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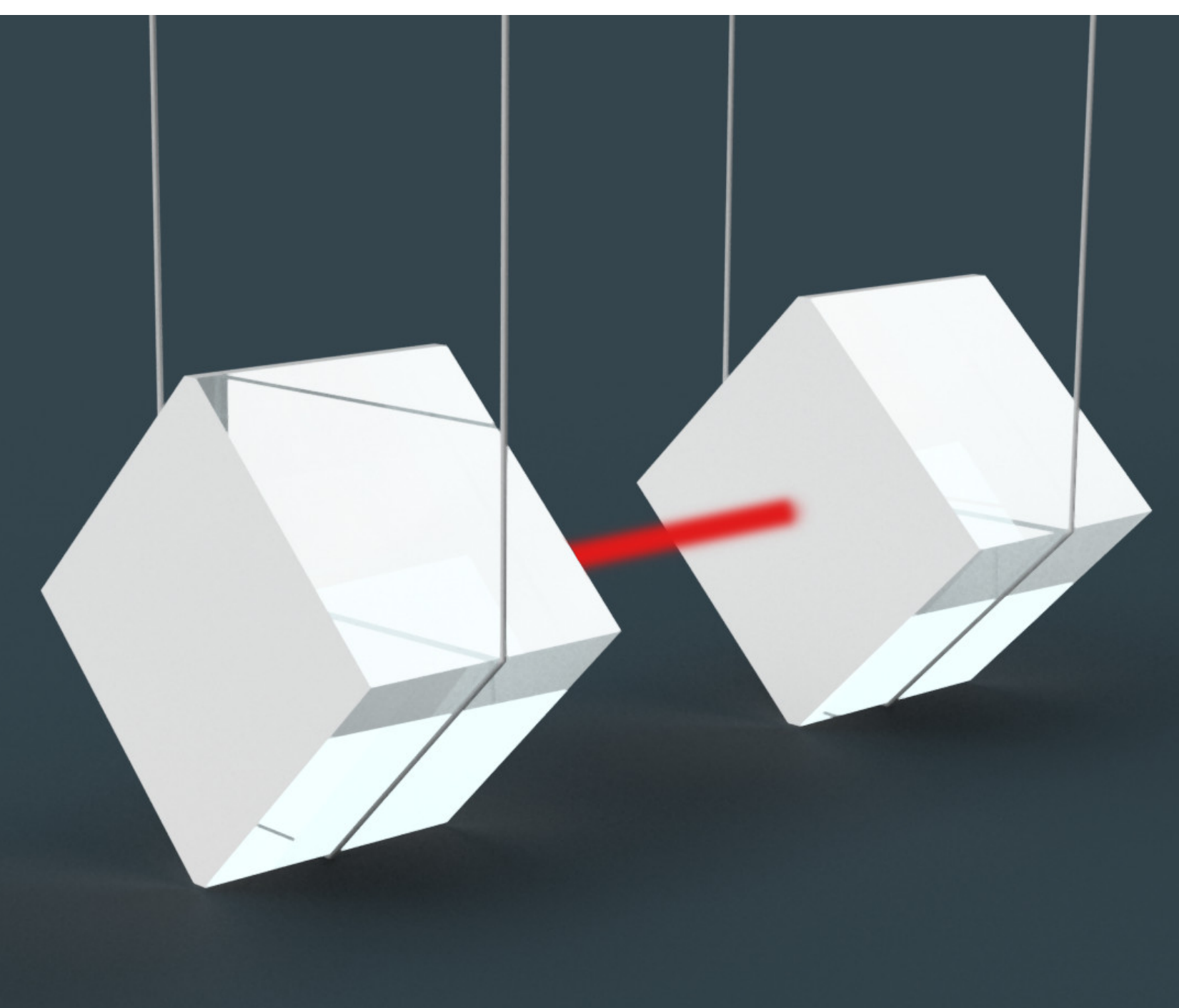
MECHANICAL PARAMETRIC FEEDBACK-COOLING FOR PENDULUM-BASED GRAVITY EXPERIMENTS



