Actuation time optimization in the Advanced Virgo mirror thermo-elastic correction

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Abstract

Heating elements surrounding the core optics of Gravitational Wave interferometer are used to correct the radius of curvature of the high reflectivity surface that can deviate from their nominal value because of manufacturing defects and the non-zero absorption of the laser power in the substrate and coatings [1]. These actuators (usually referred to as Ring Heaters, RH) requires about 10 hours to reach the steady state; this long transient makes a significant impact on the commissioning time of the interferometer.
In this work a new strategy aiming at the reduction of the actuation time of the ring heaters is presented together with the experimental results of the tests performed on the TeTis facility in the Virgo laboratory of Roma Tor Vergata. It is found that by applying a time varying voltage on the ring heater, the steady state can be reached in less than an hour.

Goal

Reduce the ring heater actuation time from $10\,\mathrm{h}$ to less than $1\,\mathrm{h}$.

Introduction

The Advanced Virgo RH is composed by two borosilicate glass rings heated by Joule effect induced by a Nickel-Crome coil and a polished copper shield which reflects back the radiadion to the mirror. The behaviour of each radiating element of the system RH+mirror is not linear and is described by:

$$P_{RAD} = A\sigma\epsilon\left(T^4 - T_e^4\right) \quad , \tag{1}$$

where T is the temperature of the element, T_e is the environment temperature, A is the radiating area, σ is the Boltzmann constant and ϵ is the emissivity.

Linear approximation hypothesis:

- Radiative emission is linear for small increments in temperature ;
- The shape of the mirror response is approximately independent from the power applied .

Starting from an Ansys simulation (Finite Element Analysis software, FEA) of the mirror response to $1\,\mathrm{W}$ step input applied to the RH, it is possible to calculate the transfer function and then compute a time varying input power needed to reach the steady state in less than $1\,\mathrm{h}$. This actuation strategy will be validated on the thermal lensing effect induced by the RH on a scaled-down version of Advanced Virgo mirror. The results are also valid for the thermo-elastic deformation since the thermal behaviour in time is the same. This choice has been driven by the higher S/N of the thermal lensing effect with respect to the thermo-elastic deformation.

The transfer function

The transfer function of the mirror is obtained by fitting the simulated curvature response to $1\,\mathrm{W}$ step power input applied to the RH (Figure 2). A good fit can be found as a sum of sinusoids and exponentials:

$$F(t) = a_1 \sin(c_1 + b_1 t)\, e^{b_2 + c_2 t} + a e^{bt} + c e^{dt} \tag{2}$$

$a = -4.376 \times 10^{-5}$	$b_1 = -2.792 \times 10^{-4}$	$c_1 = -6.53$
$a_1 = -4.967 \times 10^{-5}$	$b_2 = 2.1$	$c_2 = -0.00146$
$b = 4.967 \times 10^{-7}$	$c = -5.87 \times 10^{-5}$	$d = -1.912 \times 10^{-4}$

Figure 1: Table of the fit function coefficients .

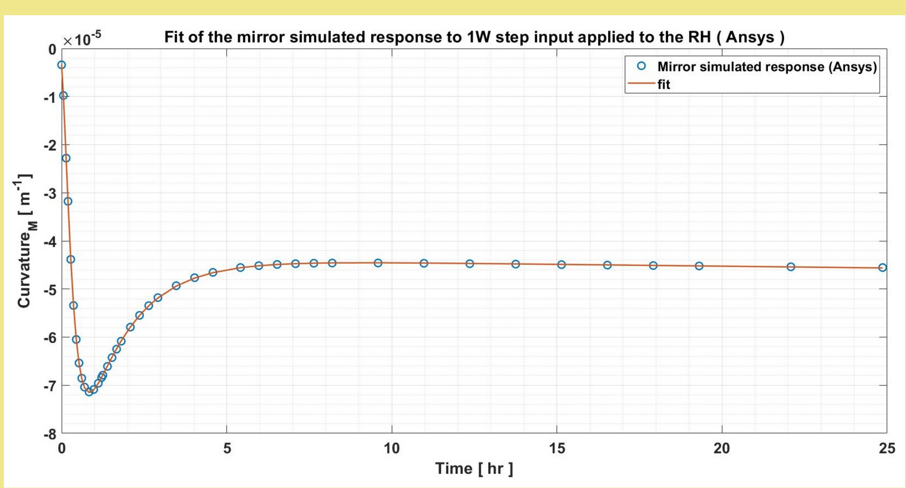

Figure 2: Fit of the mirror response obtained from the Ansys simulation with $1\,\mathrm{W}$ applied to the RH.

