Constraining The Fraction of Compact Dark Matter from Gravitational Lensing of Gravitational Waves

Soummyadip Basak[1], Apratim Ganguly[1], M K Haris[2], Shasvath Kapadia[1], Ajit Kumar Mehta[3], Parameswaran Ajith[1]

[1]International Centre for Theoretical Sciences, Bangalore, India
[2]Nikhef, Amsterdam, Netherlands
[3]Max Planck Institute for Gravitational Physics, Potsdam, Germany
soummyadip.basak@icts.res.in

Abstract

Massive astrophysical compact halo objects (MACHOs) are one of the prime candidates of dark matter. The presence of such objects in the intergalactic medium can cause deflection of gravitational waves (GWs), a phenomenon called gravitational lensing. We try to find lensing signature in the GW events detected by LIGO-Virgo during its 1st and 2nd observing runs. Non-observation of lensing signature in GW signals helps us constrain the dark matter fraction f_{DM} in the form of MACHOs.

Introduction

MACHOs can potentially bend GWs. We look for the lensing phenomenon of GWs in the wave optics regime where GW wavelength λ_{GW} is comparable to the Schwarzschild radius R_{Sch} of the lens object, i.e. $\lambda_{GW} \gtrsim R_{Sch}$. And in the LIGO frequency sensitivity band the lens mass scale is $10 \leq M_L/M_\odot \leq 10^5$. The lensed waveform is $\tilde{h}_L(f) = F(\omega, y)\,\tilde{h}_U(f)$, where $F(\omega, y)$ is frequency dependent magnification, $\omega = 8\pi M_L^z f$, M_L^z = redshifted lens mass, y = dimensionless source position from the optical axis. We also assume point mass lens model which is valid if the lens size is much smaller than the Einstein radius, and for most of the astrophysical compact objects this is applicable.

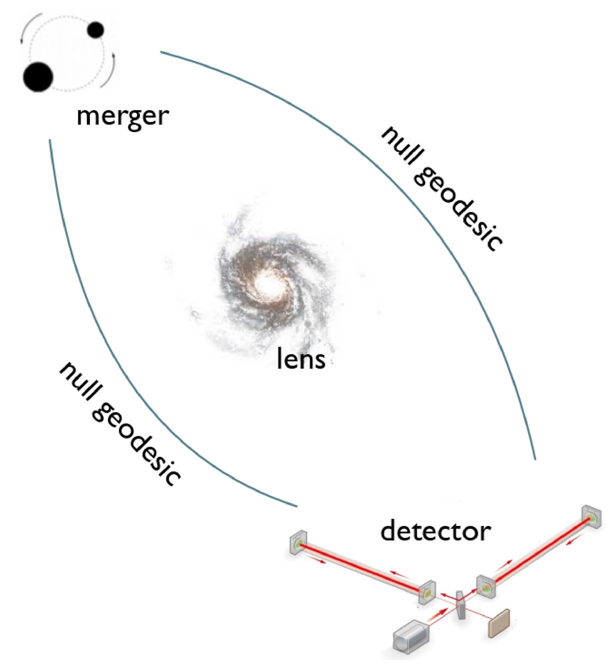

Figure 1: Ray diagram of gravitational lensing of GWs

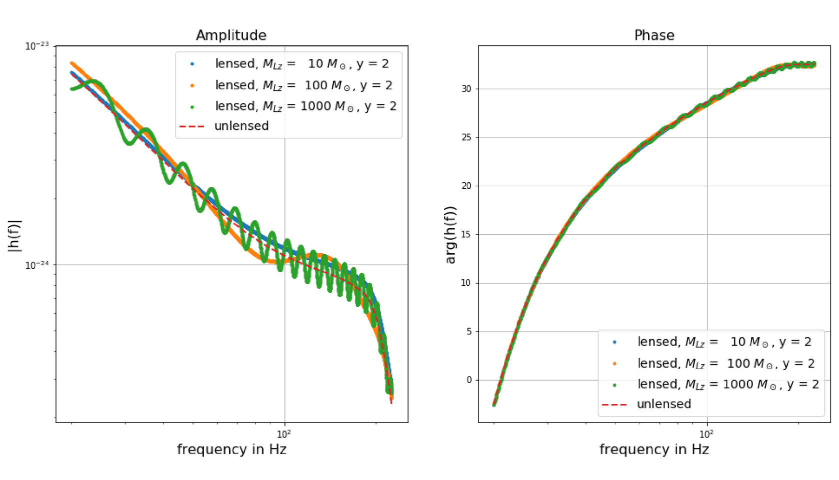

Figure 2: Lensed and unlensed waveforms

Optical depth and lensing probability

The optical depth (τ) is the measure of interaction of GWs with the intervening potential.

