Sensor Placement Optimization for Broadband Newtonian Noise Cancellation in GW Detectors

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Introduction

- Newtonian Noise (NN): Displacement noise induced in the test mass, due to terrestrial gravity perturbations in vicinity of test mass.
- Since environmental density fluctuations produce a stochastic gravitational field that couples directly to test mass, Seismic Isolation Systems cannot eliminate the effects of NN.
- Newtonian noise cancellation systems consist of seismometer networks to monitor noise source and linear filters to estimate & subtract NN.
- To achieve maximal NN subtraction, the shape of the sensor array must be optimized.
- NN subtraction must be efficient across the frequency range where NN is dominant, hence Broadband sensor placement optimization problem is analysed.

Sensor Placement Optimization

Prior Work:

- Sensor array configurations were optimized for a single seismic wave frequency. Local optima was found.[1]
- Optimization at two frequencies was analysed. Optimal sensor configuration formed by merging the optimal arrays of the two frequencies.[2]

In reality, the effect of NN is significant over $\sim (8 - 20)$ Hz.

Objective: Expand the sensor placement problem to a multi-objective optimization problem, so that the sensor positions are optimized over a broad range of frequencies.

[1] M. Coughlin et al. "Towards a first design of a Newtonian-noise cancellation system for Advanced LIGO", 2016.

[2] Badaracco F, Harms J. "Optimization of seismometer arrays for the cancellation of Newtonian noise from seismic body waves".

Seismic Models

Assume: Rayleigh surface seismic waves propagating in homogeneous and isotropic medium.

Correlation Models[3]

Cross-spectral density of normal surface displacement b/w sensors: $C_{SS}^{ij}(\vec{r}_i, \vec{r}_j; \omega) = J_0\left(\frac{2\pi}{\lambda}|\vec{r}_i - \vec{r}_j|\right) + \frac{1}{\sigma^2}\delta_{ij}$

Cross-spectral density b/w normal surface displacement at sensor & NN acceleration: $C_{SN}^{i}(\vec{r}_i; \omega) = J_1\left(\frac{2\pi}{\lambda}|\vec{r}_i|\right)\frac{x_i}{|\vec{r}_i|}$

NN variance: $C_{NN} = 0.5$

where $\vec{r} = [x; y] \in \mathbb{R}^2$ is sensor position on ground surface; $\lambda =$ seismic wavelength; $\sigma =$ Signal-to-Noise ratio of sensor and $N =$ no. of sensors

[3] Driggers et al, "Subtraction of Newtonian noise using optimized sensor arrays", Physical Review D, 2012

Noise Residual Function

- Performance of linear noise filters estimating NN is characterized by the **Noise Residual** ($R(\omega)$).
- R represents the error between actual target value and estimated NN value for the Wiener filter.

$$R(\omega) = 1 - \frac{C_{SN}^{T}(\omega) \cdot (C_{SS}(\omega))^{-1} \cdot C_{SN}(\omega)}{C_{NN}(\omega)}$$

- Clearly, the Noise Residual (R) is dependant on the position of sensors ($\vec{r}$) and seismic wavelength (λ).
- For a fixed number of seismometers, the optimal positioning of the sensors on the surface is obtained by minimizing $\sqrt{R}$.

Multi-objective Problem

To minimize R for a given frequency range of interest:

- Discretize given frequency range.
- Choose n suitable frequency points and define R for those frequencies.
- These n functions serve as the n objectives in the multi-objective optimization framework.

Multi-objective optimization problem is defined as :

$$\begin{aligned} \text{minimize} \quad & F(\vec{r}) = \begin{bmatrix} f_1(\vec{r}) \\ f_2(\vec{r}) \\ . \\ . \\ f_n(\vec{r}) \end{bmatrix} \\ \text{subject to} \quad & f_i(\vec{r}) = \sqrt{R_i(\vec{r})},\ i = 1, \dots, n \\ & ||\vec{r}||_\infty \leq 50 \end{aligned}$$

where $F : \mathbb{R}^{2N} \to \mathbb{R}^n$

Solution Approach

- Might not be feasible to find a point that optimizes all n objective functions. Obtain Pareto-optimal solution.
- **Normal Boundary Intersection** (NBI) method chosen to generate Pareto-optimal surface.[4]
- NBI method finds points on the boundary of the set of attainable objective vectors by solving an optimization problem:

$$
\begin{aligned}
\max_{z,t} \quad & t \\
\text{subject to} \quad & \phi\beta + t\hat{n} = F(z) - F^*, \\
& a \le z \le b
\end{aligned}
$$

where $t \in \mathbb{R}$

$\hat{n}$ = unit normal pointing towards origin

$\beta \in \mathbb{R}^n,\ \sum_{i=1}^{n} \beta_i = 1,\ \beta_i \geq 0$

[4] Das, Indraneel et all. "Normal-boundary intersection: A new method for generating the Pareto surface in nonlinear multicriteria optimization problems", 1998

