A quantum state for black holes and the late Universe

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Theory and Phenomenology of Fundamental Interactions
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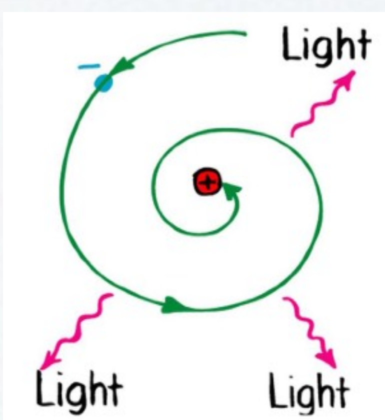
Preamble - Quantum states and classical gravity

- No data from quantum gravity = only access to classical sector of quantum theory
- Classical sector = subset of ~ orthogonal (superposition ~ 0) states $\psi_{\rm cl}$ such that

$$\langle \psi_{\rm cl} | \hat{\phi} | \psi_{\rm cl} \rangle \simeq \phi_{\rm cl}$$

- Quantum space = not all $|\psi\rangle$ may have a $\phi_{\rm cl}$ (e.g. spin $1/2$)
- Quantum space = not all $\phi_{\rm cl}$ may have a $|\psi_{\rm cl}\rangle$!! (e.g. hydrogen atom, BH?)



CED Hy lifetime $\tau \approx 10^{-11}$ sec
Actual Hy lifetime $\sim \tau \approx 10^{17}$ sec

CED off by $\sim 10^{28}$

Bound ground state

- Classical (sector of) gravity is nonlinear, long range and universal (*equivalence principle*)

Seminar plan

1) A bound on the compactness

2) Coherent state for classical geometry

3) (Non-)resolution of the black hole interior

4) Dark matter and Hubble tension

5) Conclusions

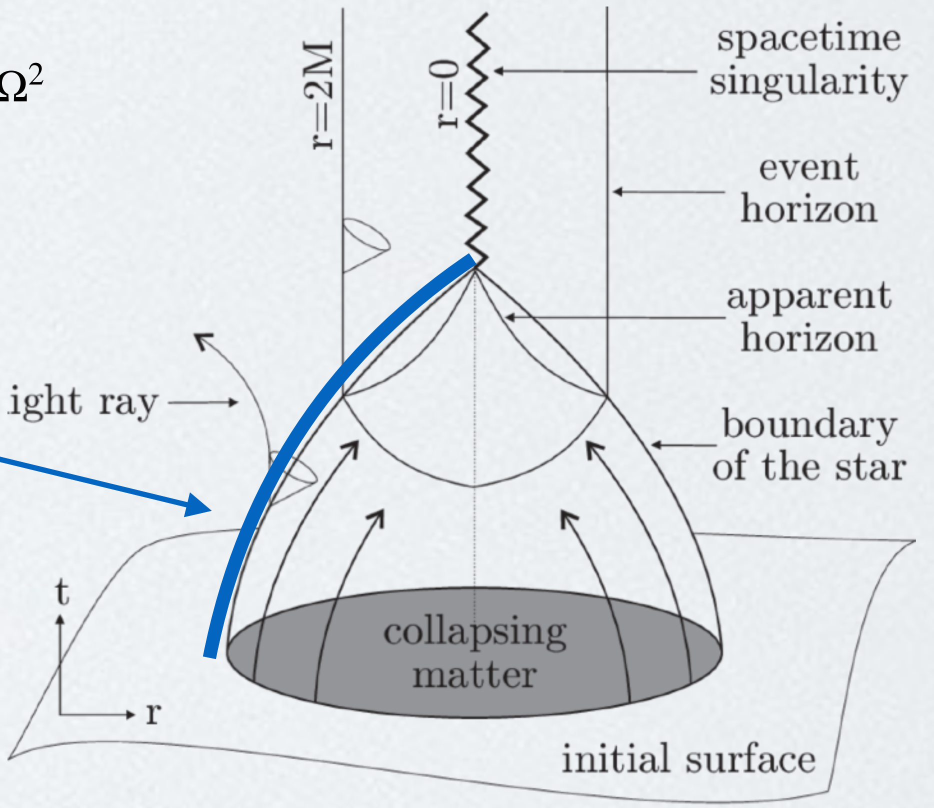
Based on:

1) R.C. *A quantum bound on the compactness,* arXiv:2103.14582
2-3) R.C. *Quantum black holes and resolution of the singularity,* arXiv:2103.00183
4) A. Giusti, S. Buffa, L. Heisenberg, R.C., *A quantum state for the late Universe,* arXiv:2108.05111

Quantum bound on compactness

- Collapsing dust*: $ds^2 = -\left(1 - \frac{2\,G_N\,M}{r}\right) dt^2 + \left(1 - \frac{2\,G_N\,M}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2$

$$\left(\frac{dR}{d\tau}\right)^2 + 1 - \frac{2\,G_N\,M}{R} = \frac{E^2}{M^2}$$



- Effective Hamiltonian:

$$H \equiv \frac{P^2}{2\,M} - \frac{G_N\,M^2}{R} = \frac{M}{2}\left(\frac{E^2}{M^2} - 1\right) \equiv \mathscr{E}$$

