PN Properties of EMRI in Non-Vacuum Environment

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Abstract:

There are many astrophysical scenarios where extreme mass ratio inspiral (EMRI) binaries can be surrounded by matter (esp. dark matter) distribution. The mass distribution can affect the dynamical properties (e.g. orbital frequency, orbital velocity, etc.) and the rate of energy radiation from the EMRI. We assume a power law dependency of mass distribution density on the radial distance from the center of mass (instead of a point mass source as used in Schwarzschild spacetime). Using the Newtonian order Poisson equation one obtains a general power law potential (instead of Kepler-Newton (KN) potential which is valid for vacuum regions). In this presentation, I would discuss the expressions for dynamical quantities and the average energy radiation rate from the circular orbit EMRI corrected up to the first post-Newtonian (1PN) order. These quantities can be significantly different in the presence of potential due to matter distribution as compared to that in the Kepler-Newton potential. The effect of mass distribution could be studied further in more realistic 3D orbits (by including eccentricities, precession, etc). Such signatures can be added to the existing templates for the gravitational waveforms which could be vital for the study using upcoming space-based detectors.

Introduction

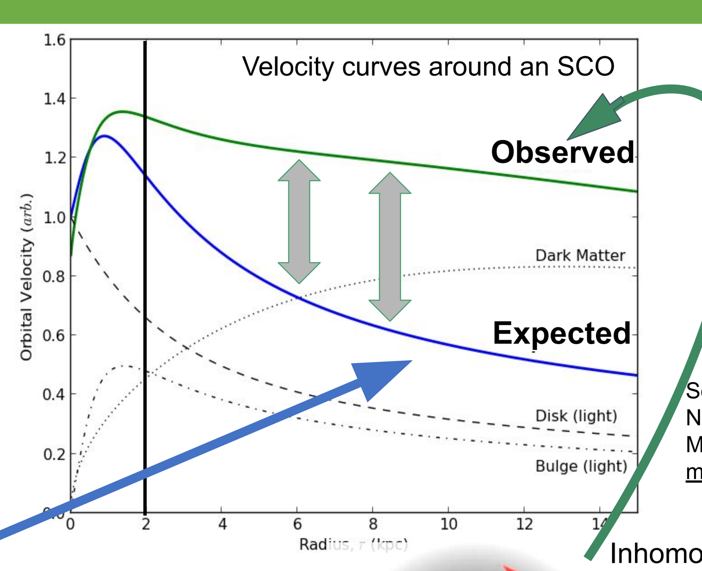
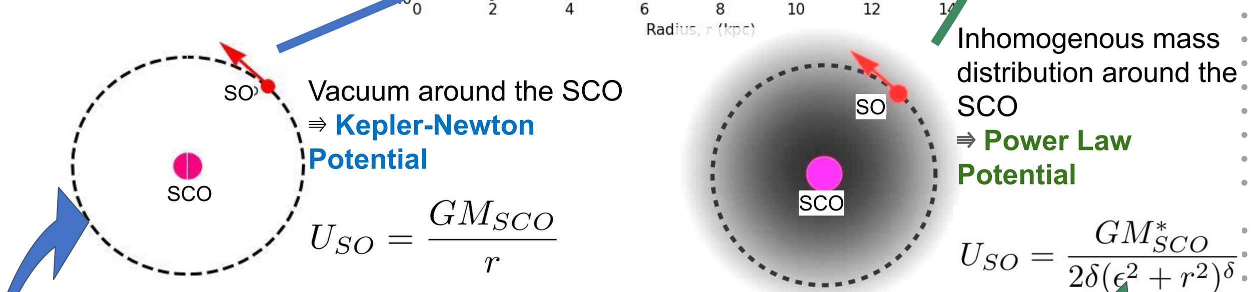


Fig 1.
Observed orbital velocity of stellar mass objects around supermassive compact objects

Source: Matthew Newby, Milkyway@home milkyway.cs.rpi.edu



The orbital velocity profile of stars around supermassive compact objects (SCO) is depicted in Fig 1. The observed profile (green) has higher values of velocity as compared to expected values (blue curve). This gap is attributed to presence of dark matter around the SCO. We extend this idea to extreme mass-ratio (EMRI) binaries of SCO and stellar mass objects (SO).

