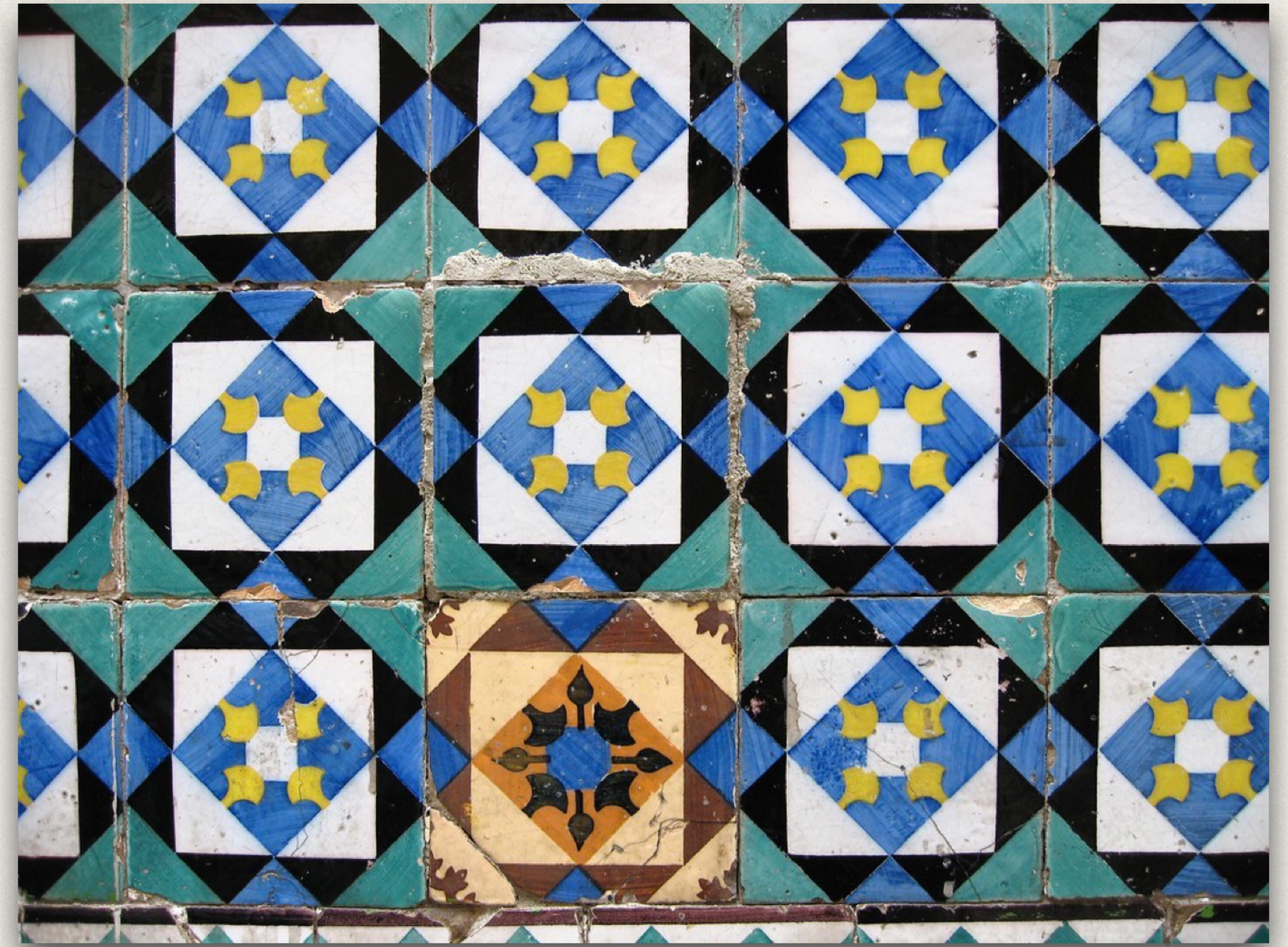


New directions for QFT in Curved Spacetime

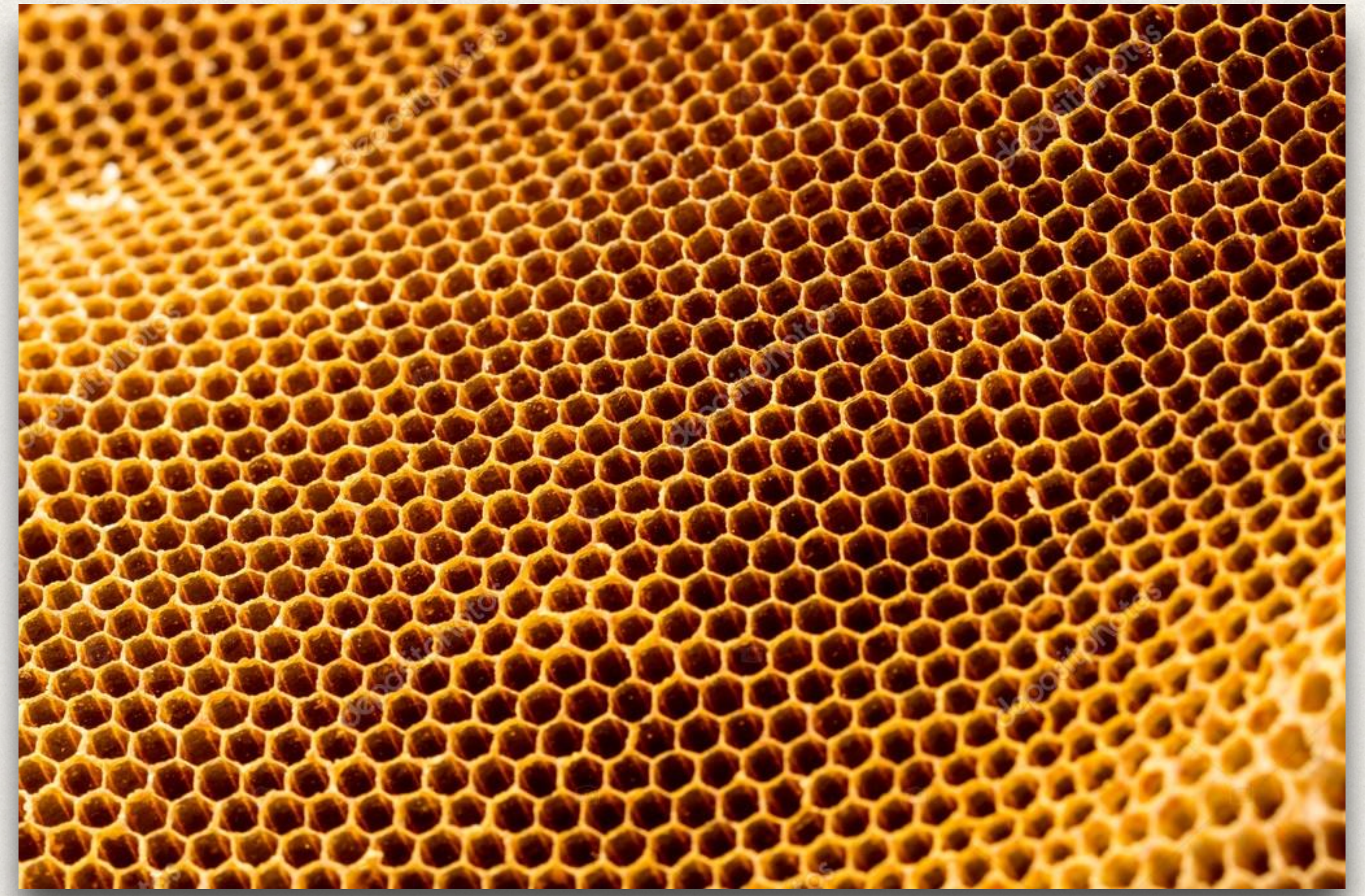
Sigrav XXIV - Urbino

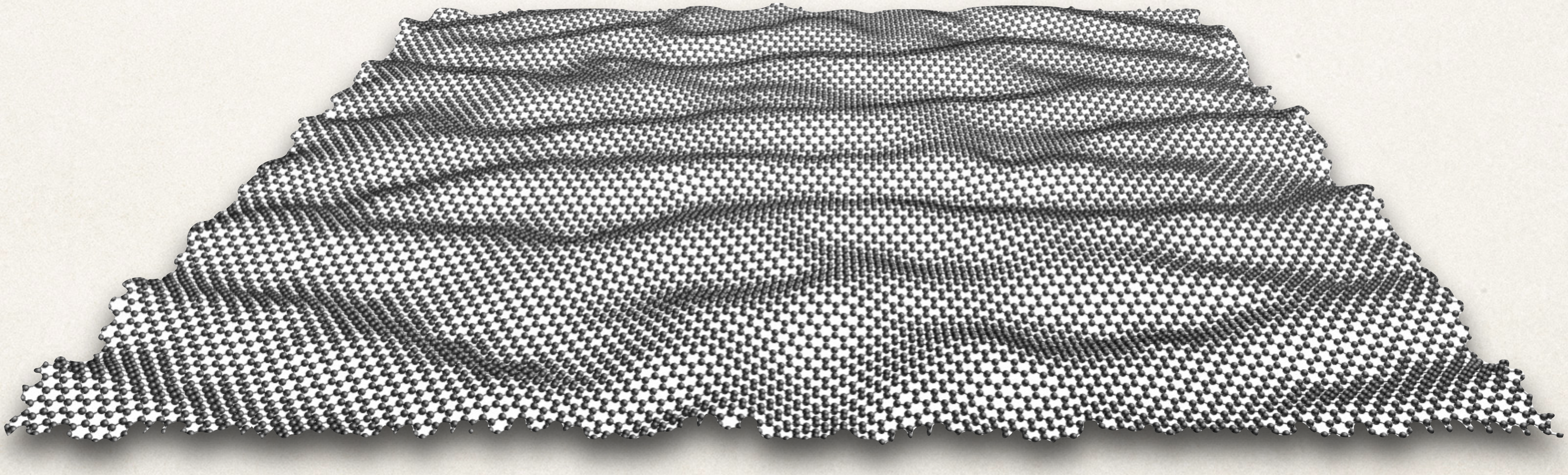
Vincenzo Vitagliano (University of Genova), 8 September 2021

