*Ranking the **Love** for the neutron star **E**quation **of** **S**tate with **third** generation detectors*

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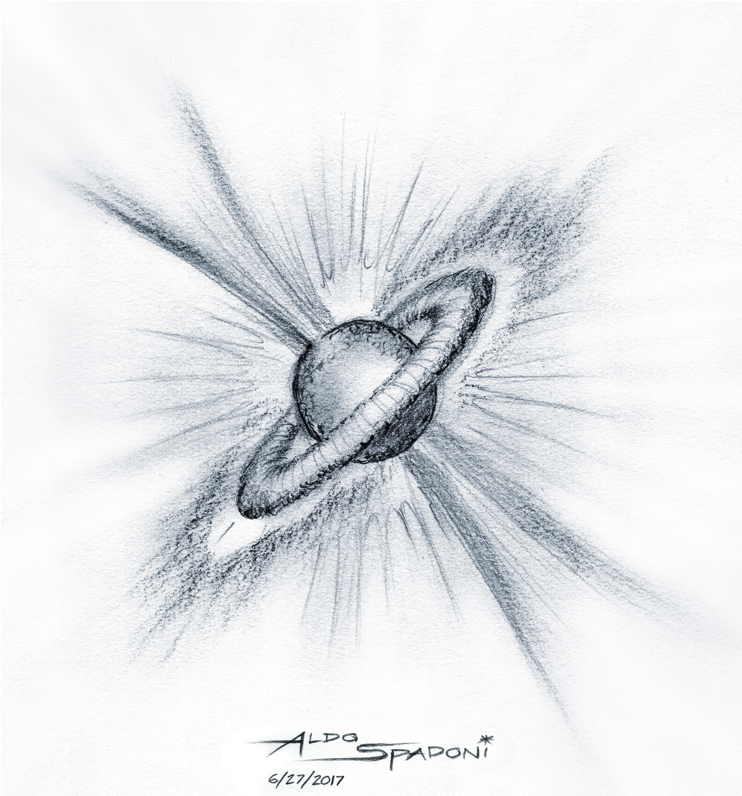
C. Pacilio, A.M., M. Fasano, P. Pani
gr-qc: 2104.10035