- Most of the current studies generally consider **vacuum environment** around an **SCO.**
  ⇒ Schwarzschild spacetime.
  ⇒ at 0-PN order, we obtain **Kepler-Newton** (KN) Potential (i.e., a special case of power law when δ=½ and $\epsilon << r$) as effective potential.
- In our study, We include **inhomogenous matter distribution** around SCO to make the study more realistic. This matter density via the Poisson equation leads to a different form of the gravitational potential.
  ⇒ **Power Law** (PL) Potential
- Considering circular orbit extreme mass-ratio inspiral (EMRI) binary system in the weak field zone, we derive the **dynamical quantities** resolved up to 1PN order.
- Using this we also derive the **average power radiation** from the EMRI and compare it with that the case of vacuum case

Methodology

Following procedure is followed to obtain the dynamical quantities and the avg. energy radiation rate accurate up to 1PN order. 0-PN terms are trivial to calculate using the 0-PN potential itself.

1. A generic form of power law potential can be written as[3,4,5]:

$$U_{SO} = \frac{GM^*_{SCO}}{2\delta(\epsilon^2 + r^2)^\delta}$$

δ < ½ = Power on radial dist. r between SCO and SO
ϵ = Scale radius of the SCO
M^*_{SCO} = Scale mass of the SCO

Using Poisson equation, it can seen that the mass density source for this potential has a power law dependence on the radial distance from the center of mass of the source. This potential is used to derive other quantities like the 0-PN velocity which is used to derive the higher order of potential

2. **1-PN** contribution to potential is due to the **Vector potential (U^i),** the **PN correction (ψ)** and the **Superpotential (X).** [2]
3. Using the derivatives of these potentials, we to obtain 1-PN **equation of motion** ⇒ the **acceleration** (a^j)[6,7]
4. Integrating over the acceleration term with the help of the 0-PN frequency (also using the conservation of angular momentum), the **orbital velocity** (v^j_{SO}) is derived, which is further use to derive the **orbital frequency** ($\dot{\phi}$)
5. These dynamical quantities are used to obtain the mass quadrupole tensor ($\dddot{I}^{\alpha\beta}$). Which is further used to obtain the **average energy radiation rate** ($\left\langle \frac{dE}{dt} \right\rangle$)

Results (contd.)

1. The orbital velocity of the SO in the EMRI using **PL potential (orange)** i.e., in presence of matter distribution is higher than that in the case of **KN Potential (blue)** i.e., in vacuum environment.
The velocity in the case of PL potential (orange) is similar to the observed velocity profile (green curve Fig.1).

