Precision tests of the AdS/CFT correspondence

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Quantum Gravity

Within the String Theory framework **Quantum Gravity** arises as:

- Closed Strings - One of the massless fluctuations of the closed string is the **graviton**. In the low energy limit ($l_s \to 0$) we obtain 10D Supergravity + string corrections (higher derivative terms).
- Holographically - String theory provides a field realization of the holographic principle,

The AdS/CFT correspondence: A gauge (non-gravity) SCFT in d-dim is dual to string theory on $\mathrm{AdS}_{d+1} \times \mathbb{K}$, or AdS_{d+1} Supergravity

bulk gravity from a boundary non-gravity theory

Strong/weak duality: A perturbative SCFT is a quantum gravity theory.

In particular, a 3D SCFT is a 4D Quantum Gravity

Tests of the correspondence I

Physical quantities at strong coupling in SCFT $\equiv$ Physical quantities in the dual perturbative supergravity description.

Physical quantities that allow for a dual description are:

- The free energy $F = \log \mathcal{Z}$
- Scattering amplitudes
- Entanglement entropy
- Correlation functions of gauge invariant, local operators
- Expectation values of BPS Wilson loops
- Defect correlators on BPS Wilson lines

Tests of the correspondence II

The AdS/CFT has already passed quite a lot of non-trivial tests. However, precision tests, that is tests beyond the leading behavior (classical string/supergravity), are still under investigation and a lot of activity points towards it.

"New" approaches to SCFTs (revival of old ones)

- **Integrability**
- **(Super)conformal Bootstrap**
- **Supersymmetric Localization**

provide Exact results in SCFTs

$\Downarrow$

the possibility to perform such tests very efficiently

Supersymmetric Localization [E. Witten (1988), V. Pestun (2007)]

$$\mathcal{Z} = \int e^{-S} \longrightarrow \mathcal{Z}(t) = \int e^{-S+tQV}\ , \qquad QS = 0,\ \ Q^2 = 0$$

Supersymmetry implies that the functional integral does not depend on t. Therefore, we compute it at $t \to +\infty$ where it localizes at the loci $QV = 0$.

The semiclassical approximation becomes exact $\Longrightarrow$ **Matrix Model**

$$\langle O \rangle = \int e^{-S+tQV}\ O$$

O = local operator or a Wilson loop

Precision tests of $\mathrm{AdS}_4/\mathrm{CFT}_3$

In the rest of the talk I will present a recent example of **precision test** of the correspondence.

Well-known formulations of the correspondence

- 4D $\mathcal{N} = 4$ SYM dual to type IIB string theory on $\mathrm{AdS}_5 \times S^5$
- 3D $\mathcal{N} = 6$ Chern-Simons-matter theory dual to IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$
- .... lower dimensional versions

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$\mathcal{N} = 6$ ABJM theory

Aharony, Bergman, Jafferis, Maldacena, JHEP10 (2008)

Aharony, Bergman, Jafferis, JHEP11 (2008)

$U(N)_k \times U(N)_{-k}$ CS-gauge vectors A_μ, $\hat{A}_\mu$ minimally coupled to

$SU(4)$ complex scalars C_I, $\bar{C}^I$ and fermions ψ_I, $\bar{\psi}^I$, $I = 1, \ldots, 4$

in the (anti)bifundamental representation of the gauge group with non-trivial potential.

$$S_{\text{ABJM}} = S_{\text{CS}} + S_{\text{mat}} + S^{\text{bos}}_{\text{pot}} + S^{\text{ferm}}_{\text{pot}}$$

$$S_{\text{CS}} = \frac{k}{4\pi i} \int d^3x \, \varepsilon^{\mu\nu\rho} \Big\{ \text{Tr} \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} i A_\mu A_\nu A_\rho \right) - \text{Tr} \left(\hat{A}_\mu \partial_\nu \hat{A}_\rho + \frac{2}{3} i \hat{A}_\mu \hat{A}_\nu \hat{A}_\rho \right) \Big\}$$

$$S_{\text{mat}} = \int d^3x \, \text{Tr} \Big[D_\mu C_I D^\mu \bar{C}^I - i \bar{\Psi}^I \gamma^\mu D_\mu \Psi_I \Big]$$