- Schrödinger equation:

$$\hat{H}\,\Psi_n = \mathscr{E}_n\,\Psi_n$$

- Spectrum of bound states ($n \geq 1$):

$$\frac{\mathscr{E}_n}{M} \simeq -\frac{G_N^2\,M^4}{2\,\hbar^2\,n^2} = -\frac{1}{2\,n^2}\left(\frac{M}{m_p}\right)^4 = \frac{1}{2}\left(\frac{E_n^2}{M^2} - 1\right)$$

"GR"

$$R_n \equiv \langle \Psi_n \,|\, R \,|\, \Psi_n \rangle \simeq n^2\,\ell_p \left(\frac{m_p}{M}\right)^3$$

Newtonian spectrum

* R = (no fundamental) "collective" d.o.f.

Quantum bound on compactness

- Allowed spectrum*: $0 \leq \frac{E_n^2}{M^2} \simeq 1 - \frac{1}{n^2}\left(\frac{M}{m_\text{p}}\right)^4$

$$\longrightarrow \quad n \geq N_M \simeq \left(\frac{M}{m_\text{p}}\right)^2$$

$$\mathscr{E}_n \geq \mathscr{E}_{N_M} \simeq -\frac{M}{2}$$

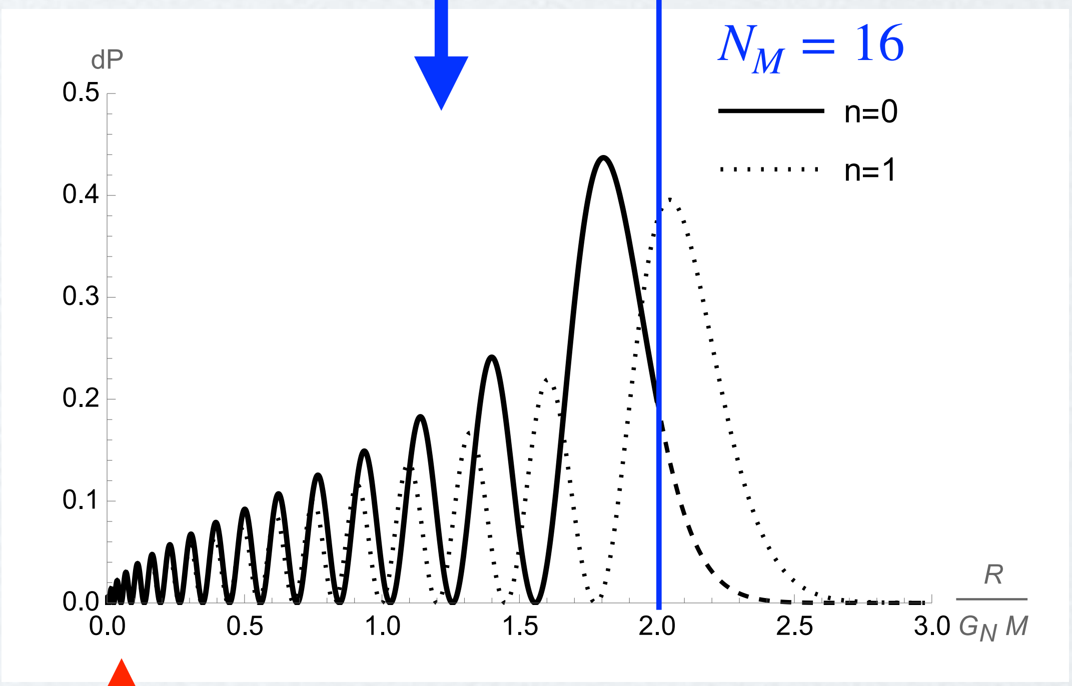
$$R_n \geq R_{N_M} \simeq G_\text{N}\, M = \ell_\text{p}\,\frac{M}{m_\text{p}}$$

- "Energy" levels:

$$|\mathscr{E}_{n+1} - \mathscr{E}_n| \simeq m_\text{p}\,\frac{m_\text{p}}{M} \ll m_\text{p}$$