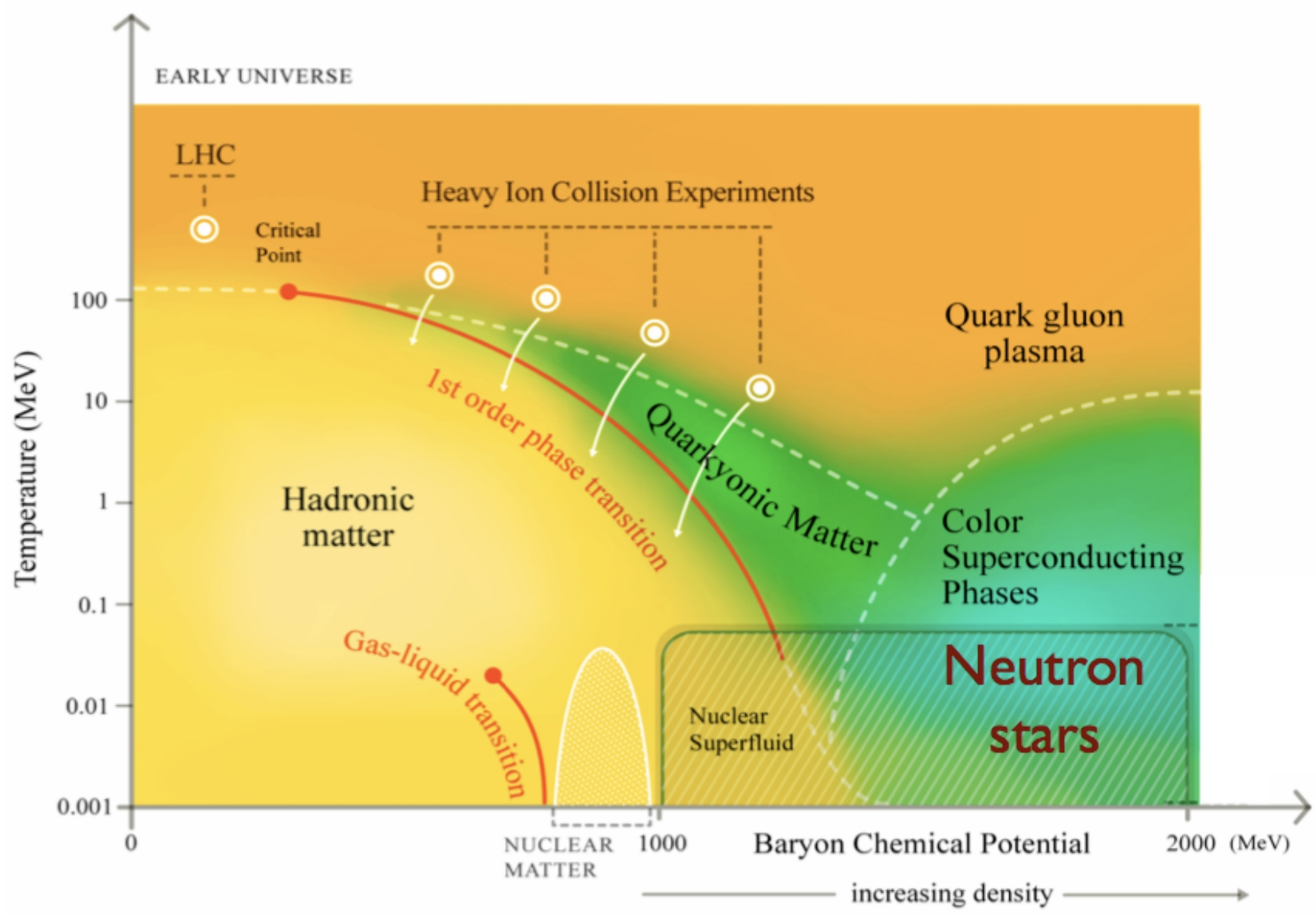


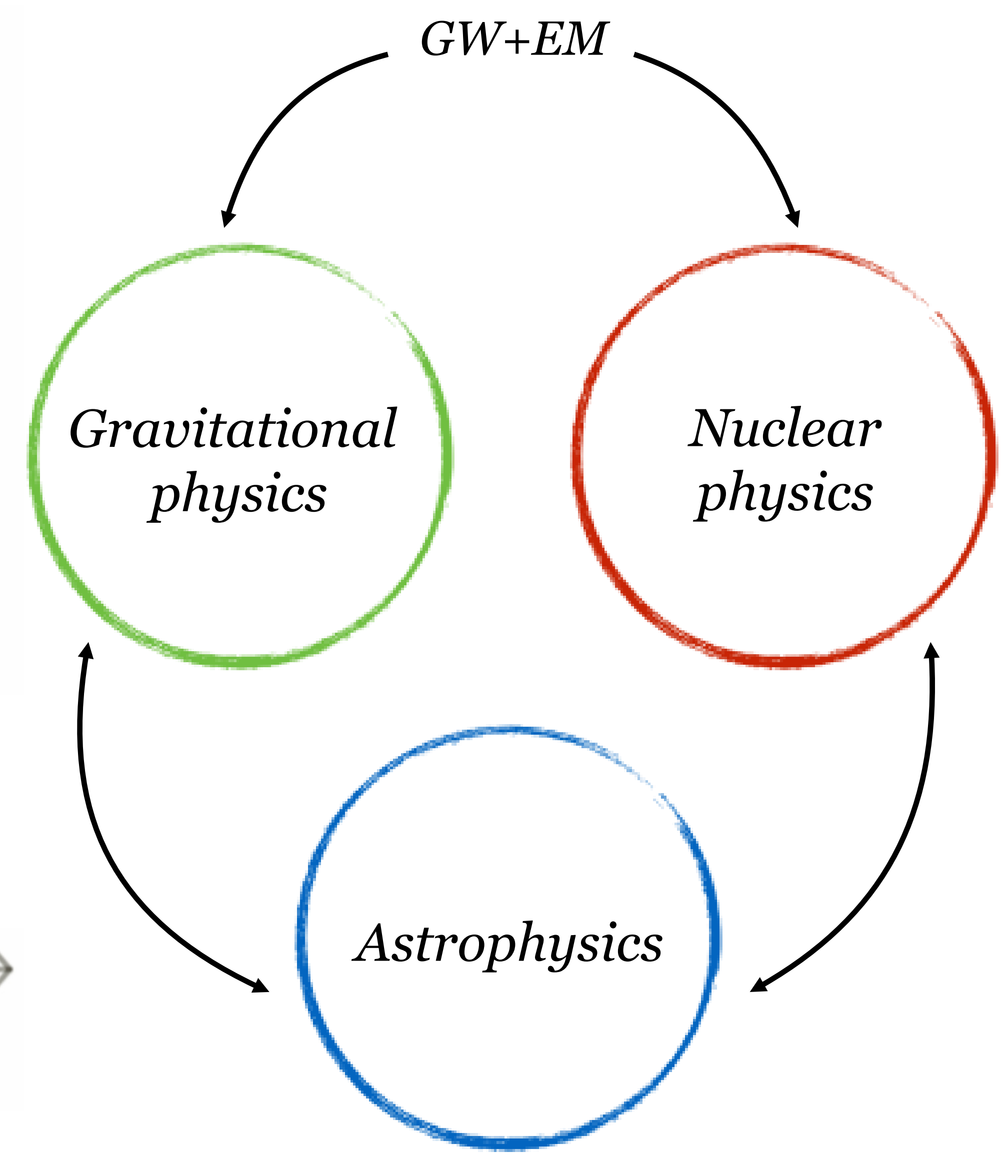
Andrea Maselli

GRAN SASSO SCIENCE INSTITUTE
CENTER FOR ADVANCED STUDIES
INFN

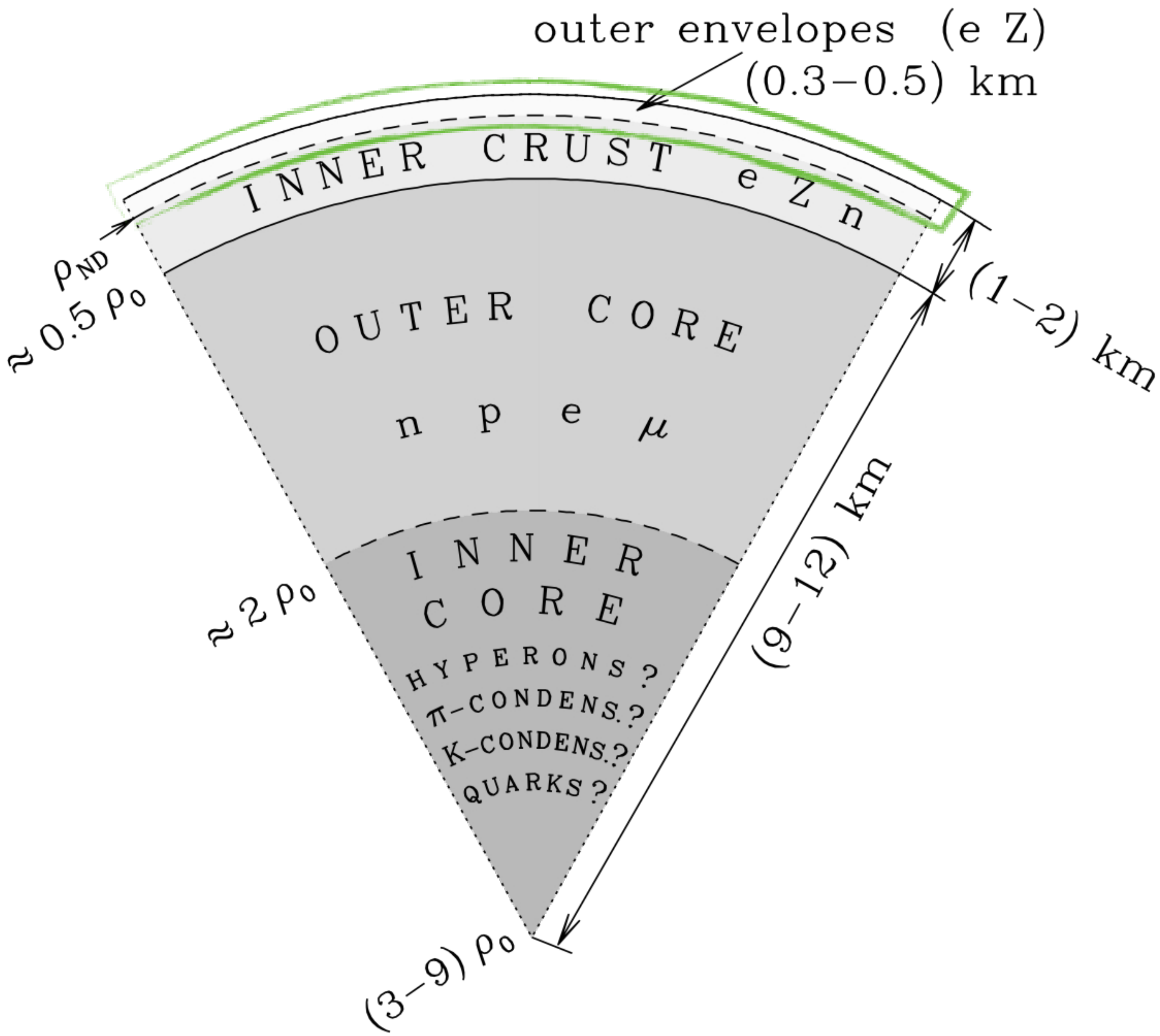
The astro-Lab

Magnifying lenses of fundamental forces





The elephant in the core



$$\rho_0 \simeq 2.67 \times 10^{14}\mathrm{g\ cm^{-3}}$$

Nuclei lattice within e⁻ gas

Lattimer & Prakas, Phys. Rep. 621, 2016
Caplan & Horowitz, PRL 121, 2018

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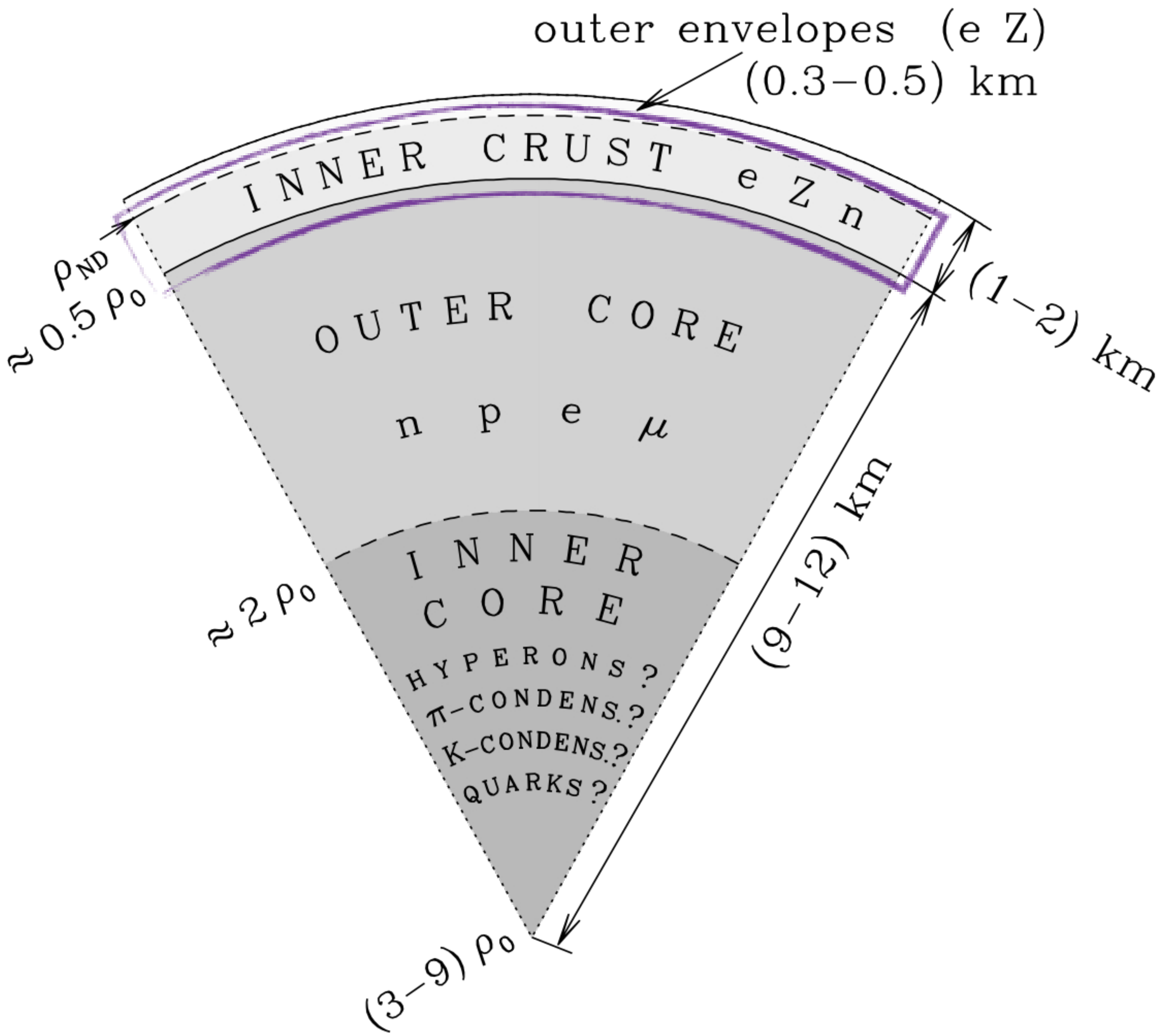

