Enhanced noise suppression for LISA using arm and cavity locking

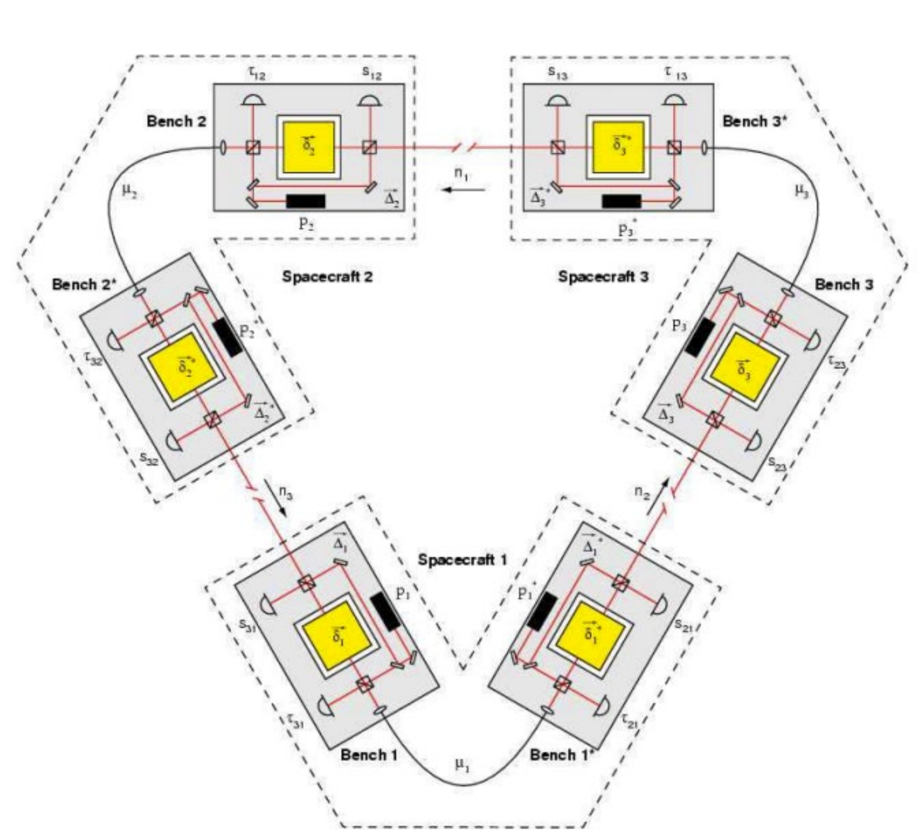
- JOBIN THOMAS VALLIYAKALAYIL
- DANIEL SHADDOCK
- KIRK McKENZIE

ANU CENTRE FOR GRAVITATIONAL ASTROPHYSICS

OzGrav
ARC Centre of Excellence for Gravitational Wave Discovery

Background

- LISA – Laser Interferometer Space Antenna
  - Space-based interferometer
  - Three spacecrafts separated by 2.5 million kms. [1]
- Interferometry done in low frequency band
  - From 0.1 mHz to 1 Hz.
  - Achieve better sensitivities than earth-based interferometer
  - Sensitivity of 10 $\mathrm{pm}/\sqrt{\mathrm{Hz}}$ [1] -> requires 14 orders of suppression from free-running laser

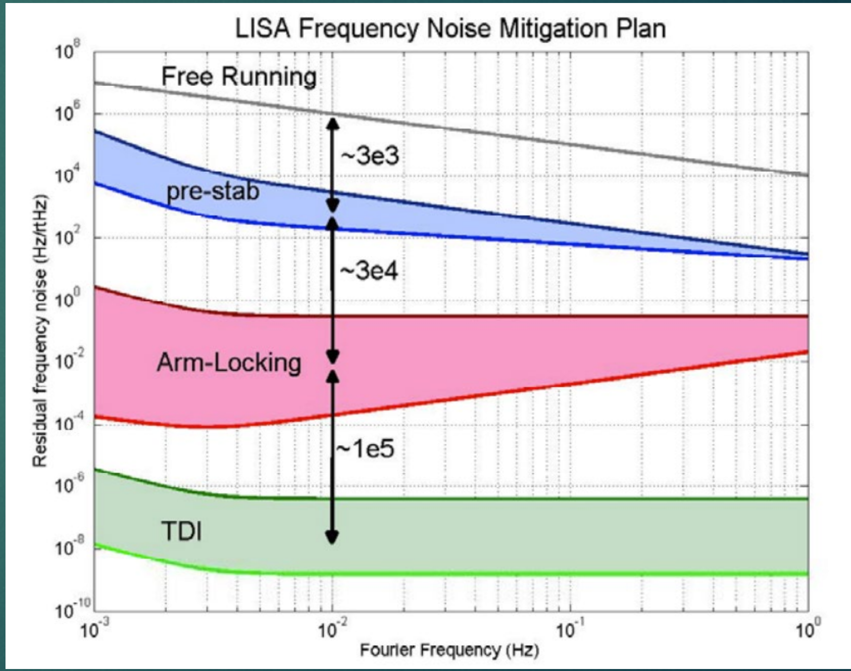


1. K. Danzmann et al. "LISA L3 mission proposal document", (2017)

Background

- Current Baseline stabilization of LISA includes cavity locking with TDI
  - Cavity locking – Technique to stabilize the laser with respect to a resonant cavity
    - PDH locking [2], reduces the residual noise to 30 $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$
  - Time-Delay Interferometry (TDI)

  Post-processing technique that mimics an equal-armlength Michelson response by applying appropriate delays to phase measurements. It can suppress the residual noise to the LISA Requirement [3].