$\tau = \frac{3}{2} f_{DM}\Omega_{DM}\frac{c}{H_0}\int_0^{z_S} dz_L \frac{(1+z_L)}{E(z_L)}\chi_L(z_L)(1-\frac{\chi_L(z_L)}{\chi_S(z_S)})\int_0^{y_0} dy\, 2y$

And lensing probability is $P_L = 1 - e^{-\tau(f_{DM}, z_S, y_0)}$

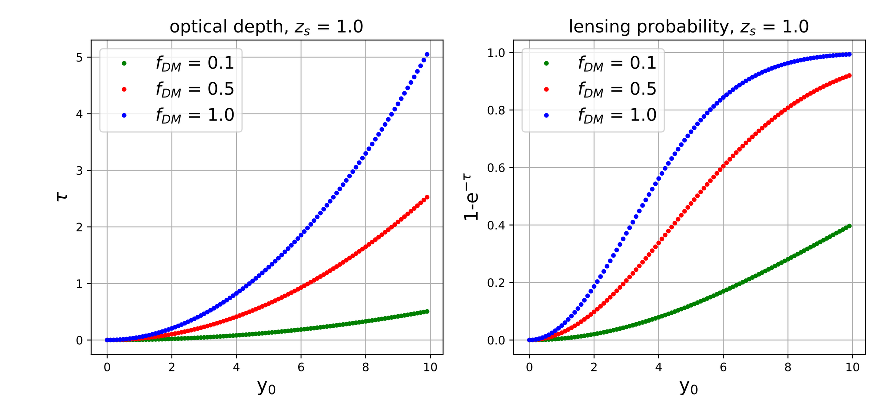

Figure 3: optical depth and lensing probability

Methodology

- The knowledge of optical depth helps us find the number of lensed events in astrophysical simulations.
  $N_L = P_L * N_T$, where N_L is the no. of lensed events and N_T is the number of total events (lensed + unlensed) in our simulations.
- In reality, y_0 (maximum value of y after which no lensing will be detectable) never reaches infinity as physically, after a certain value of y, the effect of lensing becomes insignificant. And to find the true lensing signature of any event we perform a Bayesian inference of two hypotheses:
  H_L: The signal is lensed
  H_U: The signal is unlensed.
  And we try to find the odds ratio of the posterior probabilities of these two hypotheses, i.e.
  $\mathcal{O}_U^L = \frac{P(H_L|data)}{P(H_U|data)}$
  For lensed signals, $\mathcal{O}_U^L >> 1$
- Parameter estimation done on GW events from the O1 and O2 data has found no significant lensing signature in them. [maximum of $\log_{10}$[Bayes factor] (ln[Bayes factor]) = 0.2 (0.5)]

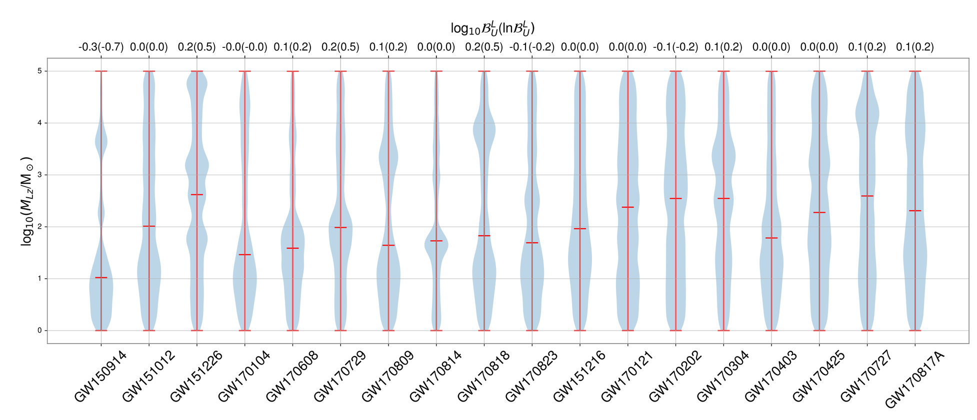

Figure 4: Study of lensing signature of GWs in the O1 and O2 runs

- So we take this as our cut-off value to define lensing signature in our simulation.