Taking the expansion at the $2nd$ order of $\sin(c_1 + b_1 t)$, the Laplace transform of the fit function $\mathcal{L}\{F(t)\} = \mathcal{F}(s)$ was analitically calculated in order to find the transfer function:

$$\mathcal{F}(s) = \frac{a_1 \sin(c_1)\, e^{b_2}}{(s - c_2)} + \frac{a_1 b_1 \cos(c_1)\, e^{b_2}}{(s - c_2)^2} + \frac{a}{(s - b)} + \frac{c}{(s - d)} \quad , \tag{3}$$

$$tf(s) = s * \mathcal{F}(s) \quad . \tag{4}$$

The output function

A challenging goal was chosen for the mirror response, that is the following output of the system:

$$\mathbf{r_{goal} = 4 \times 10^{-5} e^{-t/250} - 4.47 \times 10^{-5}} \quad . \tag{5}$$

- $\tau = 250\ \mathrm{s} = 4\,\mathrm{min}$;
- The mirror curvature will reach the $95\,\%$ of the steady state value $\mathbf{C = -4.47 \times 10^{-5}\ m^{-1}}$ in about $12\,\mathrm{min}$.

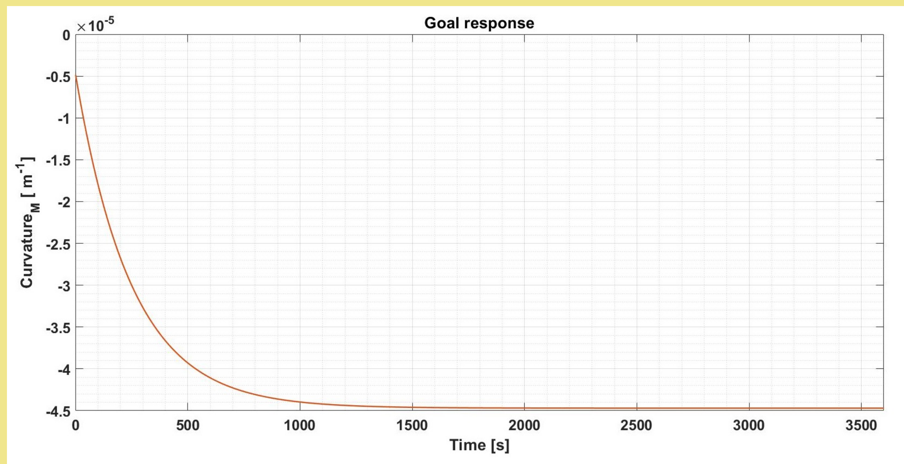

Figure 3: Goal response.

The Input function

- Knowing $\mathbf{tf_{step}}$ and $\mathbf{r_{goal}}$, the input function $P(t)_{1W}$ was computed (shown in fig 4);
- The computation was done sampling at $\Delta t = 4.475\,\mathrm{s}$ (acting time) in order to obtain the goal response in fig. 3.

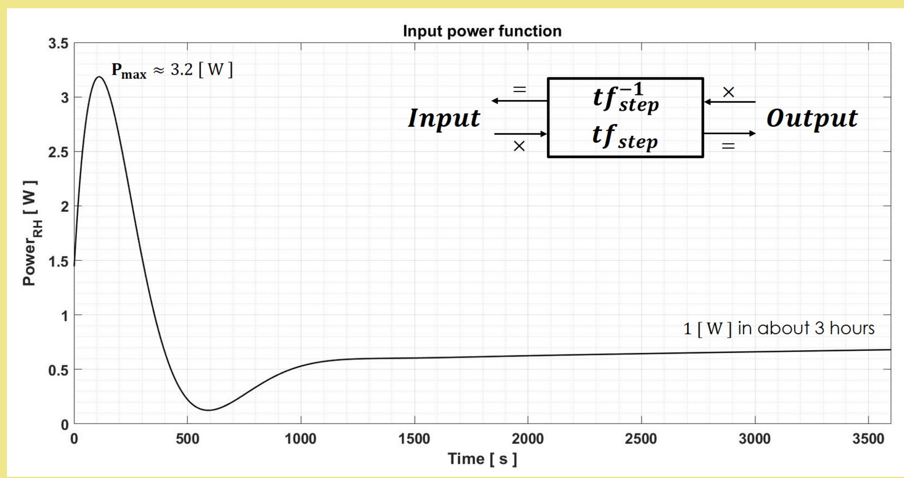

Figure 4: The time varying power to be applied to the RH.

Experimental setup

- The thermal lensing effect was measured on-axis with double passage and using an Hartmann wavefront sensor that has the same sensitivity as the one used in Virgo [2];
- The RH power supply is remotely controlled ($\Delta t = 4.475\,\mathrm{s}$).