Università di Genova

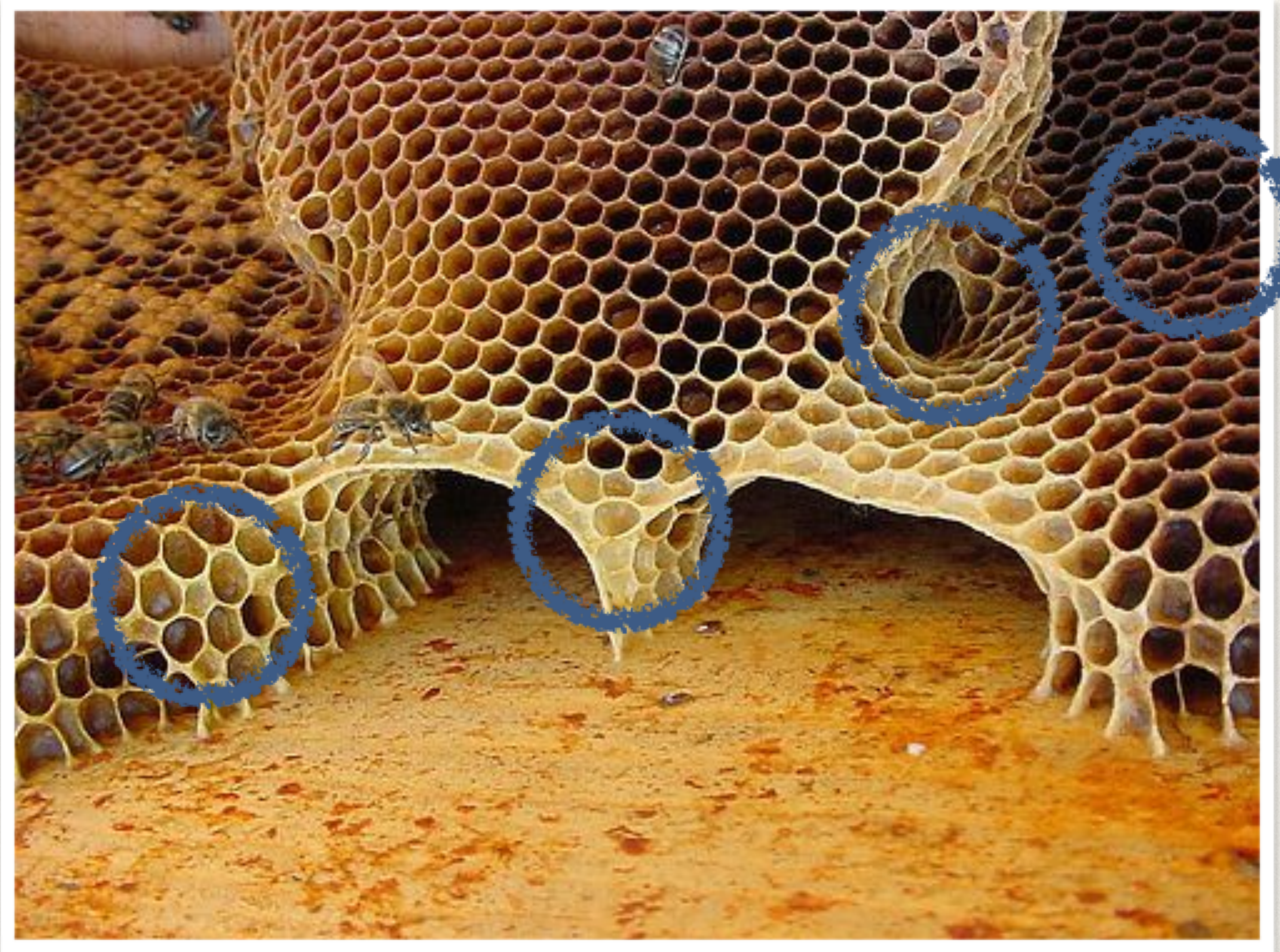
Take home

Symmetry breaking mechanisms are modified in curved spaces by effective masses of purely geometrical origin





Fasolino et al, Nature Materials (2011)