$$D_\mu C_I = \partial_\mu C_I + i A_\mu C_I - i C_I \hat{A}_\mu$$

Coupling constant $\lambda = N/k$

Dual Geometry

For $N \gg k^5$: Dual to M–theory on $\mathrm{AdS}_4 \times S^7/Z_k$

For $k \ll N \ll k^5$: Dual to Type IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$

$$ds^2 = R^2(ds^2_{\mathrm{AdS}_4} + ds^2_{\mathbb{CP}^3})$$

$$e^{2\phi} = 4\frac{R^2}{k^2} \qquad F^{(4)} = \frac{3}{2}kR^2\mathrm{vol}(\mathrm{AdS}_4) \qquad F^{(2)} = \frac{k}{4}dA$$

$$\boxed{\frac{R}{l_s} = \sqrt{\pi}(2\lambda)^{\frac{1}{4}}} \qquad \boxed{g_s = \sqrt{\pi}\,\frac{(2\lambda)^{\frac{5}{4}}}{N}}$$

Precision tests with BPS Wilson loops

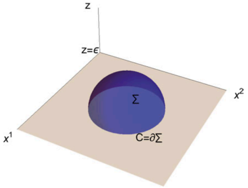
$$W(C) = \mathrm{Tr}_{fund} P e^{-i\int_C \mathcal{L}} \qquad \mathcal{L} = A_\mu dx^\mu + \cdots$$

$\cdots \longrightarrow$ include couplings to matter fields tuned such that W is invariant under a fraction of supersymmetries (BPS)

Precision tests based on the **identification** [J. Maldacena, PRL (1998); S-J Rey, J-T Yee, EPJ C (1998)]

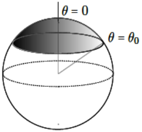
$$\boxed{\langle W(C)\rangle \equiv \int e^{-S_{\mathrm{SCFT}}} W(C) = \mathcal{Z}_{\mathrm{string}}}$$

$\mathcal{Z}_{\mathrm{string}} = \int e^{-S}$ partition function of string theory on AdS $\times \mathbb{K}$ with an *open string* worldsheet given by a disk topology ending on C at the AdS boundary



Latitude Wilson loops

$$W_F(\nu) = \mathrm{Tr} P \exp\left[-i \int_\Gamma d\tau \mathcal{L}(\nu,\tau)\right] \qquad \boxed{\nu = \sin\theta_0}$$



$$\mathcal{L}(\nu,\tau) = \begin{pmatrix} \dot{x}^\mu A_\mu - \frac{2\pi i}{k}|\dot{x}| M_J^{\ I}(\nu) C_I \bar{C}^J & -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\, \eta_I(\nu)\, \bar{\psi}^I \\ -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\, \psi_I\, \bar{\eta}^I(\nu) & \dot{x}^\mu \hat{A}_\mu - \frac{2\pi i}{k}|\dot{x}| M_J^{\ I}(\nu) \bar{C}^J C_I \end{pmatrix}$$

It preserves 1/6 of susy charges $\Longrightarrow$ **1/6 BPS**

Enhancement of SUSY $W_F(\nu = 1) \Longrightarrow$ **1/2 BPS** [Drukker,Trancanelli (2010)]

Exact result in SCFT

Applying **supersymmetric localization**, the ABJM partition function on S^3 reduces to a finite Matrix Integral [M.S. Bianchi, Griguolo, Mauri, SP, Seminara (2018)]

$$\langle W_F(\nu)\rangle_\nu \sim \mathrm{Re}\left(e^{i\frac{\pi\nu}{2}}\langle W_B(\nu)\rangle_\nu\right)$$

$$\langle W_B(\nu)\rangle_\nu = \int \prod_{a=1}^{N} d\lambda_a\ e^{i\pi k\lambda_a^2} \prod_{b=1}^{N} d\mu_b\ e^{-i\pi k\mu_b^2} \quad \left(\frac{1}{N}\sum_{a=1}^{N} e^{2\pi\sqrt{\nu}\,\lambda_a}\right)$$
$$\times \frac{\prod_{a<b}^{N} \sinh\sqrt{\nu}\pi(\lambda_a-\lambda_b)\sinh\frac{\pi(\lambda_a-\lambda_b)}{\sqrt{\nu}} \prod_{a<b}^{N} \sinh\sqrt{\nu}\pi(\mu_a-\mu_b)\sinh\frac{\pi(\mu_a-\mu_b)}{\sqrt{\nu}}}{\prod_{a=1}^{N}\prod_{b=1}^{N}\cosh\sqrt{\nu}\pi(\lambda_a-\mu_b)\cosh\frac{\pi(\lambda_a-\mu_b)}{\sqrt{\nu}}}$$