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INTRODUCTION

Precision experiments on the gravitational field such as gravitational-wave detectors [1] or measurements of the gravitational constant [2] rely on complex pendulum-like suspensions to minimize non-gravitational coupling to movements in the environment [3]. The pendulum's resonance frequency needs to be smaller than the measurement frequencies, at which the pendulum mass is then quasi-free in the direction of pendulum motion. At their resonance frequencies, however, these pendulums are very susceptible to external disturbances. Passive frictional damping results in a stronger coupling to the thermal bath. Consequentially, with increasing friction, the thermally excited motion of the test mass at frequencies *above* resonance also increases [4].

EVADING THERMAL NOISE

Using frictional damping to increase the natural damping rate and therefore reduce the susceptibility to disturbance at the resonance frequency has the disadvantage of introducing additional thermal noise at off-resonant frequencies. With parametric damping on the other hand, the coupling to the thermal bath is not increased.

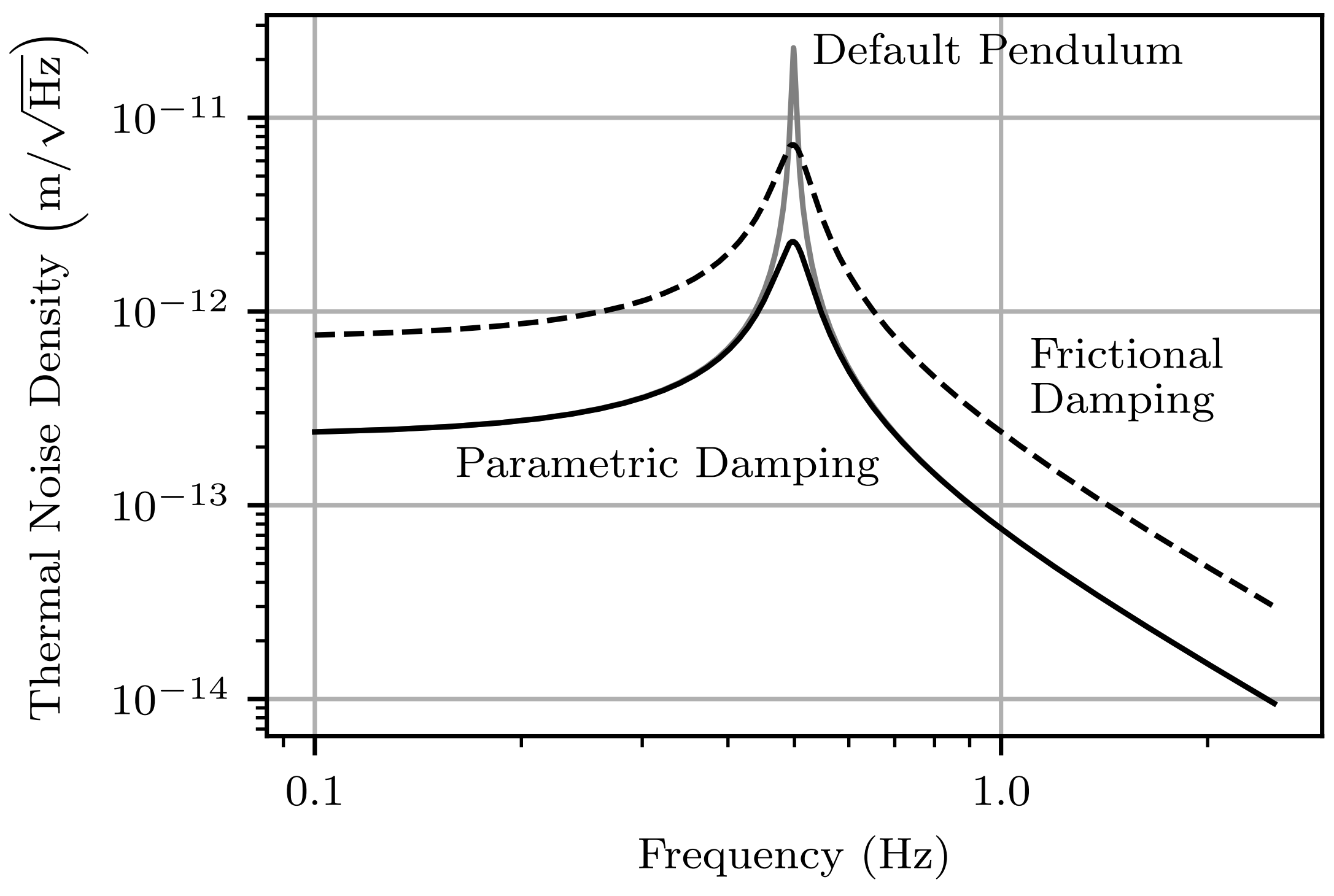
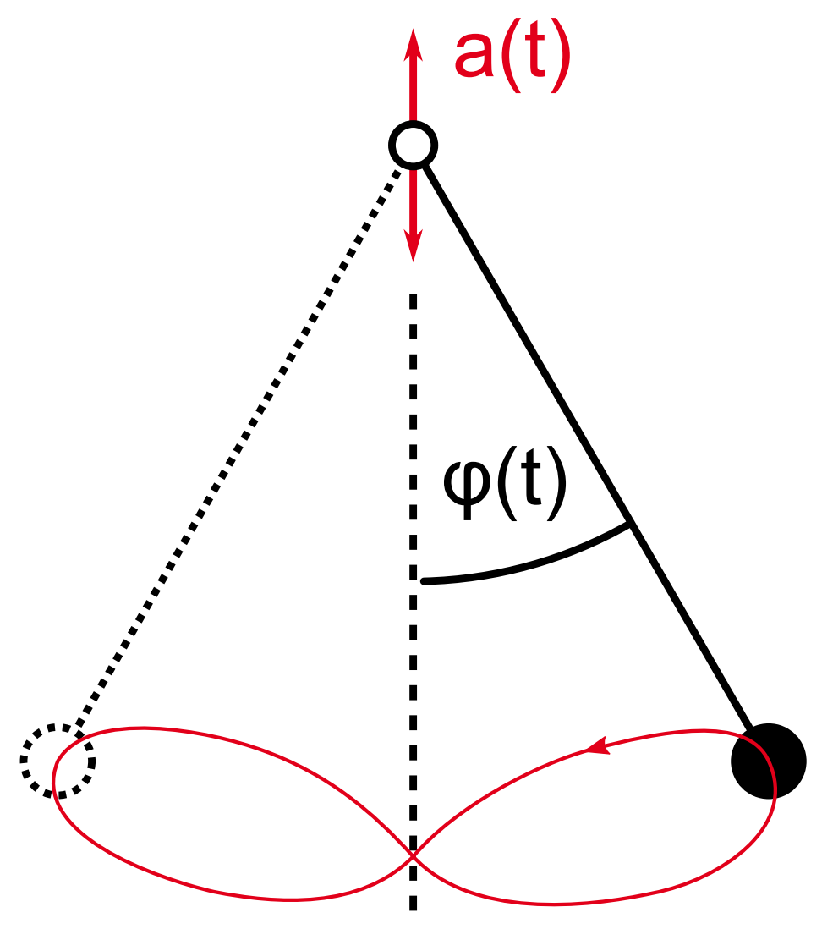


Figure: Simulated single-sided thermal noise spectral density for a pendulum with a length of 1 m, a natural Q-factor of 100 and a mass of 100 kg at room temperature (gray). The lowest curve shows the same pendulum with parametric damping that increases the damping rate by a factor of 10. For comparison, the dashed curve shows the resulting spectral density if the same damping rate was achieved using additional frictional damping. In this case, the off-resonance thermal noise is increased, raising the noise floor for measurements at these frequencies.

Parametric damping reduces the on-resonance thermal excitation while preserving the thermal noise level elsewhere. If the pendulum is in a thermal-noise-limited state of motion, parametric damping effectively cools the mode to a temperature below the thermal environment.