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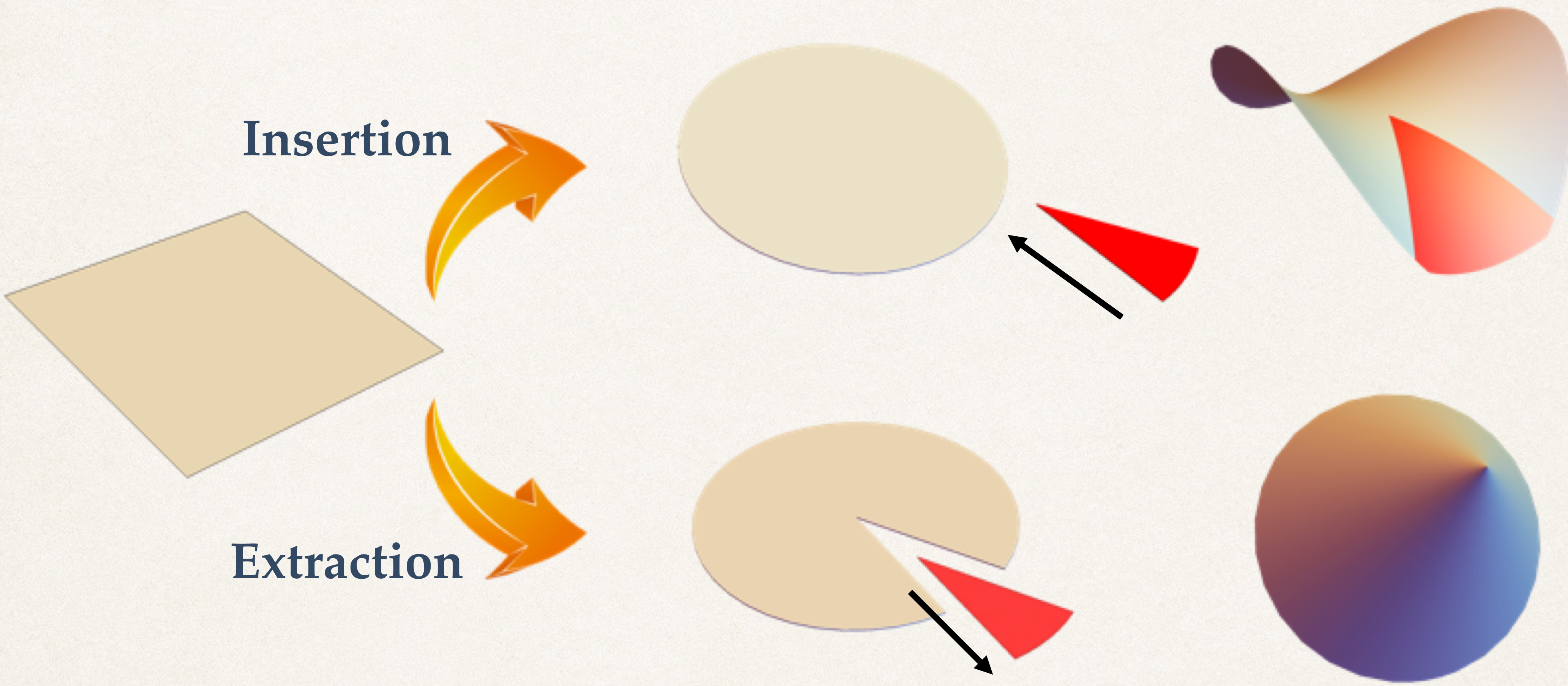
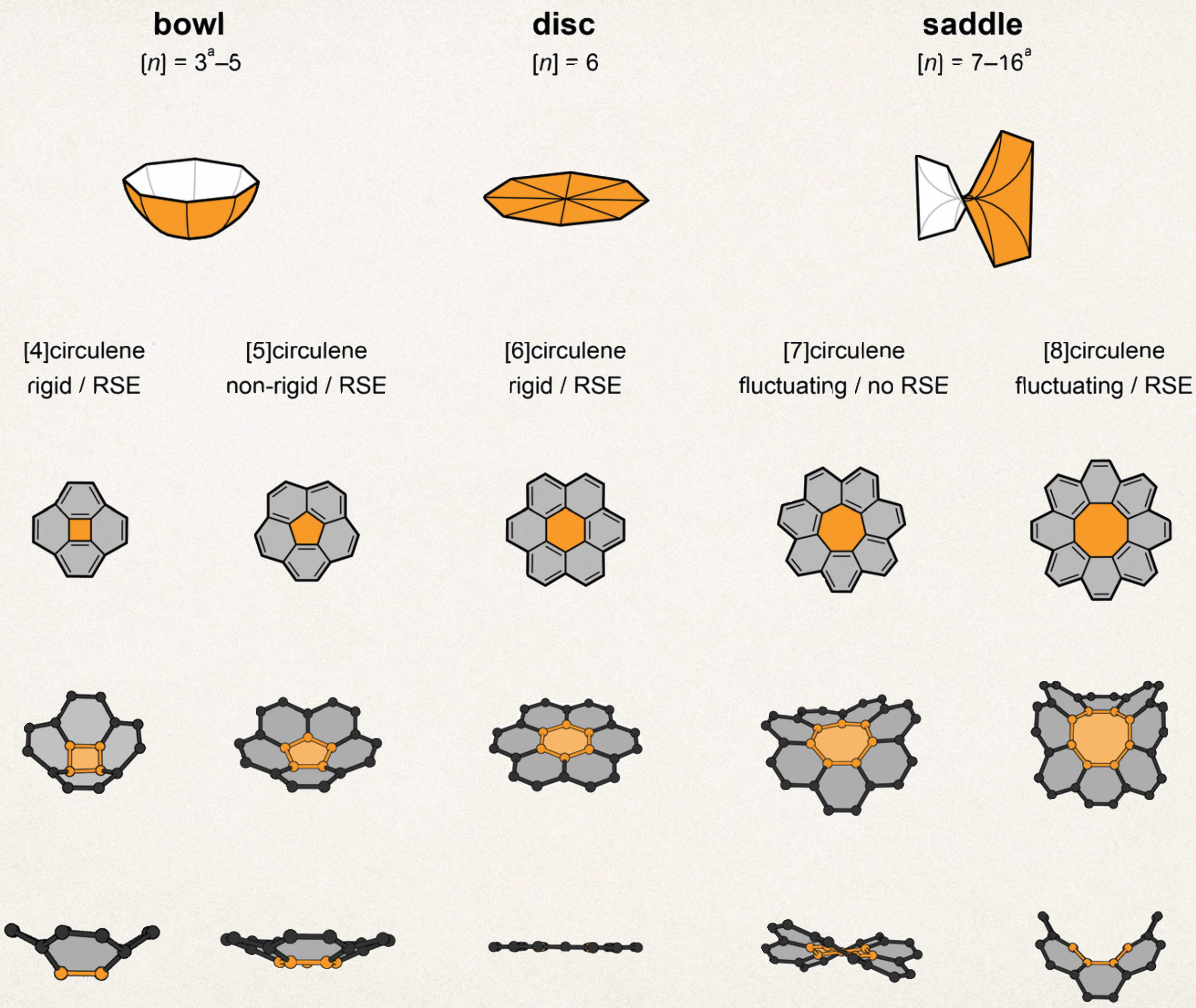


