Thermodynamics of Scalar-Tensor gravity

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Our Approach

$(G = c = 1)$

Scalar-Tensor gravity (Jordan frame)

$$S = \int d^4x \sqrt{-g}\,\mathcal{L}(g_{ab}, \phi) + S^{(\mathrm{m})} \implies$$

Field Equations

$$\frac{\delta S}{\delta g^{ab}} = 0 \qquad \frac{\delta S}{\delta \phi} = 0$$

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G_{\mathrm{eff}}(\phi)\, T^{(\mathrm{m})}_{ab} + 8\pi T^{(\phi)}_{ab}$$

(Eff. Einstein's Eq.s)

$$\Box\phi = \cdots$$

$T^{(\phi)}_{ab}$ stress-energy tensor of an effective fluid

Compute σ_{ab} , θ , ω_{ab} for the ϕ-fluid $\implies$

Thermodynamic analogy
from the effective constitutive relations
for the fluid

Effective Fluid Approach to Scalar-Tensor gravity

$(G = c = 1)$

"First generation" Scalar-Tensor theories (Jordan frame)

$$S_{\mathrm{ST}} = \int d^4x \frac{\sqrt{-g}}{16\pi}\left[\phi R - \frac{\omega(\phi)}{\phi}\nabla^c\phi\nabla_c\phi - V(\phi)\right] + S^{(\mathrm{m})}$$

$\omega(\phi)$ Brans-Dicke "coupling"

$G_{\mathrm{eff}} = 1/\phi\,, \quad \phi > 0$

Field Equations

$$R_{ab} - \frac{1}{2}g_{ab}R = \frac{8\pi}{\phi}T^{(\mathrm{m})}_{ab} + \frac{\omega}{\phi^2}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi\right) + \frac{1}{\phi}\left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right) - \frac{V}{2\phi}$$

$$\Box\phi = \frac{1}{2\omega+3}\left(\frac{8\pi T^{(\mathrm{m})}}{\phi} + \phi\frac{dV}{d\phi} - 2V - \frac{d\omega}{d\phi}\nabla^c\phi\nabla_c\phi\right)$$

$$\underbrace{R_{ab} - \frac{1}{2} g_{ab} R}_{\text{Einstein tensor}} = \frac{8\pi}{\phi} T^{(\mathrm{m})}_{ab} + \underbrace{\frac{\omega}{\phi^2}\left(\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi\right) + \frac{1}{\phi}\left(\nabla_a \nabla_b \phi - g_{ab} \Box \phi\right) - \frac{V}{2\phi}}_{8\pi T^{(\phi)}_{ab}}$$

Einstein tensor

$8\pi G_{\text{eff}} T^{(\mathrm{m})}_{ab}$

$8\pi T^{(\phi)}_{ab}$

Effective ϕ-fluid

$$\Longrightarrow \qquad R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G_{\text{eff}}\, T^{(\mathrm{m})}_{ab} + 8\pi\, T^{(\phi)}_{ab}$$

Goal: draw a connection between the properties of $T^{(\phi)}_{ab}$ and some sort of Thermodynamics of Irreversible Processes

Characterizing the effective scalar field fluid

[V. Faraoni, J. Coté, Phys. Rev. D 98 (2018) 084019]

$$8\pi T_{ab}^{(\phi)} = \frac{\omega(\phi)}{\phi^2}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi\right) + \frac{1}{\phi}\left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right) - \frac{V}{2\phi}$$

A natural fluid interpretation is possible if $\nabla_a\phi$ is timelike

4-velocity of the fluid: $u^a = \dfrac{\nabla^a\phi}{\sqrt{-\nabla^e\phi\nabla_e\phi}}$ $\qquad u^a u_a = -1$

3+1 splitting: time direction u^a + 3d space of the comoving observers

$$h_{ab} = g_{ab} + u_a u_b$$ (push-forward of the induced metric)

Projection operator: $h^a{}_b = \delta^a{}_b + u^a u_b$

$$h^a{}_b u^b = h^a{}_b u_a = 0$$
$$h^a{}_b h^b{}_c = h^a{}_c$$
$$h^a{}_a = 3$$

Continuing with the kinematics of the ϕ-fluid ...

$$h_{ab} = g_{ab} - \frac{\nabla_a\phi\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}$$

Velocity gradient

$$\nabla_b u_a = \frac{1}{\sqrt{-\nabla^e\phi\nabla_e\phi}}\left(\nabla_a\nabla_b\phi - \frac{\nabla_a\phi\nabla^c\phi\nabla_b\nabla_c\phi}{\nabla^e\phi\nabla_e\phi}\right)$$