$$|E_{n+1} - E_n| \simeq m_\text{p}$$

$(n \to N_M + n)$



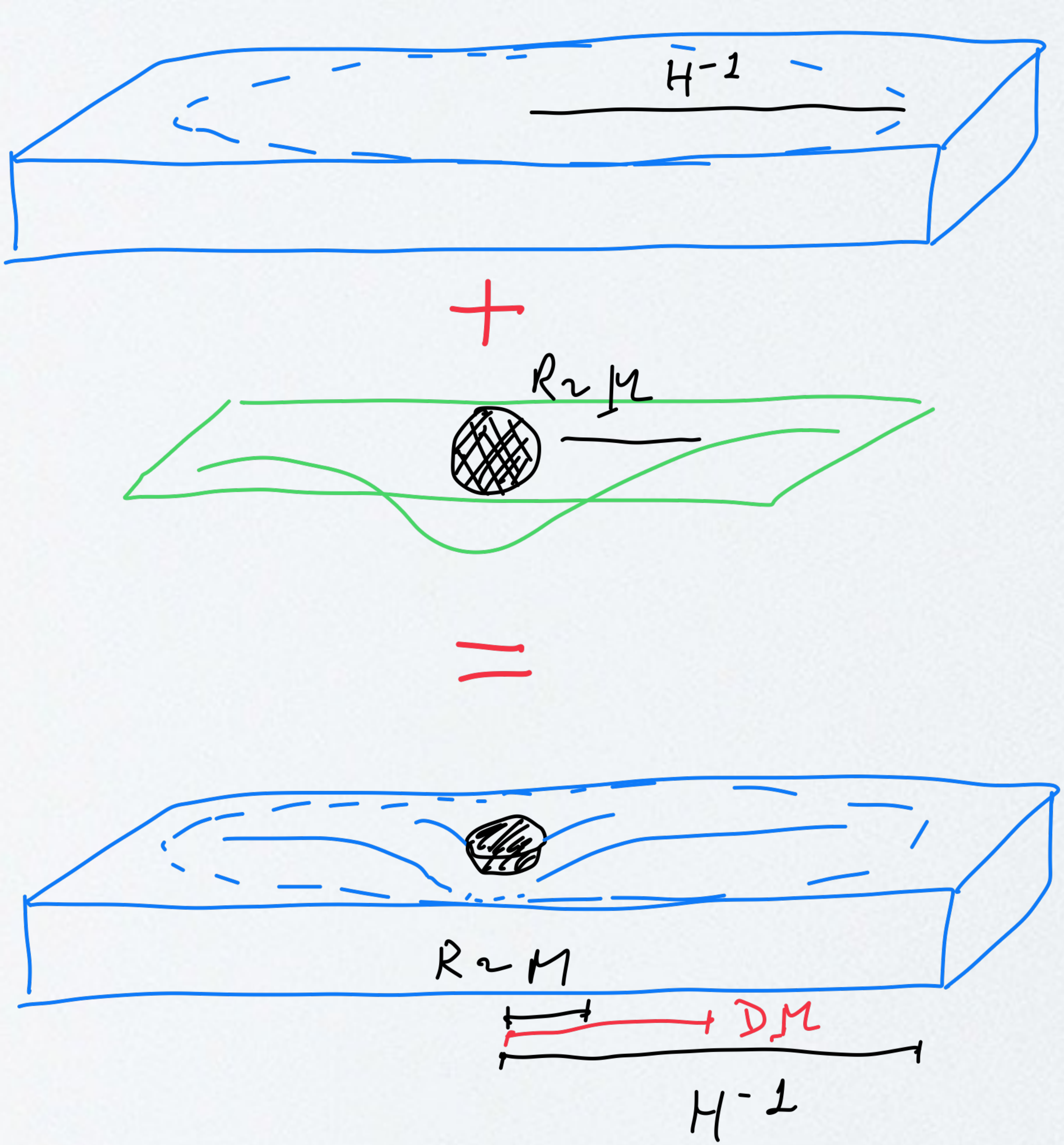
- Bounded compactness: $\frac{G_\text{N}\, M}{R_n} \lesssim 1$

$$R_{n=1} \sim \ell_\text{p}\left(\frac{m_\text{p}}{M}\right)^3$$

- Outer geometry?

* *Classicalization* in action

Coherent state for classical geometry



Empy "de Sitter" Universe

N_Λ gravitons

$\lambda_\Lambda \sim H^{-1}$

Localised matter

$\lambda_M \sim M$

ΛCDM

(MOND)

$\lambda_M < \lambda < \lambda_\Lambda$

Coherent state for classical geometry

- SdS geometry:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2$$

$$f = 1 + 2\,V_{\mathrm{SdS}} = 1 - \frac{2\,G_{\mathrm{N}}\,M}{r} - \frac{\Lambda\,r^2}{3}$$

$$V_{\mathrm{SdS}} = V_M + V_\Lambda = -\frac{G_{\mathrm{N}}\,M}{r} - \frac{\Lambda\,r^2}{6}$$

- Horizons ($f = 0 \leftrightarrow 2\,V = -1$):