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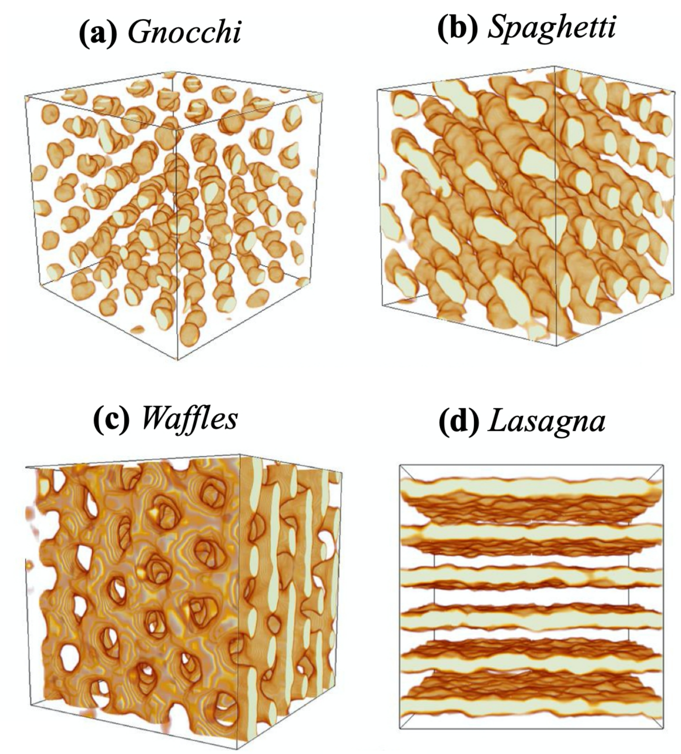
Pasta phases



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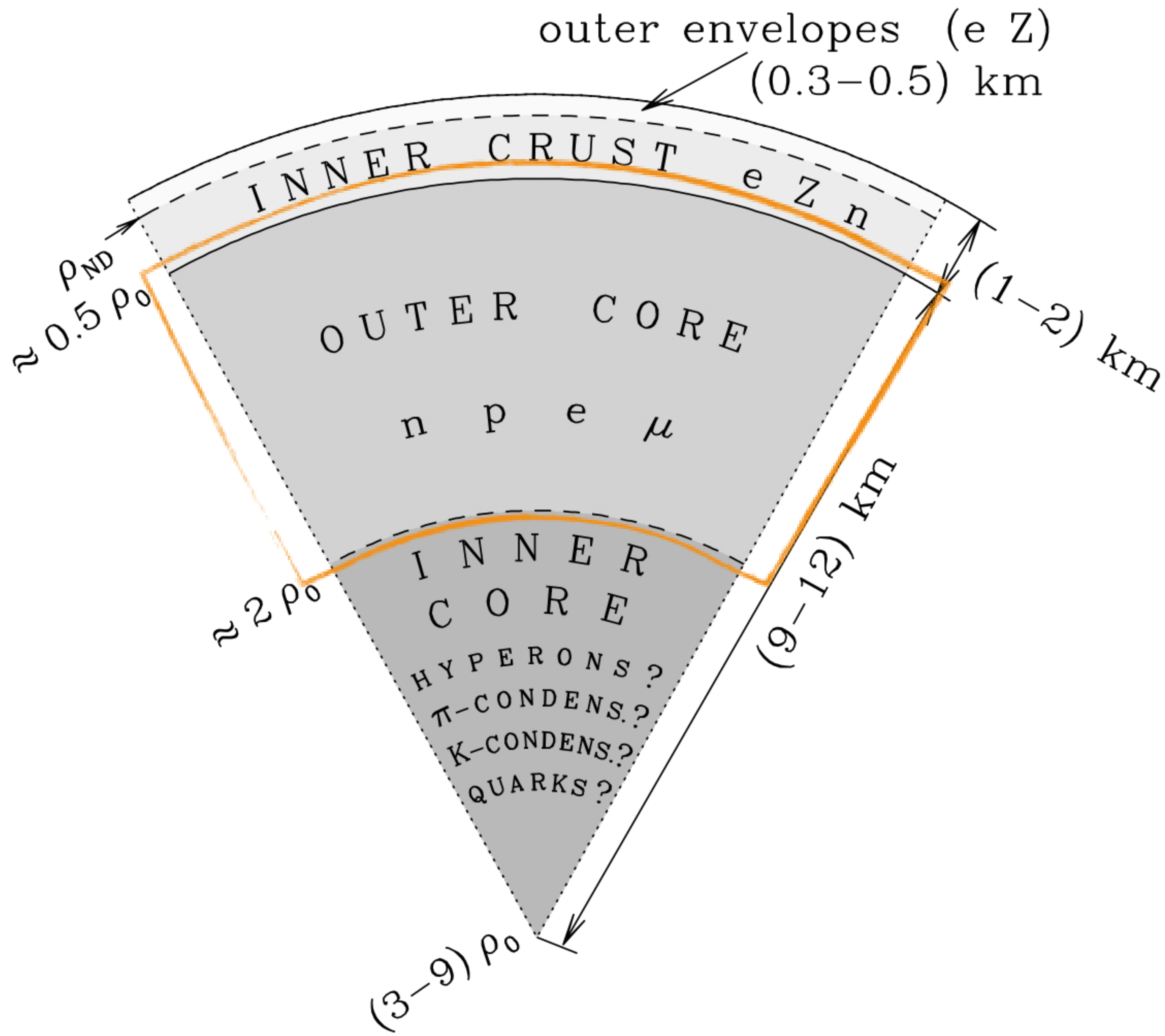

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constraints at supranuclear density

Nucleonic matter in $\beta-$ equilibrium

Lattimer & Prakas, Phys. Rep. 621, 2016
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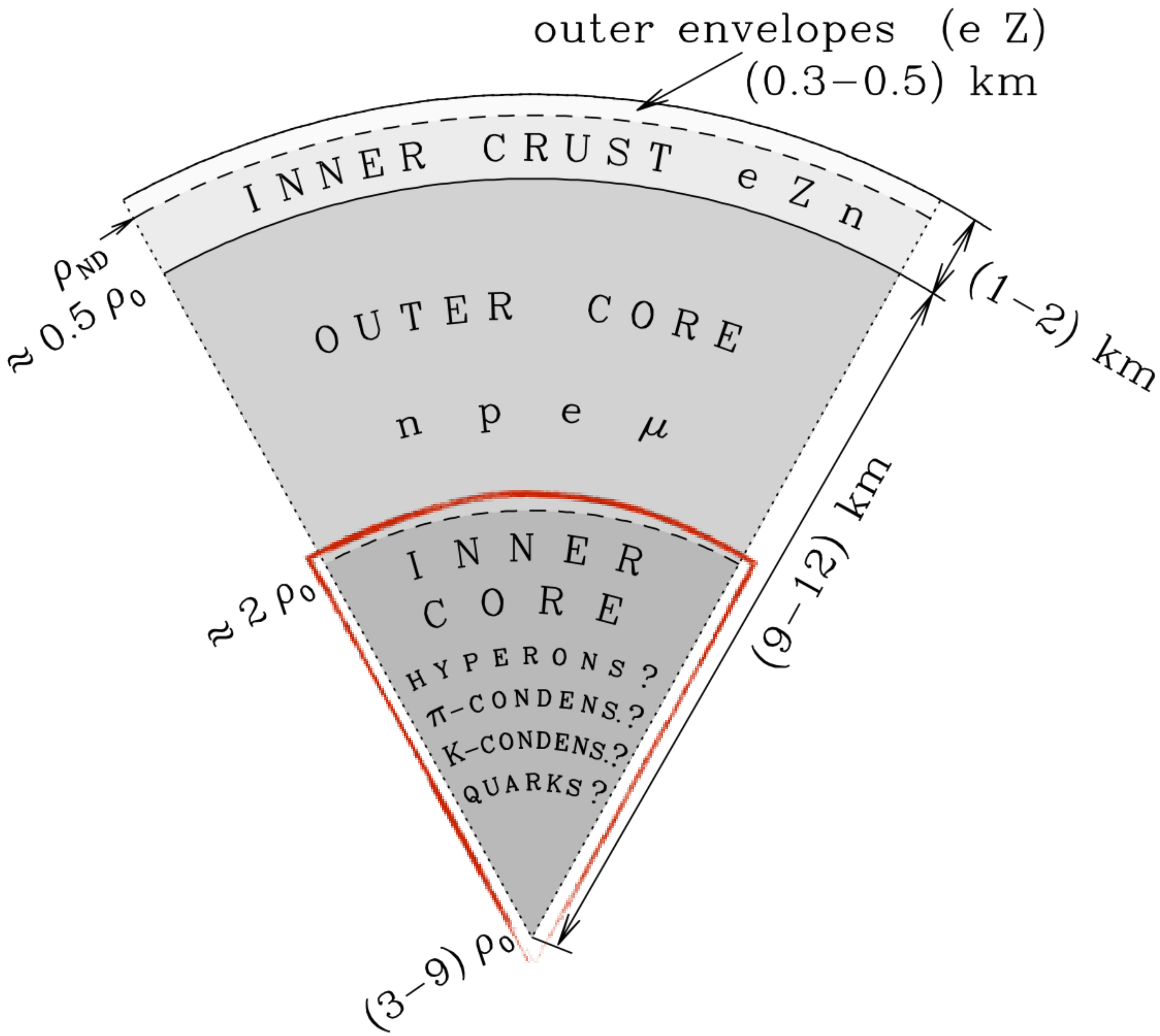
The elephant in the core



