PN Properties of EMRI in Non-Vacuum Environment

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Methodology

Following procedure is followed to obtain the dynamical quantities and the avg. energy radiation rate accurate up to 1PN order. 0-PN terms are trivial to calculate using the 0-PN potential itself.

1. A generic form of power law potential can be written as:

\[ U_{SO} = \frac{GM_{SO}}{2(\alpha + \beta)} \]

- \( \alpha = \frac{\delta}{\epsilon} \) Power on radial dist. r between SCO and SO
- \( \beta = \frac{\epsilon}{\delta} \) Scale radius of the SCO
- \( \epsilon = \) Scale mass of the SCO

Using Poisson equation, it can be seen that the mass density source for this potential has a power law dependence on the radial distance from the center of mass of the source. This potential is used to derive other quantities like the 0-PN velocity which is used to derive the higher order of potential.

2. 1-PN contribution to potential is due to the Vector potential (U), the PN correction (\( \phi \)) and the Superpotential (\( \chi \)).

3. Using the derivatives of these potentials, we obtain 1-PN equation of motion = acceleration (\( \rho \)).

4. Integrating over the acceleration term with the help of the 0-PN frequency (also using the conservation of angular momentum), the orbital velocity (\( v_{SO} \)) is derived, which is further used to derive the orbital frequency (\( \Omega \)).

5. These dynamical quantities are used to obtain the mass quadrupole tensor (\( \rho^{\alpha\beta} \)). Which is further used to obtain the average energy radiation rate (\( \frac{\Delta E}{\Delta t} \)).

\[ \frac{\Delta E}{\Delta t} = -2GM_{SO}M_{EMRI}\left[\frac{\Omega_{SO}}{\Omega_{EMRI}}\left(\frac{\rho^{\alpha\beta}}{\rho^{\alpha\beta}_{SO}}\right)\right] \]

\[ \frac{\Delta E}{\Delta t} = \frac{1}{2} \frac{GM_{SO}M_{EMRI}}{\rho^{\alpha\beta}_{SO}} \]

Results

- Circular orbit EMRI considered = Simplified calculations.
- Dynamical variables of the SCO:
  - Acceleration:
    \[ a_{SO} = \frac{GM_{SO}}{r^{3+\delta}} \left[ 1 - \frac{\rho_{SO}}{r^{3+\delta}} \right] \]
  - Orbital velocity:
    \[ v_{SO} = \sqrt{\frac{GM_{SO}}{r^{3+\delta}} \left[ 1 + \frac{1}{2} \frac{GM_{SO}}{r^{3+\delta}} \right]} \]
  - Orbital frequency:
    \[ \Omega = \frac{1}{r} \sqrt{\frac{GM_{SO}}{r^{3+\delta}} \left[ 1 + \frac{1}{2} \frac{GM_{SO}}{r^{3+\delta}} \right]} \]

- And the average energy radiation rate is obtained as:

\[ \frac{\Delta E}{\Delta t} = -2GM_{SO}M_{EMRI}\left[\frac{\Omega_{SO}}{\Omega_{EMRI}}\left(\frac{\rho^{\alpha\beta}}{\rho^{\alpha\beta}_{SO}}\right)\right] \]

- We compare these for an EMRI in
  - Vacuum region (\( \delta = 0 \)) i.e., KN Potential
  - In inhomogenous mass distribution (\( \delta \geq 0.27 < 1 \)) i.e., PL Potential

It can be noted that by substituting the value of \( \delta = 1/2 \) (i.e., KN potential) we obtain the same expressions as expected for quantities derived using KN potential.

Future Scopes

To have more enhanced and astrophysical study of the system, we can include:
- eccentricity
- higher order PN terms
- spins of the bodies
- damping effect due to the matter distribution (i.e., when the deviation is very high from \( \delta = 1/2 \)).
- orbital decay
- tidal effects on the stellar mass object

The tidal effects on the star might emit electromagnetic radiations. Hence, using individually resolvable sources, multimessenger astronomy can be performed.

References