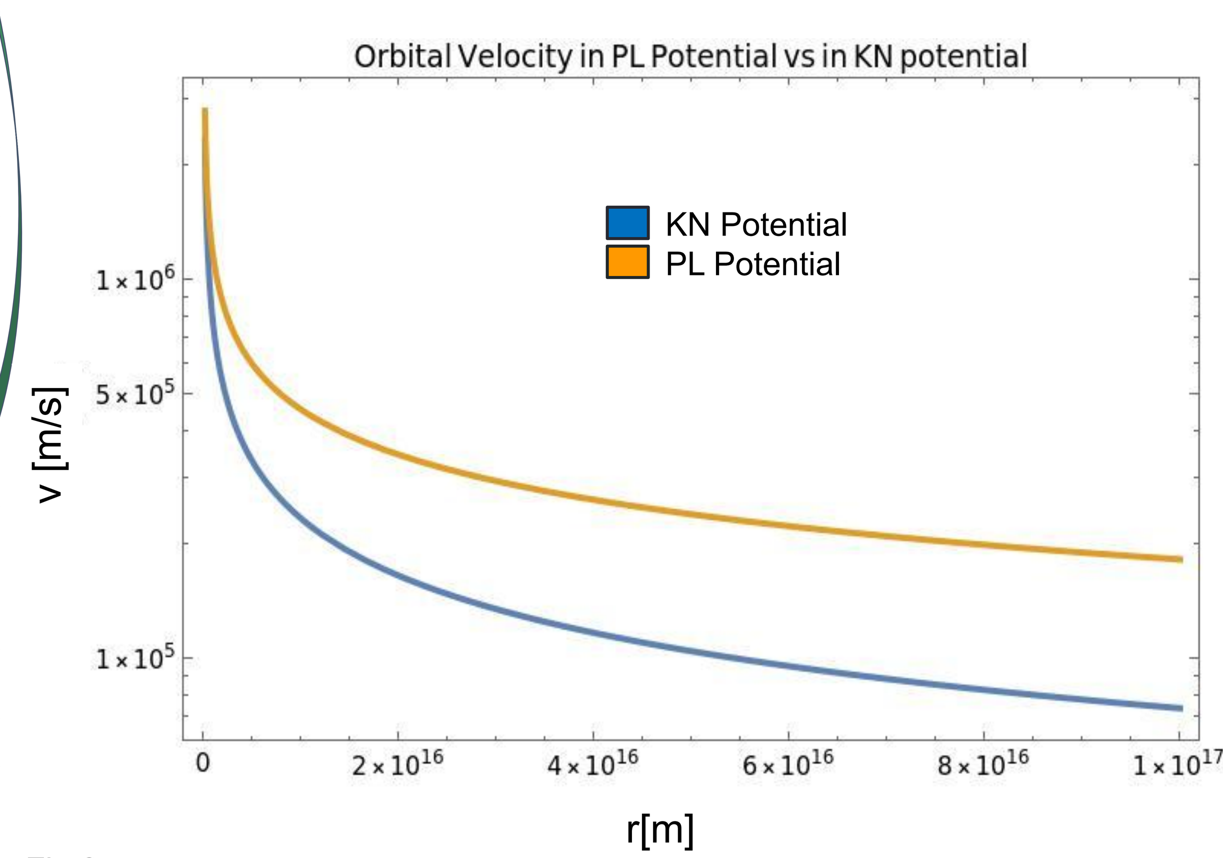


Fig 2.
Orbital velocity comparison. Orange represents the velocity profile in the case of PL potential and blue represents the KN potential case

Results

- Circular orbit EMRI considered ⇛ Simplified calculations.
- Dynamical variables of the SCO:
  - Acceleration:

$$a_{SO} = -\frac{GM_{SCO}\epsilon^{2\delta-1}}{r^{2\delta+1}}\left[1 - \frac{v^2_{SO;0PN}}{c^2}\frac{(2-\delta)}{\delta}\right]$$

  - Orbital velocity

$$v_{SO} = \sqrt{\frac{GM_{SCO}\epsilon^{2\delta-1}}{r^{2\delta}}}\left[1 - \frac{1}{c^2}\frac{GM_{SCO}\epsilon^{2\delta-1}}{r^{2\delta}}\frac{(2-\delta)}{\delta}\right]$$

  - Orbital frequency

$$\dot{\phi} = \frac{1}{r}\sqrt{\frac{GM_{SCO}\epsilon^{2\delta-1}}{r^{2\delta}}}\left[1 - \frac{1}{c^2}\frac{2GM_{SCO}\epsilon^{2\delta-1}}{\delta r^{2\delta}}(2-\delta)\right]$$

- And the average energy radiation rate is obtained as:

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{32G^4M^3_{SCO}M^2_{SO}\epsilon^{3(2\delta-1)}}{5c^5r^{2(3\delta+1)}}\left[1 - \frac{1}{c^2}\frac{2GM_{SCO}\epsilon^{2\delta-1}}{\delta r^{2\delta}}(2-\delta)(3+2\delta)\right]$$

- We compare these for an EMRI in
  vacuum region (δ=½) i.e., KN Potential
  and in inhomogenous mass distribution (δ =0.27< ½) i.e., PL Potential

It can be noted that by substituting the value of δ=½ (i.e., KN potential) we obtain the same expressions as expected for quantities derived using KN potential.

Results (contd.)

2. The **average energy radiation rate** from the EMRI using **PL potential (orange)** is significantly more than that in the case of **KN Potential (blue)** i.e., in vacuum environment.
Signatures of this effect would be seen in the amplitude and the frequency of the GW signals from such a system.