COOLING PRINCIPLE



By modulating the height of the pendulum suspension axis vertically, we introduce a time-dependence in the parameters of the pendulum equation

$$\ddot{\phi}(t)+\gamma\dot{\phi}(t)+\frac{1}{l}(g+a(t))\phi(t)=0,$$

where $\phi \ll 1$ is the deflection angle of the pendulum, γ is the natural damping rate, defined as the reciprocal of the $1/e$ energy damping time, l is its effective length and g is the gravitational acceleration, which define the angular resonance frequency $\omega_0 = \sqrt{g/l}$. $a(t) = a_0\Omega^2 \sin \Omega t$ is the vertical acceleration of the suspension point with amplitude a_0, which acts as a parametric drive. If the angular actuation frequency Ω matches twice the angular natural resonance frequency $2\omega_0$, a phenomenon called parametric resonance occurs, which allows for driving or cooling the excitation of the pendulum mode, depending on the relative phase between vertical actuation and horizontal movement of the pendulum. In practice, this phase needs to be continuously maintained by a feedback loop.

PROOF-OF-PRINCIPLE EXPERIMENT

We parametrically control the excitation of a pendulum with angular resonance frequency $\omega_0 = 2\pi \cdot 1.3585$ Hz by vertical actuation of the suspension point at angular frequency $\Omega = 2\omega_0$.