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constraints at supranuclear density

- *Phase transitions*
- *Hyperons/mesons*

$$n + e^- \rightarrow \Sigma^- + \nu_e$$

(udd) *(dds)*

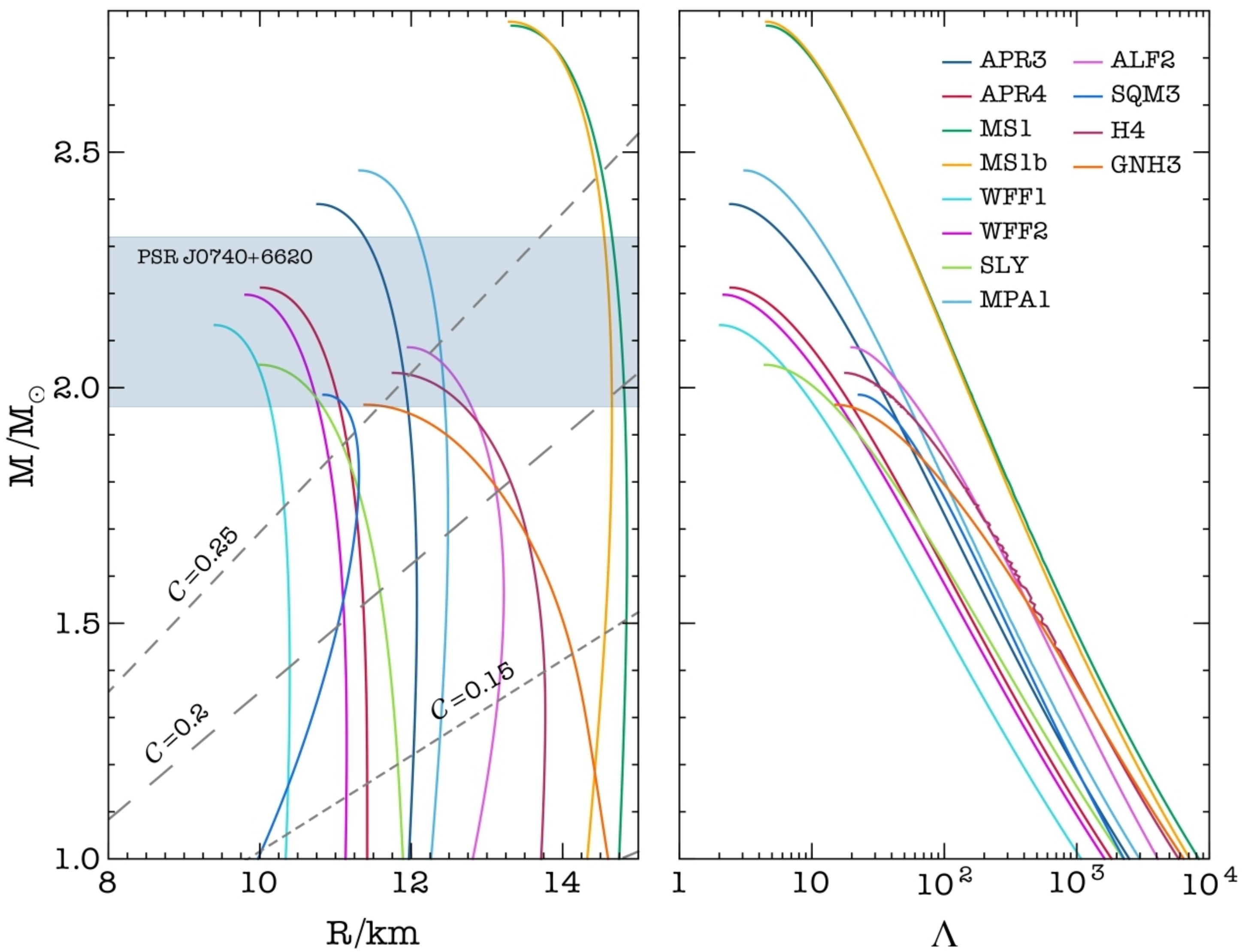
- *Quark deconfinement*

Lattimer & Prakas, Phys. Rep. 621, 2016
Caplan & Horowitz, PRL 121, 2018

From macro to micro

*Microscopic properties, the **E**quation **o**f **S**tate, reflect into macroscopic stellar observables*

radius *Love numbers*



EoS	family	particles
ALF2	nmbt+bag	$npe\mu + Q$
APR3	nmbt	$npe\mu$
APR4	nmbt	$npe\mu$
GNH3	rmft	$npe\mu + H$
H4	rmft	$npe\mu + H$
MPA1	rmft	$npe\mu$
MS1	rmft	$npe\mu$
MS1b	rmft	$npe\mu$
SLY	rmft	$npe\mu$
SQM3	rmft+bag	$npe\mu + H + Q$
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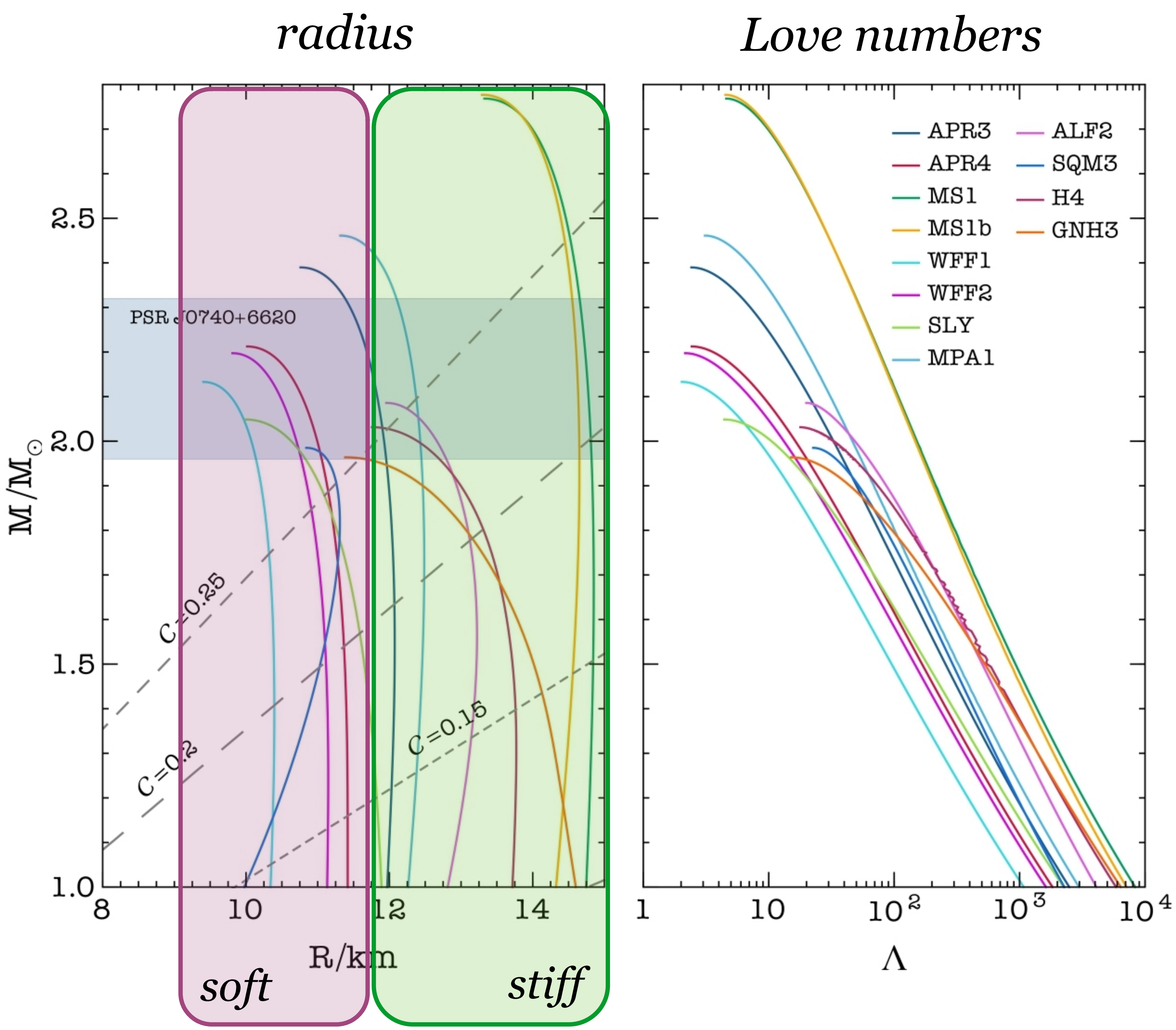