2. E. D. Black. (2001). "An introduction to Pound–Drever–Hall laser frequency stabilization" (PDF). Am J Phys. 69 (1): 79–87
3. D.A. Shaddock, M. Tinto, F.B. Estabrook, J.W. Arm-strong Phys. Rev. D, 68 (2003), p. 061303(R)
4. J C Livas et al 2009 Class. Quantum Grav. 26 094016

Motivation

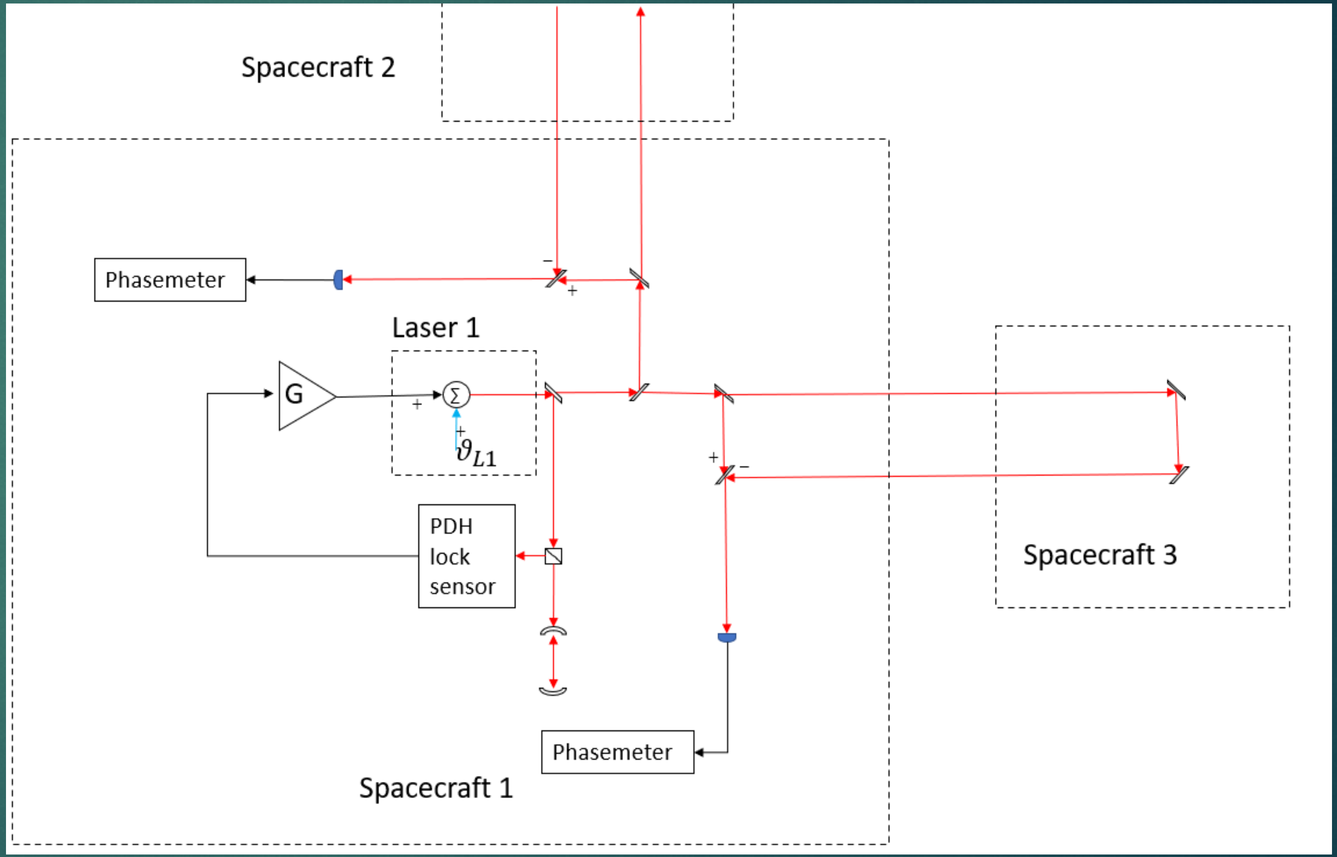
- TDI is a powerful technique, but it is difficult to verify on ground without the complexity of the system in space
  - If TDI fails to meet the requirements, there is a potential risk of losing out GW data.
- But what about arm locking? Stabilising the laser with respect to the arm length of the interferometer - the most stable reference in LISA.
  - LIGO also uses arm locking to stabilize their laser
    - The arm lengths are very small (in $< 30\mu s$), while LISA has large armlengths (~16.67s).
    - The relative motion of the spacecrafts introduce Doppler shifts -> not a problem in LIGO.

Goal – Investigate the arm locking stabilization to relax TDI requirements with no hardware changes to LISA baseline design.

What are we doing?

