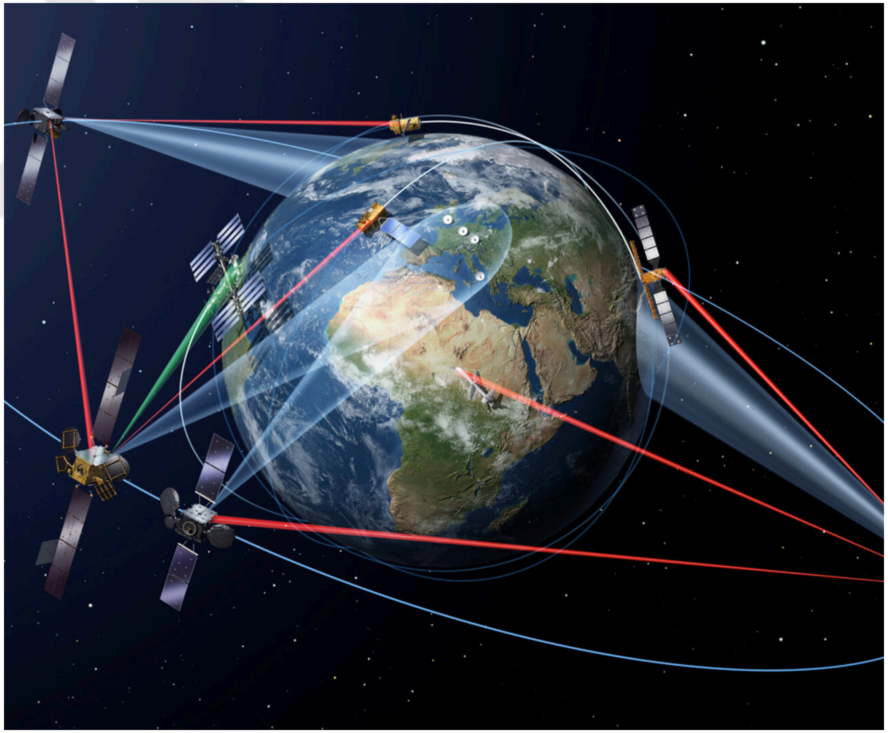
Spacetime effects on wavepackets of coherent light



Dr. David Edward Bruschi

PGI-12, Forschungszentrum Jülich

Germany

Figure by ESA: https://www.esa.int/Enabling_Support/Preparing_for_the_Future/Discovery_and_Preparation/Space_internet_to_enhance_Earth_observation

XXIV SIGRAV Conference on General Relativity and Gravitation

Quantum technologies: Earth or Space

Earth based

Earth advantages:
- Cheap(er);
- Reproducible;
- Upgradable.

Earth disadvantages:
- Many sources of noise;
- Small distances;
- Bound to surface.

Satellite based

Space advantages:
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- Microgravity;
- Less noise;

Space disadvantages:
- Very expensive;
- Few-shot experiments;
- Not very flexible once launched;

Relativity?

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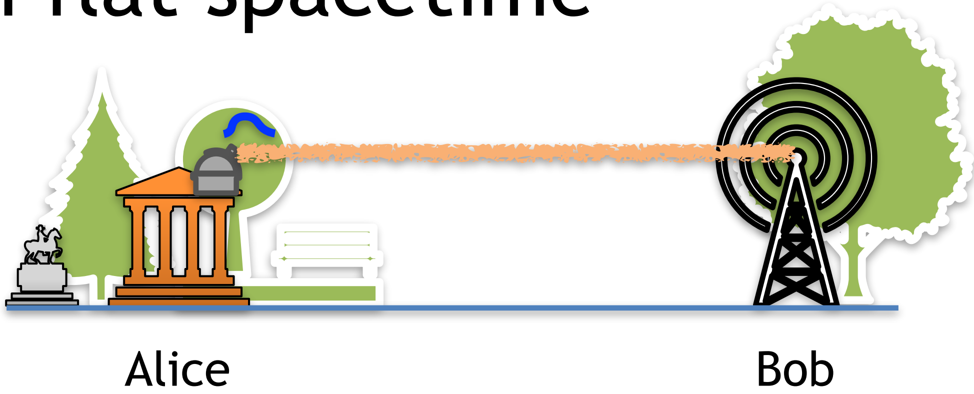
Relativity?