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The Love number

Tidal interactions leave the footprint of the NS structure on the GW signal

Hinderer, The Astrop. J. 677, 2008; Binnington & Poisson Phys. Rev. D 80, 2009
Damour & Nagar, Phys. Rev. D 80, 2009

- *Deformation properties encoded within the Love numbers*

star's quadrupole

$$\mathcal{Q}_{ij} = \frac{2}{3} k_2 R^5 \mathcal{E}_{ij} = \lambda \mathcal{E}_{ij}$$

external tidal field

- λ *depends on the EoS only for a given compactness* $M_{\rm NS}/R_{\rm NS}$
- λ *enters within the gravitational waveform*

Where do we look for Love?

Tidal effects add linearly to the GW phase $h(f) = \mathcal{A} e^{i[\psi_{\rm PP}(f) + \psi_{\rm T}(f)]}$

$$\psi_{\rm PP} \propto \left[1 - \left(\frac{743}{336} + \frac{11}{4}\eta \right) \frac{(m\pi f)^{2/3}}{c^2} + \frac{\dots}{c^3} + \ \dots \ + \frac{\dots}{c^7} \right]$$

Newt *1 PN* *3.5 PN*

$$\psi_{\rm T} \propto \frac{1}{26} \left[\left(1 + 12 \frac{m_2}{m_1} \right) \lambda_1 + \left(1 + 12 \frac{m_1}{m_2} \right) \lambda_2 \right] \frac{(m\pi f)^{10/3}}{c^{10}} + \frac{\dots}{c^{12}}$$

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5 PN: small term!

- *Relevant at high frequencies*

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$\approx$

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$\mathcal{O}(10^4)$ *5 PN: small term!*

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$\wr$

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$\mathcal{O}(10^4)$ *5 PN: small term!*

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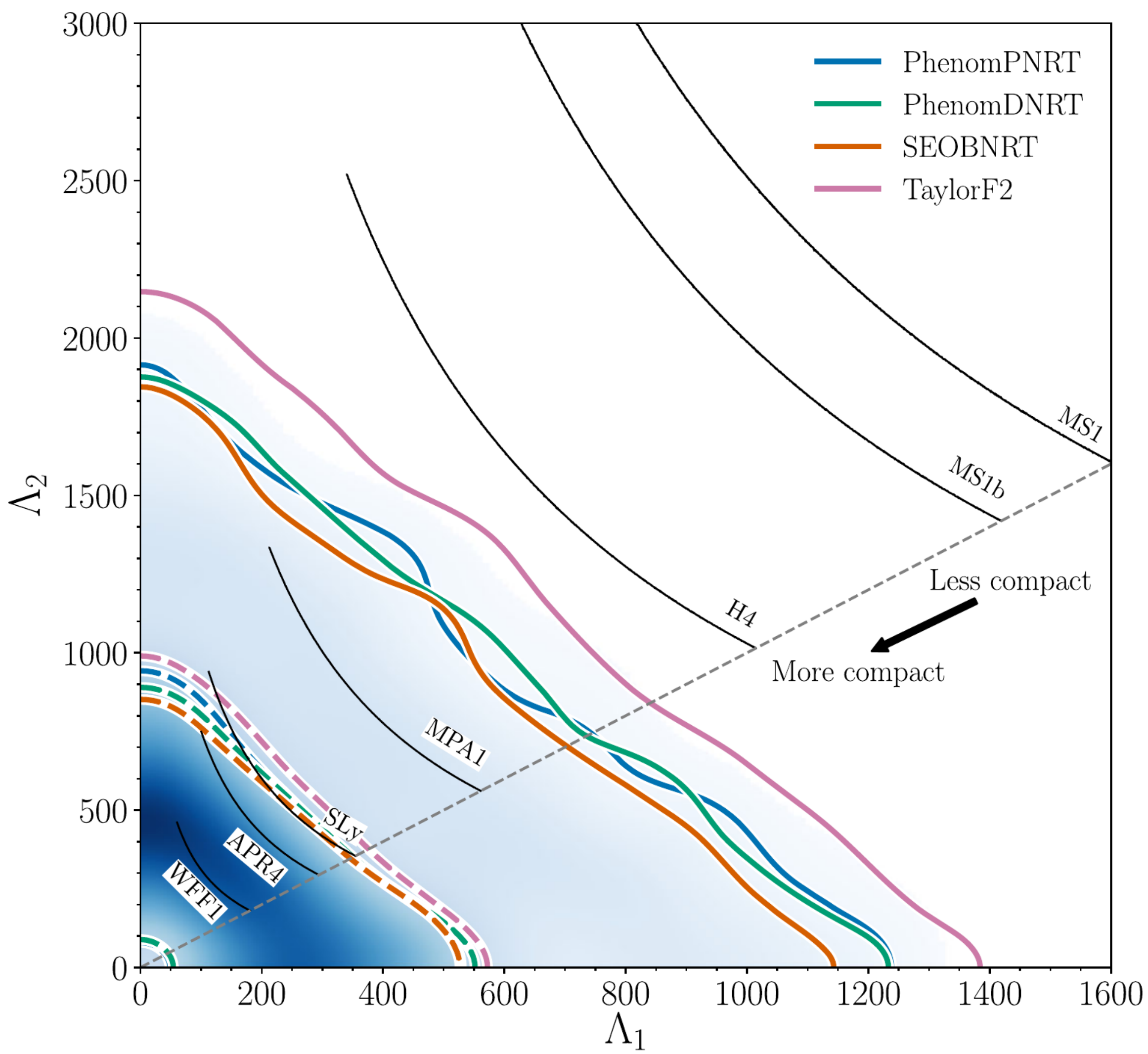
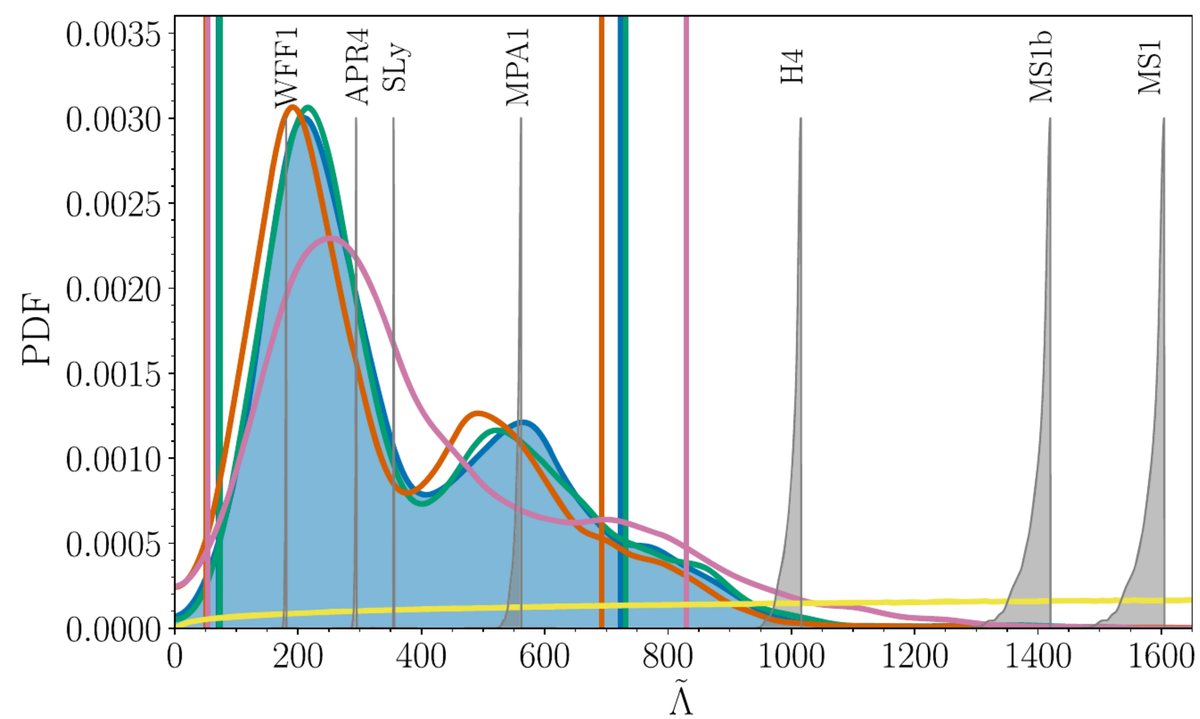
The observables: GWs

Measurements of the ***Love Numbers*** *from GW170817*

LVC, Phys. Rev. Lett. 121, 2018
LVC, Phys. Rev. X 9, 011001, 2019





Ranking the EoS

Hierarchical Bayesian test which rank different EoS given GW binary NS observation(s)

- *Degree of belief that 2 NS of a binary obey a particular EoS*

$$\mathcal{Z}(\mathcal{D}|\mathrm{EoS}) = \int_a^b dp^{(1)} \int_a^b dp^{(2)}\, \mathcal{P}(\mathcal{M}, \eta, \tilde{\Lambda}|\mathcal{D}) \times \mathcal{P}(p^{(1)}|\mathrm{EoS})\, \mathcal{P}(p^{(2)}|\mathrm{EoS})\,,$$

GW measured chirp, Love..