4-acceleration

$$\dot{u}^a \equiv u^c\nabla_c u_a = \frac{\nabla^b\phi}{(-\nabla^e\phi\nabla_e\phi)^2}\Big[(-\nabla^e\phi\nabla_e\phi)\nabla_a\nabla_b\phi + \nabla^c\phi\nabla_b\nabla_c\phi\nabla_a\phi\Big]$$

The (double) projection of the velocity gradient

$\equiv \nabla_a u^a$ expansion scalar

$$h_a{}^c\, h_b{}^d\, \nabla_d u_c = \sigma_{ab} + \frac{\theta}{3} h_{ab} + \omega_{ab}$$

rotation tensor

(symmetric, trace-free) shear tensor

From which we find...

$$\theta = \frac{\Box\phi}{\left(-\nabla^e\phi\nabla_e\phi\right)^{1/2}} + \frac{\nabla_a\nabla_b\phi\nabla^a\phi\nabla^b\phi}{\left(-\nabla^e\phi\nabla_e\phi\right)^{3/2}}$$

$$\begin{aligned}\sigma_{ab} = \left(-\nabla^e\phi\nabla_e\phi\right)^{-3/2}\Bigg[&-\left(\nabla^e\phi\nabla_e\phi\right)\nabla_a\nabla_b\phi - \frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\,\nabla^c\phi\nabla_c\phi\right)\Box\phi \\ &-\frac{1}{3}\left(g_{ab} + \frac{2\nabla_a\phi\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}\right)\nabla_c\nabla_d\phi\nabla^d\phi\nabla^c\phi + \left(\nabla_a\phi\nabla_c\nabla_b\phi + \nabla_b\phi\nabla_c\nabla_a\phi\right)\nabla^c\phi\Bigg]\end{aligned}$$

$\omega_{ab} = 0 \quad \Longleftrightarrow \quad$ *irrotational*

Going back to the effective stress-energy tensor of the ϕ-fluid

$$8\pi\, T_{ab}^{(\phi)} = \frac{\omega(\phi)}{\phi^2}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}\, g_{ab}\nabla_c\phi\nabla^c\phi\right) + \frac{1}{\phi}\left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right) - \frac{V}{2\phi}$$

Imperfect fluid form

bulk viscous pressure

$$T_{ab}^{(\phi)} = \rho^{(\phi)} u_a u_b + q_a^{(\phi)} u_b + q_b^{(\phi)} u_a + \Pi_{ab}^{(\phi)}\,, \qquad \Pi_{ab}^{(\phi)} = \left(P^{(\phi)} + \bar{p}^{(\phi)}\right) h_{ab} + \pi_{ab}^{(\phi)}$$

Effective: energy density heat flux density (isotropic) pressure viscous stress tensor

Going back to the effective stress-energy tensor of the ϕ-fluid

$$8\pi\, T^{(\phi)}_{ab} = \frac{\omega(\phi)}{\phi^2}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}\, g_{ab}\nabla_c\phi\nabla^c\phi\right) + \frac{1}{\phi}\left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right) - \frac{V}{2\phi}$$

Imperfect fluid form

$$T^{(\phi)}_{ab} = \rho^{(\phi)} u_a u_b + q^{(\phi)}_a u_b + q^{(\phi)}_b u_a + \Pi^{(\phi)}_{ab}\,, \qquad \Pi^{(\phi)}_{ab} = (P^{(\phi)} + \bar{p}^{(\phi)})h_{ab} + \pi^{(\phi)}_{ab}$$

This splitting turns out to be purely arbitrary in this analogy, thus we choose to set the bulk viscous pressure to zero, hence

$$\Pi^{(\phi)}_{ab} = (P^{(\phi)} + \bar{p}^{(\phi)})h_{ab} + \pi^{(\phi)}_{ab} \longrightarrow \boxed{\Pi^{(\phi)}_{ab} = P^{(\phi)} h_{ab} + \pi^{(\phi)}_{ab}}$$