Propagation in flat spacetime

Bob: $\hat{a}^\dagger_{\omega_0} := \int_0^{+\infty} d\omega F_{\omega_0}(\omega) e^{-i\omega(r_B - t_B)} \hat{a}^\dagger_\omega$



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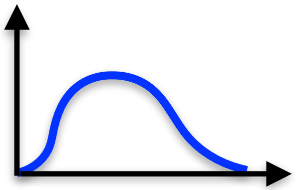

We use QFT in curved spacetime

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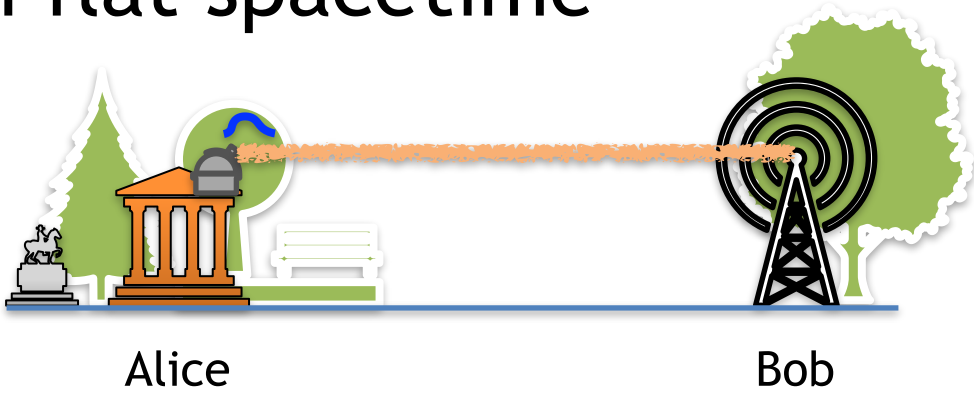

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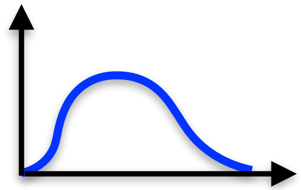

Approximation: pulse 1-dimenstional

$$\int d^3k F_{k_0}(k_x, k_y, k_z) \approx \int d\omega F_{\lambda_0}(\omega)$$

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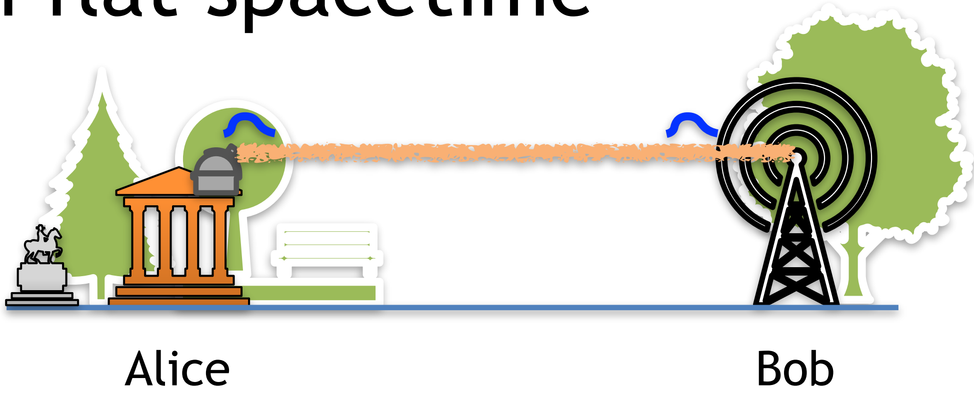