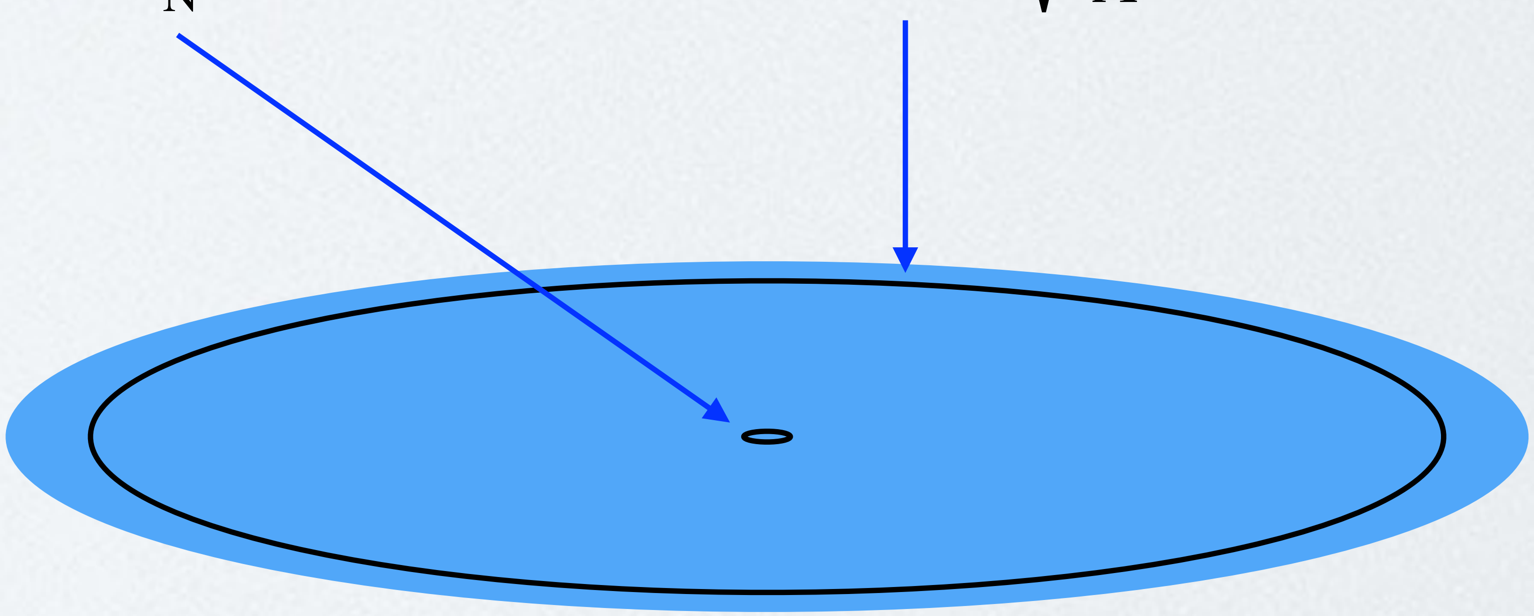
$1/\sqrt{\Lambda} \gg G_{\mathrm{N}}\,M$

Localised source gravitational radius

$$R_{\mathrm{H}} \simeq 2\,G_{\mathrm{N}}\,M$$

Cosmological horizon

$$H^{-1} = L \simeq \sqrt{\frac{3}{\Lambda}}$$



Coherent state for classical geometry

- Corpuscular scaling law* (Bekenstein's area law):

$$\mathscr{A} \sim \ell_{\rm p}^2 \, N_{\rm G}$$

$$N_M \simeq \left(\frac{M}{m_{\rm p}}\right)^2 \simeq N_{\rm G}$$

- Massless scalar field:

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)\right]\Phi(t,r) = 0$$

$$u_k(t,r) = e^{-i\,k\,t}\, j_0(k\,r)$$

$$4\,\pi \int_0^\infty r^2\, dr\, j_0(k\,r)\, j_0(p\,r) = \frac{2\,\pi^2}{k^2}\,\delta(k-p)$$

- Normal mode expansion of operators:

$$\hat{\Phi}(t,r) = \int_0^\infty \frac{k^2\, dk}{2\,\pi^2}\sqrt{\frac{\hbar}{2\,k}}\left[\hat{a}_k\, u_k(t,r) + \hat{a}_k^\dagger\, u_k^*(t,r)\right]$$

$$\hat{\Pi}(t,r) = i\int_0^\infty \frac{k^2\, dk}{2\,\pi^2}\sqrt{\frac{\hbar\, k}{2}}\left[\hat{a}_k\, u_k(t,r) - \hat{a}_k^\dagger\, u_k^*(t,r)\right]$$

$$\left[\hat{\Phi}(t,r), \hat{\Pi}(t,s)\right] = \frac{i\,\hbar}{4\,\pi\, r^2}\,\delta(r-s)$$

$$\left[\hat{a}_k, \hat{a}_p^\dagger\right] = \frac{2\,\pi^2}{k^2}\,\delta(k-p)$$

* G. Dvali, C. Gomez

Coherent state for classical geometry

- Coherent state: $\hat{a}_k \,|\, g \rangle = g_k \, e^{i\,\gamma_k(t)} \,|\, g \rangle$

$$\langle g \,|\, \hat{\Phi}(t,r) \,|\, g \rangle = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2} \sqrt{\frac{2\,\ell_\mathrm{p}\, m_\mathrm{p}}{k}}\, g(k)\, \cos[\gamma_k(t) - k\,t]\, j_0(k\,r)$$

- "Classical" coherent state:

$$\sqrt{\frac{\ell_\mathrm{p}}{m_\mathrm{p}}}\, \langle g \,|\, \hat{\Phi}(t,r) \,|\, g \rangle = V(r)$$

$$g(k) = \sqrt{\frac{k}{2}}\, \frac{\tilde{V}(k)}{\ell_\mathrm{p}}$$

$$\gamma_k = k\,t$$

$$V = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\, \tilde{V}(k)\, j_0(k\,r)$$

- Occupation number ~ distance from vacuum:

$$|\, g \rangle = e^{-N_\mathrm{G}/2} \exp\left\{ \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\, g(k)\, \hat{a}_k^\dagger \right\} |\, 0 \rangle$$

$$N_\mathrm{G} = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\, g^2(k)$$

$$\langle k \rangle = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\, k\, g^2(k)$$