That implies,

$$
\begin{aligned}
8\pi\rho^{(\phi)} &= -\frac{\omega}{2\phi^2}\,\nabla^e\phi\nabla_e\phi + \frac{V}{2\phi} + \frac{1}{\phi}\left(\Box\phi - \frac{\nabla^a\phi\nabla^b\phi\nabla_a\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}\right)\,, \\
8\pi q_a^{(\phi)} &= \frac{\nabla^c\phi\nabla^d\phi}{\phi\left(-\nabla^e\phi\nabla_e\phi\right)^{3/2}}\left(\nabla_d\phi\nabla_c\nabla_a\phi - \nabla_a\phi\nabla_c\nabla_d\phi\right), \\
8\pi\Pi_{ab}^{(\phi)} &= \left(-\frac{\omega}{2\phi^2}\,\nabla^c\phi\nabla_c\phi - \frac{\Box\phi}{\phi} - \frac{V}{2\phi}\right)h_{ab} + \frac{1}{\phi}\,h_a{}^c h_b{}^d\nabla_c\nabla_d\phi\,, \\
8\pi P^{(\phi)} &= -\frac{\omega}{2\phi^2}\,\nabla^e\phi\nabla_e\phi - \frac{V}{2\phi} - \frac{1}{3\phi}\left(2\Box\phi + \frac{\nabla^a\phi\nabla^b\phi\nabla_b\nabla_a\phi}{\nabla^e\phi\nabla_e\phi}\right)\,, \\
8\pi\pi_{ab}^{(\phi)} &= \frac{1}{\phi\nabla^e\phi\nabla_e\phi}\left[\frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\nabla^c\phi\nabla_c\phi\right)\left(\Box\phi - \frac{\nabla^c\phi\nabla^d\phi\nabla_d\nabla_c\phi}{\nabla^e\phi\nabla_e\phi}\right)\right. \\
&\left. +\nabla^d\phi\left(\nabla_d\phi\nabla_a\nabla_b\phi - \nabla_b\phi\nabla_a\nabla_d\phi - \nabla_a\phi\nabla_d\nabla_b\phi + \frac{\nabla_a\phi\nabla_b\phi\nabla^c\phi\nabla_c\nabla_d\phi}{\nabla^e\phi\nabla_e\phi}\right)\right]
\end{aligned}
$$

Comparing the imperfect fluid description with the kinematic quantities

Kinematic

$$\dot{u}^a = \frac{\nabla^b\phi}{(-\nabla^e\phi\nabla_e\phi)^2}\left[(-\nabla^e\phi\nabla_e\phi)\nabla_a\nabla_b\phi + \nabla^c\phi\nabla_b\nabla_c\phi\nabla_a\phi\right]$$

$$\sigma_{ab} = (-\nabla^e\phi\nabla_e\phi)^{-3/2}\left[-\left(\nabla^e\phi\nabla_e\phi\right)\nabla_a\nabla_b\phi - \frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\nabla^c\phi\nabla_c\phi\right)\Box\phi\right.$$
$$\left. -\frac{1}{3}\left(g_{ab} + \frac{2\nabla_a\phi\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}\right)\nabla_c\nabla_d\phi\nabla^d\phi\nabla^c\phi + \left(\nabla_a\phi\nabla_c\nabla_b\phi + \nabla_b\phi\nabla_c\nabla_a\phi\right)\nabla^c\phi\right]$$

Imperfect fluid

$$8\pi q_a^{(\phi)} = \frac{\nabla^c\phi\nabla^d\phi}{\phi\left(-\nabla^e\phi\nabla_e\phi\right)^{3/2}}\left(\nabla_d\phi\nabla_c\nabla_a\phi - \nabla_a\phi\nabla_c\nabla_d\phi\right),$$

$$8\pi\pi_{ab}^{(\phi)} = \frac{1}{\phi\nabla^e\phi\nabla_e\phi}\left[\frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\nabla^c\phi\nabla_c\phi\right)\left(\Box\phi - \frac{\nabla^c\phi\nabla^d\phi\nabla_d\nabla_c\phi}{\nabla^e\phi\nabla_e\phi}\right)\right.$$
$$\left. +\nabla^d\phi\left(\nabla_d\phi\nabla_a\nabla_b\phi - \nabla_b\phi\nabla_a\nabla_d\phi - \nabla_a\phi\nabla_d\nabla_b\phi + \frac{\nabla_a\phi\nabla_b\phi\nabla^c\phi\nabla_c\nabla_d\phi}{\nabla^e\phi\nabla_e\phi}\right)\right]$$

Comparing the imperfect fluid description with the kinematic quantities

One finds...

$$\boxed{q_a^{(\phi)} = \mathcal{K}\mathcal{T}\,\dot{u}_a} \quad \text{with} \quad \mathcal{K}\mathcal{T} = \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi}$$

$$\boxed{\pi_{ab}^{(\phi)} = -2\eta\,\sigma_{ab}} \quad \text{with} \quad \eta = -\frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{16\pi\phi}$$

...that resemble two of the constitutive equations of Eckart's approach to the Thermodynamics of Irreversible Processes (TIP).