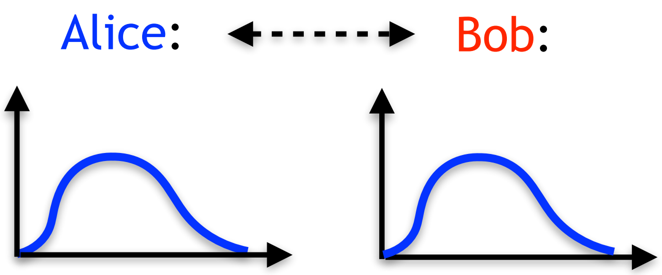

Alice's wave packet as measured **locally** by Bob

$$\boxed{F_{\omega_0}(\omega) = F_{\omega_0}(\omega)}$$

Propagation in curved spacetime

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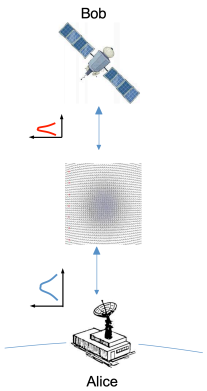

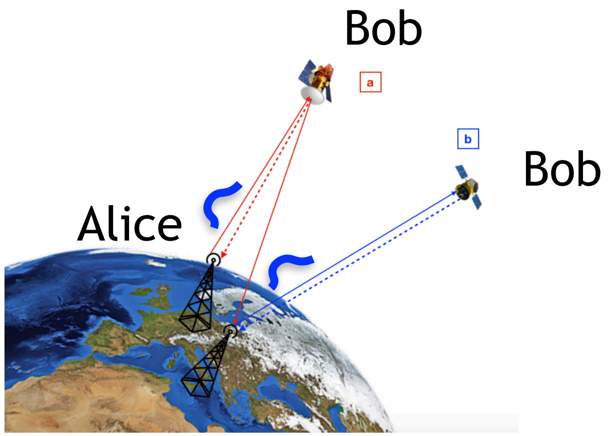

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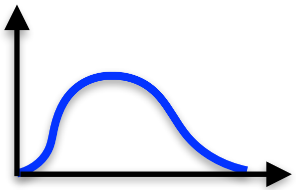

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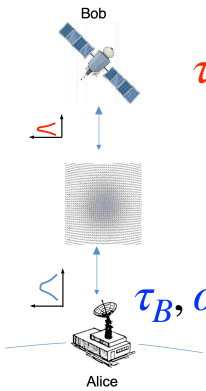
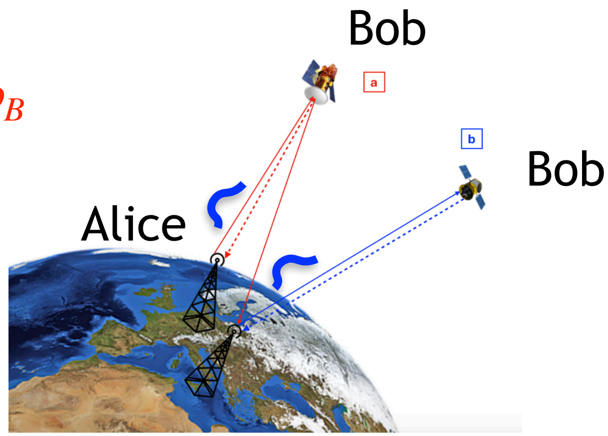


τ_B, ω_B

τ_B, ω_B

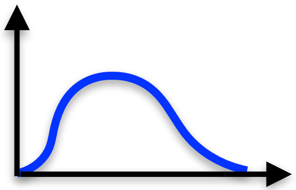


Alice's wave packet as measured **locally** by Bob

$$\tau_B = \frac{f(r_B)}{f(r_A)}\tau_A \qquad \omega_B = \frac{f(r_A)}{f(r_B)}\omega_A$$

$$f(r_A) = 1 - \frac{r_S}{r_A}$$

Alice:



Propagation in curved spacetime

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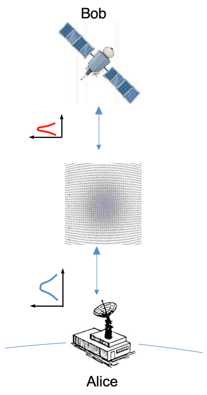
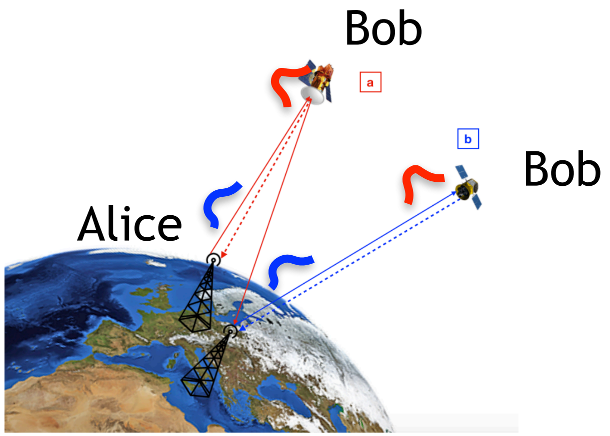
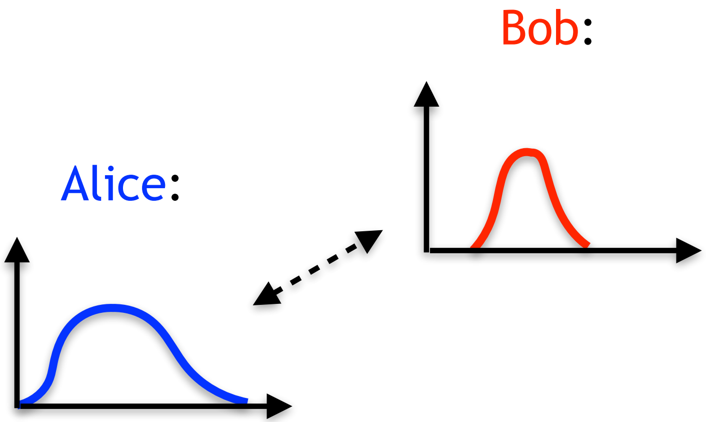




Alice's wave packet as measured **locally** by Bob

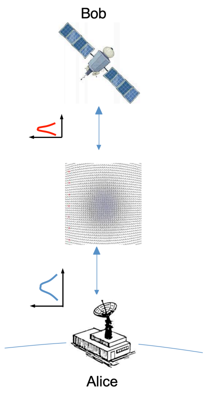
$$\boxed{F'_{\omega_0'}(\omega) = \sqrt[4]{f(r_A, r_B)}\, F_{\omega_0}\left(\sqrt{f(r_A, r_B)}\,\omega\right)}$$

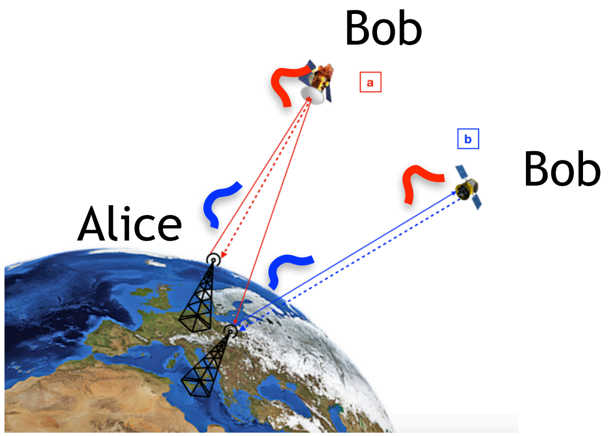
$$f(r_A, r_B) = \frac{1 - \frac{3M}{r_B}}{1 - \frac{2M}{r_A}}$$



Propagation in curved spacetime

It is important to note that "any" protocol operated between Alice and Bob will result in the receiver (e.g., Bob) to witness effects that depend on the overlap Δ of the two wave packets, i.e.,





$$\Delta := \left| \int_{-\infty}^{+\infty} d\omega F'_{\omega_0'}(\omega) F^*_{\omega_0}(\omega) \right|$$

This overlap, for example, measures how well can we distinguish two single photons.

