Theoretical Effective Emissivity for the LIGO Voyager test masses

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SUMMARY

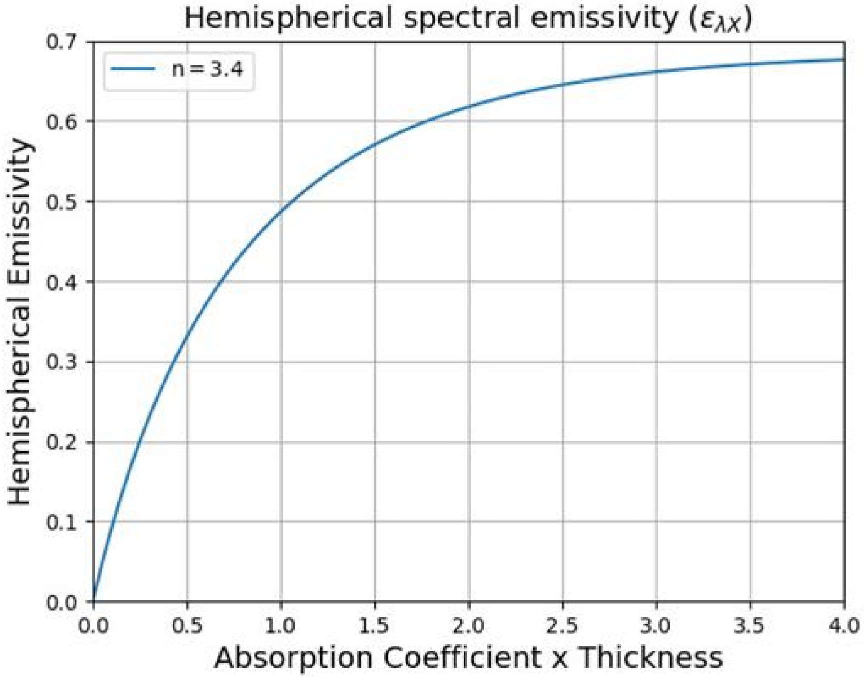
1. Introduction;
2. Calculation of mean thickness;
3. Calculation of the absorption;
4. Results and Future Prospects.

INTRODUCTION

- LIGO Voyager: Si test masses at 123 K;
- Si is semi-transparent around the Black Body peak at 123K;
- Thus, emissivity becomes a bulk phenomenon.
- According to Gardon (1956):

$$\epsilon_{\lambda x} = 2 \cdot \int_0^{\frac{\pi}{2}} \left(1 - e^{-\frac{\alpha_\lambda \chi}{\cos\gamma}}\right) \Upsilon' \sin\beta \cos\beta d\beta$$

- This method was first suggested to us by Martin Fejer.



OUR GOAL

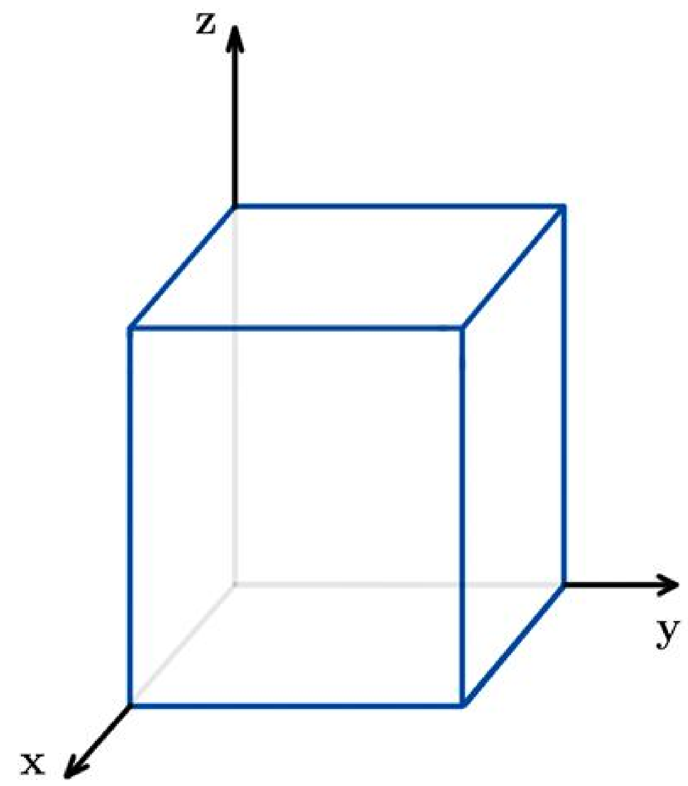
- Present you the calculations of the effective emissivity for the LIGO Voyager test masses;
- For illustration, we will describe, first, the calculations for a parallelepiped shaped piece;
- This piece was provided by the Caltech group (Rana, Koji, Aaron, and Chris).
- 70x30x10.34 mm
- Refractive index of n = 3.4
- Wavelength from 16.7 μm to 33.4 μm.