Coherent state for classical geometry (M)

- Localised source: $$V_M = -\frac{G_N M}{r} \qquad \tilde{V} = -4\pi G_N \frac{M}{k^2}$$

- Mass scaling*: $$N_M = \frac{4M^2}{m_p^2}\int_0^\infty \frac{dk}{k} \to \frac{4M^2}{m_p^2}\int_{k_{IR}}^{k_{UV}} \frac{dk}{k} = \frac{4M^2}{m_p^2}\ln\left(\frac{k_{UV}}{k_{IR}}\right)$$

- Compton length scaling: $$\langle k \rangle = \frac{4M^2}{m_p^2}\int_0^\infty dk \to \frac{4M^2}{m_p^2}\int_{k_{IR}}^{k_{UV}} dk = \frac{4M^2}{m_p^2}\left(k_{UV} - k_{IR}\right)$$

- Quantum bounded compactness (BH = ground state): $k_{UV}^{-1} \simeq R_{N_M} \simeq G_N M$

* Cut-offs = existence condition for quantum state: $g(k < k_{IR}) = g(k > k_{UV}) = 0$!

Coherent state for classical geometry (M)

- Localised source:

$$V_M = -\frac{G_N M}{r}$$

- Localised source in dS:

$$k_{UV}^{-1} \simeq R_{N_M} \simeq G_N M$$

$$k_{IR}^{-1} \simeq L$$

$$V_{QM} \equiv \sqrt{\frac{\ell_p}{m_p}} \langle g \,|\, \hat{\Phi}(t, r) \,|\, g \rangle \simeq V_M(r) \qquad R_H \lesssim r \lesssim L$$

(*Observable?*) additional effects

* Cut-offs = conditions on quantum state: $g(k < k_{IR}) = g(k > k_{UV}) = 0$!

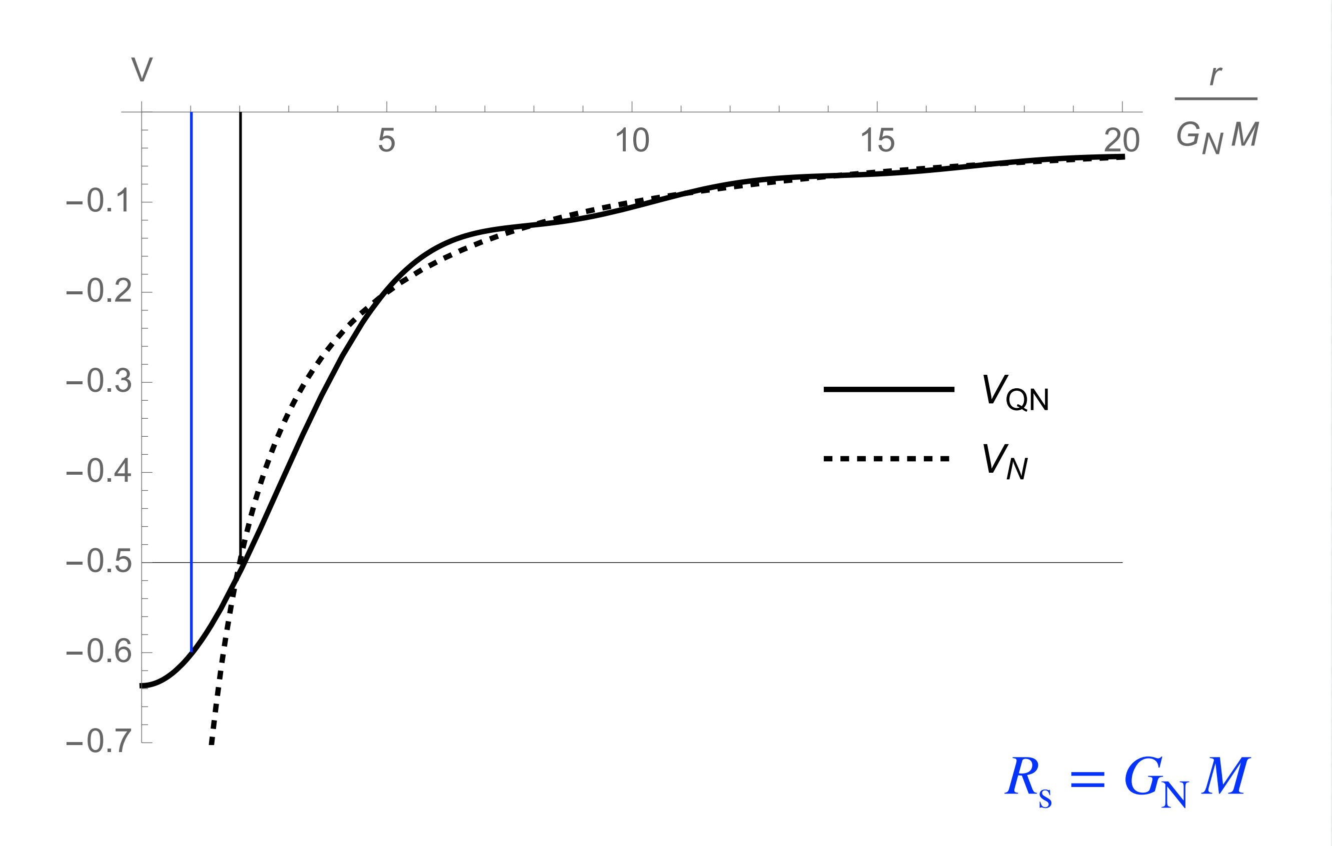
Singularity (non-)resolution

- Corpuscular scaling laws*:

$$N_M \sim \frac{M^2}{m_p^2}$$

$$\lambda_M \simeq \frac{N_M}{\langle k \rangle} \sim R_s \sim G_N M$$

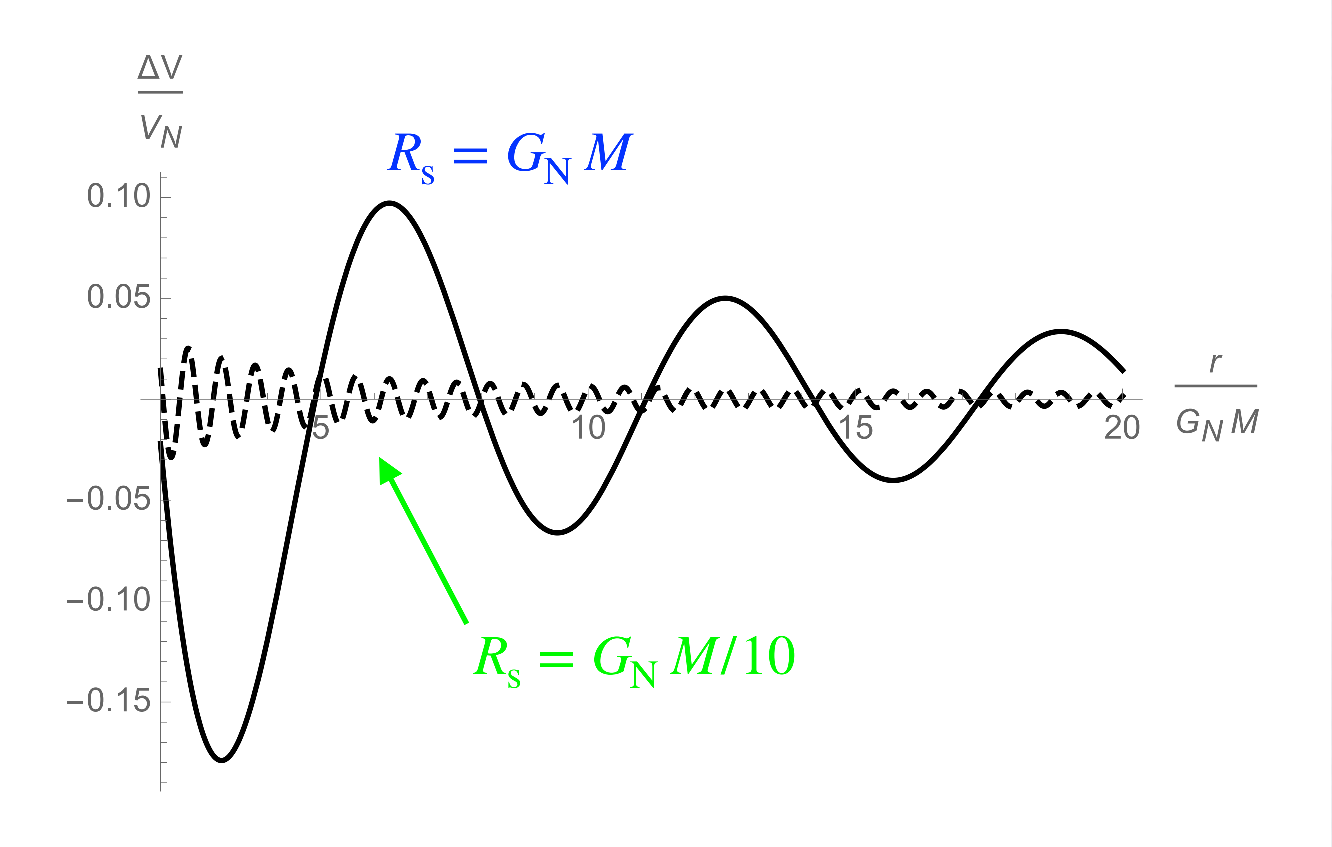
(logs → proper time dependence)



- Quantum deviations (near horizon):

$$ds^2 \simeq -\left(1 + 2\,V_{QM}\right) dt^2 + \frac{dr^2}{1 + 2\,V_{QM}} + r^2\, d\Omega^2$$

(*integrable* singularity)



* G. Dvali and C. Gomez

Coherent state for classical geometry (SdS)

- SdS geometry:

$$V_{\mathrm{SdS}} = V_M + V_\Lambda = -\frac{G_{\mathrm{N}}\,M}{r} - \frac{\Lambda\,r^2}{6}$$