Potential wave packet: Gaussian profile

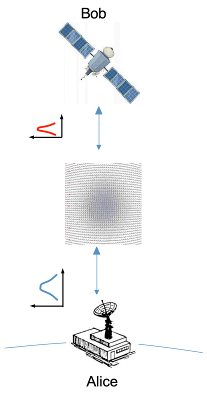
$$F_{\omega_0}(\omega) = C\, e^{-\frac{(\omega-\omega_0)^2}{4\sigma^2}}$$

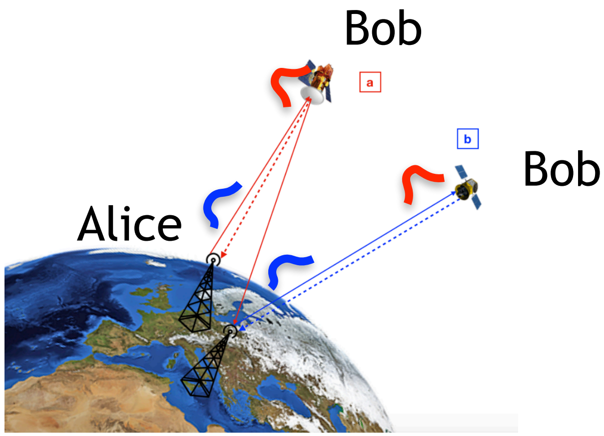
Propagation in curved spacetime

We apply our results to realistic protocols (more later):

$$\delta := \frac{1-\frac{3M}{r_B}}{1-\frac{2M}{r_A}} - 1 \ll 1$$

$$\Delta \approx 1 - \frac{\omega_0^2}{8\sigma^2}\delta^2$$





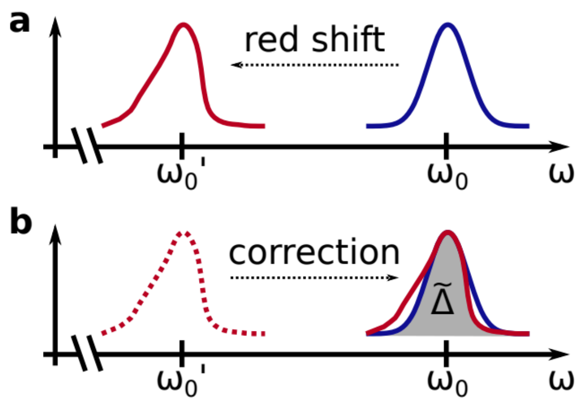
This regime yields viable results when

$$\delta^2 \ll \frac{\omega_0^2}{\sigma^2}\delta^2 \ll 1$$

In our case this occurs

$$\delta \sim 10^{-10}; \quad \omega_0 \sim 4 \times 10^{14}\text{Hz}$$
$$\sigma \sim 10^6\text{Hz}$$

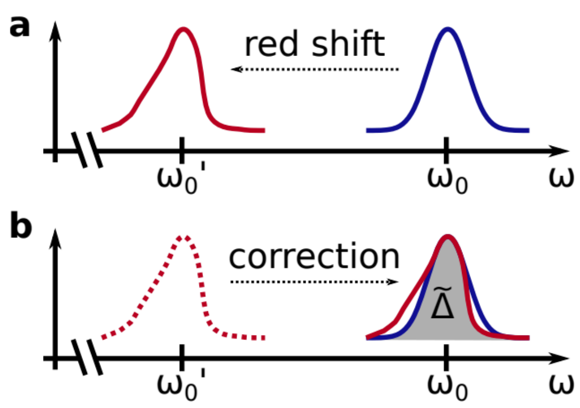
Genuine distortion vs. rigid shift



Main question:

Can we distinguish between genuine distortion and rigid translation?