MEAN THICKNESS

- Code in Python;
- The code calculates the distance between a point inside the piece and another one from the six faces;
- It adds all distances and takes the average;

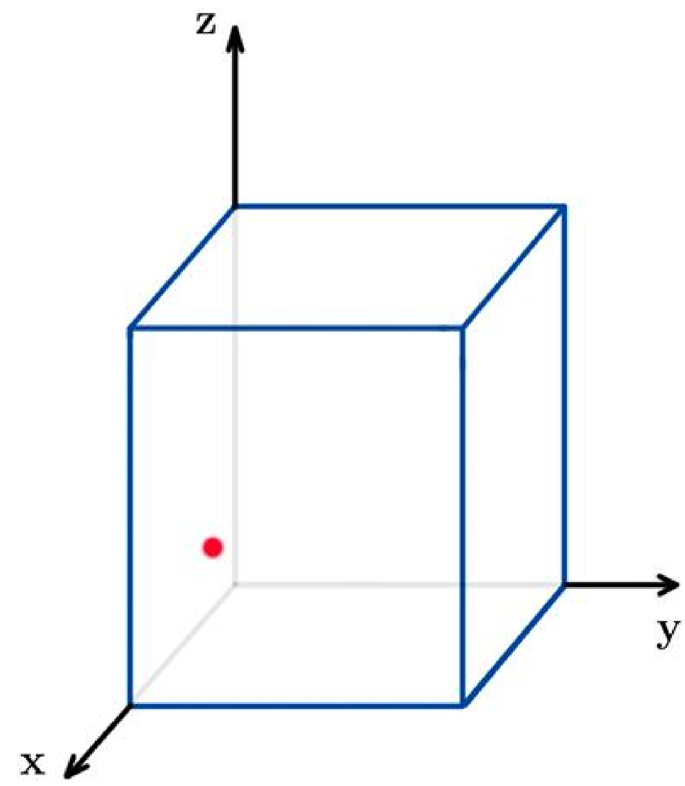
MEAN THICKNESS

- Code in Python;
- Describe the shape of the piece;



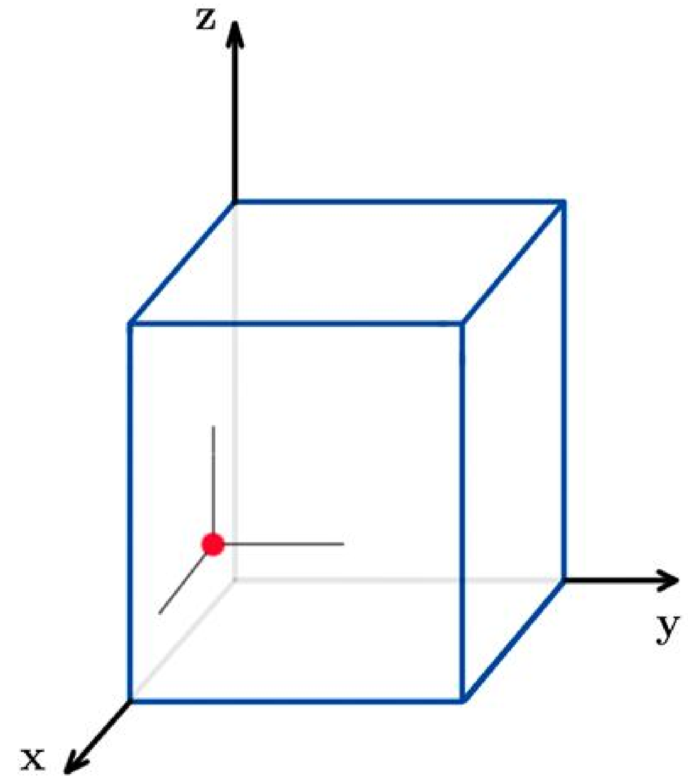
MEAN THICKNESS

- Code in Python;
- Describe the shape of the piece;
- Take a point inside it;



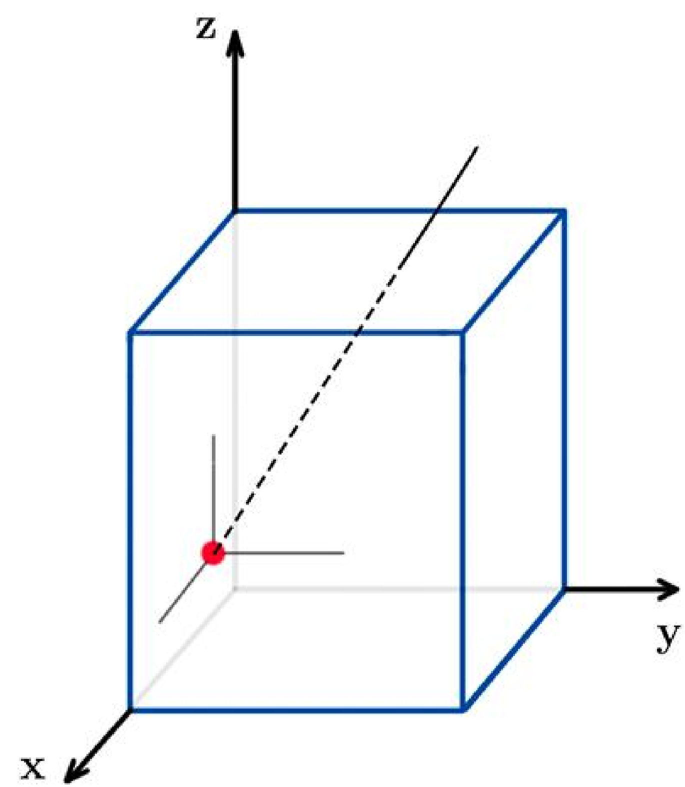
MEAN THICKNESS

- Code in Python;
- Describe the shape of the piece;
- Take a point inside it;
- Spherical Coordinate System;