LISA baseline design

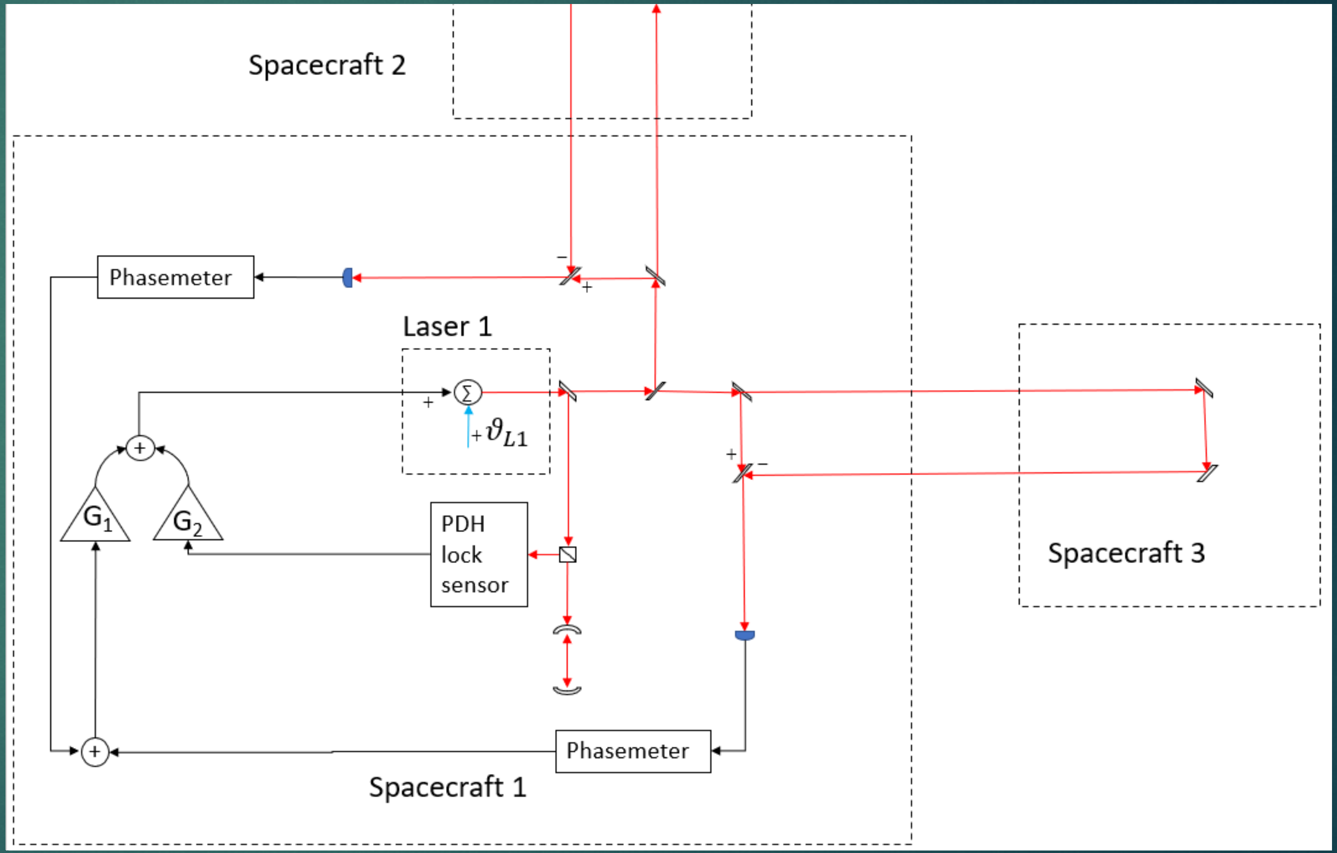
Pre-stablised laser with FP cavity is sent to the spacecrafts and measurement is taken using Phasemeter and TDI is applied later in the post-processing data analysis.



What are we doing?

Combine both the arm sensor and the PDH sensor and feed it back to the laser, using appropriate controllers.

This could utilize the best parts of both sensors



Previous work and challenges

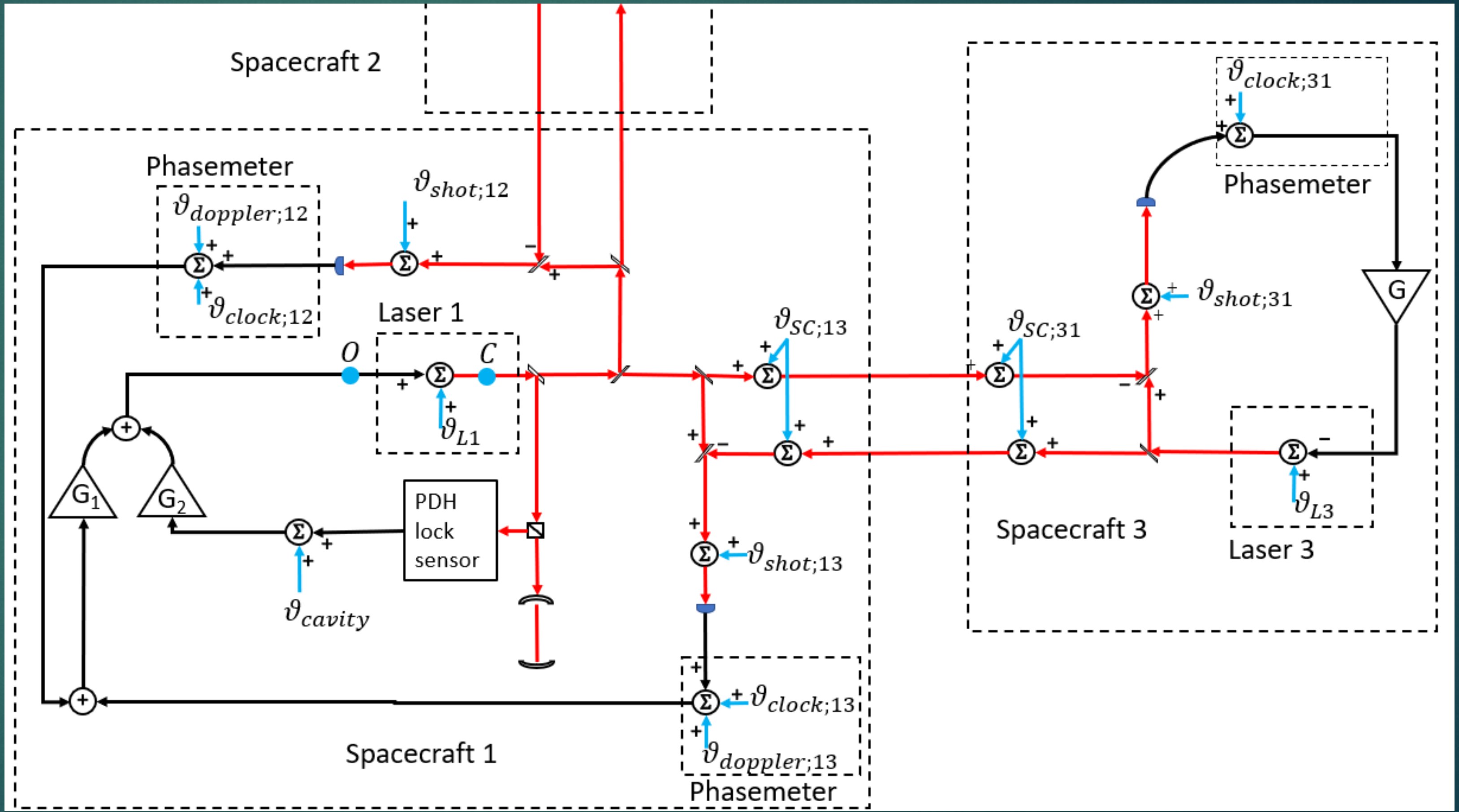
- Previous Proposals
  - Single arm locking[5], Use a single arm roundtrip for stabilization
  - More complex schemes have been explored using two arms.[6][7]
  - Combination of the arm with Mach-Zehnder and Fabry-Pérot pre-stabilisation has been investigated [7].
- No technique are compatible with the current LISA baseline design.
- Additional challenge: **Doppler pulling** [7][8]
  - Received light is Doppler shifted by 10 MHz due to relative speed of the spacecrafts.
  - Sensor has zero response at DC, and so the Doppler shifts must be cancelled
  - Error in the Doppler frequency knowledge will lead to ramp in laser frequency over time, causing potential problems.