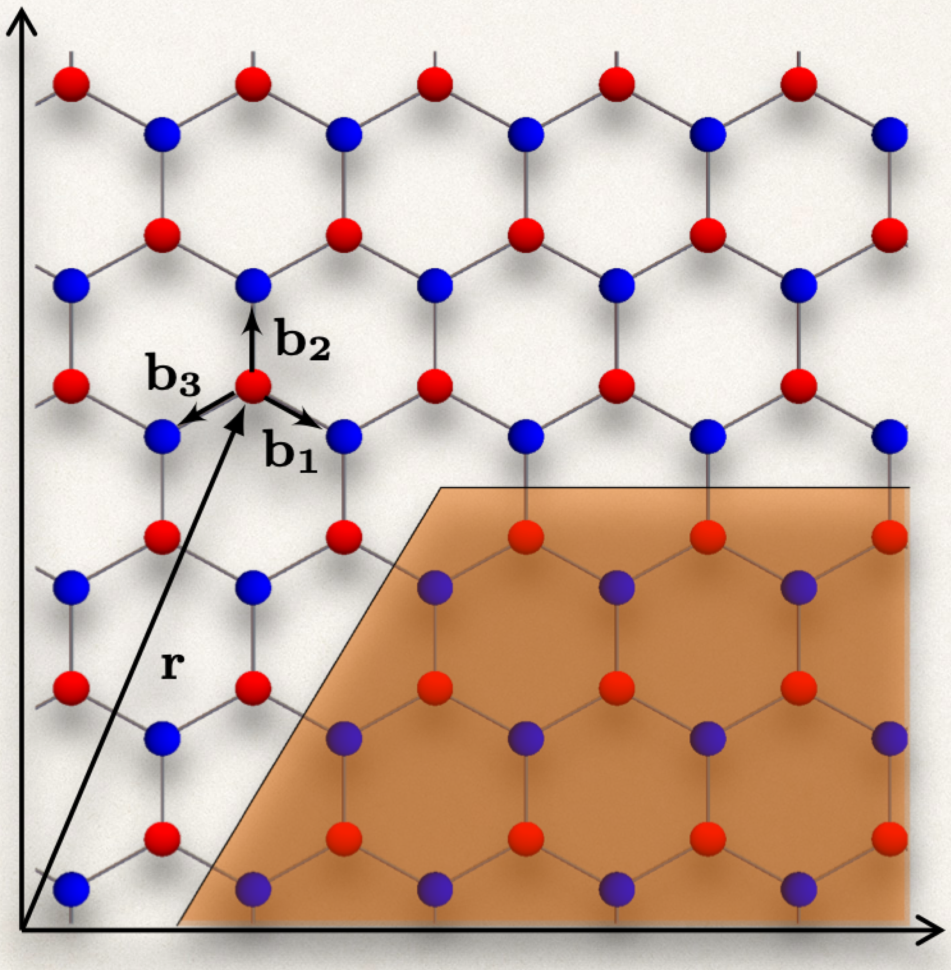
Castro, Flachi, Ribeiro & me, PRL (2018)
Flachi & me, PRD (2019)



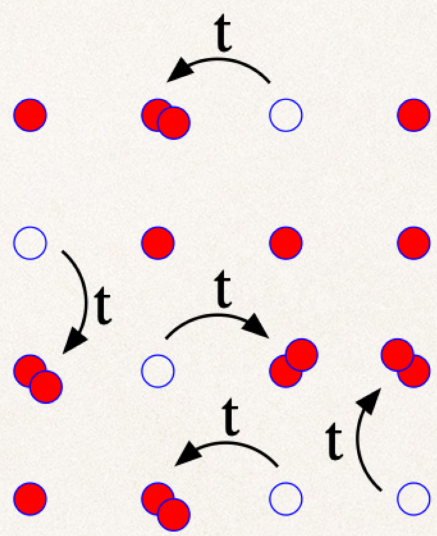
Rickhaus et al., Chemical Society Reviews (2017)

Hubbard model

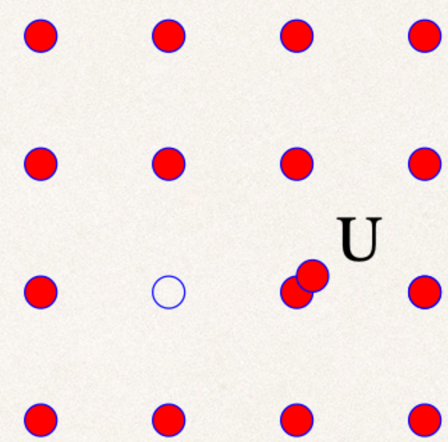
$$\mathsf{H} = -t \sum_{\mathbf{r},\ i,\ \sigma=\pm} u_\sigma^\dagger(\mathbf{r}) v_\sigma(\mathbf{r}+\mathbf{b}_i) + \text{H.C.} + \frac{U}{4} \sum_{\mathbf{r},\sigma,\sigma',i} \left(n_\sigma(\mathbf{r}) n_{\sigma'}(\mathbf{r}) + n_\sigma(\mathbf{r}+\mathbf{b}_i) n_{\sigma'}(\mathbf{r}+\mathbf{b}_i)\right)$$



Hubbard model



Hopping



Interaction

Scalettar, Quantum Materials (2016)

Hubbard model

Bosonization

$$\mathscr{L} = \bar{\psi}_\sigma \imath \not{\partial} \psi_\sigma + \left(\sigma \bar{\psi}_\sigma \phi \psi_\sigma\right) + \frac{\phi^2}{2\lambda} \quad ; \qquad \sigma = \pm$$

$$\psi_\sigma^T \;=\; \left(\psi_\sigma^{A1}, \psi_\sigma^{B1}, \psi_\sigma^{A2}, \psi_\sigma^{B2}\right)$$

$$\psi_\sigma^{IJ} = \int d^2 p \, e^{-\imath \mathbf{p}\cdot\mathbf{r}} z_\sigma^{IJ}(\mathbf{p})$$

e.g. Weng et al, PLB[R] (1990); Schultz, PRL (1990)

The metric

A conical metric…

$$ds^2 = d\tau^2 + dr^2 + \alpha^2 r^2 d\varphi^2$$

…and its regularisation

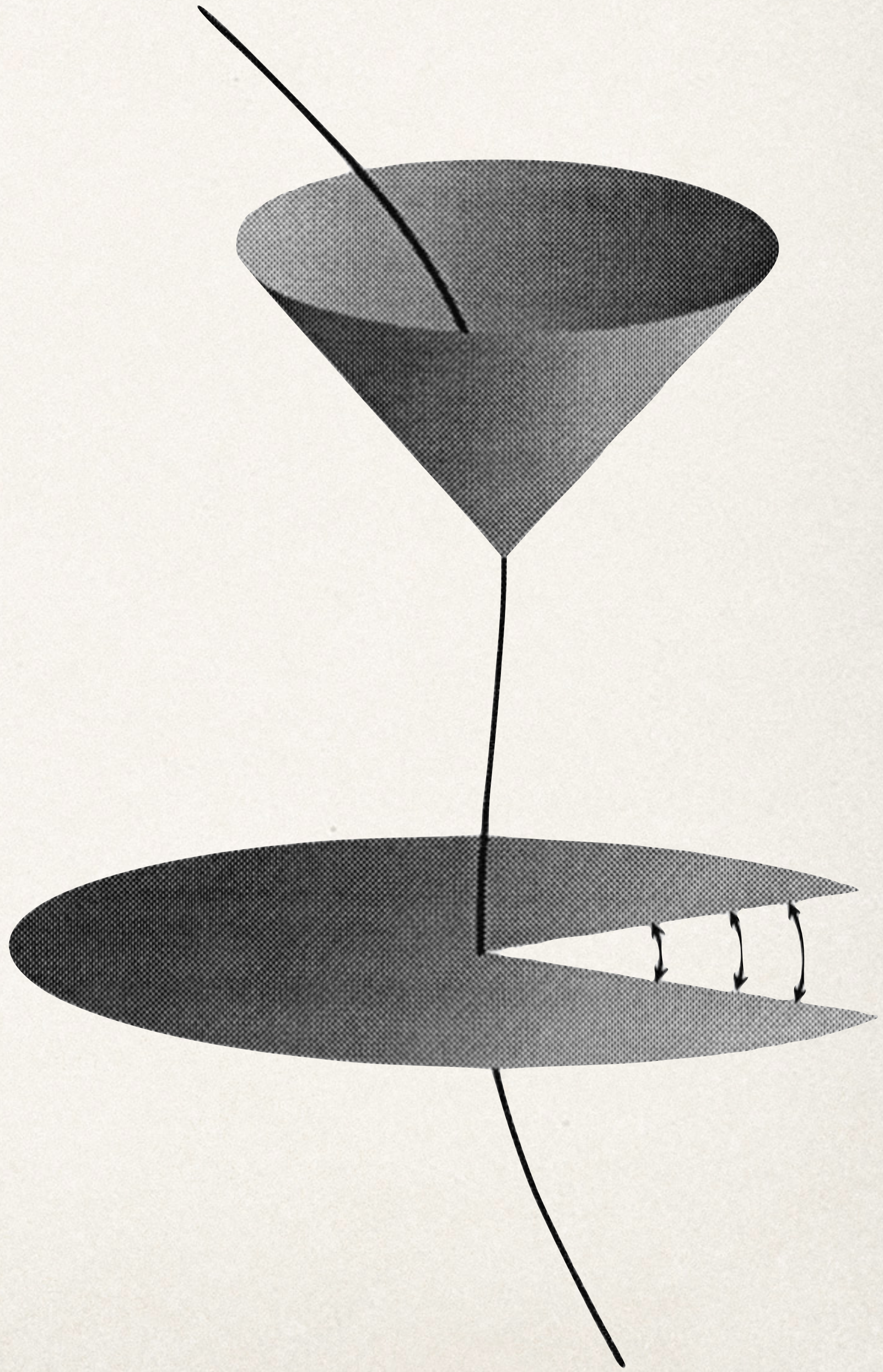
$$d\tilde{s}^2 = d\tau^2 + f_\epsilon(r) dr^2 + \alpha^2 r^2 d\varphi^2$$