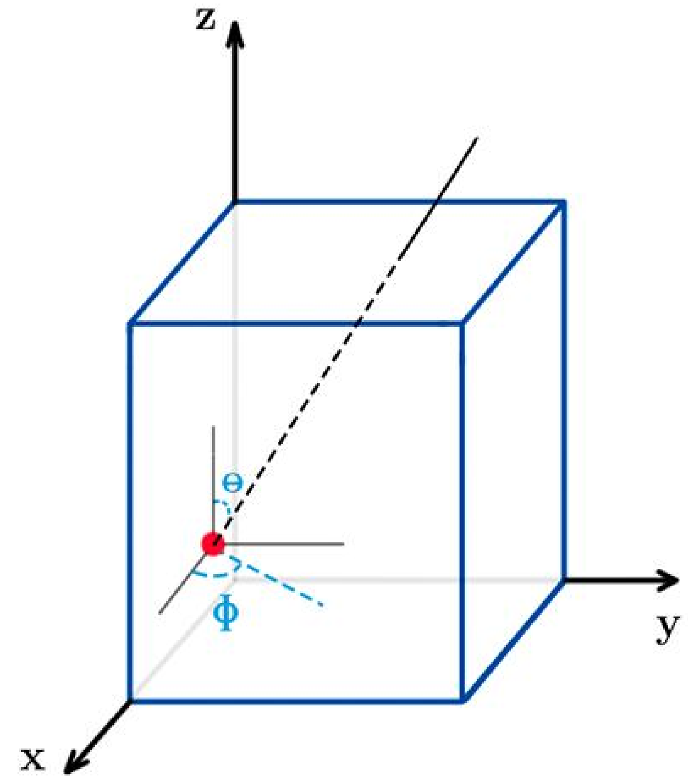
MEAN THICKNESS

- Code in Python;
- Describe the shape of the piece;
- Take a point inside it;
- Spherical Coordinate System;
- Take a direction;



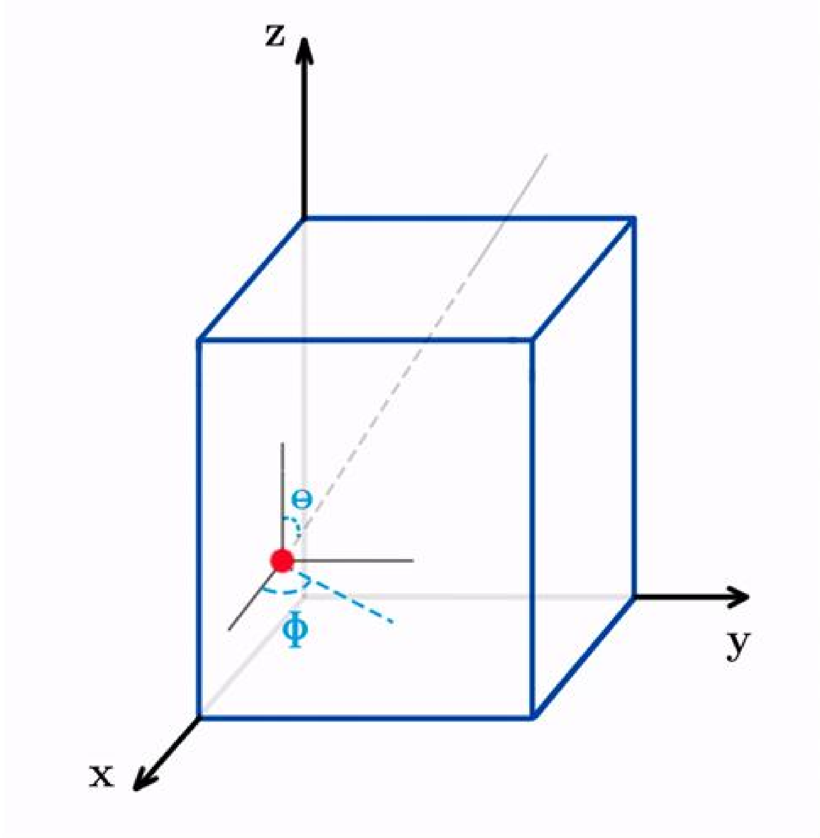
MEAN THICKNESS

- Code in Python;
- Describe the shape of the piece;
- Take a point inside it;
- Spherical Coordinate System;
- Take a direction (θ, φ);