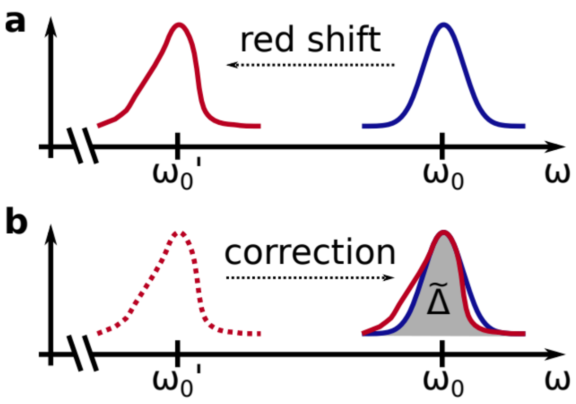
Genuine distortion vs. rigid shift



We have:

$$F'(\omega)_{\omega_0'} = \chi F\left(\frac{\chi^2\omega - \omega_0}{\sigma}\right) \qquad \chi^2\omega - \omega_0 = \chi^2\left[(\omega - \omega_0) + \frac{\chi^2 - 1}{\chi^2}\omega_0\right]$$

Genuine distortion vs. rigid shift

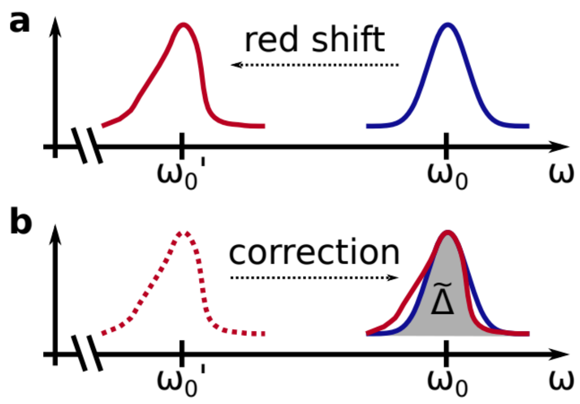


Assume that it lis locally possible to perform: $\omega \to \omega + \delta\omega$ $(z := \omega/\sigma)$

$$\Delta \to \tilde{\Delta} := \int dz\, f(\chi z + \bar{z}) f(z/\chi) e^{-i(\psi(\chi z + z_0 + \bar{z}) - \psi(z/\chi + z_0))}$$

$F(z) := (\sigma)^{-1/2} f(z) e^{-i\psi(z)}$ $\bar{z} := (\chi^2 - 1) z_0 + \chi^2 \delta z$

Genuine distortion vs. rigid shift



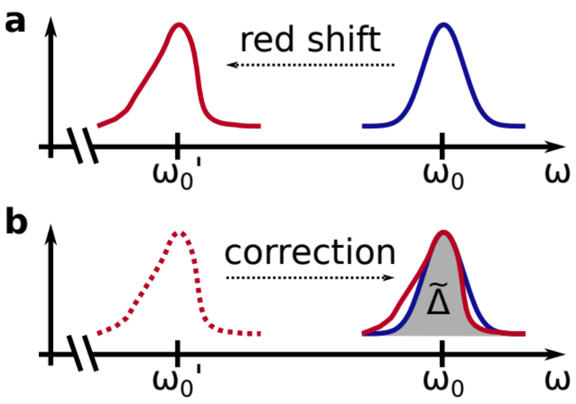
Optimize $\tilde{\Delta} := \int dz\, f(\chi z + \bar{z}) f(z/\chi) e^{-i(\psi(\chi z + z_0 + \bar{z}) - \psi(z/\chi + z_0))}$ with respect to $\bar{z}$:

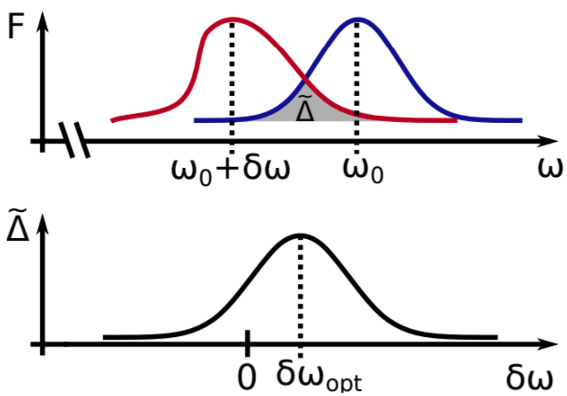
i) obtain **effective redshift** δz_{opt} ;

ii) obtain **genuine redshift** $\tilde{\Delta}_{\text{opt}}$.

$$\bar{z} := (\chi^2 - 1) z_0 + \chi^2 \delta z$$

Genuine distortion vs. rigid shift





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Genuine distortion vs. rigid shift

Pure state: $\hat{\rho}_{\mathrm{p}} := |1_A\rangle\langle 1_A|$

Mixed state: $\hat{\rho}_{\mathrm{m}}$

$$\boxed{F(z) = 1/\sqrt[4]{2\pi}\exp[-z^2/4 - i\,\tilde{\phi}\,z].}$$

$$\boxed{\tilde{\phi} z \to \tilde{\phi}^2 z^2}$$

$$\tilde{\Delta}^{\mathrm{Ga}}_{\mathrm{p,opt}} = \frac{\sqrt{2}\,\chi}{\sqrt{1+\chi^4}}\, e^{-\frac{(\chi^2-1)^2}{\chi^4+1}\tilde{\phi}^2}$$

$$\tilde{\Delta}^{\mathrm{Ga}}_{\mathrm{m,opt}} = \frac{\sqrt{2}\,\chi}{\sqrt{1+\chi^4}}.$$

$$\tilde{\Delta}^{\mathrm{Ga}}_{\mathrm{p,opt}} \approx 1 - (1 + 2\tilde{\phi}^2)\,\delta_1^2$$

$$\tilde{\Delta}^{\mathrm{Ga}}_{\mathrm{m,opt}} \approx 1 - \delta_1^2,$$

$$\tilde{\Delta}^{\mathrm{Ga}}_{\mathrm{p,opt}} = \frac{\sqrt{2}\,\chi}{\sqrt{1+\chi^4}}\, \frac{e^{-4\,\frac{(\chi^2-1)^2}{\chi^4+1}\,\frac{\phi^4}{\xi(\tilde{\phi})}\,z_0^2}}{\sqrt{\xi(\tilde{\phi})}}\, e^{256\,\frac{a_1^2(\tilde{\phi})}{a_2(\tilde{\phi})}}$$

$$\tilde{\Delta}^{\mathrm{Ga}}_{\mathrm{m,opt}} = \frac{\sqrt{2}\,\chi}{\sqrt{1+\chi^4}}.$$

$$\tilde{\Delta}^{\mathrm{Ga}}_{\mathrm{p,opt}} \approx 1 - (1 + 32\tilde{\phi}^4 + 8\tilde{\phi}^4 z_0^2)\delta_1^2 + 2^9\,\frac{\tilde{\phi}^4 z_0^2 \delta_1^4}{1 + 16\tilde{\phi}^4}$$

$$\tilde{\Delta}^{\mathrm{Ga}}_{\mathrm{m,opt}} \approx 1 - \delta_1^2.$$

Curved spacetime as a mode-mixer

$$\hat{A}_{\omega_0} := \int d\omega F_{\omega_0}(\omega) e^{-i\psi(\omega)} \hat{a}_\omega$$

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$$\hat{A}_{\omega_0} := \int d\omega F_{\omega_0}(\omega) e^{-i\psi(\omega)} \hat{a}_\omega$$