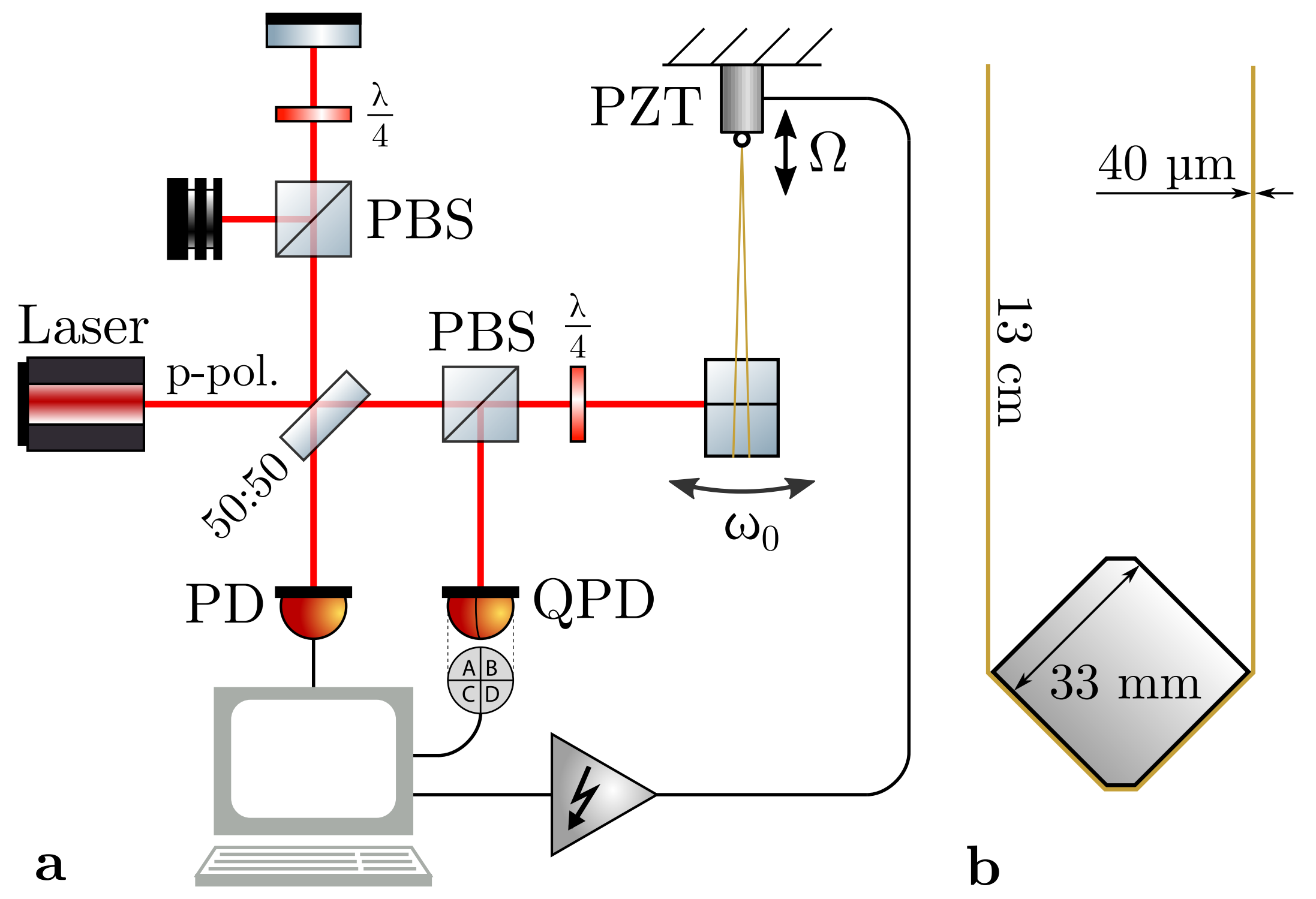


Figure: a The pendulum oscillation at frequency $f = 1.3585$ Hz was parametrically cooled by vertical actuation of the suspension point at frequency $2f$ using using a piezoelectric actuator (PZT). Motion of the pendulum test mass was measured in two ways. First, it was interferometrically read out using a photodiode (PD). Second, deflection of the reflected beam was measured using a quadrant photodiode (QPD) on a part of the reflected beam that was branched off using a polarizing beam splitter (PBS). The interferometer offered a higher resolution, but the signal was ambiguous, when the oscillation amplitude was greater than half of a wavelength. **b** Test mass front view. The cubic test mass hanged from a pair of tungsten suspension wires of 40 µm thickness. The left and right edges of the cube provided break-off points for the wires.

RESULTS

After exciting the pendulum to a certain amplitude, we recorded the decay of the amplitude with and without the use of parametric damping. We found that the Q-factor was reduced by a factor of 5.7 from 2740 to 480 by the use of parametric damping. The minimally reachable amplitude was limited by seismic noise and sensor resolution.