5. S. Sheard, M. B. Gray, D. E. McClelland, and D. A. Shaddock, Phys. Lett. A 320, 9 (2003).
6. A. Sutton and D. A. Shaddock, Phys. Rev. D 78, 082001 (2008)
7. K. McKenzie, R. Spero and D. Shaddock, Phys. Rev. D 80, 102003 (2009)
8. D. A. Shaddock and et al, LISA Frequency Control WhitePaper, LISA Project technical note LISA-JPL-TN-823(2009)

Model

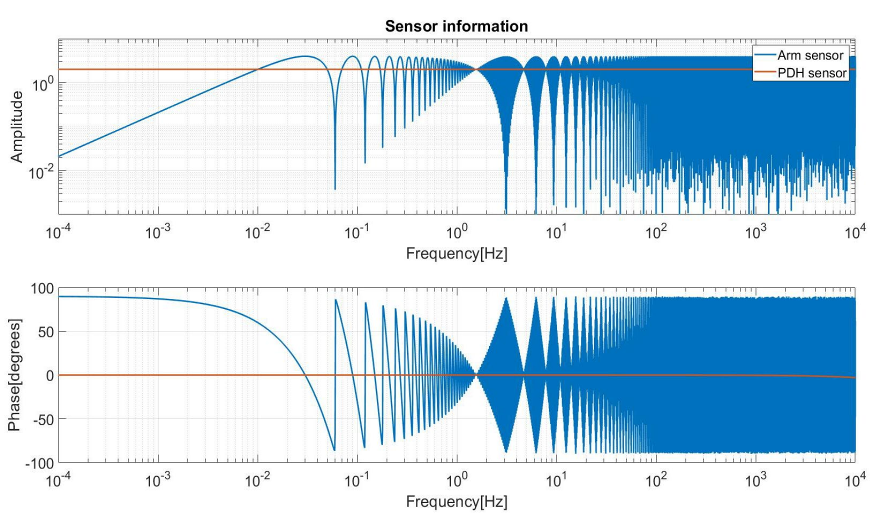
Assumption

High gain transponder for Controller in Spacecrafts 2 and 3 -> Allows for Spacecrafts 2 and 3 to act as a reflective mirror.