With

1) $\lim_{\epsilon \to 0} f_\epsilon(r) = 1$;

2) $f_\epsilon(r) \approx 1$ for $r \gg \epsilon$;

3) $f_\epsilon(r) = \text{const}$ for $r = 0$



The effective action

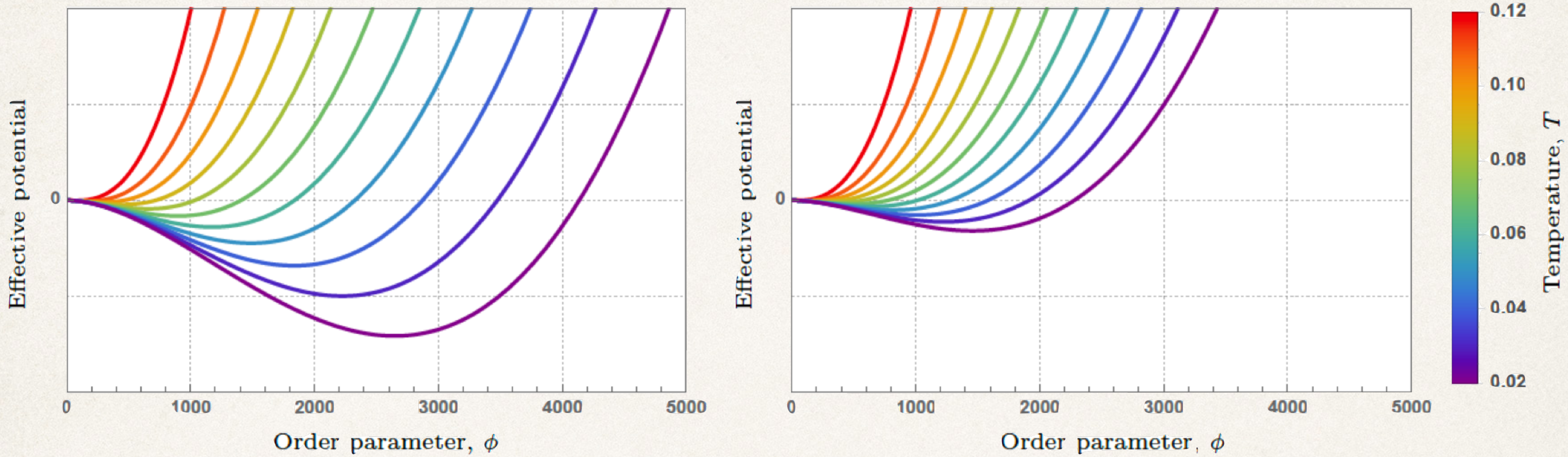
$$\tilde{\Gamma}\left[\phi\right] = -\int d^3x \sqrt{\tilde{g}}\,\frac{\phi^2}{2\lambda} + \mathrm{Tr}\log\left(\imath\gamma^\mu \tilde{D}_\mu \pm \phi\right)$$

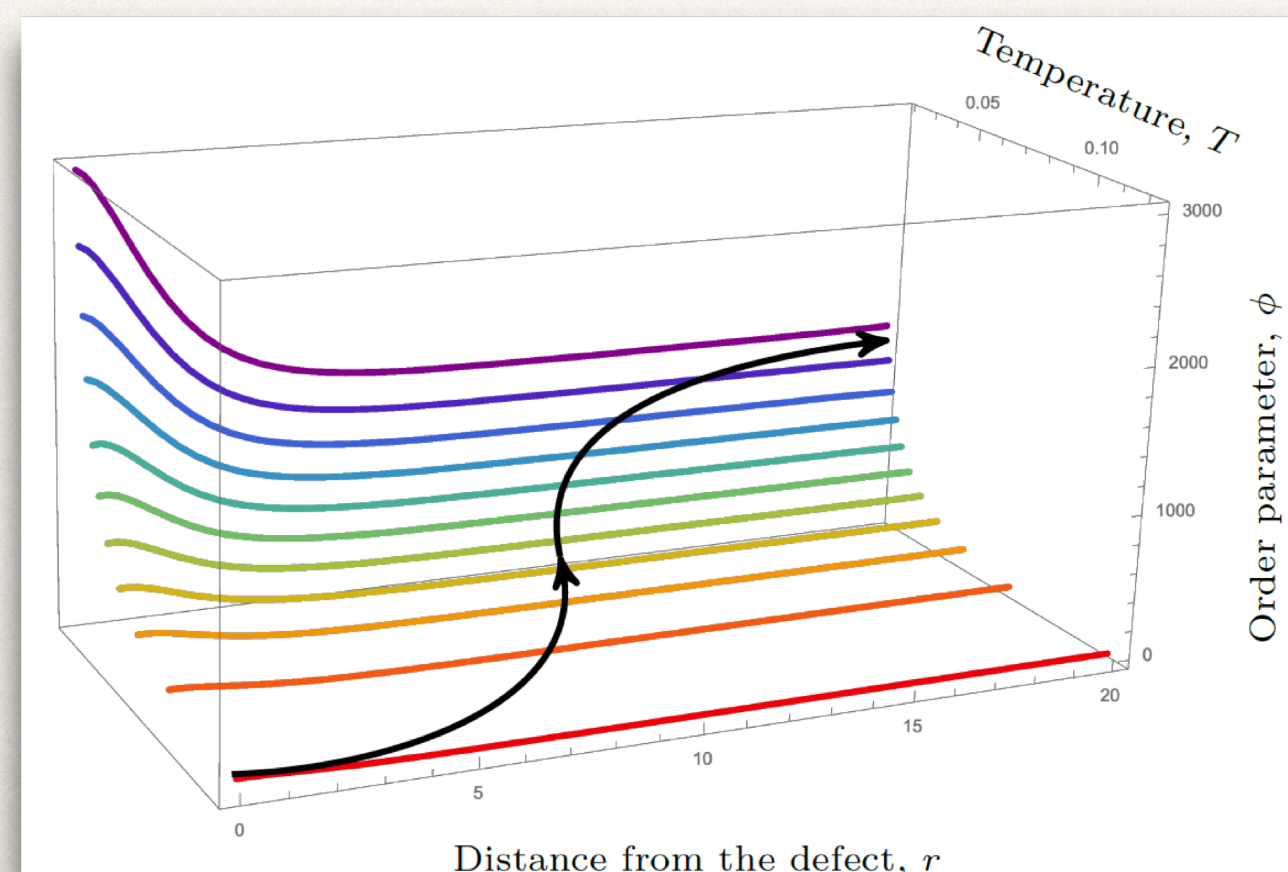
$$\tilde{\Gamma}\left[\phi\right] = -\int d^3x \sqrt{\tilde{g}}\,\frac{\phi^2}{2\lambda} + \frac{1}{2}\sum_{p=\pm}\log\det\left(\tilde{\Box} + \frac{\tilde{R}}{4} + \phi^2 \pm \sqrt{\tilde{g}^{rr}}\,\phi'\right)$$

