Higher-order Hermite-Gauss modes for thermal noise reduction

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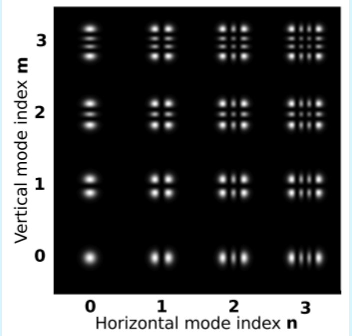
Thermal noise of the test masses is one of the limiting noise sources in Advanced detectors. It is expected to remain a limiting noise source in future detectors, despite radical changes to the design including cryogenic operations, new materials and the use of longer laser wavelengths. This poster covers progress towards verifying higher-order Hermite-Gauss laser modes as a possible alternative or complementary technology to reduce thermal noise.

Thermal noise of higher-order HG modes

Several so-called "flat beams" have been studied for thermal noise reduction in gravitational wave detectors. They help by better averaging over the random mirror surface fluctuations caused by thermal motions. Laguerre-Gauss modes were studied extensively, but fell out of favor due to their fragility against astigmatism in mirrors [1,2].



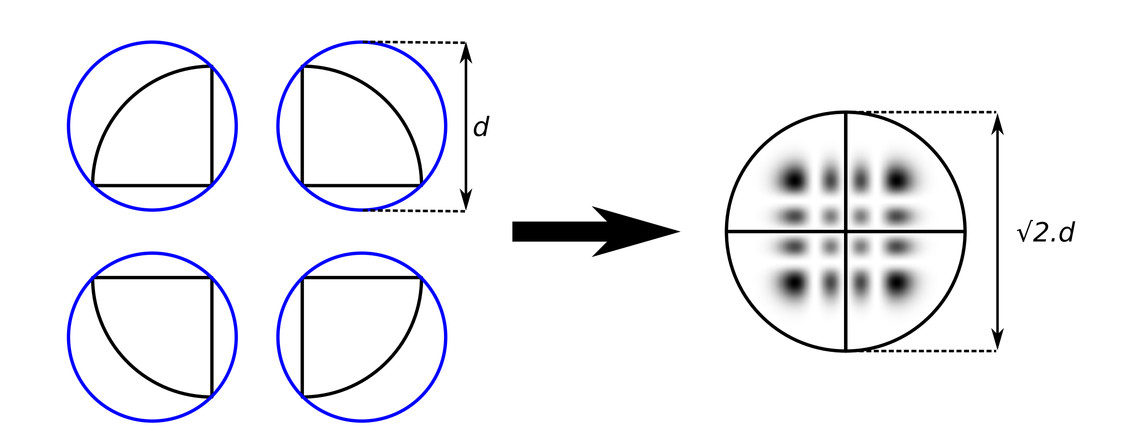
In experiments, LG modes appeared to break up into HG modes of the same mode order. This led to the idea to try using HG modes as a flat beam instead, despite their more modest thermal noise improvement for circular mirrors. Intensity profiles of HG_{nm} modes up to HG_{33} are shown to the left.

The table to the right shows coating Brownian thermal noise *power* spectral density improvement factors over the HG_{00} mode (after adjusting beam size to keep 1ppm clipping losses on a circular mirror).

n \ m	0	1	2	3	4	5
0	1	1.10	1.11	1.08	1.05	1.02
1	1.10	1.29	1.33	1.40	1.30	1.27
2	1.10	1.33	1.40	1.41	1.41	1.39
3	1.08	1.32	1.41	1.44	1.45	1.45
4	1.05	1.30	1.41	1.45	1.47	1.47
5	1.02	1.27	1.39	1.45	1.47	1.48

HG modes with segmented mirrors

Odd-ordered HG modes also have the interesting property of intensity nulls along the axes of symmetry. This might make them more suitable for use with segmented mirrors, as illustrated below.



If the maximum mirror size is limited by the available diameter of the substrate material, a 4-segmented mirror would allow an increase in mirror diameter of up to a factor √2.