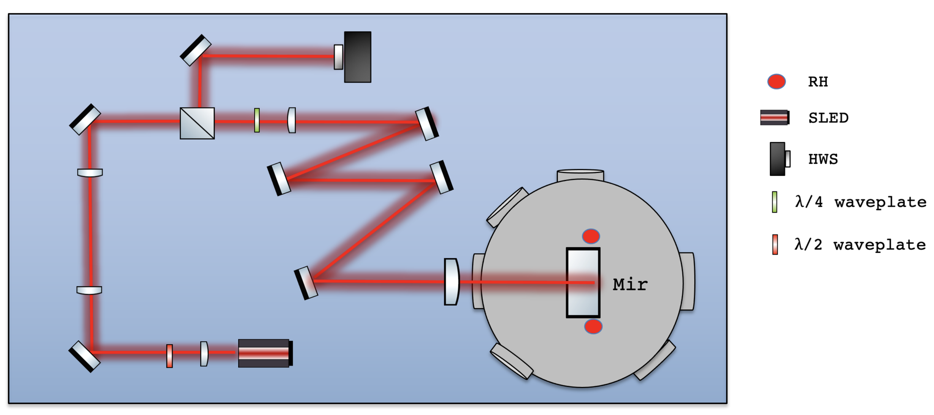

Figure 5: Sensing optical layout.

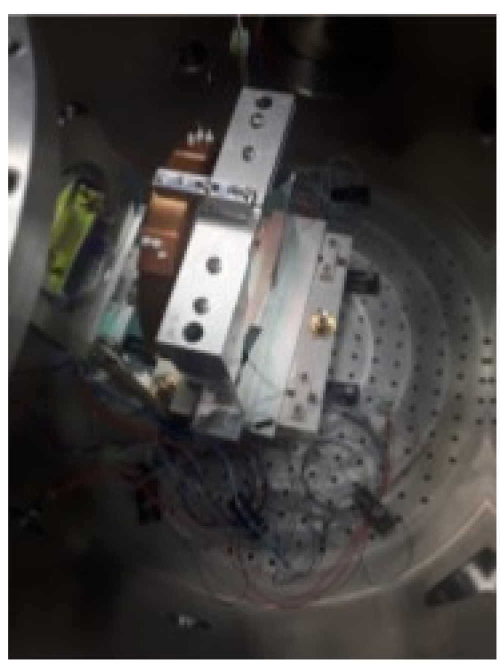

Figure 6: Scaled down ring heater.

Wavefront analysis

- The wavefronts were decomposed in Zernike polynomials, and the fourth coefficient is taken to obtain the curvature in time ;
- The defocus contribution due to the environmental temperature change was subtracted.

Results - $P(t)_{1W}$

The thermal behaviour of the glass rings by applying $P(t)_{1W}$ is in figure 7.

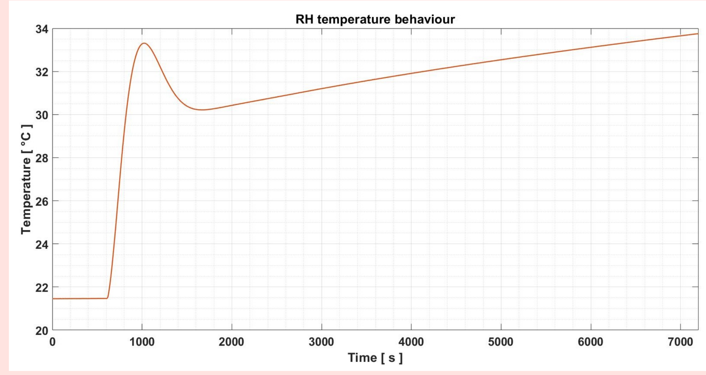

Figure 7: Temperature behaviour for 1W steady power.

The curvature response shown in figure 8 agrees with the model, as the $95\,\%$ of the steady state is reached in about $\sim 15\,\mathrm{min}$.