Eckart's Thermodynamics of Scalar-Tensor gravity

[V. Faraoni and A. Giusti, Phys.Rev.D 103 (2021) L121501]

Eckart's first order thermodynamics: constitutive equations

$$\begin{aligned} \bar{p} &= -\zeta\,\theta\,, \\ q_a &= -\mathcal{K}\left(h_{ab}\nabla^b\mathcal{T} + \mathcal{T}\dot{u}_a\right)\,, \\ \pi_{ab} &= -2\eta\,\sigma_{ab} \end{aligned}$$

ζ bulk viscosity

$\mathcal{K}$ thermal conductivity

η shear viscosity

Comparing with our effective fluid we find:

$$\boxed{\zeta = 0}$$

$$\eta = -\frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{16\pi\phi} = -\frac{\mathcal{K}\mathcal{T}}{2} < 0$$

$$\mathcal{K}\mathcal{T} = \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi} > 0$$

$$\boxed{h_{ab}\nabla^b\mathcal{T} = 0}$$

Perks of this analogy?

We can solve explicitly the system:

$$\mathcal{K}\mathcal{T} = \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi}$$

$$h_{ab}\nabla^b\mathcal{T} = 0$$

$$\Longrightarrow$$

For instance,

$$\mathcal{T} = \frac{1}{\phi} = G_{\text{eff}} \qquad \mathcal{K} = \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi}$$

GR $\Longrightarrow$ $\phi = \text{constant}$ ($\phi = 1$ coupling to matter) $\Longrightarrow$ $\mathcal{T}_{\text{GR}} = G_{\text{N}}\,, \quad \mathcal{K} = 0$

GR $\simeq$ "perfect insulator" limit of the ϕ-fluid

The approach to equilibrium

Effective heat equation for the ϕ-fluid

$$\frac{d\left(\mathcal{K}\mathcal{T}\right)}{d\tau} = 8\pi\left(\mathcal{K}\mathcal{T}\right)^2 - \theta\,\mathcal{K}\mathcal{T} + \frac{\Box\phi}{\sqrt{-\nabla^e\phi\nabla_e\phi}} \qquad \frac{d}{d\tau} \equiv u^a\nabla_a$$

substantially different from the standard result of Eckart's first-order thermodynamics.

Physical interpretation? Consider a simplified scenario: electrovacuum + $\omega = \text{const.}$ + $V(\phi) = 0$

$$\Longrightarrow \quad \Box\phi = 0$$

1) If $\theta < 0 \Longrightarrow$ $\mathcal{K}\mathcal{T}$ diverges (at a finite "time") away from GR

$\Longrightarrow$ near spacetime singularities deviations of scalar-tensor gravity from GR are extreme

2) If $\theta > 0 \Longrightarrow$ $-\theta\mathcal{K}\mathcal{T}$ could dominate over $\left(\mathcal{K}\mathcal{T}\right)^2 \Longrightarrow$ ST relaxes to GR

An Example: Scalar-Tensor Black Holes

[S.W. Hawking, Commun. Math. Phys. 25 (1972) 167]
[T.P. Sotiriou, V. Faraoni, Phys.Rev.Lett. 108 (2012) 081103]

Faraoni-Hawking-Sotiriou theorem

All asymptotically flat, stationary, axially symmetric black holes in vacuum scalar-tensor gravity reduce to GR black holes provided that $V(\phi)$ has a minimum at the constant value ϕ_0 at which the scalar stabilizes outside the horizon.

Gravitational collapse to form a BH in ST gravity $\Longrightarrow$ $\phi(\tau) \xrightarrow{\tau \to \tau_{\mathrm{BH}}^-} \phi_0 \Longrightarrow \mathcal{K}\mathcal{T} \searrow 0$ outside the horizon

Once the BH is formed $\mathcal{K}\mathcal{T} = 0$ outside the horizon, however the singularity inside the horizon becomes hot and deviates from GR!

Eckart's Thermodynamics of Horndeski gravity

[A Giusti, S Zentarra, L Heisenberg, V Faraoni, arXiv:2108.10706]

"Surviving subclass" of Horndeski (i.e. $c_{\mathrm{GW}} = 1$) $[G_5 = 0\,,\ G_{4X} = 0]$

$$\bar{\mathcal{L}} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R$$

$$X \equiv -\nabla^c\phi\nabla_c\phi/2 > 0$$

$$G_{i\phi} \equiv \partial G_i/\partial\phi$$

$$G_{iX} \equiv \partial G_i/\partial X$$

Kinematics is the same as in the first-generation ST case (theory independent).