The benefits for thermal noise are clear: bigger beam sizes means better averaging, and lower coating thermal noise by a factor √2:

$$\delta\tilde{h}_{\mathrm{CTN}} \propto w^{-1} \propto D^{-1}$$

But consider also the benefits for radiation pressure noise. if we maintain the same test mass aspect ratio, the volume and therefore mass will increase by the cube of the diameter increase. Since the radiation pressure noise scales inversely with the mirror mass

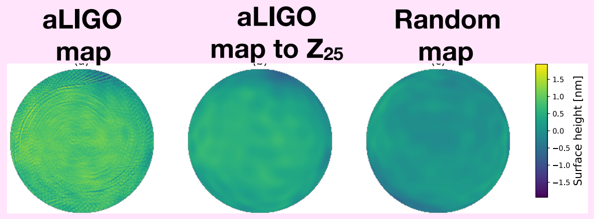
$$\delta\tilde{h}_{\mathrm{RP}} \propto M^{-1} \propto D^{-3}$$

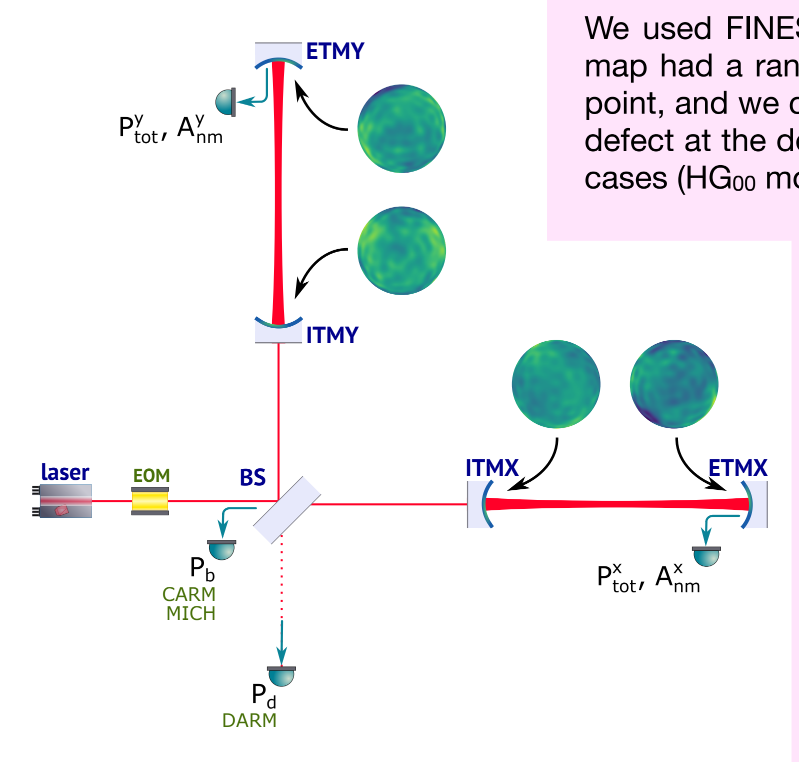
We could get an improvement of $2^{3/2}$ = 2.83 in radiation pressure noise compared to the smaller non-segmented mirror.

HG modes compatibility with realistic imperfect mirrors

Our first test for HG modes was to see if they performed better with imperfect mirrors, where LG modes had struggled. We pursued a simulation study comparing HG_{33}, LG_{22} and HG_{00} modes in terms of their intra-cavity losses, and contrast defect, in a Fabry-Perot Michelson interferometer with realistic mirror surface maps [3].

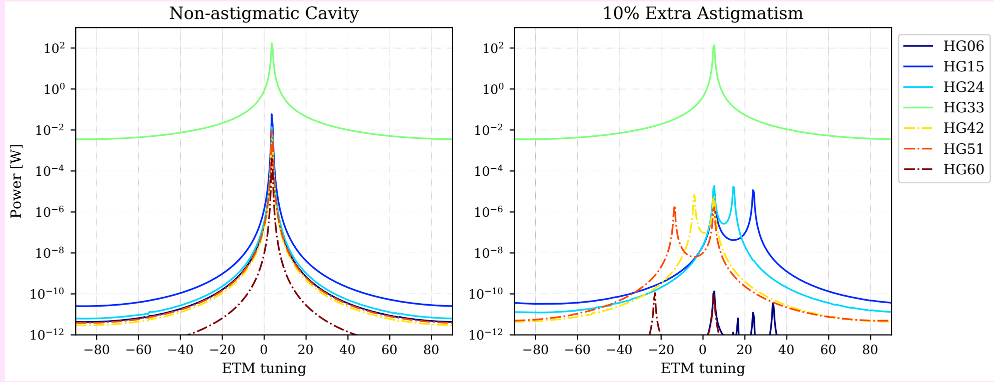
Realistic random maps were generated by decomposing measured aLIGO mirror maps into Zernike polynomials, and then building new maps with randomized Zernike amplitudes, but the same spatial frequency characteristics as the measured maps.