Castro, Flachi, Ribeiro & me, PRL (2018)

Flachi & me, PRD (2019)

Results

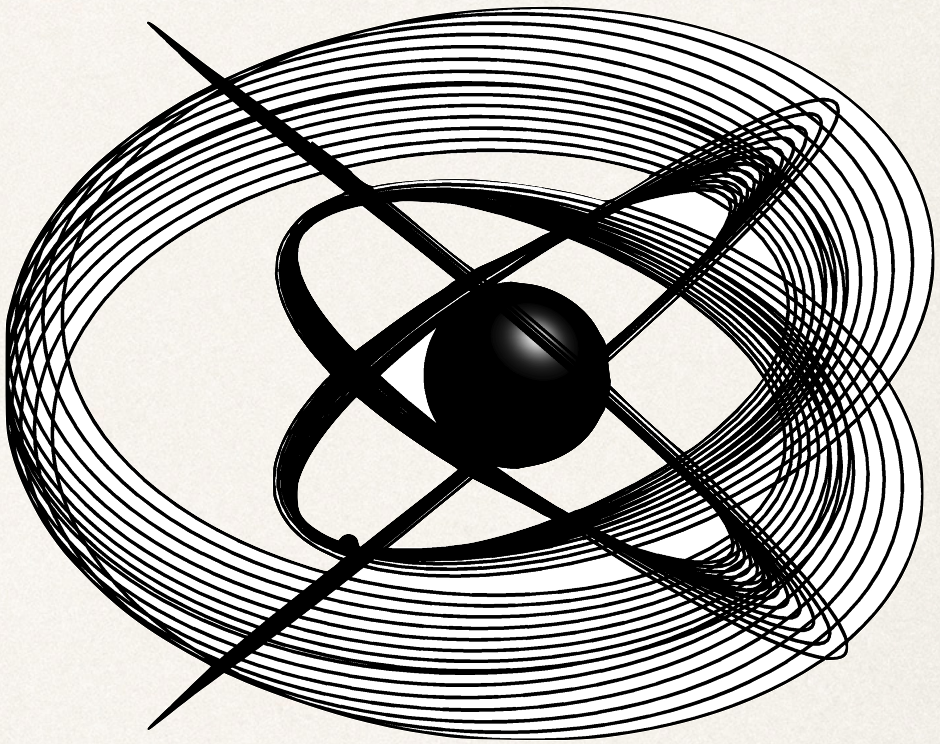




Castro, Flachi, Ribeiro & me, PRL (2018)
Flachi & me, PRD (2019)

How does the interplay between strong interactions and gravity work?

BHs outskirts:curvature effects comparable to Lambda_QCD



$T_{BH} \sim 1/m_{BH}$

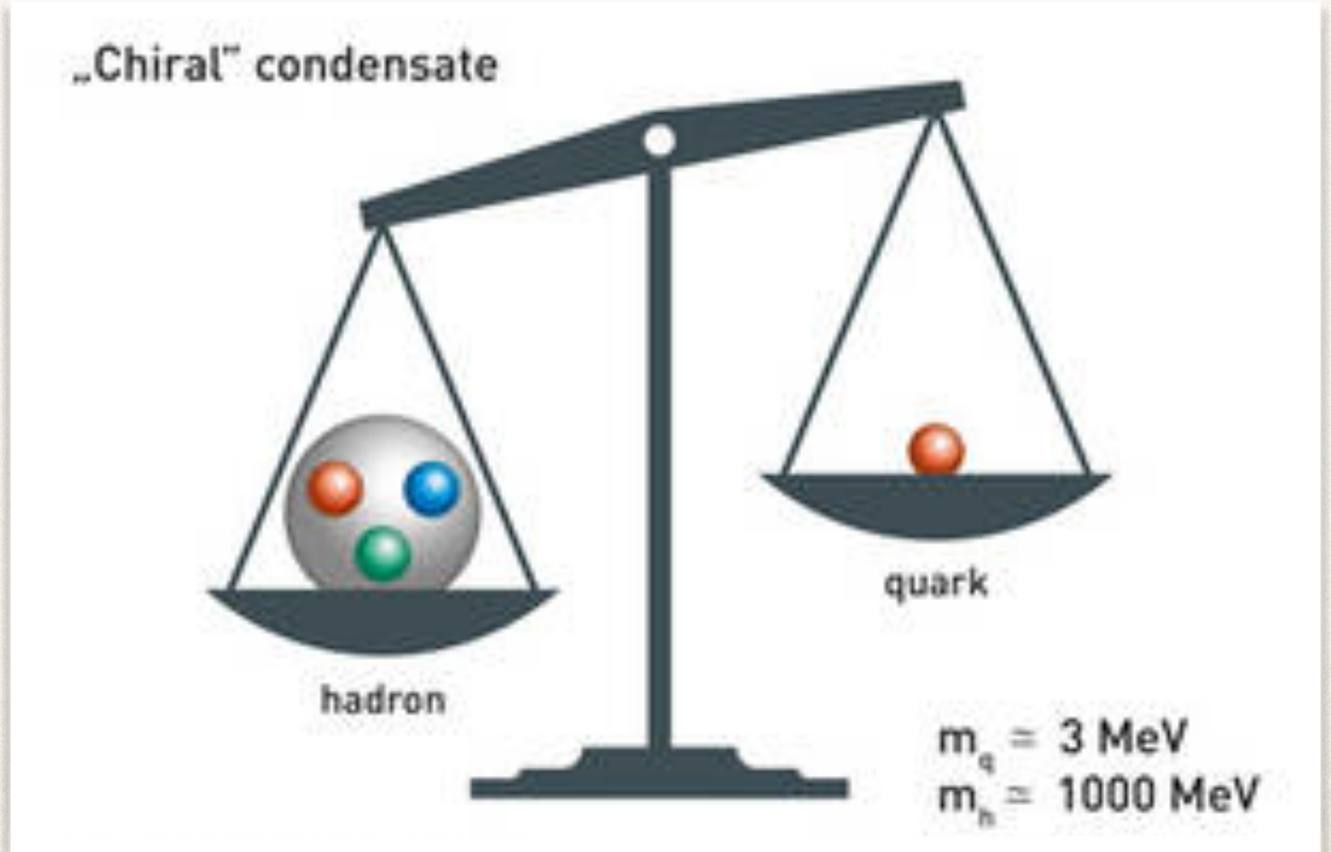
Photons, neutrinos and gravitons… → **…electrons…** → **…muons, pions and heavier hadrons**

Effective field theory models

Massive fermions: spontaneously broken symmetry

$$S_{\rm NJL} = \int d^4x\sqrt{g}\left\{\bar{\psi}i\gamma^\mu\nabla_\mu\psi + \frac{\lambda}{2N}(\bar{\psi}\psi)^2\right\}$$

Generation dynamical effective mass $M_{\rm eff} \sim \langle\bar{\psi}\psi\rangle$



The *gourmet* recipe

NJL + Large N approximation

+ Hubbard-Stratonovich transf.