- Normalization:

$$g(k) = \sqrt{\frac{k}{2\,\ell_{\mathrm{p}}^2}}\left[\tilde{V}_M(k) + \tilde{V}_{\mathrm{dS}}(k)\right] = g_M(k) + g_\Lambda(k)$$

$$N_{\mathrm{G}} = \int \frac{k^2\,dk}{2\,\pi^2}\left[g_M(k) + g_\Lambda(k)\right]^2 = N_M + N_\Lambda + N_{M\Lambda}$$

- Three contributions:

$$N_M \simeq \frac{M^2}{m_{\mathrm{p}}^2}\int_{k_{\mathrm{IR}}=L^{-1}}^{k_{\mathrm{UV}}=R_{\mathrm{H}}^{-1}} \frac{dk}{k} = \frac{M^2}{m_{\mathrm{p}}^2}\ln\left(\frac{L}{R_{\mathrm{H}}}\right)$$

$$N_\Lambda \simeq \frac{L^2}{\ell_{\mathrm{p}}^2}\ln\left(\frac{L}{R_{\mathrm{H}}}\right)$$

$$N_{M\Lambda} \simeq \frac{M\,L}{m_{\mathrm{p}}\,\ell_{\mathrm{p}}}\ln\left(\frac{L}{R_{\mathrm{H}}}\right)$$

- Hierarchy:

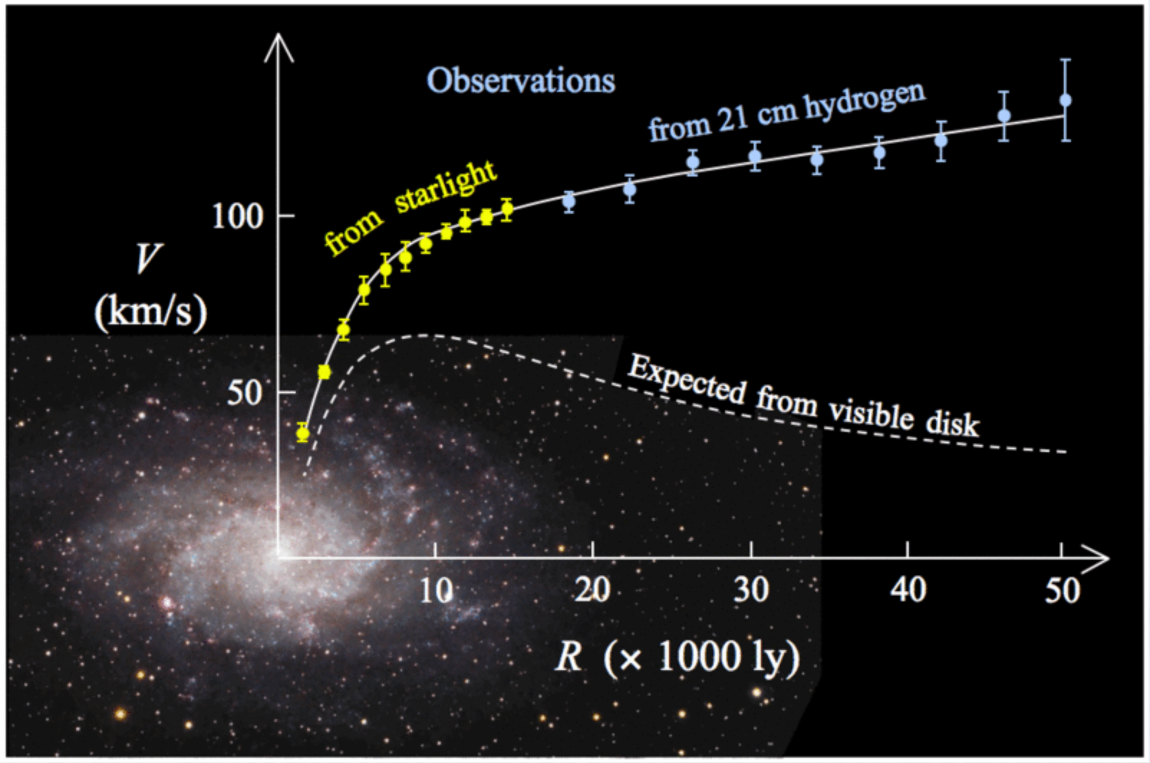
$$N_M \ll N_{M\Lambda} \ll N_\Lambda$$

Dark Matter

- MOND potential:

$$V_{\rm MOND} = \frac{G_{\rm N}\,M}{\ell}\,\ln\left(\frac{r}{\ell}\right)$$

$$\ell = \sqrt{G_{\rm N}\,M\,L}$$



- Normalization:

$$N_{\rm MOND} \sim \frac{G_{\rm N}\,M}{\ell_{\rm p}\,\ell}\int_{k_{\rm IR}=L^{-1}}^{k_{\rm UV}\to\infty}\frac{dk}{k^3} = \frac{M\,L}{m_{\rm p}\,\ell_{\rm p}}$$