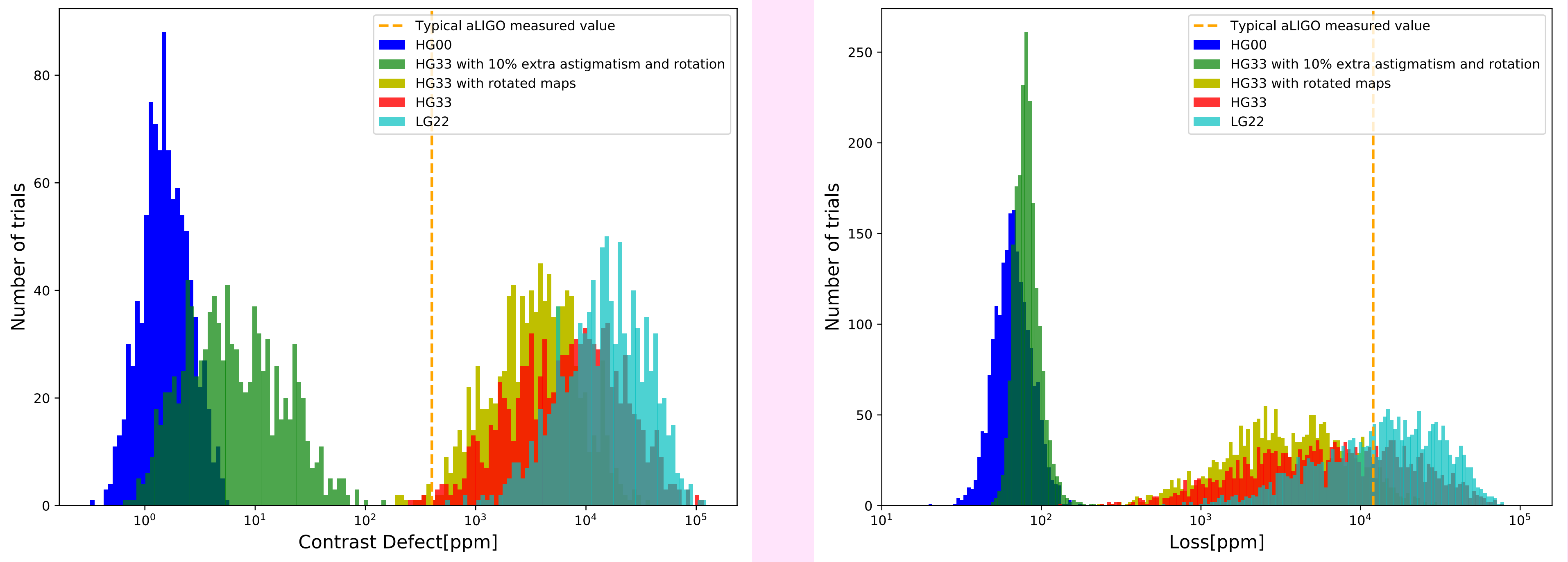


We used FINESSE to simulate an aLIGO-like Fabry-Perot Michelson detector. Each test map had a random mirror map applied. The detector was tuned to an optimal operating point, and we calculated the losses and mode purity in each cavity, as well as the contrast defect at the detector dark port. This simulation was run ~1000 times, for several different cases (HG_{00} mode, HG_{33} mode, LG_{22} mode, and modified HG_{33} mode cases).

Deliberately adding astigmatism to the mirror maps vastly improved the performance of the HG_{33} mode. This is because the astigmatism further separates otherwise "pseudo-degenerate" modes of equal order in the cavities, as shown below.



The results were compiled in histograms, to show the typical scale and spread of the figures of merit for all different cases. As shown below, adding 10% extra astigmatism to the mirrors improved their performance from "not-much-better-than-LG_{22}" to "almost-as-good-as-HG_{00}". Performance at the level shown here should be perfectly adequate for future detectors, since it is already orders of magnitude better than current detectors.



