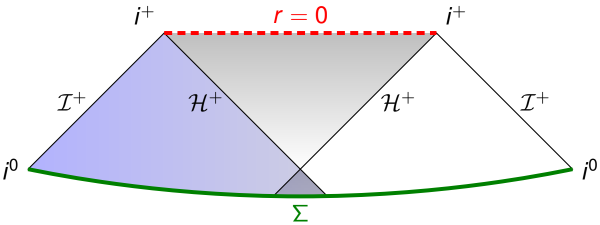

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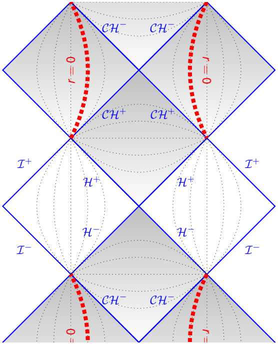
There exist Cauchy horizons $\Rightarrow$ Kerr *maximal analytic extension* is *larger* than Kerr *maximal Cauchy development*



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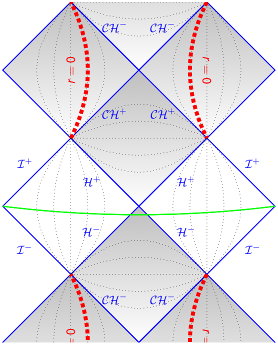

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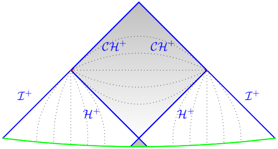

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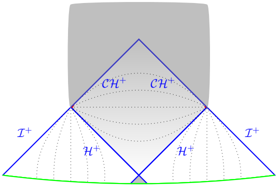


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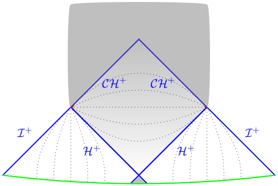


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