Local readout at gravitational wave detectors

- gravitational wave detectors are currently limited by seismic & technical noise at low frequencies
- pendulum suspension (passive) and actuators (active) mitigates seismic noise on the mirror suspensions of GWD

Figure: Sensitivity difference of current GW-detectors (LIGO & Virgo) and the proposed Einstein-Teleskope. Source: Conor Mow-Lowry

May 12, 2021 — Tobias Eckhardt
Deep-Frequency Modulated Interferometry (DFMI)

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- Multi-fringe readout with simple optical setup using frequency modulated laser

\[ P_{\text{out}}(t) = P_0 \sin(\omega_{\text{DFM}} \cdot t) \]

\[ \omega_{\text{DFM}} = \omega_0 + \Delta \omega \cdot \sin(\omega_m t_0 + \psi) \]

- allows for very compact design

- precision of down to \( 10^{-14} m/\sqrt{\text{Hz}} \)

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Source: Gerberding & Isleif 2021

source

Measured noise from a DFM-Interferometer setup.
Source: Isleif et al. 2019
Fundamental readout limits

- Compare limit of readout of different interferometer types via Cramer-Rao lower bound \( \implies \) minimal variance of readout

- **Cramer-Rao Lower Bound (CRLB)** given by

\[
\text{var}(\hat{\theta}) \geq \frac{1}{E \left[ \left( \frac{\partial (\ln \rho(X; \theta))}{\partial \theta} \right)^2 \right]}
\]

\[
\text{Fisher-information}
\]

\( \rho \) probability density function of the (noisy) measured signal

\( \text{var} \) variance

\( X \) measured signal

\( \theta \) real phase

\( \hat{\theta} \) readout / phase estimate

\( \mathbb{E} \) expectation value

\[
\begin{align*}
\text{Fundamental readout limit:} \\
\Delta L^2 & \geq \\
\text{single output} & \\
\text{homodyne} & = \frac{2}{1 + \cos \theta} S + \frac{8}{1 - \cos 2\theta} (A + D) \\
\text{heterodyne} & = 2S + 8(A + D) \\
\text{deep modulation} & = \frac{2}{1 + J_0(m) \cdot \cos \theta} S + \frac{8}{1 - J_0(2m) \cdot \cos 2\theta} (A + D) \\
\text{quadrature} & = 2S + 16(A + D)
\end{align*}
\]

with

\[
S = \frac{\lambda^2 q_e}{2\pi^2 R_{\text{PD}} \kappa P_0}, \quad A = \left( \frac{7 \lambda}{2\pi R_{\text{PD}} \kappa P_0} \right)^2, \quad D = \frac{\lambda^2}{24\pi^2 f_S \alpha^2 4 \text{NOB}}
\]
DFM-Readout Algorithm

The measured signal:

\[ P_{\text{measured}}(t) \approx P_0 \sin (m \cdot \sin(\omega_m t + \psi) + \varphi) \implies \tilde{P}_{\text{measured}}(\omega) \approx P_0 \sum_{n \in \mathbb{Z}} J_n(m) e^{i(n\psi + \varphi)} \delta(\omega - n\omega_m) \]

Parameter of interest: \((\varphi, m, \psi, P_0)\)

\[ J_{n-1}(m) + J_{n+1}(m) = \frac{2n}{m} J_n(m) \]

- faster algorithm
- near optimal precision
- no need for starting values (opposed to fit)

Example power spectral density of DFM signal

Figure: Runtime of the algorithm specific parts (excluding time to input data etc.) for different time-series lengths. Sampled with a sampling frequency of \(f_S = 10\text{kHz}\), \(10^3\) datapoints correspond to a readout frequency of \(f_R = 0.1\text{Hz}\) and \(10^6\) datapoints to \(f_R = 10\text{Hz}\)

Figure: Plot of a sample algorithm readout (blue) and the theoretically archivable minimum given by the CRLB (red).