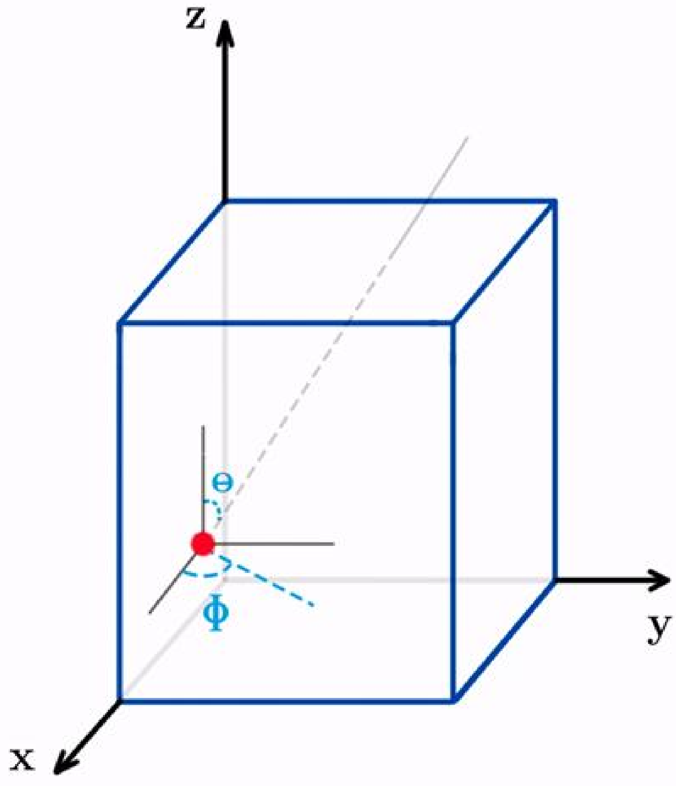
MEAN THICKNESS

- Take a direction (θ, φ);
- A vector **r** is generated in that direction with a initial length of 0.05 mm;



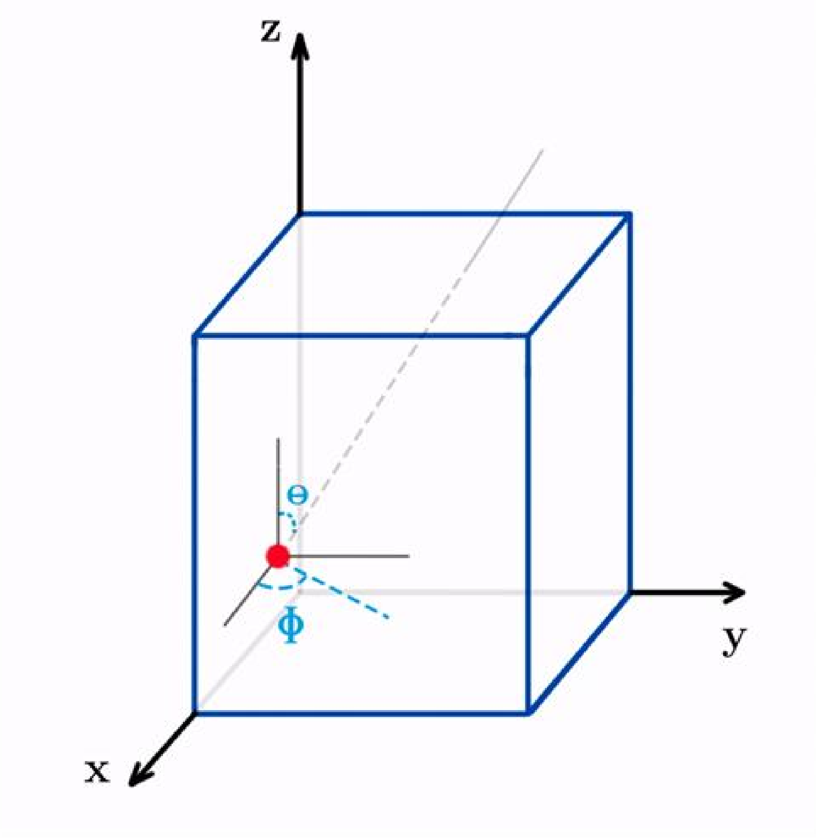
MEAN THICKNESS

- Take a direction (**θ**, **φ**);
- A vector **r** is generated in that direction with a initial length of 0.05 mm;
- This vector length increases in until it reaches one face of the piece;



MEAN THICKNESS

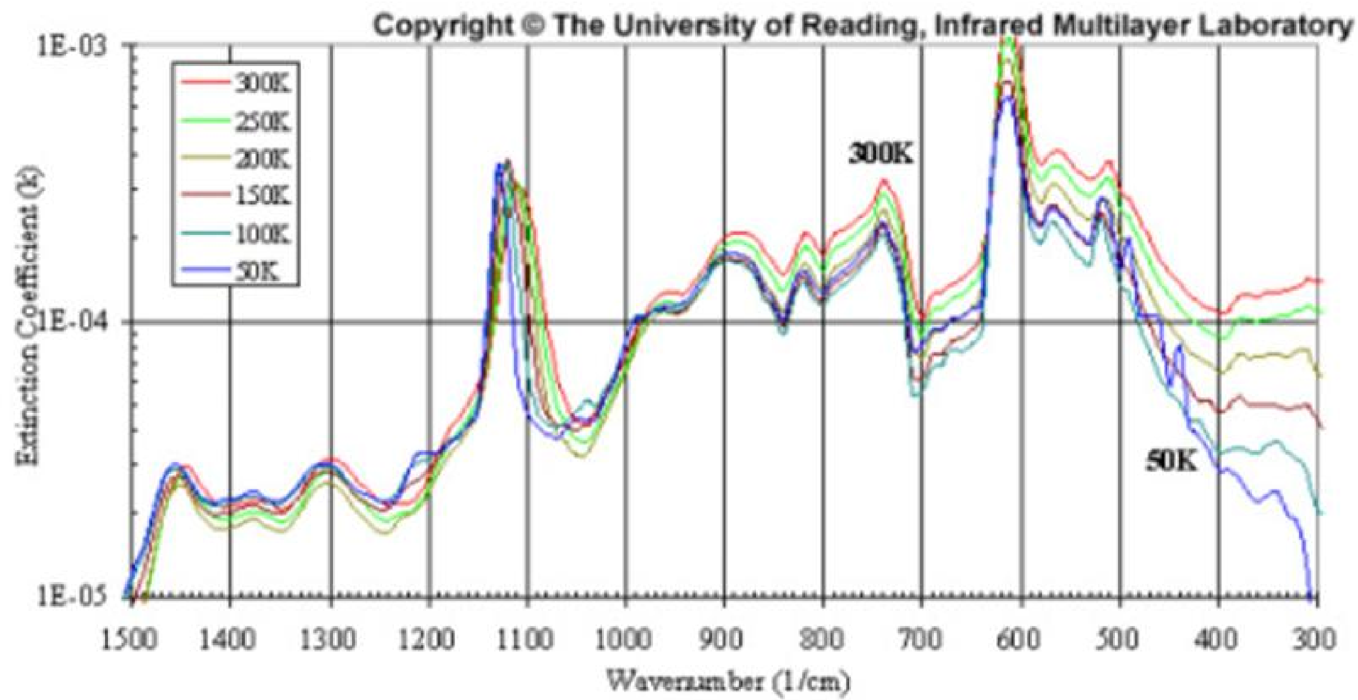
- Take a direction (**θ**, **φ**);
- A vector **r** is generated in that direction with a initial length of 0.05 mm;
- Once reached any face, the code **changes the direction** (θ, φ).