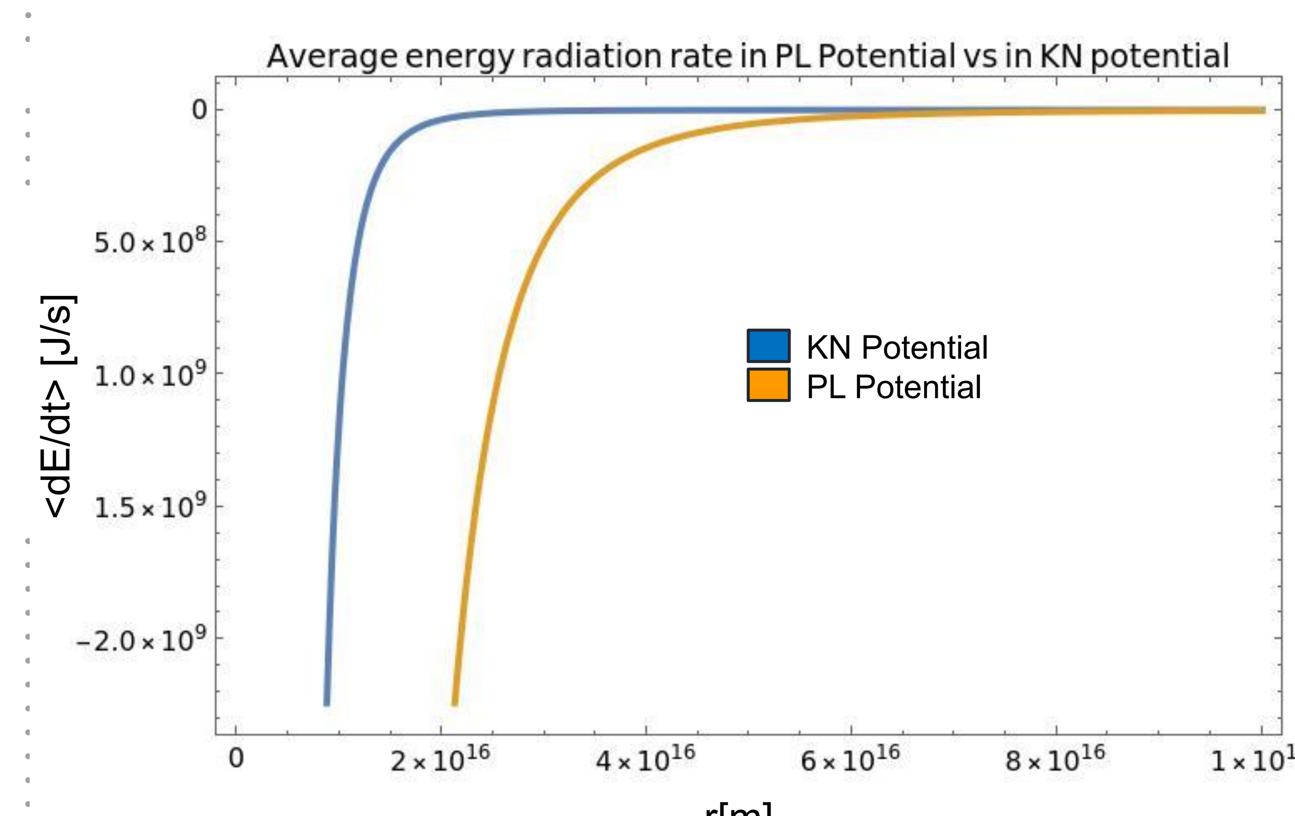


Fig 3.
The average energy radiation rate from the EMRI in the case of PL potential (Orange curve) is higher than when the EMRI is in the vacuum region i.e., where KN potential is applicable (blue curve). Note that the negative values of energy rate denote the radiation of the energy

Conclusions

1. In case of the EMRi that is surrounded by inhomogenous matter distribution:
   a. The dynamics is faster
   b. The energy radiation rate is higher
   c. The stellar mass object (SO) may plunge sooner into the supermassive compact object (SCO)
2. Signatures of these effects would be evidently apparent in the gravitational waves signals from such binaries. The calculated PN corrections provide an opportunity for performing more precise parameter estimation.
3. Hence these effects can also be included in the current waveform models. Upcoming detectors like LISA and DECIGO are will function at very high sensitive and might be able to significantly detect these signatures provided the waveform models include these effects.

Future Scopes

To have more enhanced and astrophysical study of the system, we can include:

- eccentricity
- higher order PN terms
- spins of the bodies
- damping effect due to the matter distribution (i.e., when the deviation is very high from δ=½).
- orbital decay
- tidal effects on the stellar mass object

The tidal effects on the star might emit electromagnetic radiations. Hence, using individually resolvable sources, multimessenger astronomy can be performed.

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