Alignment and mismatch tolerance and sensing with HG modes

A potential downside to using HG modes, first reported by Aaron Jones *et al.*, is the tighter tolerances that they will require in terms of mode matching [4]. Intuitively, since higher-order HG modes contain higher spatial frequencies than the HG00 mode, their coupling efficiency e.g. into optical cavities drops faster when mode mismatching is imperfect. We followed up Aaron's work to consider also misalignment, leading to the following results [5]:

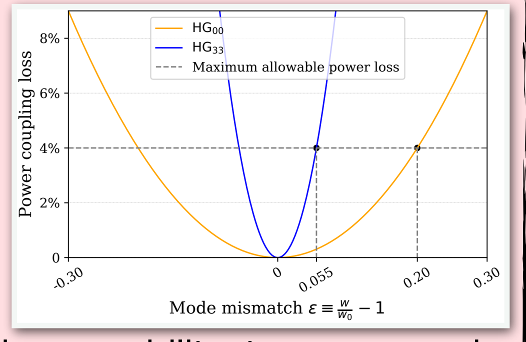
Mode mismatch induced coupling loss for HG_{nm} relative to HG_{00}:

$$\Gamma^{\Delta w_0}_{n,m} = \Gamma^{\Delta z_0}_{n,m} = \frac{n^2+n+m^2+m+2}{2}$$

Misalignment induced coupling loss for HG_{nm} relative to HG_{00}:

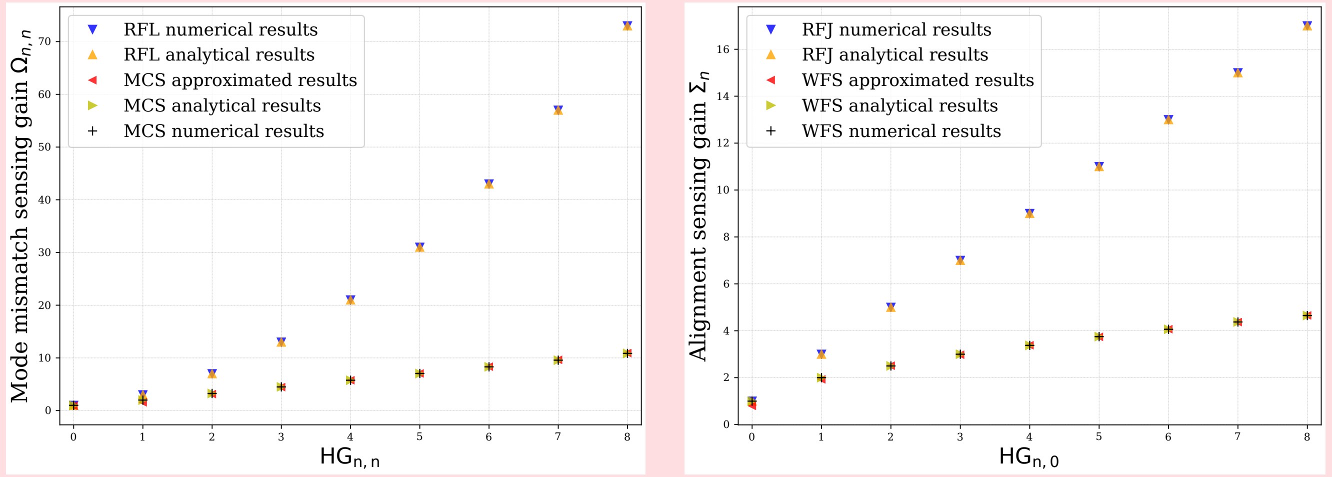
$$\Gamma^{\mathrm{offset}}_{n} = \Gamma^{\mathrm{tilt}}_{n} = 2n+1$$

The plot to the right shows how the faster drop in coupling efficiency leads to a mode matching tolerance about 3.6x tighter for the HG_{33} mode compared to the HG_{00} mode.



Another important aspect of alignment and mode matching is our ability to *sense* and therefore correct errors. We derived alignment and mode mismatch sensing signals for generic HG modes with several different sensing schemes (QPD-based alignment sensing [6], RF jitter-based alignment sensing [7], QPD-based mode mismatch sensing [8] and RF lens-based mode mismatch sensing [9]). As expected, the higher spatial frequency content of higher-order HG modes leads to an *increase* in the sensing gain compared to the HG00 mode. This increase is stronger in the RF-jitter and RF-lens schemes, because they also involve modulation of the degree of freedom being sensed.

Relative mode mismatch and alignment sensing gains for different schemes and HG modes



Conclusions and next steps

Progress is being made towards validating HG modes as a future technology for GW detectors. Several challenges remain, including producing squeezed light in HG modes (see poster from Joscha Heinze), and tabletop and prototype demonstrations of HG mode interferometry (this has been started by Stefan Ast *et al.* [10]). We also plan to look at HG mode performance in full DRFPMI configuration simulations, HG mode performance with regards to parametric instabilities, and further investigate the concept of segmented mirrors. Please let us know what other concerns you have about HG modes and we'd be happy to look at that too!

References:

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