- *Relative odds of two EoS given the data*

$$\mathcal{B}_2^1 = \frac{\mathcal{Z}(\mathcal{D}|\mathrm{EoS}_1)}{\mathcal{Z}(\mathcal{D}|\mathrm{EoS}_2)} \xrightarrow{n \text{ events}} \mathcal{B}_2^1 = \prod_{k=1}^{n} \frac{\mathcal{Z}(\mathcal{D}_k|\mathrm{EoS}_1)}{\mathcal{Z}(\mathcal{D}_k|\mathrm{EoS}_2)}$$

- *Kass-Raftery ranking*

$\log_{10} \mathcal{B}_2^1 < -2$ — *EoS 1* ***decisively*** *disfavored*

$-2 \leq \log_{10} \mathcal{B}_2^1 \leq -1$ — *EoS 1* ***strongly*** *disfavored*

The road to the EoS

Bayesian ranking of 12 realistic EoS based on microscopic calculations

- *Consistent with max mass constraint of J0740+6620*

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WFF1	nmbt	$npe\mu$
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GW170817

first constraints

extreme EoS models

Stacked events

stiff matter v.s.

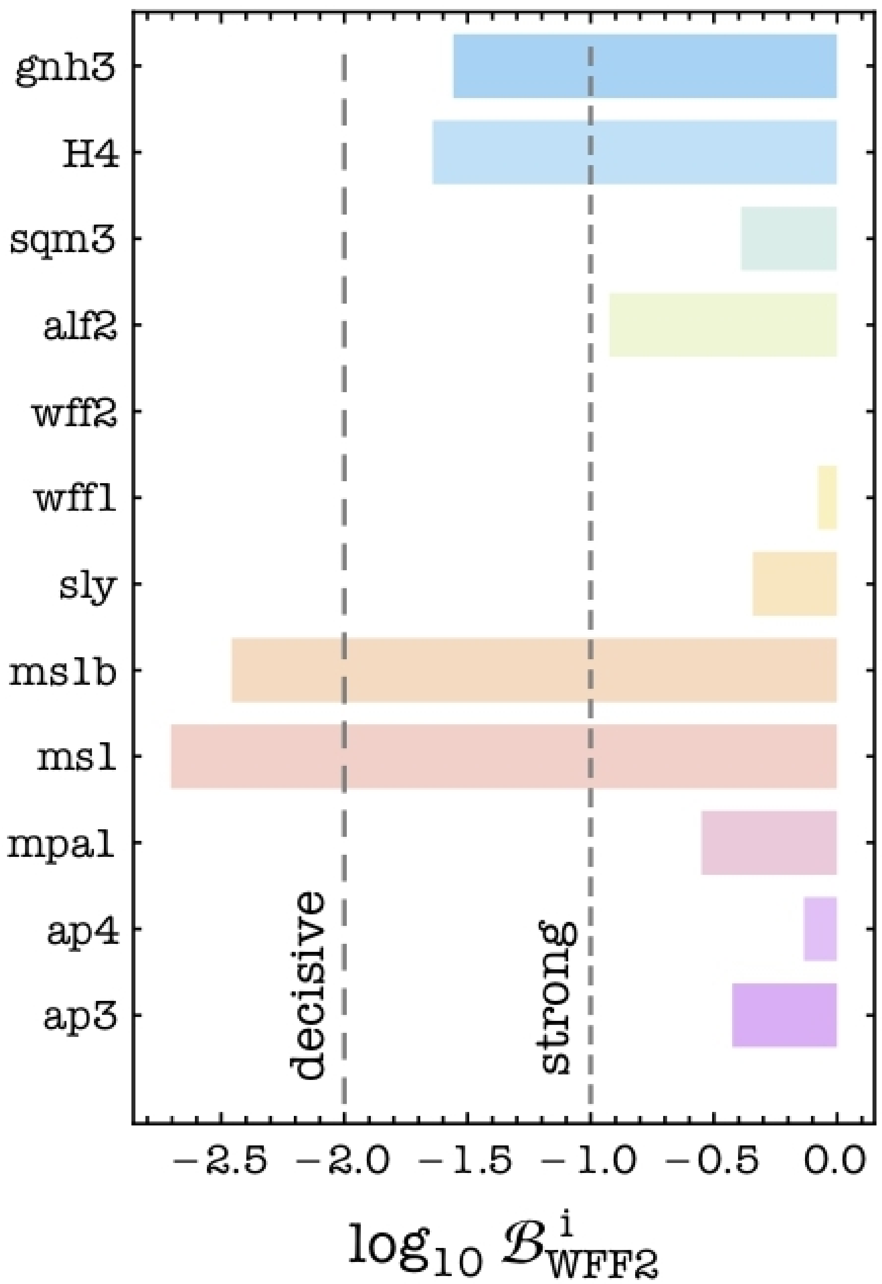
soft matter

Einstein Telescope

Discriminate among different microphysical properties of the EoS

GW170817

Bayes factor normalised to the EoS with the largest evidence (WFF2)



- *The evidence against other EoS is weak beside GNH3 and H4*
- *Decisive evidence against MS1 and MS1b*
  - *Stiffest EoS of the catalogue*

*Doing better with **more** help?*

GW170817

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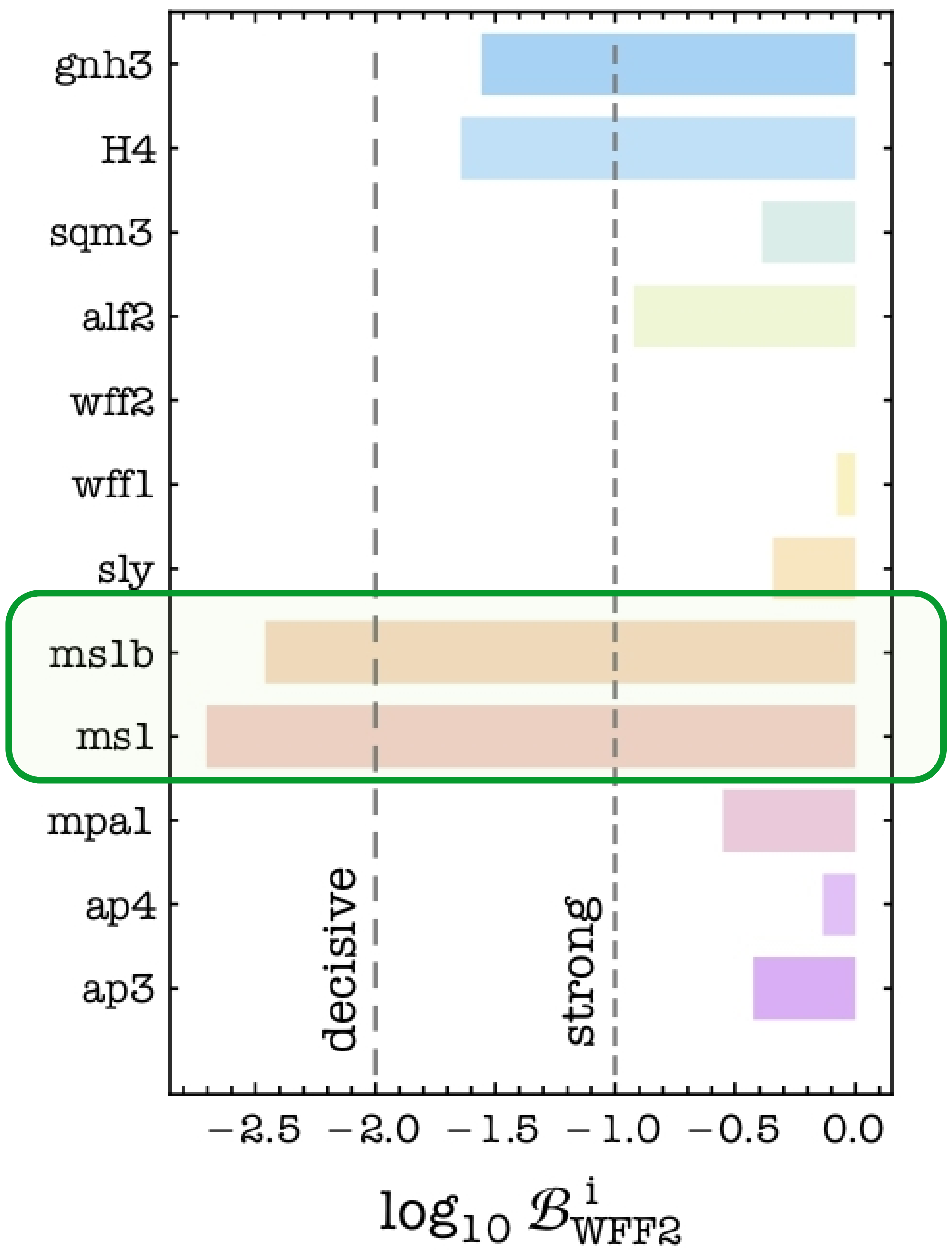