Gravitational redshift:

i) Is NOT a unitary transformation on sharp frequencies: $\hat{a}_{\chi^2\omega} \neq \hat{U}^\dagger(\chi)\hat{a}_\omega \hat{U}(\chi)$

ii) IS a unitary transformation on realistic photons: $\hat{A}_{\omega_0'} = \hat{U}^\dagger(\chi)\hat{A}_{\omega_0}\hat{U}(\chi)$

Curved spacetime as a mode-mixer

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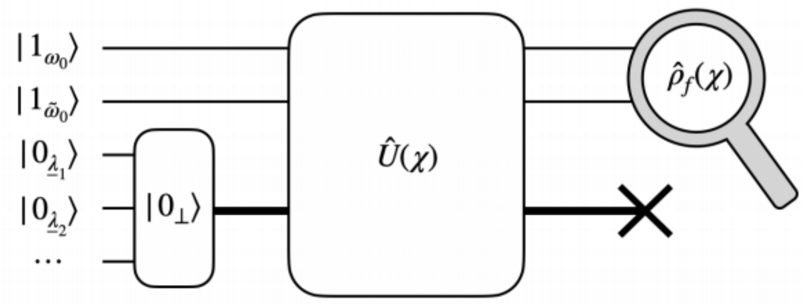

$$|\psi_0\rangle = |1_{\omega_0} 1_{\omega_0'}\rangle$$

$$\begin{aligned}\hat{\rho}_{\mathrm{f}}(\chi) = &\rho_{0000}|00\rangle\langle 00| + \rho_{0202}|02\rangle\langle 02| + \rho_{2020}|20\rangle\langle 20| \\ &+\rho_{1010}|10\rangle\langle 10| + \rho_{0101}|01\rangle\langle 01| + \rho_{1111}|11\rangle\langle 11| \\ &+\rho_{2011}|20\rangle\langle 11| + \rho_{0211}|02\rangle\langle 11| \\ &+\rho_{2001}|20\rangle\langle 02| + \rho_{1001}|10\rangle\langle 01| + \mathrm{h.c.}\end{aligned}$$

Curved spacetime as a mode-mixer

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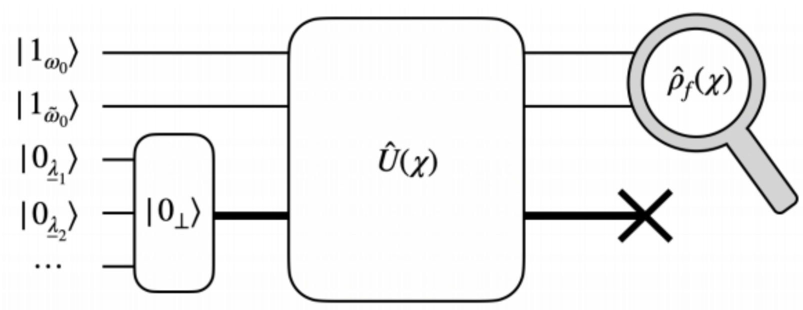

Hong-Ou-Mandel interference
Purely quantum

$$|\psi_0\rangle = |1_{\omega_0} 1_{\omega_0'}\rangle$$

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Conclusions

Gravitational effects on modes of light:

- Rigid shift of wave packets (“redshift”);
- Genuine deformation;
- NO unitarity for ideal sharp photons;
- Mode-mixer for realistic photons;
- Quantum interference due to propagation;
- **Hope for test with cubesats/nanosatellites.**

Grazie

Spacetime effects on satellite-based quantum communications	PRD 90, 045041 (2014)
Quantum estimation of the Schwarzschild space-time parameters of the Earth	PRD 90, 124001 (2014)
Quantum communications and quantum metrology in the spacetime of a rotating planet	EPJ Q Techn. 4:7 (2017)
Quantum-metrology estimation of spacetime parameters of the Earth outperforming classical precision	PRA 99, 032350 (2019)
Spacetime effects on wavepackets of coherent light	arXiv:2106.12424
Gravitational redshift induces quantum interference	arXiv:2109.00728