Careful Controller Design

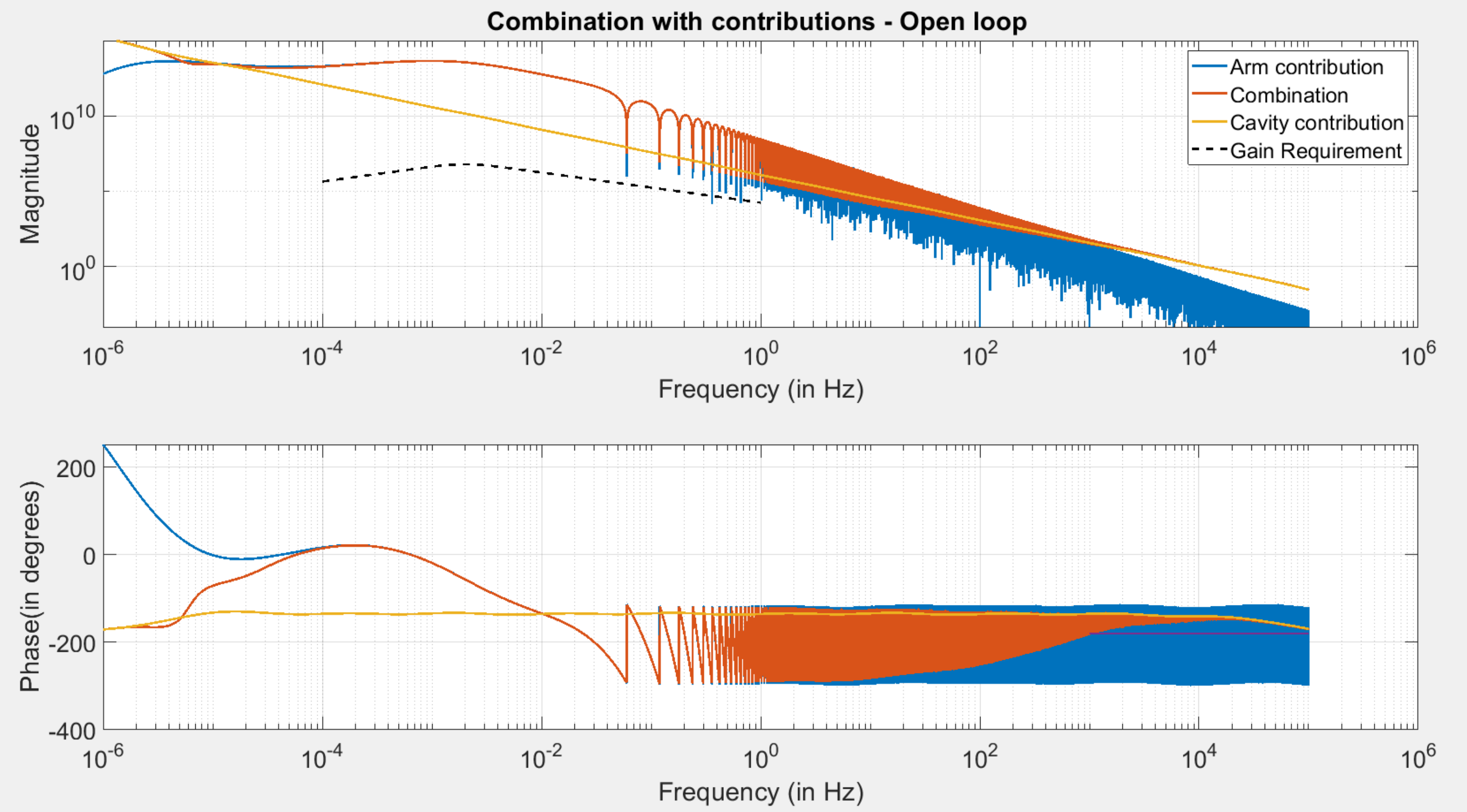
- Arm response should be dominant in the LISA band(10^{-4} to 1 Hz)
  - Arm is more stable and hence will give maximum suppression
- The cavity must be dominant in the other bands
  - Phase variations of Arm are not desirable
  - Orbital dynamics dictate that the armlength variations are periodic with half-yearly and yearly period. [9]



- Controller stability conditions
  - The phase margin at unity gain crossings must be more than 30 degrees (open loop phase more than -150 degrees).
- Maintain cavity resonance – Doppler pulling lower than the linewidth of the cavity
  - In this case a linewidth of 200 kHz → doppler pulling <15 kHz

9. Joseph C. Amato, "Flying in formation: The orbital mechanics of LISA's three spacecraft," Am. J. Phys. 87,18–23 (2019)

Controller Design



Unity Gain Frequency ~10 kHz
Phase Margin at UGF ~ 30°
Phase margin at lower gain crossover point ~52°
Arm dominant from 10 μHz to 2 kHz.

Residual Noise requirements

- Suppression requirement for implementing TDI-2, for a sensitivity of 2 pm/$\sqrt{(\mathrm{Hz})}$. TDI-2 algorithm can compensate for spacecraft motion, but is reliant upon accurate inter-satellite range knowledge (<1m) [7]

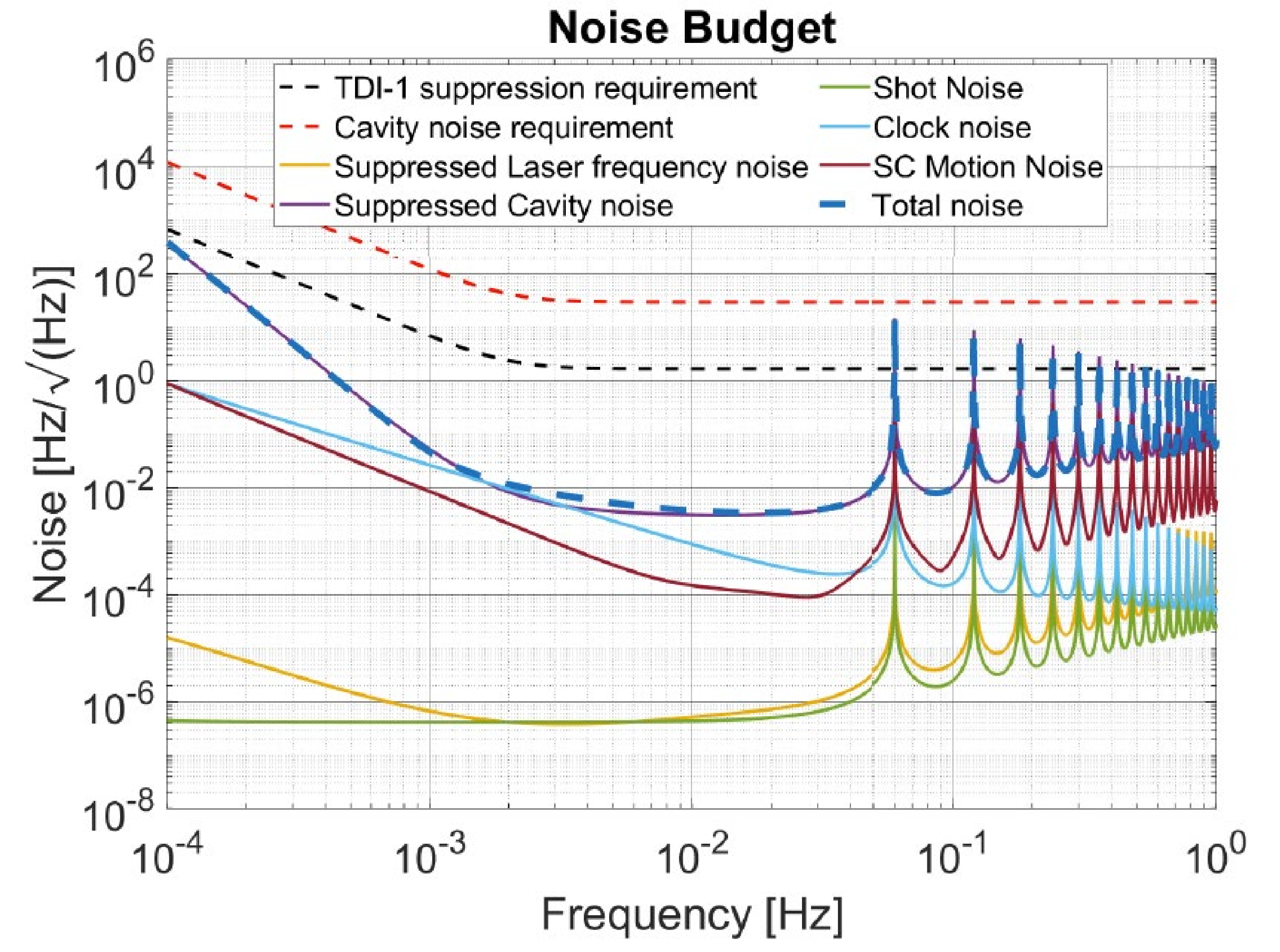
$$\nu_{TDI-2} = 282\,\frac{Hz}{\sqrt{Hz}}\sqrt{1+\left(\frac{2mHz}{f}\right)^4}$$