$$\Gamma = -\int d^4x\sqrt{g}\left(\frac{\sigma^2}{2\lambda}\right) + \mathrm{Tr}\ln(i\gamma^\mu\nabla_\mu - \sigma)$$

with $\sigma[r] \equiv -\frac{\lambda}{N}\bar{\psi}\psi$

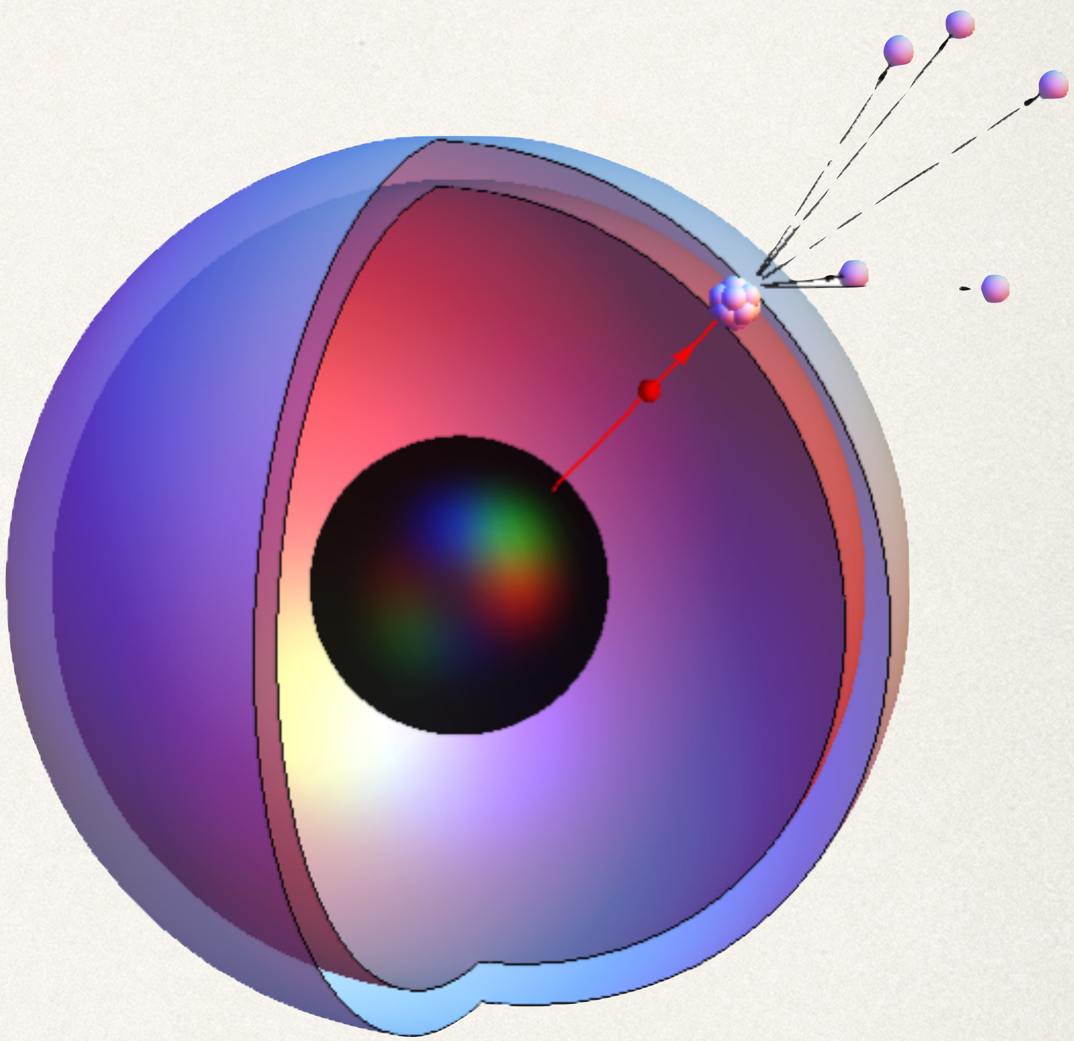
Flachi, PRL (2013)

Flachi and Fukushima, PRL (2014)

Flachi, Fukushima & me, PRL (2015)

The *chiral gap effect*

QCD phase diagram is in principle much more complicated…



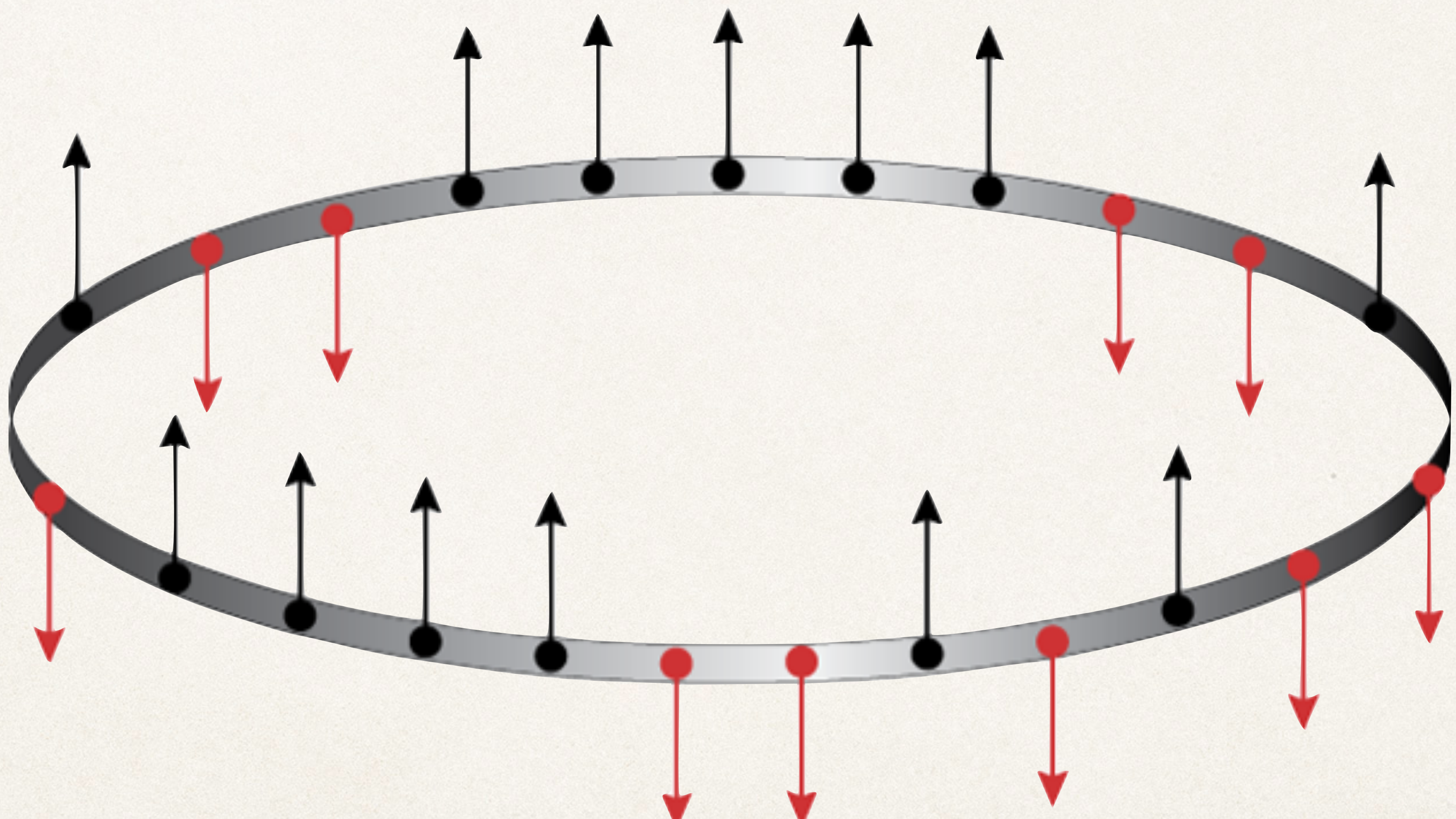
Flachi, PRL (2013)

Flachi and Fukushima, PRL (2014)

Flachi, Fukushima & me, PRL (2015)

A Boson Carousel

Cold atoms trap, $CsNiCl_3$ and NENP antiferromagnets



Flachi and me, JoPA(2021)

Corradini et al (including me), JoPA(2021)

Cf. also with Cominotti et al, PRL(2014)

Summary and some food for thought…

- Symmetry breaking response to geometrical deformations, from BHs to 2D graphene-like kirigami

- Special defects configurations?