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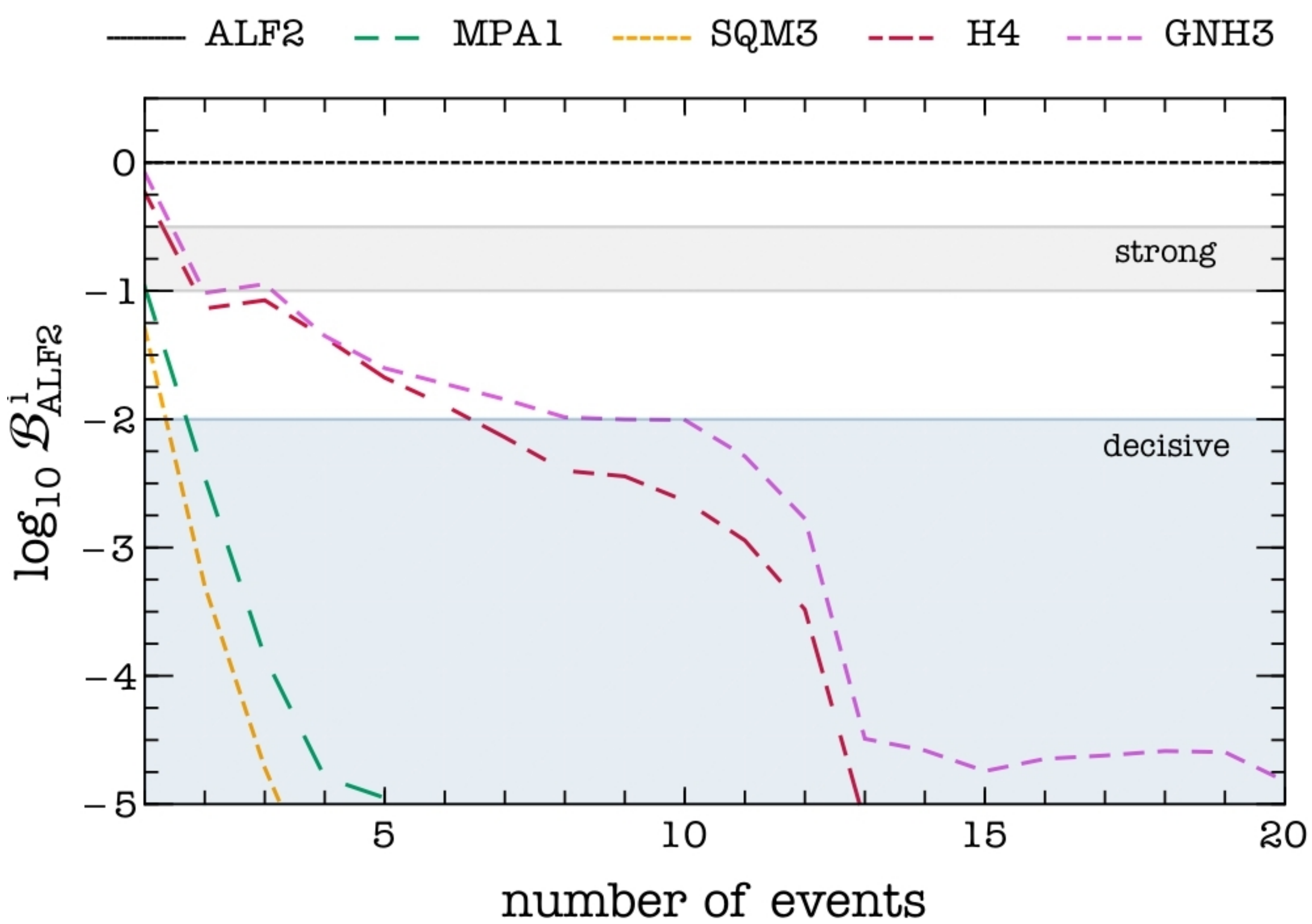
Doing better with ***more*** *help?*

Stacking at design

Bayes factor as a function of # of events detected by HLV at design [20 observations within $60 \leq d_{\mathrm{L}}/\mathrm{Mpc} \leq 210$ *]*

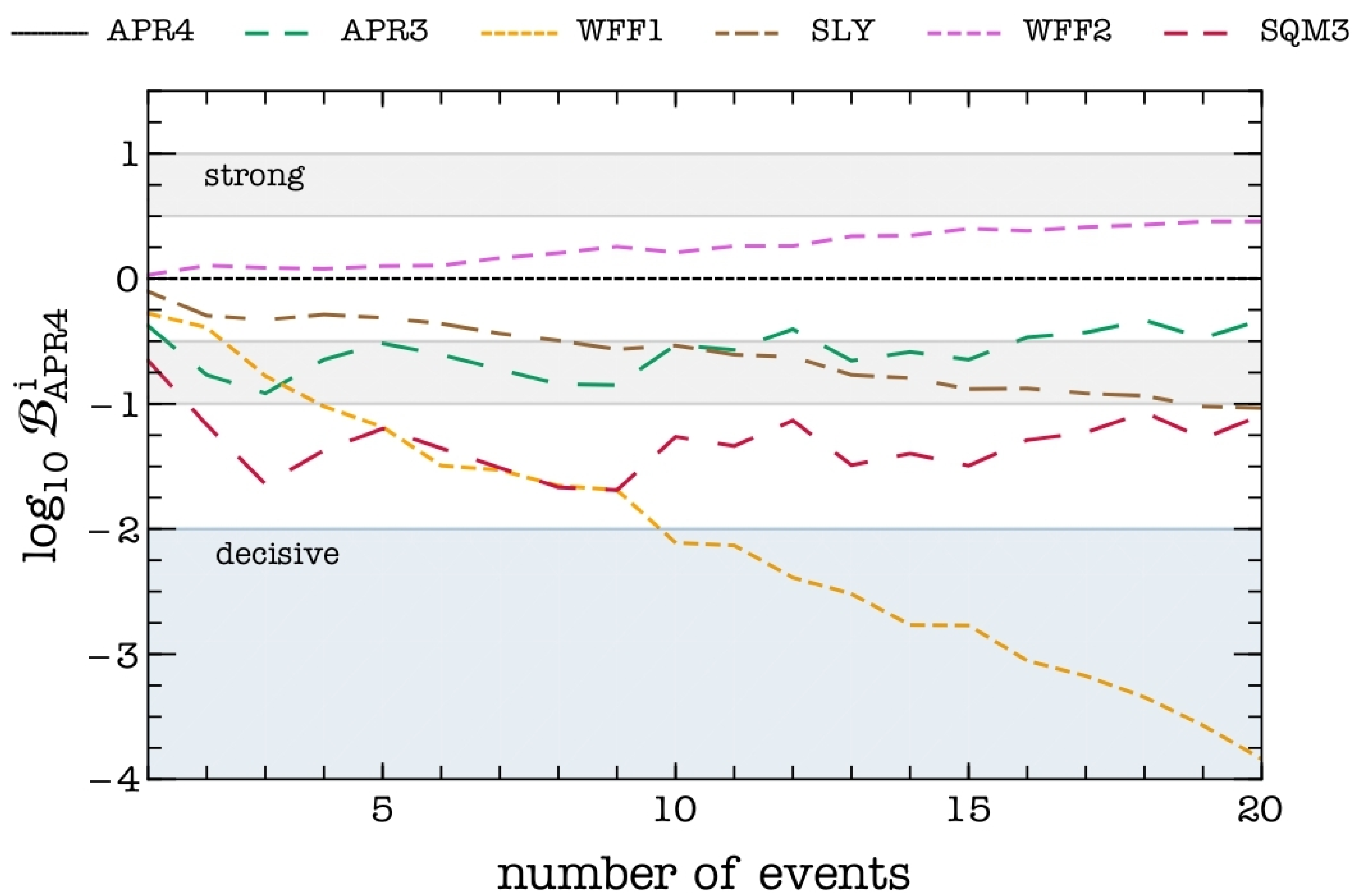
- *The case of the* ***stiff*** *EoS: ALF2*



