Local readout at gravitational wave detectors

- gravitational wave detectors are currently limited by seismic & technical noise at low frequencies
- pendulum suspension (passive) and actuators (active) mitigates seismic noise on the mirror suspensions of GWD

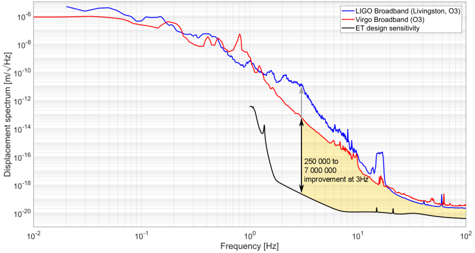
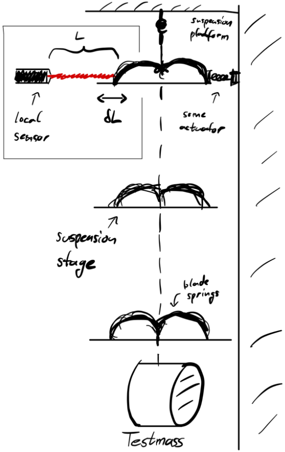
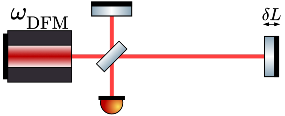


Figure: Sensitivity difference of current GW-detectors (LIGO & Virgo) and the proposed Einstein-Teleskope. Source: Conor Mow-Lowry



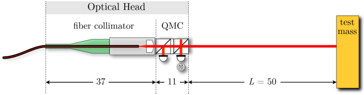
Deep-Frequency Modulated Interferometry (DFMI)



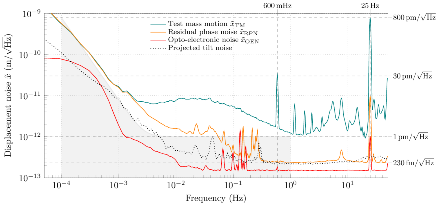
- Multi-fringe readout with simple optical setup using frequency modulated laser

$$P_{\text{out}}(t) = P_0 \sin(\omega_{\text{DFM}} \cdot t)$$
$$\omega_{\text{DFM}} = \omega_0 + \Delta\omega \cdot \sin(\omega_m t_0 + \psi)$$

- allows for very compact design



Source: Gerberding & Isleif 2021



Measured noise from a DFM-Interferometer setup.
Source: Isleif et al. 2019

- precision of down to $10^{-14} m/\sqrt{\text{Hz}}$

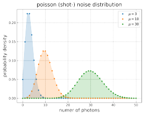
Fundamental readout limits

- Compare limit of readout of different interferometer types via Cramer-Rao lower bound $\Longrightarrow$ minimal variance of readout
- **C**ramer-**R**ao **L**ower **B**ound (CRLB) given by

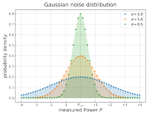
$$\mathrm{var}(\hat{\theta}) \geq \frac{1}{\underbrace{\mathbb{E}\left[\left(\frac{\partial(\ln \rho(X;\theta))}{\partial\theta}\right)^2\right]}_{\text{Fisher-information}}}$$

ρ	probability density function of the (noisy) measured signal		
var	variance	X	measured signal
θ	real phase	$\hat{\theta}$	readout / phase estimate
$\mathbb{E}$	expectation value		

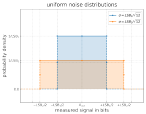
shot noise



electronic noise



quantization noise



$\Downarrow$

Fundamental readout limit:

$\widetilde{\Delta L^2} \geq$	single output
homodyne	$\frac{2}{1 \pm \cos\theta} S + \frac{8}{1 - \cos 2\theta}(A + D)$
heterodyne	$2S + 8(A + D)$
deep modulation	$\frac{2}{1 \pm J_0(m) \cdot \cos\theta} S + \frac{8}{1 - J_0(2m) \cdot \cos 2\theta}(A + D)$
quadrature	$2S + 16(A + D)$

with

$$S = \frac{\lambda^2 q_e}{2\pi^2 R_{PD}\, \kappa P_0}\,, \quad A = \left(\frac{\tilde{I}\,\lambda}{2\pi R_{PD}\, \kappa P_0}\right)^2\,, \quad D = \frac{\lambda^2}{24\pi^2 f_S\, \alpha^2\, 4^{NOB}}$$

DFM-Readout Algorithm

The measured signal:

$$P_{\text{measured}}(t) \approx P_0 \sin\left(m \cdot \sin(\omega_m t + \psi) + \varphi\right) \implies \widetilde{P}_{\text{measured}}(\omega) \approx P_0 \sum_{n\in\mathbb{Z}} J_n(m) e^{i(n\psi+\varphi)} \delta(\omega - n\omega_m)$$

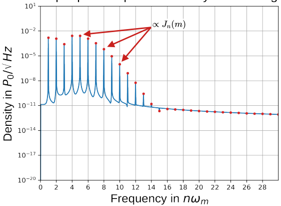
Parameter of interest: (φ, m, ψ, P_0)

$\implies$ use analytical relations between coefficient (Bessel functions)

$$J_{n-1}(m) + J_{n+1}(m) = \frac{2n}{m} J_n(m)$$

- faster algorithm
- near optimal precision
- no need for starting values (opposed to fit)

example power spectral density of DFM signal



(partial) algorithm runtime comparison

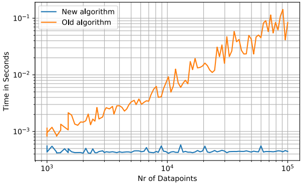


Figure: Runtime of the algorithm specific parts (excluding time to input data etc.) for different time-series lengths. Sampled with a samling frequency of $f_S = 10$kHz, 10^3 datapoints correspond to a readout frequency of $f_R = 0.1$Hz and 10^6 datapoints to $f_R = 10$Hz

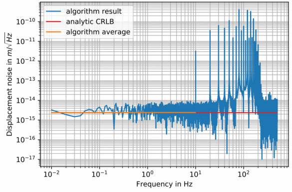


Figure: Plot of a sample algorithm readout (blue) and the theoretically archivable minimum given by the CRLB (red).