ABSORPTION

- The **absorption** can be expressed as:

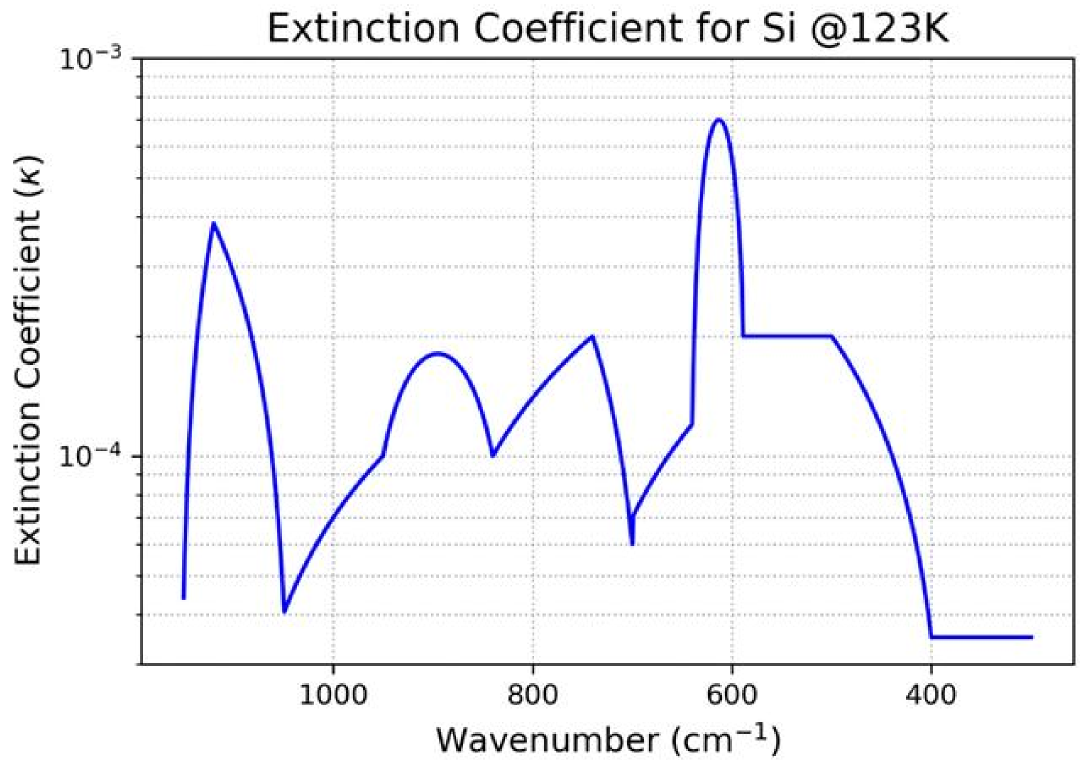
$$\alpha = \frac{4\pi\kappa}{\lambda}$$



ABSORPTION

- Estimated curve for κ;

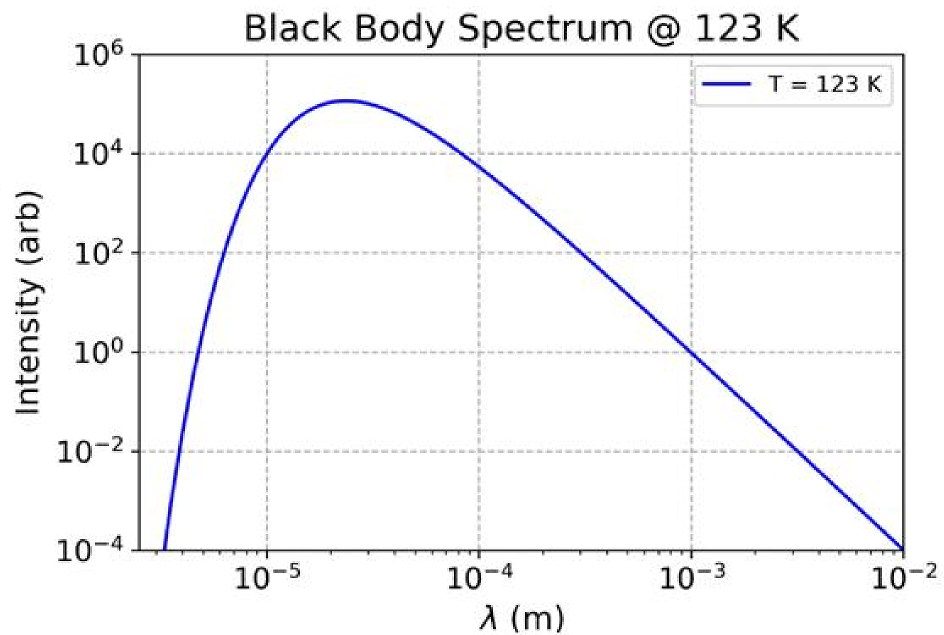
- This curve was estimated taken the mean between the values for 100K and 150 K.