- *EoS with stiffness different from ALF2 are immediately ruled out*
- *After ~ 10 events EoS with stiffness similar to ALF2 are ruled out*

Stacking at design

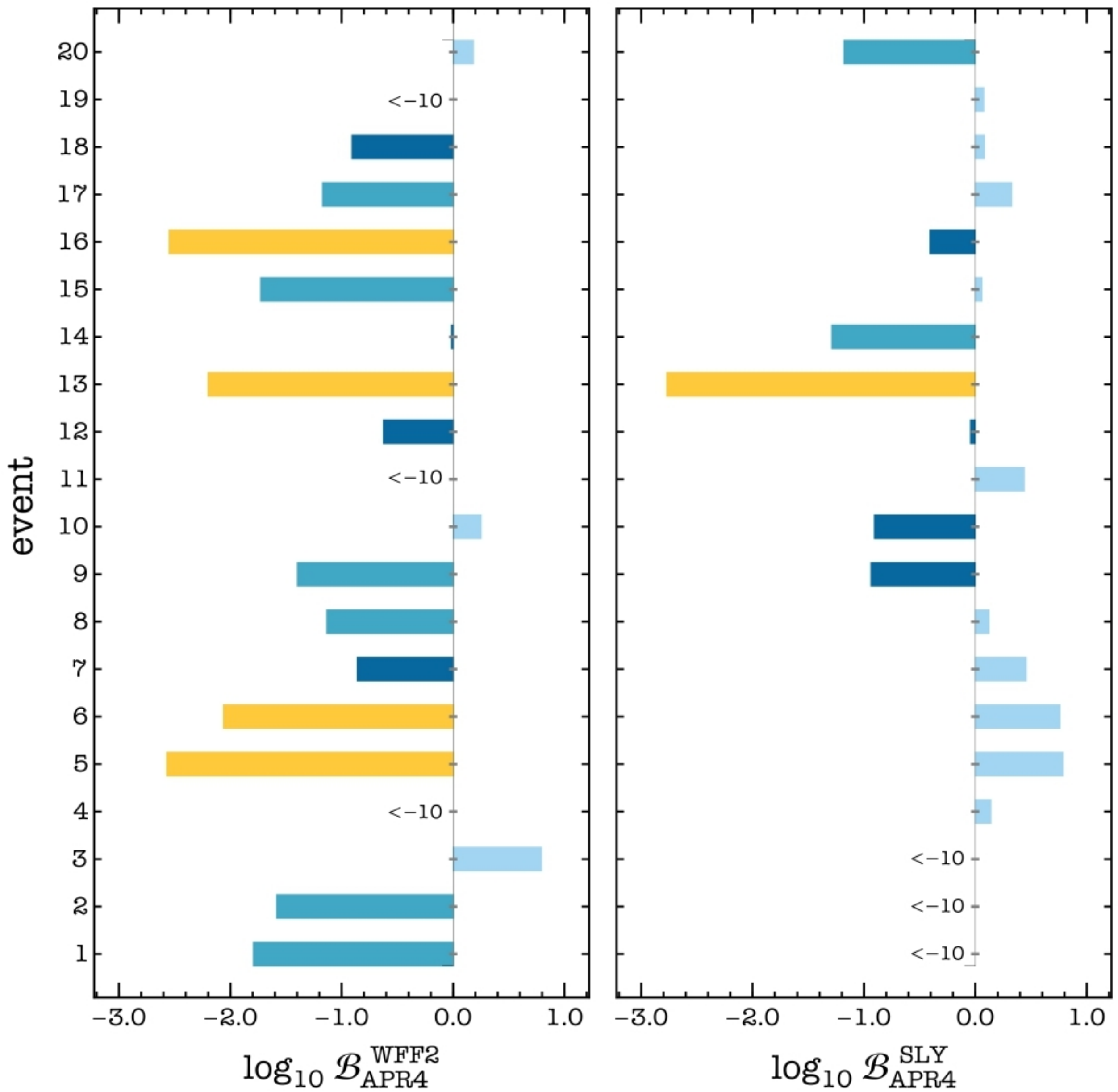
- *The case of the **soft** EoS: APR4*



- *Challenging to discriminate among EoS with similar stiffness*
  - *Even multiple detections are not enough to discriminate models built with different methods and particle content*

The message from ET

- *Let's see again the case of the **soft** EoS: APR4*



- *Just 2 EoS in the dataset survive to the selection*

- *Combining > 3 events rules out almost EoS other than APR4*

*ET can distinguish **stiffness** and **micro-physics***

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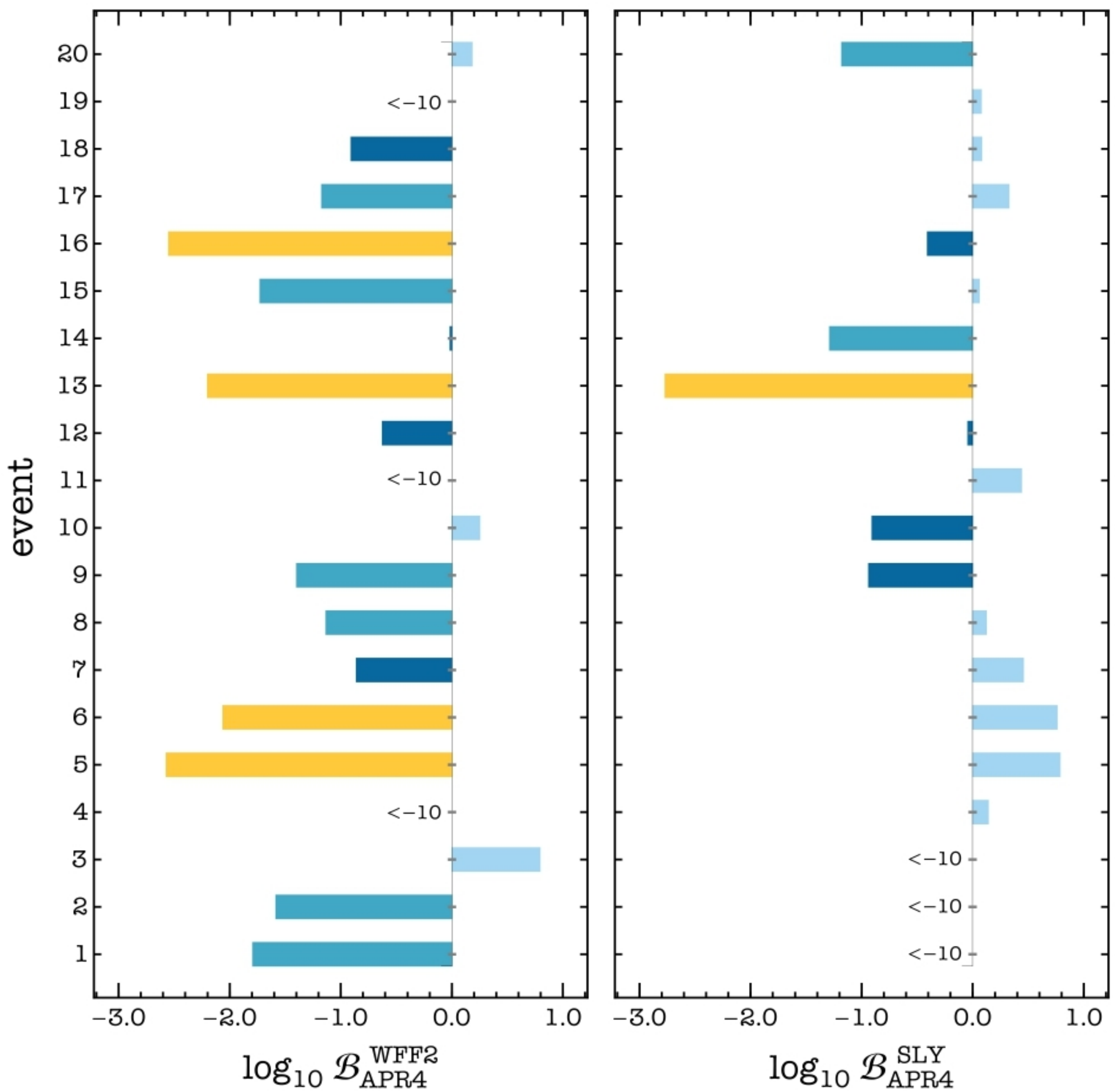

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different N-N interactions