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On a new Einstein field equation with possible application to black hole cosmology

Alice Roitberg
a.roitberg@campus.unimib.it
Università degli Studi di Milano-Bicocca
Joint work with Renzo L. Ricca

BECs in cosmology

An example: two-phase model

Boson gas
Global Dark Matter

Slow and steady condensation

BEC
Dark Energy

Collapse

Localized objects
Localized DM

(Fukuyama & Morikawa, 2008)

Analogue Models

1974 Hawking
Black hole explosions?

1981 Unruh
Experimental black hole evaporation?

2001 Visser, Liberati, Barceló
Analogue gravity from Bose-Einstein condensates

...

Acoustic waves trapped by vortex defects described by acoustic metric

Light waves trapped by black holes described by spacetime geometry

Relativistic BECs

(Khlopov et al, 1985; Fukuyama & Morikawa, 2006; Fagnocchi et al, 2010; Matos & Gomez, 2015; ...)

Starting from the Lagrangian:

$$\mathcal{L} = g^{\mu\nu}\partial_\mu \Phi^* \partial_\nu \Phi - \frac{m^2c^2}{\hbar^2}\Phi^*\Phi - \frac{\lambda}{2\hbar^2}(\Phi^*\Phi)^2$$

we get a Klein-Gordon equation:

$$\Box\Phi + \frac{m^2c^2}{\hbar^2}\Phi + \frac{\lambda}{\hbar^2}(\Phi^*\Phi)\Phi = 0$$

that, by substituting $\Phi = \Psi e^{-imc^2t/\hbar}$, gives the relativistic GPE:

$$i\hbar\partial_t\Psi - \frac{\hbar^2}{2mc^2}\partial_{tt}^2\Psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + \frac{\lambda}{2m}|\Psi|^2 + U_{\text{ext}}\right)\Psi$$

Relativistic BECs

(Roitberg & Ricca, J. Phys. A: Math. Theor. **54**, 2021)

Using again Madelung transform $\Psi = \sqrt{\rho} e^{i \frac{m}{\hbar} \theta}$ and equating real and imaginary parts we get

From imaginary part: $\partial_t \rho = \partial_\mu \rho \partial^\mu \theta + \rho \partial_\mu \partial^\mu \theta = \partial_\mu (\rho \partial^\mu \theta)$

From real part: $\partial_t \theta = -\frac{\hbar^2}{2m^2} \frac{\Box \sqrt{\rho}}{\sqrt{\rho}} + \frac{1}{2} \partial_\mu \theta \partial^\mu \theta - \frac{\lambda}{2m^2} \rho - \frac{U_{ext}}{m}$

Similar results are obtained by applying the explained procedure, thus providing a set of dynamical laws for cosmological models based on condensate physics.

Momentum conservation law

(Roitberg & Ricca, J. Phys. A: Math. Theor. **54**, 2021)

$$\partial_t(\rho \boldsymbol{u}) = -\nabla \cdot (\mathcal{D} + \Pi - \tau + \mathcal{G}) + \chi$$

$$\mathcal{D}_{ij} = \rho u_i u_j, \quad \Pi_{ij} = \wp g_{ij}, \quad \boxed{\mathcal{G}_{ij} = \frac{\hbar^2 \rho}{4m^2} G_{ij},} \quad \boxed{\chi = -\frac{\hbar^2}{8m^2} R \,\mathrm{d}\rho}$$

χ breaks the standard form of a conservation law, but in some particular cases it can be written as divergence of some pseudotensor.

Steady state with constant curvature

(Roitberg & Ricca, J. Phys. A: Math. Theor. **54**, 2021)

If *R* is constant then we can write

$$\chi = -\nabla \cdot \left(\frac{\hbar^2}{8m^2} g_{ij} R\rho \right)$$

and the momentum conservation law takes the standard form of a continuity equation.