Solution Approach

- Finding global optima of NBI problem will give us a pareto-optimal point.
- To solve the above non-linear, non-convex optimization problem we have adapted the **Multi-level Single Linkage** (MLSL) algorithm.
- Obtain multiple points on Pareto-optimal surface by solving NBI problem for different values of β.
- To choose desirable pareto-optimal point, its preferable to focus on regions that present a high trade-off in the objective functions.
- Smart filter method used to reduce obtained solution set with emphasis on regions of significant trade-off.[5]

[5] Mattson et all. "Smart Pareto filter: obtaining a minimal representation of multiobjective design space", 2004

Results

- Noise residual of the sensor configuration plotted against frequency is chosen as the performance indicator; used to select desirable solution from reduced pareto-optimal set.
- Results are verified for various cases by setting the number of sensors and objective functions.
- Desired solution compared with Single objective case, Maximum of residuals, Sum of residuals and Merged case.
- Implementation: Matlab 2019

  Desired frequency range = $(8 - 20)$ Hz
  SNR $(\sigma) = 100$ for each sensor
  Rayleigh wave speed decreases from 1.5 km/s at 1 Hz to 250 m/s at 50 Hz

Results

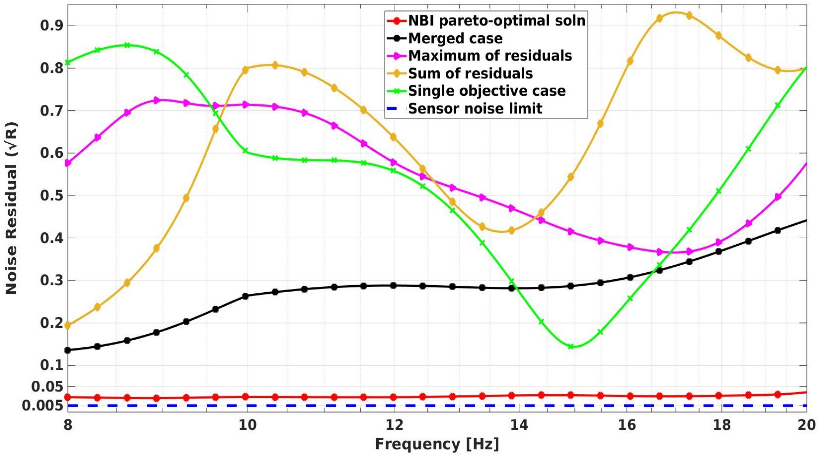

Figure 1: Comparison of optimal solutions for $N = 4$ sensors & $n = 4$ objectives

Results

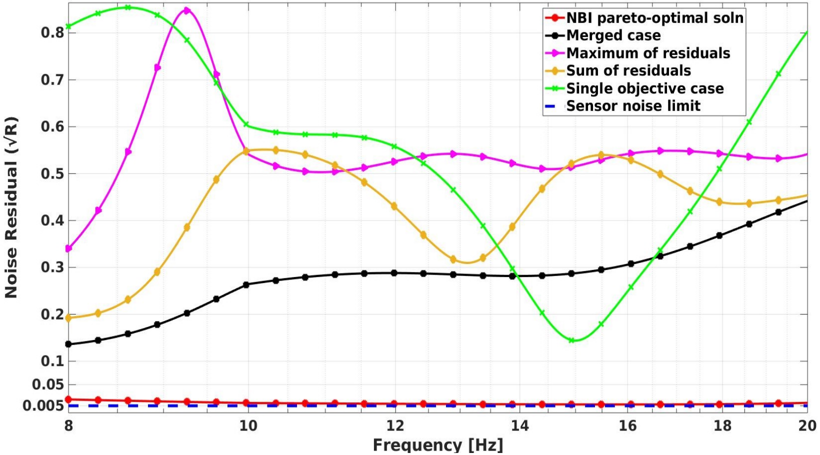

Figure 2: Comparison of optimal solutions for $N = 4$ sensors & $n = 8$ objectives

Conclusion & Future Work

- Obtained optimal seismometer array configurations for Newtonian noise cancellation over a broadband frequency range.
- Results show significant improvement from the single objective case and the error is limited only by seismometer self noise.
- More complex correlations models to be used.
- Use some advanced methods to directly obtain the desired Pareto-optimal solution.