$$G_{ab} = \frac{1}{G_4} T^{(\mathrm{m})}_{ab} + \boxed{T^{(\mathrm{eff})}_{ab}} \quad \Longrightarrow \quad q^{(\mathrm{eff})}_a = -\frac{\sqrt{2X}(G_{4\phi} - XG_{3X})}{G_4}\dot{u}_a$$

$$\pi^{(\mathrm{eff})}_{ab} = \frac{G_{4\phi}\sqrt{2X}}{G_4}\sigma_{ab}$$

Still has an imperfect fluid form

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$$G_{i\phi} \equiv \partial G_i/\partial\phi$$

$$G_{iX} \equiv \partial G_i/\partial X$$

Comparing with Eckart's constitutive relations one gets:

$$\zeta = 0 \qquad \eta = -\frac{\sqrt{X}\,G_{4\phi}}{\sqrt{2}\,G_4} \qquad h_{ab}\nabla^b\mathcal{T} = 0 \qquad \mathcal{K}\mathcal{T} \equiv \frac{\sqrt{2X}(G_{4\phi} - XG_{3X})}{G_4}$$

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$$X \equiv -\nabla^c\phi\nabla_c\phi/2 > 0$$

$$\bar{\mathcal{L}} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R$$

$$G_{i\phi} \equiv \partial G_i/\partial\phi$$

$$G_{iX} \equiv \partial G_i/\partial X$$

Alternatively:

$$\zeta = 0 \qquad \eta = -\frac{\sqrt{X}\,G_{4\phi}}{\sqrt{2}\,G_4} \qquad \mathcal{T} \equiv \frac{1}{G_4} \qquad \mathcal{K} \equiv \sqrt{2X}\,\left(G_{4\phi} - X\,G_{3X}\right)$$

Eckart's Thermodynamics of Horndeski gravity

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$$G_{iX} \equiv \partial G_i/\partial X$$

Expanding $\bar{\mathcal{L}}$ on a <u>spatially flat, homogeneous, isotropic</u> background
→ 4 parameters controlling the dynamics of perturbations

$$\mathcal{T} \propto \frac{1}{M_\star^2} \qquad \mathcal{K} \propto -\alpha_{\rm B} \qquad \Longleftarrow \qquad \mathcal{T} \equiv \frac{1}{G_4} \qquad \mathcal{K} \equiv \sqrt{2X}\,(G_{4\phi} - X\,G_{3X})$$

Eff. Planck Mass

Braiding

Eckart's Thermodynamics of Horndeski gravity

[A Giusti, S Zentarra, L Heisenberg, V Faraoni, arXiv:2108.10706]

General Horndeski

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2 \,, \\
\mathcal{L}_3 &= -G_3 \,\Box\phi \,, \\
\mathcal{L}_4 &= G_4 \, R + G_{4X} \left[(\Box\phi)^2 - (\nabla_a \nabla_b \phi)^2 \right] \,, \\
\mathcal{L}_5 &= G_5 \, G_{ab} \, \nabla^a \nabla^b \phi \\
&\quad - \frac{G_{5X}}{6} \left[(\Box\phi)^3 - 3 \,\Box\phi \, (\nabla_a \nabla_b \phi)^2 + 2 \, (\nabla_a \nabla_b \phi)^3 \right] \,,
\end{aligned}$$

$$T_{ab}^{(\mathrm{eff})} \supset f(\phi, X) \, R_{acbd} \nabla^c \phi \nabla^d \phi$$

$$\Longrightarrow \qquad \pi_{ab}^{(\mathrm{eff})} \supset f(\phi, X) \, R_{aebf} \nabla^e \phi \nabla^f \phi - \frac{f(\phi, X)}{3} R_{ef} \nabla^e \phi \nabla^f \phi \not\propto \sigma_{ab}$$

Analogy with Eckart's theory no longer works!

Conclusions

- Starting Point: Effective Fluid approach to Scalar-Tensor Gravity
- Correspondence with the constitutive equations of Eckart's first order thermodynamics
- Analogy → Define: Temperature, bulk viscosity and shear viscosity of modified gravity
- GR = "perfect insulator" limit
- Approach to equilibrium governed by a generalized heat equation
- Horndeski: "luminal" subclass works as "first generation" ST theories

  beyond this class, the thermodynamic analogy breaks down.

Thank You!

A bunch of useful references

V. Faraoni, J. Coté, Phys. Rev. D 98 (2018) 084019
V. Faraoni, A. Giusti, Phys. Rev. D 103 (2021) L121501
A. Giusti, S. Zentarra, L. Heisenberg, V. Faraoni, arXiv:2108.10706 [gr-qc]