- Suppression requirement for implementing TDI-1, assuming the worst-case scenario when the maximum relative velocity is 10 m/s [1]

Goal →

$$\boxed{\boldsymbol{\nu_{\mathrm{TDI-1}} = 1.7\,\frac{Hz}{\sqrt{Hz}}\sqrt{1+\left(\frac{2mHz}{f}\right)^4}}}$$

Residual Noise Results



Suppression requirement for LISA can be realized with this controller setup and can perform better than the cavity alone.

But can we maintain lock to the cavity?

Doppler pulling –Lock acquisition

- Step response corresponding to turn on of the controllers.

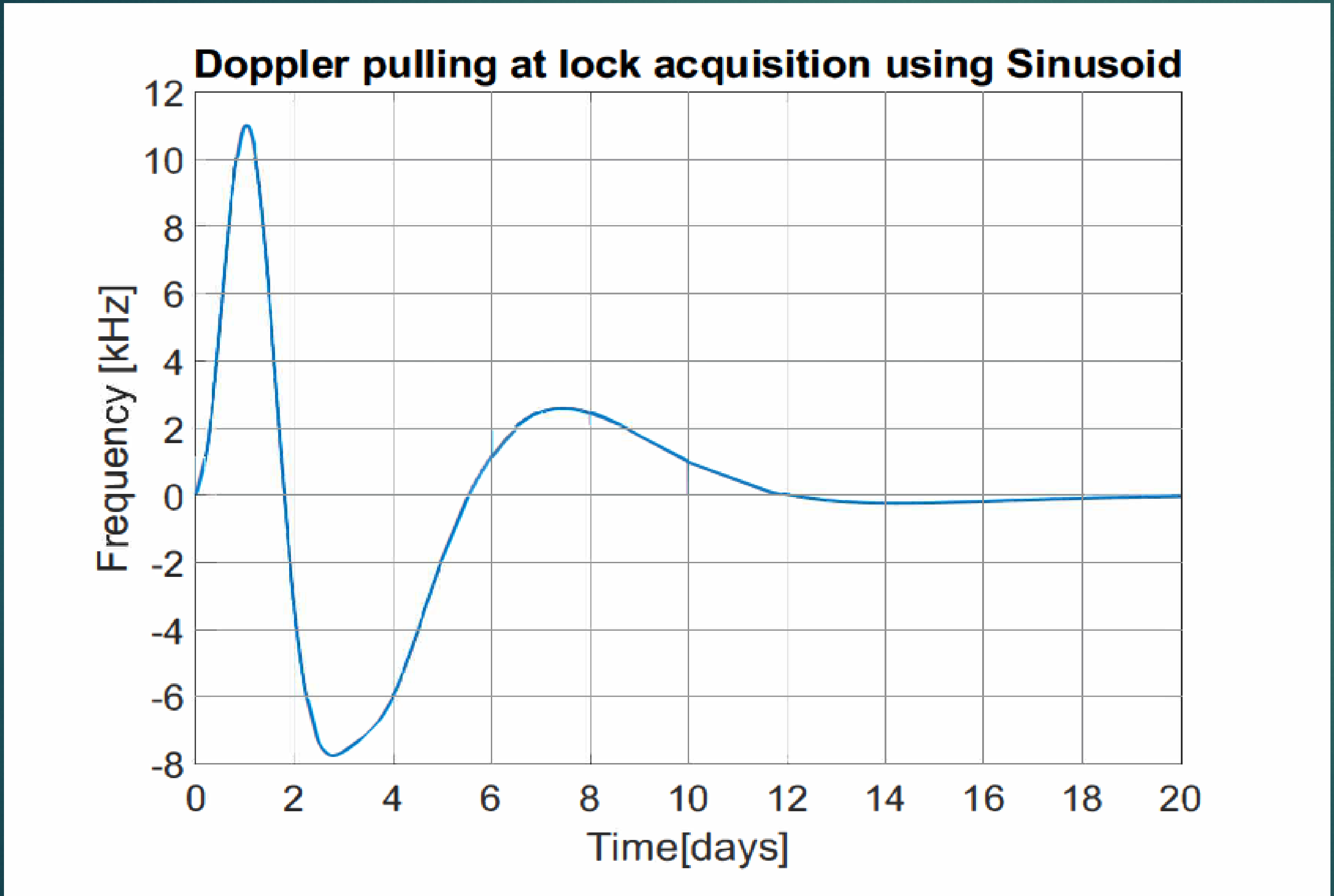
$$v_C(t) \ = L^{-1}\left[\frac{\left(\left[v_{doppler;+}(s) - v_{doppler;lock}(s)\right]V(s)\right)}{s}\right]$$