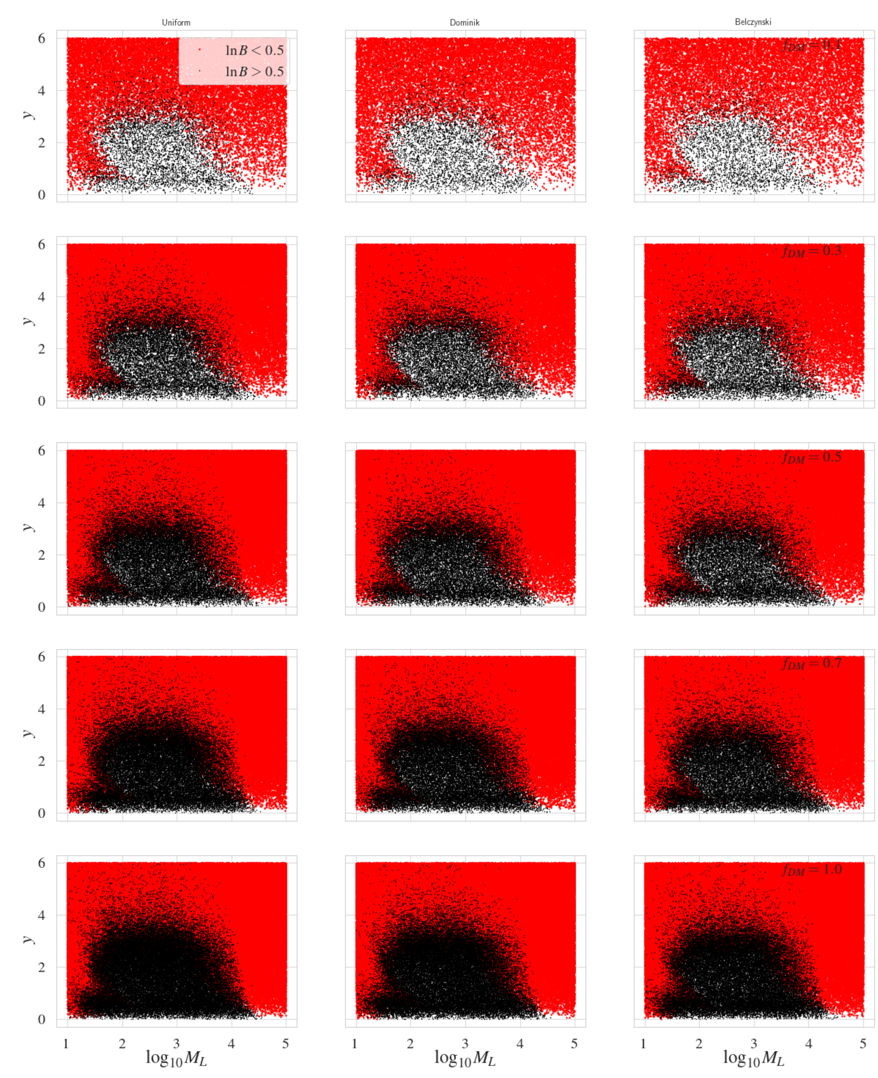

Figure 5: Lensed (black dots) and unlensed events (red dots) in the lens mass (M_L) and y plane

- From our astrophysical simulations, we can relate $u := \frac{N_L}{N_T}$ with f_{DM} and a knowledge of posterior distribution of u ($p(u|data)$) can relate to ($p(f_{DM}|data)$) via the relation:

$$p(f_{DM}|data) = p(u|data)\left|\frac{du}{df_{DM}}\right|$$

Here data refers to the fact that we have 18 number of GW events from the O1 and O2 runs.