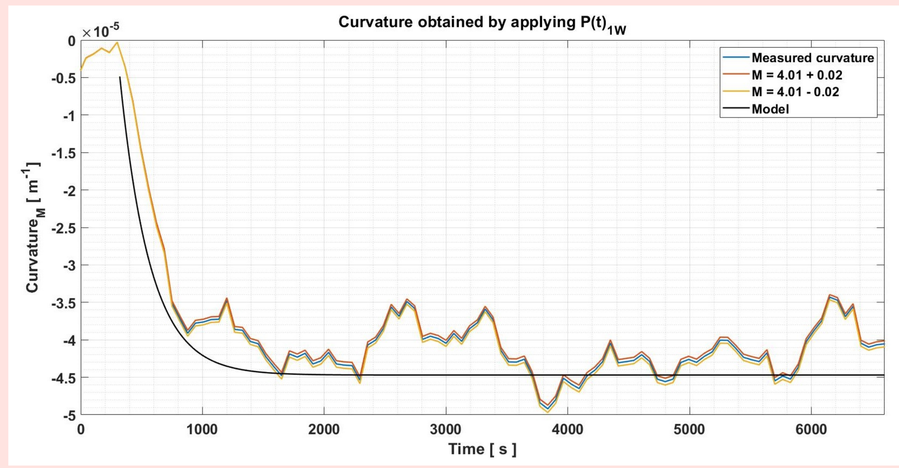

Figure 8: Mirror curvature response to the computed power with $1\,\mathrm{W}$ of steady value.

The red and yellow curves are obtained considering the error on the magnification of the telescope between the sensor and the mirror $M = 4.01 \pm 0.02$. Due to the low applied power, the sensor noise clearly appears in the curvature measurement.

Results - $P(t)_{2W}$

The same measurements was performed by applying $2\,\mathrm{W}$.

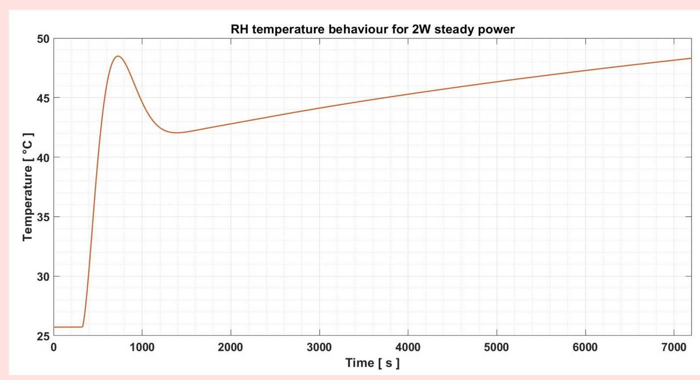

Figure 9: Temperature behaviour for 2W steady power.

The thermal behaviour of the glass rings by applying $P(t)_{2W}$ is in figure 9. The curvature response shown in figure 10 is in agreement with the model, and the $95\,\%$ of the steady state is reached in about $\sim 17\,\mathrm{min}$.

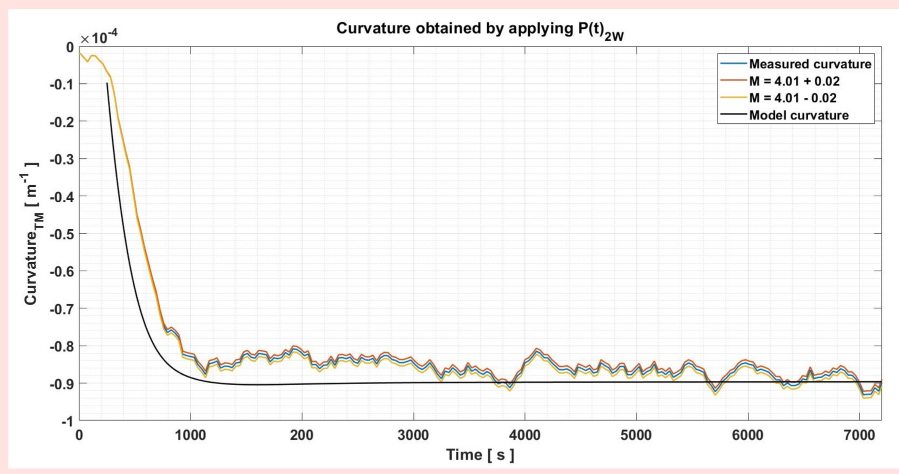

Figure 10: Mirror curvature response to the computed power with $2\,\mathrm{W}$ of steady state value.

Increasing the applied power improves the SNR of the measurement (as expected)

Conclusions

- The obtained power curves P_{1W} and P_{2W} bring the system to the steady state in less than $20\,\mathrm{min}$;
- The curvatures measured at P_{1W} and P_{2W} are in a very good agreement with the analytical model, allowing a fair reduction of the settling time of the RH ;
- A deeper investigation is needed to check the behaviour at higher power where non-linear effects related to the thermal radiation are likely to become relevant .

References

[1] The Virgo Collaboration. *Advanced Virgo Technical Design Report*. Tech. rep. VIR-0128A-12, 2012.

[2] I. Nardecchia. "Control of optical aberrations in advanced interferometric gravitational wave detectors, PhD thesis in Astronomy, Astrophysics and Space Science". PhD thesis. 2016.

GWADW
Gravitational Wave Advanced Detector Workshop, May 17 th 21 st , 2021 Remote