- Higher dimensions?

- 3D lattice structures and nature of the defects?

HS transformation

$$H = -t \sum_{\langle i,j \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.}) + U \sum_j n_{i\uparrow} n_{i\downarrow}$$

$$n_{i\uparrow} n_{i\downarrow} = \frac{\rho_i^2}{4} - (S_i^z)^2$$

$$\rho_i = n_{i\uparrow} + n_{i\downarrow}$$

$$S_i^z = \frac{1}{2} \sum_\sigma c_{i\sigma}^\dagger \sigma_z c_{i\sigma}$$

$$e^{U \sum_i n_{i\uparrow} n_{i\downarrow}} = \int \prod_i \frac{d\phi_i \, d\Delta_i \, d^2 \mathbf{n}_i}{4\pi^2 U} \exp \sum_i \left(\frac{\phi_i^2}{U} + i\phi_i \rho_i + \frac{\Delta_i^2}{U} - 2\Delta_i \mathbf{n}_i \cdot \mathbf{S}_i \right)$$

$$Z = \int \prod_i \frac{dc_i^\dagger dc_i \, d\phi_i \, d\Delta_i \, d^2 \mathbf{n}_i}{4\pi^2 U} \exp \left(-\int_0^\beta L(\tau) \right)$$

$$L(\tau) = \sum_{i\sigma} c_{i\sigma}^\dagger \partial_\tau c_{i\sigma} - t \sum_{\langle i,j \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.}) + \sum_i \left(\frac{\phi_i^2}{U} + (i\phi_i - \mu)\rho_i + \frac{\Delta_i^2}{U} - 2\Delta_i \mathbf{n}_i \cdot \mathbf{S}_i \right)$$

Effective action calculation

$$D = \log\det\left(\tilde{\Box} + E_p\right)$$

$$\zeta_p(s) = \frac{1}{\Gamma(s)}\int_0^\infty \frac{d\tau}{\tau^{s-1}}\mathrm{Tr}\, e^{-\tau(\tilde{\Box}+E_p)}$$

$$\mathscr{K}(\tau) = \frac{e^{-\tau E_p}}{(4\pi\tau)^{3/2}}\sum_k a_k\tau^k$$

$$a_0 = 1,\ a_1 = 0, a_2 = \frac{1}{180}\left(\tilde{R}_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta} - \tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu}\right) - \frac{1}{30}\tilde{\Delta}\tilde{R} + \frac{1}{6}\tilde{\Delta}E_p + \frac{1}{12}W^{\mu\nu}W_{\mu\nu}$$

$$D = \int_{\mathrm{vol}}\left(\zeta'(0) + \zeta(0)\log\ell^2\right)$$

Boundary conditions calculation

$$\imath\bar{\psi}\gamma^\mu\nabla_\mu\psi + \frac{\lambda}{2\mathcal{N}}(\bar{\psi}\psi)^2 = \imath\psi'^\dagger A^\dagger\gamma^0\gamma^\mu\nabla_\mu(A\psi') + \frac{\lambda}{2\mathcal{N}}(\psi'^\dagger A^\dagger\gamma^0 A\psi')^2 =$$

$$= \imath\psi'^\dagger A^\dagger\gamma^0\gamma^\mu(\nabla_\mu A)\psi' + \imath\psi'^\dagger\gamma^0\gamma^\mu\nabla_\mu\psi' + \frac{\lambda}{2\mathcal{N}}(\overline{\psi'}\psi')^2 =$$

$$= \imath\psi'^\dagger\gamma^0\gamma^\mu A^\dagger(\nabla_\mu A)\psi' + \imath\overline{\psi'}\gamma^\mu\nabla_\mu\psi' + \frac{\lambda}{2\mathcal{N}}(\overline{\psi'}\psi')^2 =$$

$$= \imath\overline{\psi'}\gamma^\mu\left(-\imath\delta^\phi_\mu\frac{N_d}{4}R\right)\psi' + \imath\overline{\psi'}\gamma^\mu\nabla_\mu\psi' + \frac{\lambda}{2\mathcal{N}}(\overline{\psi'}\psi')^2$$

$$\imath\overline{\psi'}\gamma^\mu\left(\nabla_\mu - \imath\mathcal{B}_\mu\right)\psi' + \frac{\lambda}{2\mathcal{N}}(\overline{\psi'}\psi')^2 \equiv \imath\overline{\psi'}\gamma^\mu\mathcal{D}_\mu\psi' + \frac{\lambda}{2\mathcal{N}}(\overline{\psi'}\psi')^2$$

Large-N expansion

$$S = \frac{1}{2}\int d^d x \left\{ \sum \left|\partial_\mu n_i\right|^2 - \frac{\alpha(x)}{\sqrt{N}}\left(\sum |n_i|^2 - \frac{N}{f}\right)\right\}$$

$$\mathcal{Z} = \int \mathcal{D}\alpha \mathcal{D}n \exp[-S] = \int \mathcal{D}\alpha \mathcal{D}n \exp\left\{-\frac{1}{2}\int d^d x \left\{ \sum \left|\partial_\mu n_i\right|^2 - \frac{\alpha(x)}{\sqrt{N}}\left(\sum |n_i|^2 - \frac{N}{f}\right)\right\}\right\}$$

$$\mathcal{Z} = \int \mathcal{D}\alpha \exp[-S_{eff}] = \int \mathcal{D}\alpha \exp\left\{-\left[\frac{N}{2} Tr \log\left(-\partial^2 + \frac{\alpha(x)}{\sqrt{N}}\right) - \int d^d x \cdot \frac{\alpha(x)\sqrt{N}}{2f}\right]\right\}$$

$$S_{eff} = \frac{N}{2} Tr \log(-\partial^2 + M^2) - \int d^d x \cdot \frac{N}{2f} M^2 + \text{terms in } \delta\alpha \text{ with lower order in N}$$

Effective action calculation

$$S^E_{eff} = -\frac{N}{2}\int d^{D-1}x\cdot\left(\zeta(0)\log\Lambda^2 + \zeta'(0)\right) - \int_0^\beta d\tau \int d^D x \cdot M^2 \cdot r$$

$$\zeta(s) = \sum_k \sum_{n=-\infty}^{\infty} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \left(\mathbf{q}^2 + p_k^2 + 4\pi^2 n^2/\beta^2\right)^{-s} \qquad \left(-\frac{\partial^2}{\partial x_1^2} + M^2\right) f_k = p_k^2 f_k$$

$$\zeta(s) = \frac{1}{(4\pi)^{\frac{D-1}{2}}} \frac{1}{\Gamma(s)} \int_0^\infty K(t) \times \Theta(t) \frac{dt}{t^{1+\frac{D-1}{2}-s}}$$

$$K(t) = \sum_k e^{-tp_k^2} = \frac{\ell}{\sqrt{4\pi t}} e^{-tM^2}\left(1 + \delta K(t)\right)$$

$$\Theta(t) \equiv \sum_{n=-\infty}^{\infty} e^{-4\pi^2 n^2 t/\beta^2} = \frac{\beta}{\sqrt{4\pi t}}\left[1 + 2\sum_{n=1}^{\infty}\left(e^{-\frac{\beta^2 n^2}{4t}}\right)\right]$$