ABSORPTION

- For the absorption we used an **average value**;
- We considered a range of wavelengths around the black-body peak and weighted the mean value by the **black-body curve at 123 K**.

$$\bar{\alpha} = \frac{\int_{\lambda_0}^{\lambda} B(\lambda, T) \cdot \alpha(\lambda)\, d\lambda}{\int_{\lambda_0}^{\lambda} B(\lambda, T)\, d\lambda}$$



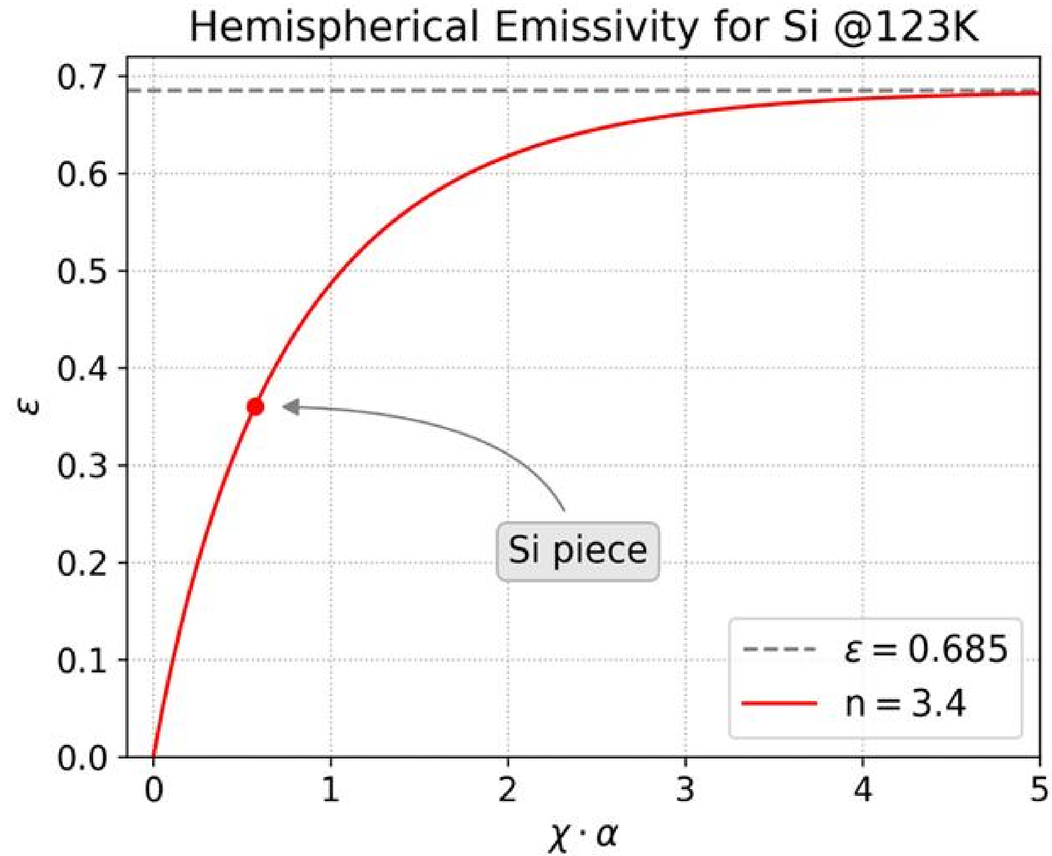
RESULTS

- According to our calculations:

$$\chi = 10.3963\ (mm)$$

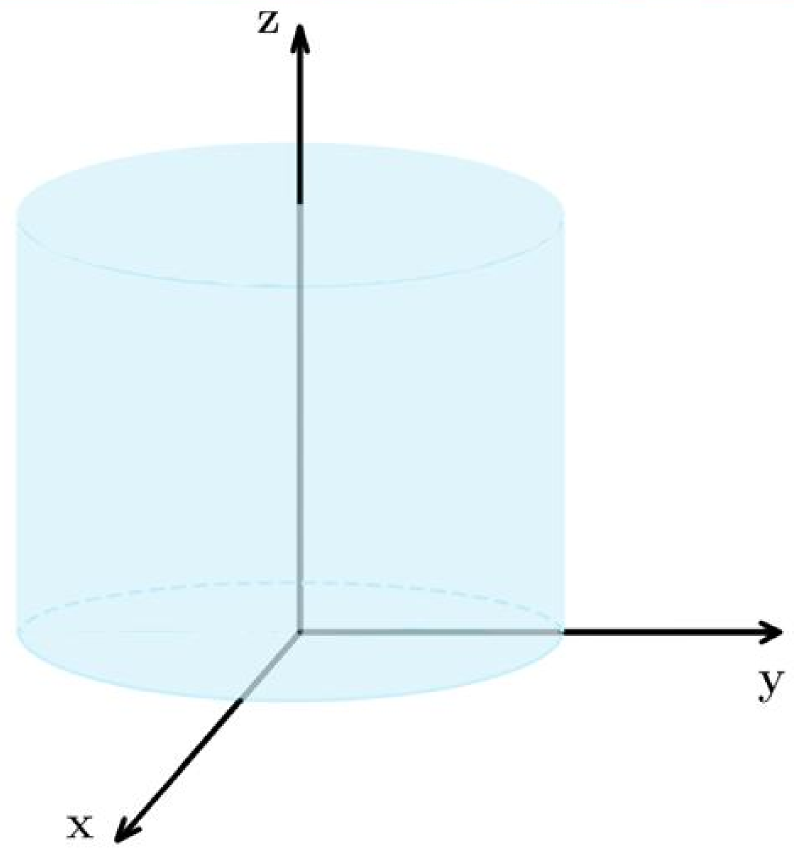
$$\alpha = 55.1731\ (1/m)$$

$$\varepsilon = 0.35601$$



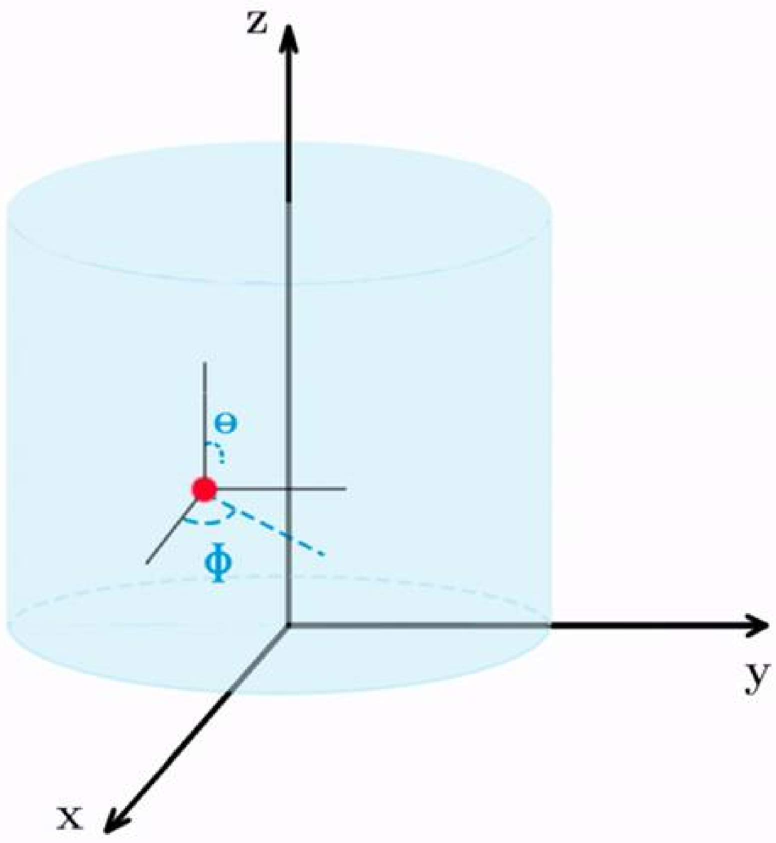
APPLICATION TO LIGO VOYAGER

- The LIGO Voyager will have Si test masses;
- Thus we used the same absorption for these calculations;
- The mean thickness was calculated by a code in Python.



APPLICATION TO LIGO VOYAGER

- Cylinder with:
  - 400 mm of height;
  - 450 mm of radius.
- It uses the same method to calculate the distances between the point and the surfaces.