$v_{doppler;lock}(s)$ can be a sinusoidal approximation as shown below

$$v_{doppler;lock}(s) = v_0 + \gamma_0 t + \iint \alpha(t)dtdt'$$

$$\alpha(t) = \alpha_1 \sin(\omega_1 t + \phi_1) + \alpha_2 \sin(\omega_2 t + \phi_2)$$

Doppler pulling –Lock acquisition



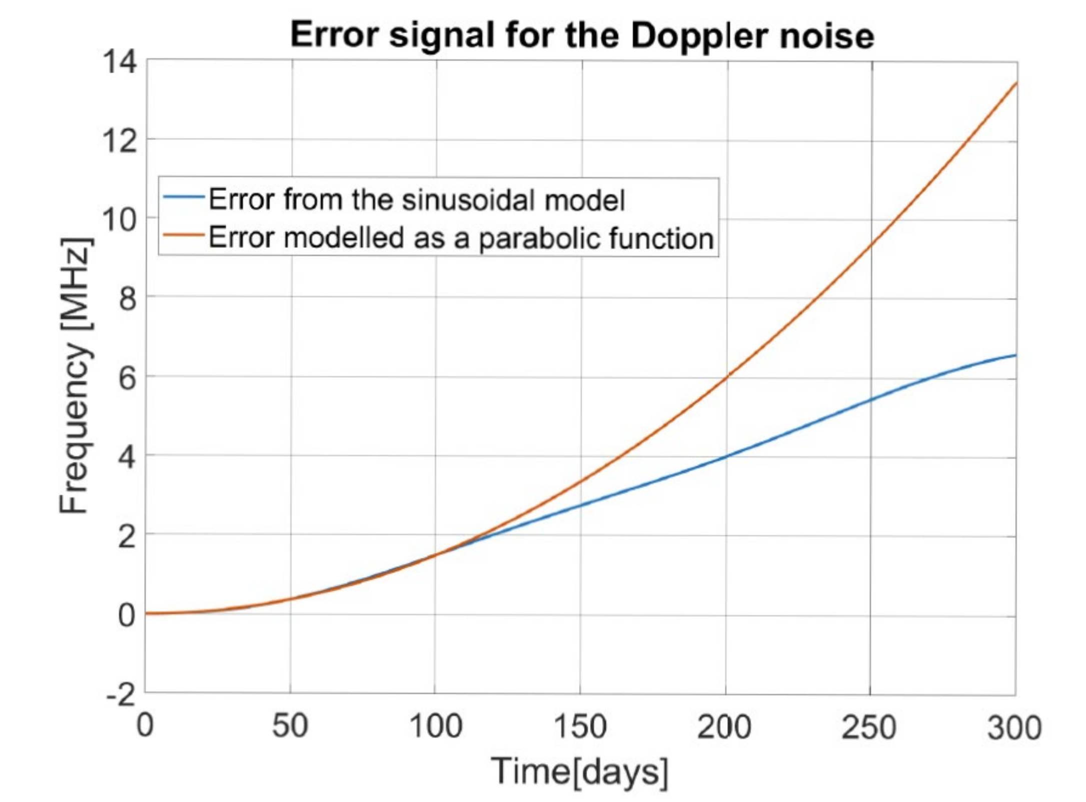
Cavity linewidth ~ 200 kHz
Doppler requirement < 15 kHz