$\{\lambda_a\}$ and $\{\mu_b\}$ are the eigenvalues of the Cartan matrices of the two $U(N)$'s.

Matrix Model result at $\lambda \gg 1$

$$\langle W_F(\nu)\rangle_\nu = \sum_g g_s^{2g-1} \langle W_F(\nu, g)\rangle_\nu \; , \qquad g_s = \frac{2\pi i}{k}$$

Finite!

At genus 0

$$\langle W_F(\nu)\rangle_\nu \Big|_{g=0} = -i \, \frac{2^{-\nu-2}\, \xi^\nu \, \Gamma\left(-\frac{\nu}{2}\right)}{\sqrt{\pi}\, \Gamma\left(\frac{3-\nu}{2}\right)} \qquad \lambda = \frac{\log^2 \xi}{2\pi^2} + \frac{1}{24} + \mathcal{O}(\xi^{-2})$$

It is convenient to consider the ratio (for $\xi \gg 1$)

$$\boxed{\frac{\langle W_F(1)\rangle_1}{\langle W_F(\nu)\rangle_\nu}\Big|_{g=0} = e^{\pi\sqrt{2\lambda}(1-\nu)} \, \frac{\Gamma(1+\nu)\Gamma\left(\frac{3-\nu}{2}\right)}{\Gamma\left(\frac{1+\nu}{2}\right)}}$$

[M.S. Bianchi, Griguolo, Mauri, SP, Seminara (2018)]

The string dual prediction at $\lambda \gg 1$

Semiclassical approach to compute the string partition function with

$$X^m \to X^m + \epsilon Y^m \,, \qquad m = 0, \cdots, 9$$

X^m = classical string solution minimizing the string worldsheet
Y^m = small fluctuations (only transverse, $Y^0 = Y^1 = 0$)

Compute $\mathcal{Z}_{\rm string}$ at quadratic order in the fluctuations

$$\frac{\langle W_F(1)\rangle_1}{\langle W_F(\nu)\rangle_\nu}\Big|_{g=0} = \frac{\mathcal{Z}_{\rm string}(1)}{\mathcal{Z}_{\rm string}(\nu)} = e^{-(S(1)-S(\nu))} \underbrace{\left(\frac{\det \mathcal{F}(1) \det \mathcal{B}(\nu)}{\det \mathcal{B}(1) \det \mathcal{F}(\nu)}\right)^{1/2}}_{\text{one-loop string corrections}}$$

$$\frac{\langle W_F(1)\rangle_1}{\langle W_F(\nu)\rangle_\nu}\Big|_{g=0} = e^{\pi\sqrt{2\lambda}(1-\nu)} \, \frac{\Gamma(1+\nu)\Gamma\left(\frac{3-\nu}{2}\right)}{\Gamma\left(\frac{1+\nu}{2}\right)}$$

[Aguilera-Damia, Faraggi, Pando Zayas, Rathee, Silva (2018); Medina-Rincon (2019); David, de Leon Ardon, Faraggi, Pando Zayas, Silva (2019); Giombi, Tseytlin (2020)]

Comments & Perspectives

This is a highly **non-trivial test**, since

- The Matrix Model has been guessed
- The string calculation involves divergences which need to be regularized. They eventually cancel to match the finite field theory result if suitable boundary conditions for fermionic string modes are chosen.
  The understanding and control of these divergences is crucial for precision holography.

What's next?

- Go beyond the quadratic approximation
- Generalization to m-wound Wilson loops
- Generalization to Wilson loops in different representations
- Wilson loops at the intersection between localization, integrability and superconformal bootstrap