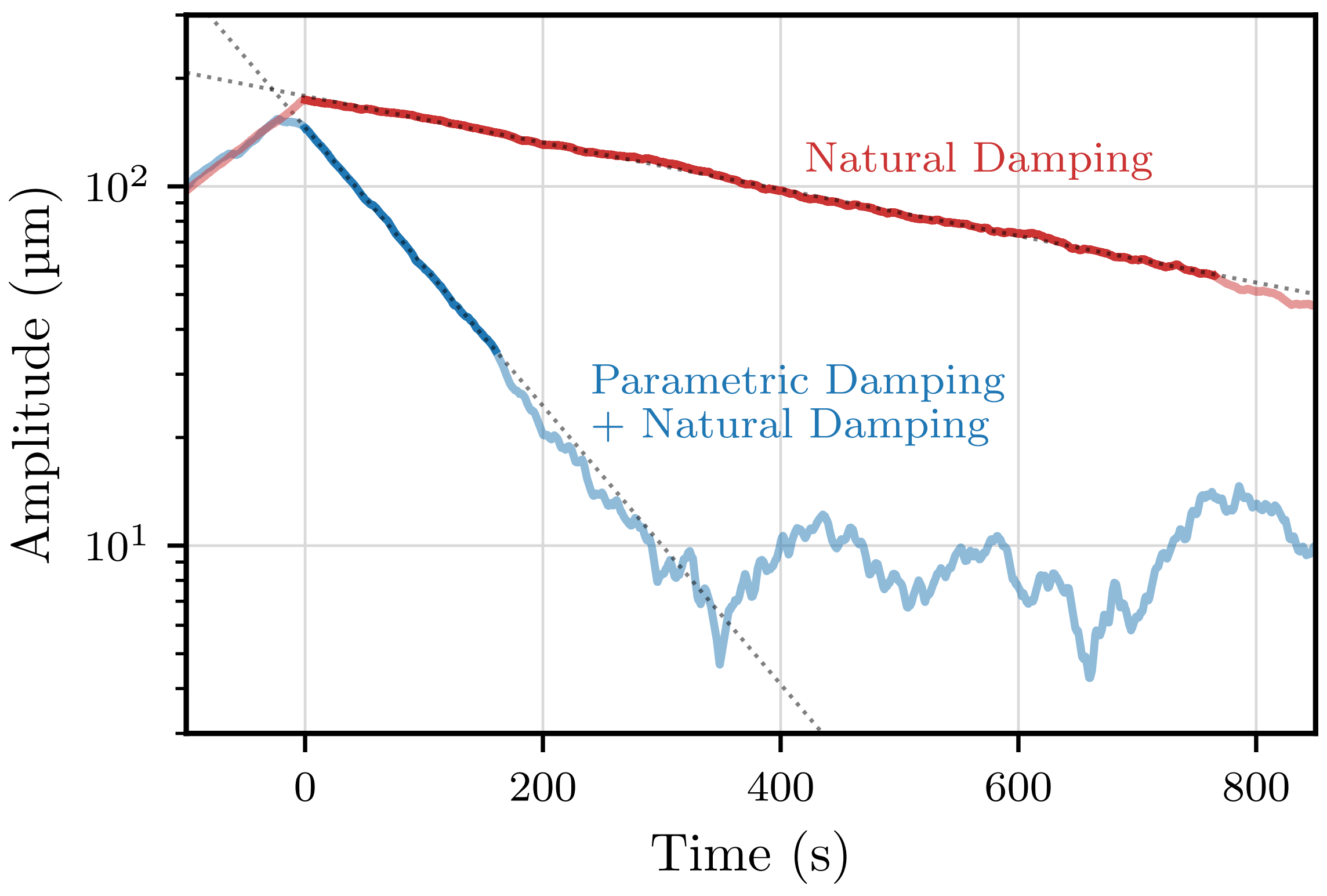


Figure: Example ringdown measurements. The pendulum amplitude is recorded over time. In the beginning, the pendulum is parametrically excited, then the parametric damping feedback is either enabled (bottom curve) or left disabled (top curve). The slope during the damping phase can be used to reconstruct the damping rate. This slope is found by fitting a linear function (dashed). With parametric feedback enabled, the damping rate is the sum of parametric and natural damping rates ($\mu + \gamma \approx 17.9 \times 10^{-3}\,\mathrm{s}^{-1}$), without it, it is solely the natural damping rate ($\gamma \approx 3.1 \times 10^{-3}\,\mathrm{s}^{-1}$).

OUTLOOK

Vibration isolation systems and vibration mitigation of cryogenic cooling are limited in their performances. Therefore, high-Q-factor modes often need to be damped. In our proof-of-principle experiment, we increased the natural damping rate of a pendulum by a factor of 5.7 using mechanical parametric cooling. If our pendulum was excited by thermal energy, the factor achieved corresponded to cooling of the pendulum mode from room temperature to about 50 K. Hypothetically, starting from a Q-factor of 4×10^9, a cryo-cooled 1 Hz pendulum at 2 mK could be cooled near its ground state if the pendulum is parametrically damped to a final Q-factor of 100.

According to theory, thermal noise in measurements performed with a pendulum being parametrically cooled does not increase at any frequency, which is an advantage over frictional damping. Furthermore, unlike damping with linear feedback, parametric damping only creates a monochromatic disturbance in the measurement. Generally, the applicability of mechanical parametric cooling should not be limited to pendulum modes.

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ACKNOWLEDGEMENTS

European Research Council
Established by the European Commission

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 339897 until 2019, and by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy EXC 2121 "Quantum Universe"—390833306.

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