Results

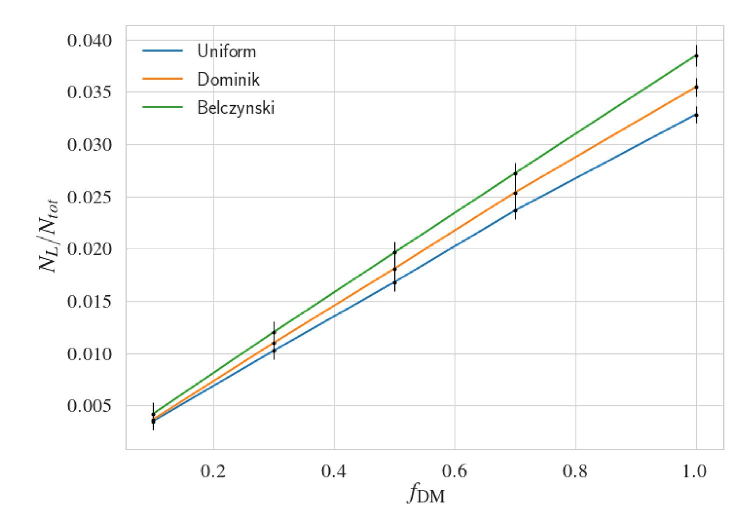

Figure 6: Lensed events ratio as a function of f_{DM} for different source distribution models

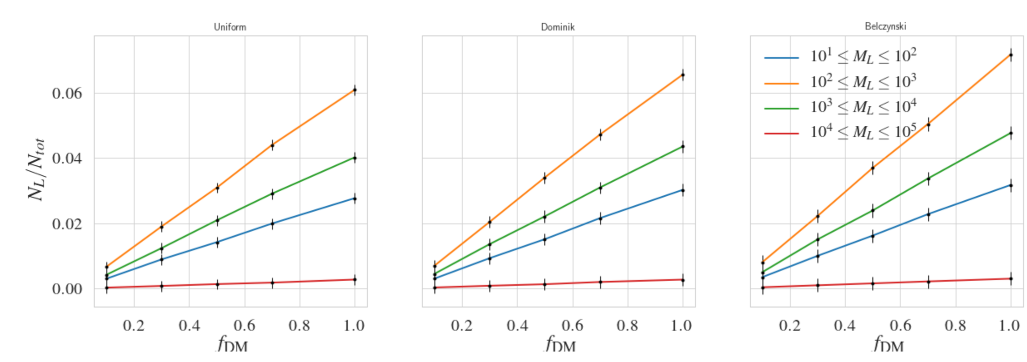

Figure 7: Lensed events ratio as a function of f_{DM} in different mass bins for different source distribution models

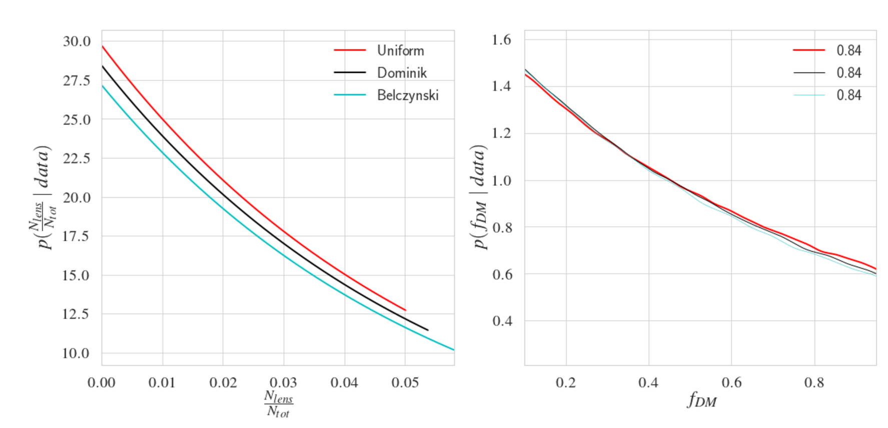

Figure 8: Posterior on $\frac{N_L}{N_T}$ is translated into posterior on f_{DM}

90% cutoff on $p(\frac{N_L}{N_T}|data)$ translates f_{DM} = 0.84 with a 90% credible interval.

Conclusions

With 18 number of GW events carrying no lensing signature, we put an upper limit on MACHOs abundance as f_{DM} = 0.84. Better constraints can be achieved with more number of GW events in future.