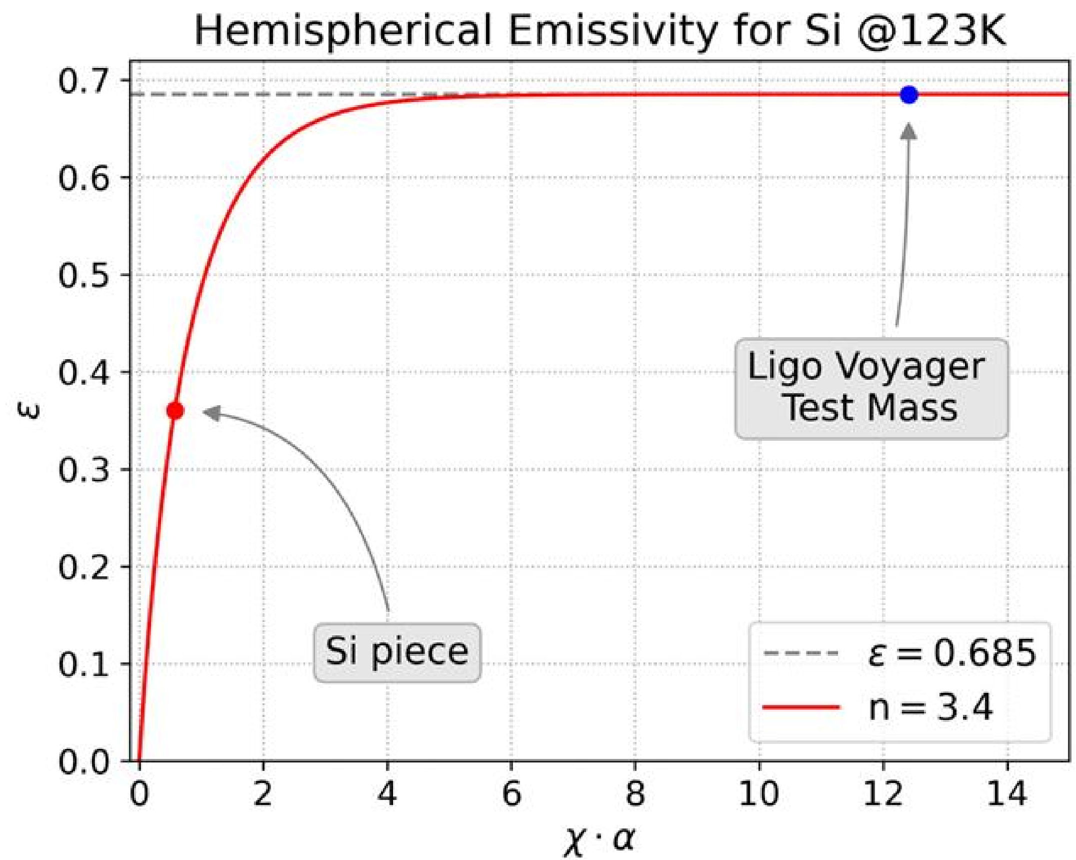
RESULTS

- For the LIGO Voyager test masses:

$$\chi = 224.631 \text{ (mm)}$$

$$\alpha = 55.1731 \text{ (1/m)}$$

$$\varepsilon = 0.68492$$



CONCLUSIONS

- We calculated:

  I. the mean thickness through a code in Python;

  II. the absorption weighted by the Black Body's curve;

  III. the effective emissivity for a Si piece.

  IV. the effective emissivity for LIGO Voyager test masses and obtained $\sim \mathbf{0.7}$

FUTURE PROSPECTS

- So far we have calculated the mean thickness **without** conside-ring internal reflections;

- Our **next step** is to implement these **internal reflections** in our code.

Thank You!

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HEMISPHERICAL EMISSIVITY

- By definition (Gardon, 1956):

$$\varepsilon_{\lambda\chi} = \frac{W_{\lambda e\chi}}{W_{B\lambda}}$$

where W represents the radiant flux (or hemispherical emissive power). So we can conclude that the hemispherical spectral emissivity have no unity.

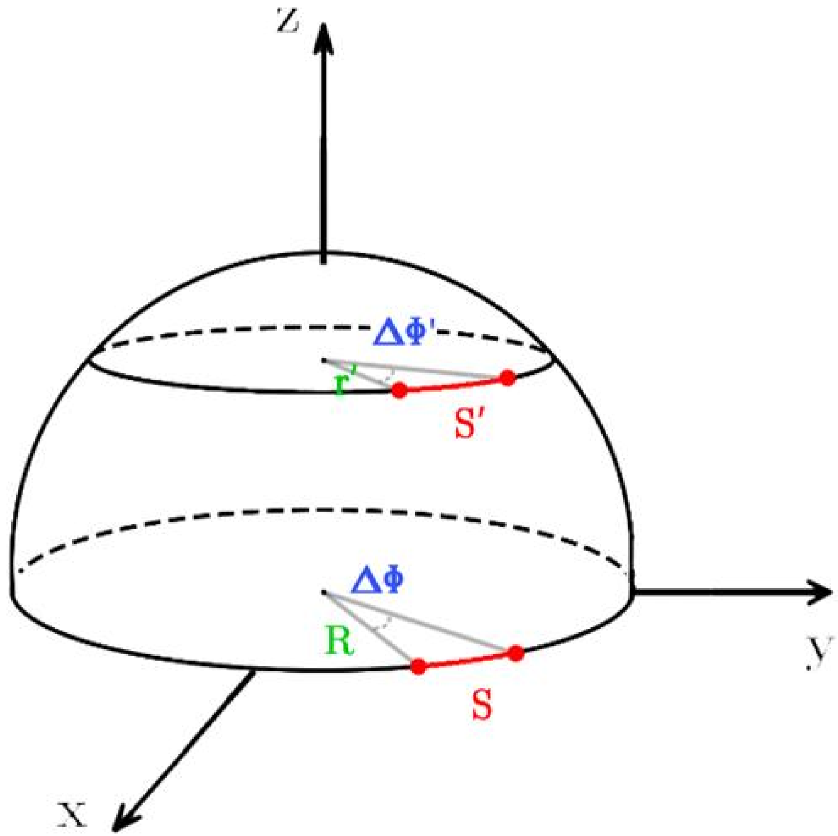
MEAN THICKNESS CODE

$$S' = \frac{2 \cdot \pi \cdot r}{n}$$

$$S' = r \cdot \Delta\phi$$

$$\Delta\phi = \frac{2\pi}{N_0 \sin(\theta)}$$

$$\therefore \; \Delta\phi = \frac{\Delta\phi_e}{\sin(\theta)}$$



MEAN THICKNESS CODE

$$S_\theta = R \cdot \Delta\theta$$

$$S_\phi = r \cdot \Delta\phi$$

$$\begin{aligned} S_\theta &= S_\phi \\ R \cdot \Delta\theta &= r \cdot \Delta\phi \\ \Delta\theta &= \sin(\theta) \cdot \Delta\phi \\ \Delta\theta &= \sin(\theta) \cdot \frac{\Delta\phi_e}{\sin(\theta)} \end{aligned}$$

$$\Delta\phi_e = \Delta\theta$$