If we impose the steady state condition we get a condition on Ricci tensor:

$$R_{ij} = -\frac{4m^2}{\hbar^2} u_i u_j - 8\pi a_s g_{ij} \rho + \mathrm{Hess}_{ij}(\ln \rho)$$

Steady state condition:
New Einstein type field equation

(Roitberg, J.Phys: Conf. Ser. **1730**, 2021)

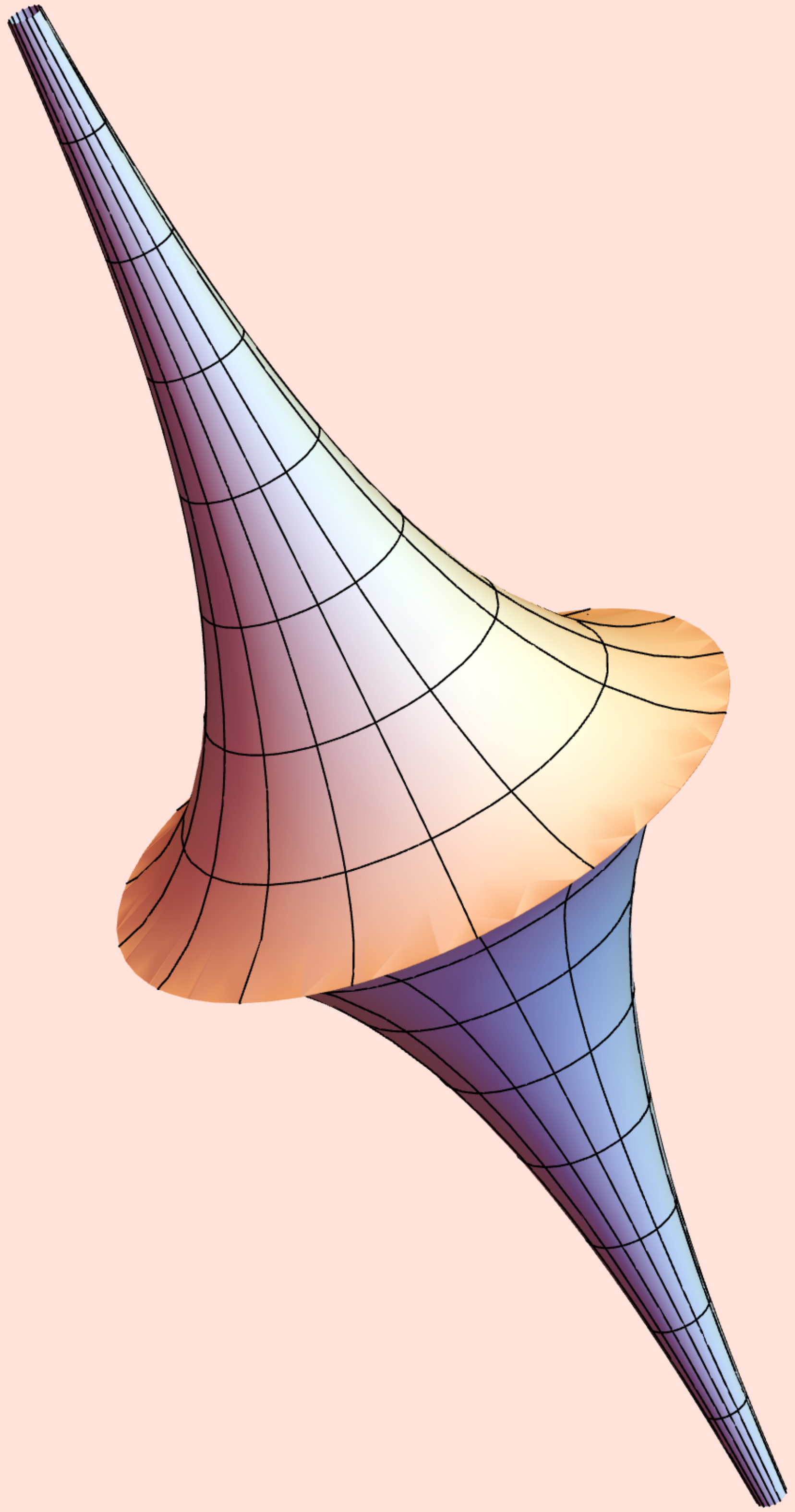
We can also impose the steady state condition by simultaneously asking for these conditions: $\text{(i)} \quad \mathcal{D} - \tau + \mathcal{G} = 0 \qquad \text{(ii)} \quad \chi = \nabla \cdot \Pi$

and we obtain a new type of Einstein field equations:

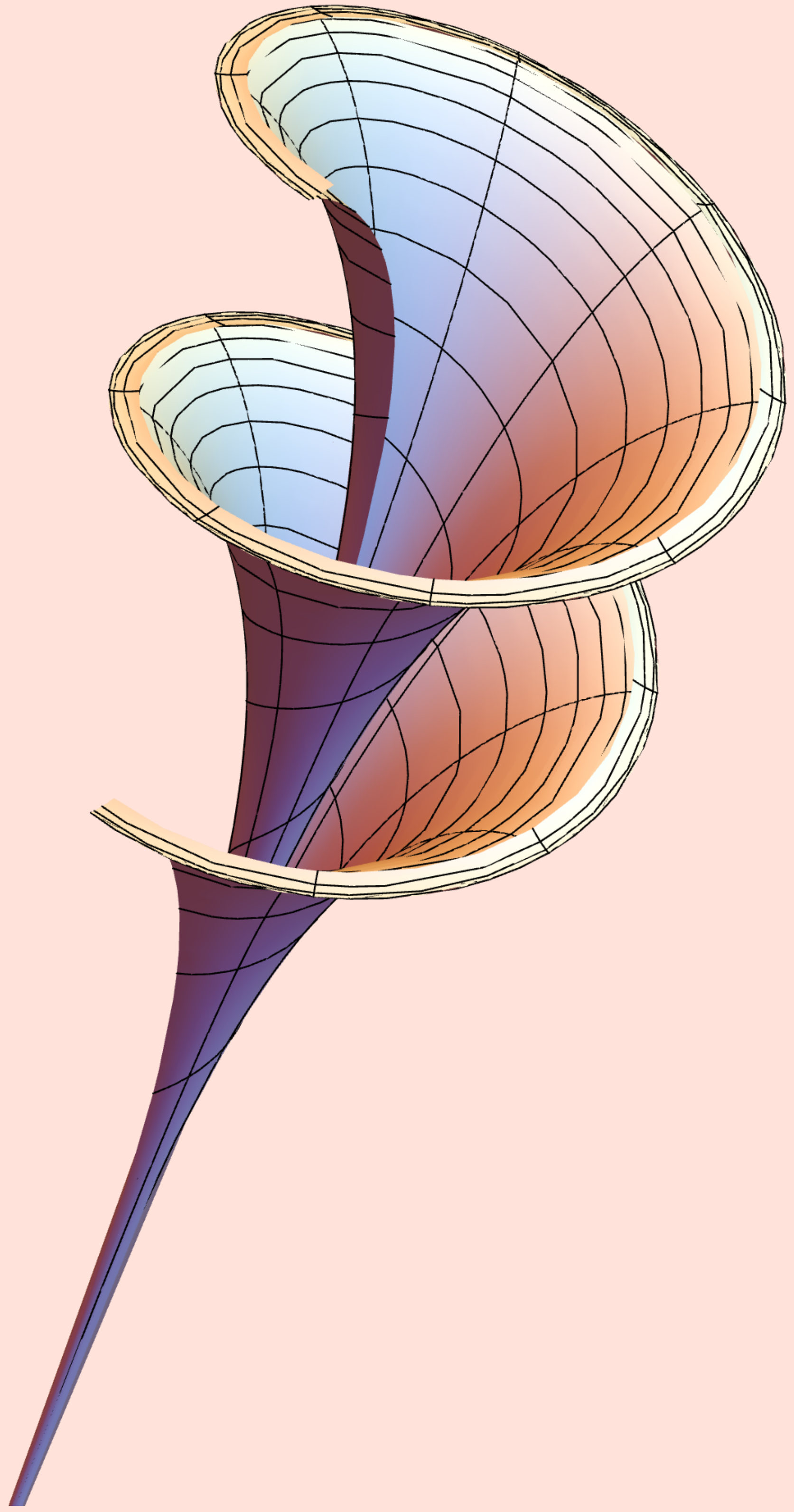
$$G_{ij} = -\frac{4m^2}{\hbar^2} u_i u_j + \mathrm{Hess}_{ij}(\ln \rho)$$

$$R = -32\pi a_s \rho = -\frac{8}{\xi^2}$$

What next?



- Role of negative curvature
- Connection with sine-Gordon equation
- Role of minimal surface geometry
- Role of superimposed twist phase



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Thank you for your attention

Alice Roitberg

a.roitberg@campus.unimib.it