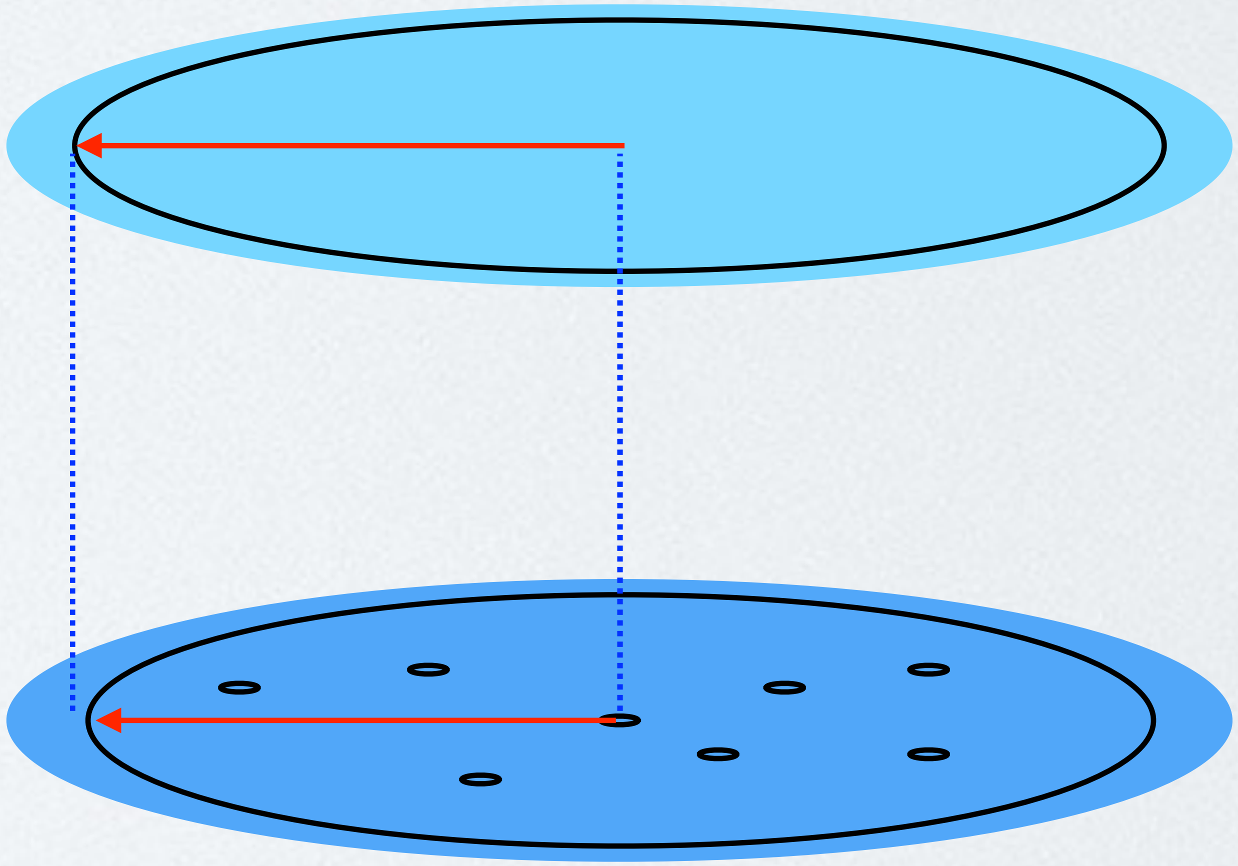
$$N_{M\Lambda} \simeq \frac{M\,L}{m_{\rm p}\,\ell_{\rm p}}\,\ln\left(\frac{L}{R_{\rm H}}\right)$$

M. Cadoni, R.C., A. Giusti, W. Mück, M. Tuveri, PLB 776 (2018) 242
M. Cadoni, R.C., A. Giusti, M. Tuveri, PRD 97 (2018) 044047

Hubble tension

- Empty dS universe:

$$N_{\bar{\Lambda}} \sim \frac{\bar{L}^2}{\ell_{\mathrm{p}}^2}$$



- dS universe with matter:

$$N_{\bar{\Lambda}} = N_{\Lambda} + N_{M\Lambda} + N_M$$

$$\bar{L}^2 = L^2 + L\, G_{\mathrm{N}} \sum^{n_{\mathrm{g}}} M_i + G_{\mathrm{N}}^2 \sum^{n_{\mathrm{g}}} M_i^2$$

$$\bar{L} \simeq L \left(1 + \frac{G_{\mathrm{N}} \sum M_i}{L} \right)$$

H_{CMB}^{-1} (→ $\bar{L}$)

H_{SN}^{-1} (→ L)

$$\frac{G_{\mathrm{N}} \sum M_i}{L} \simeq 5\,\%$$

↓

$$\frac{H_{\mathrm{SN}} - H_{\mathrm{CMB}}}{H_{\mathrm{CMB}}} \simeq 5\,\%$$

Conclusions

- Black holes as (macroscopic) quantum objects (*ground state* very far from vacuum)
- Singularity is not resolved (fuzzy geometry)
- Universe geometry emerges from *ground state* for matter and gravity
- Dark Matter arises from Universe finite size (*N*-body analysis)
- Hubble tension *relieved* (time evolution must be properly accounted for)