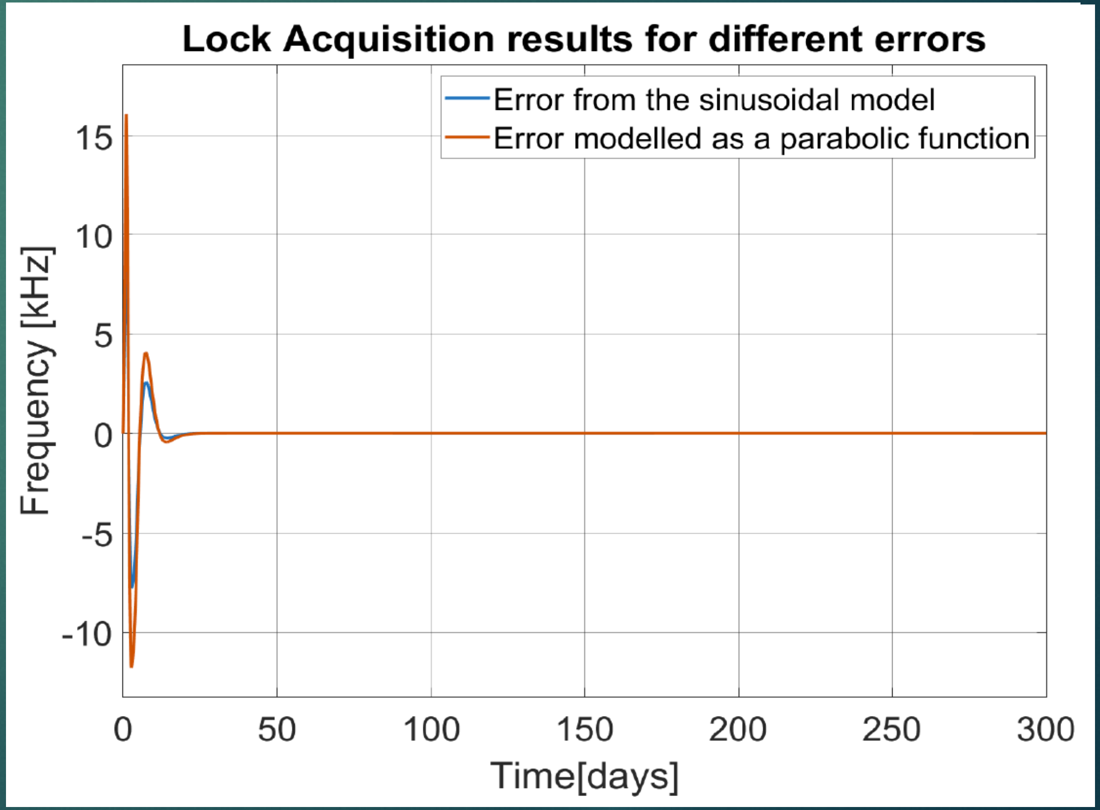
$$\widetilde{\nu_0} < 10\ Hz$$
$$\widetilde{\gamma_0} < 100\ \mu Hz/s$$
$$\widetilde{\alpha_1} < 30\ nHz/s^2$$
$$\widetilde{\alpha_2} < 30\ nHz/s^2$$
$$\widetilde{\omega_1} < 2\pi 10^{-10} rad/s$$
$$\widetilde{\omega_2} < 2\pi 10^{-10} rad/s$$
$$\widetilde{\phi_1} < 2\pi 10^{-5}\ rad$$
$$\widetilde{\phi_2} < 2\pi 10^{-5}\ rad$$

Piecing all the Doppler information

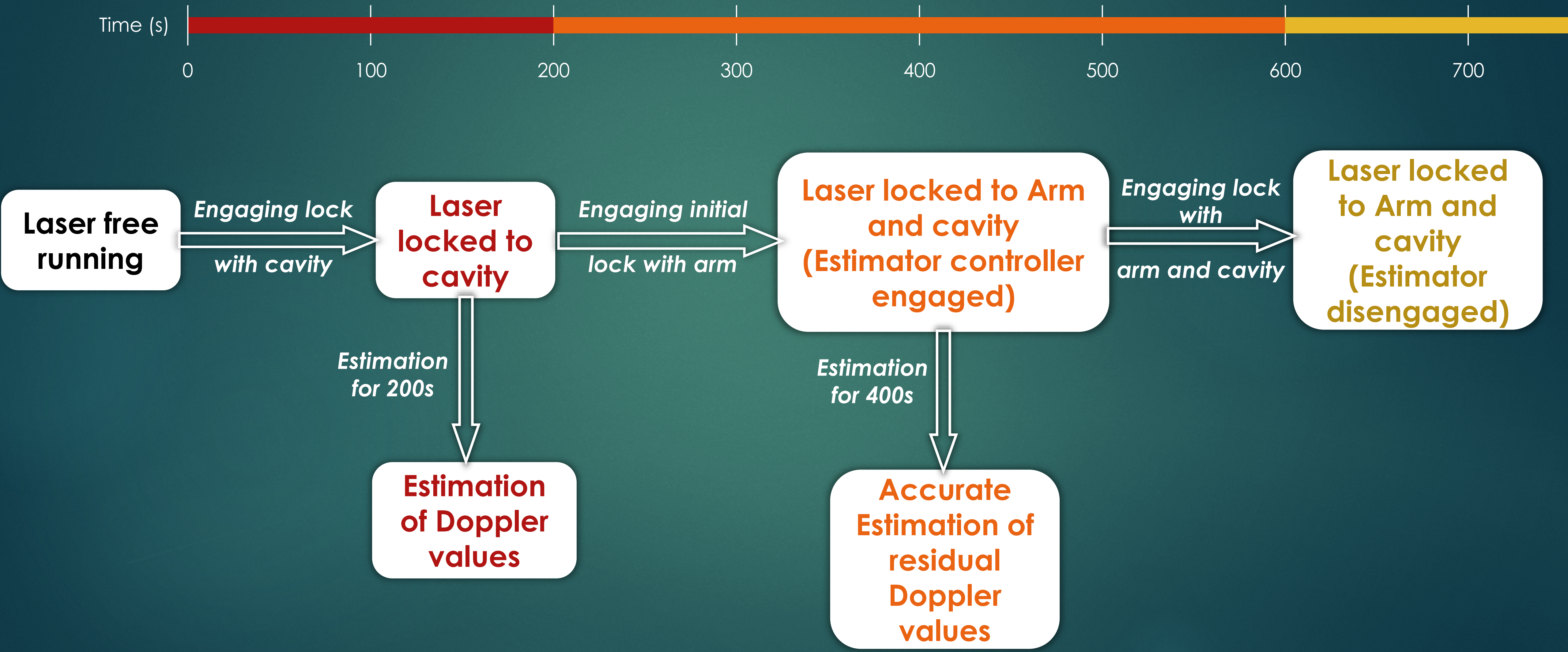
- Need to transition from lock acquisition to steady state (upto 300 days).
- Investigated amount of deviation possible to allow for transition.

This shows we can go from lock acquisition to steady state operation without losing cavity lock

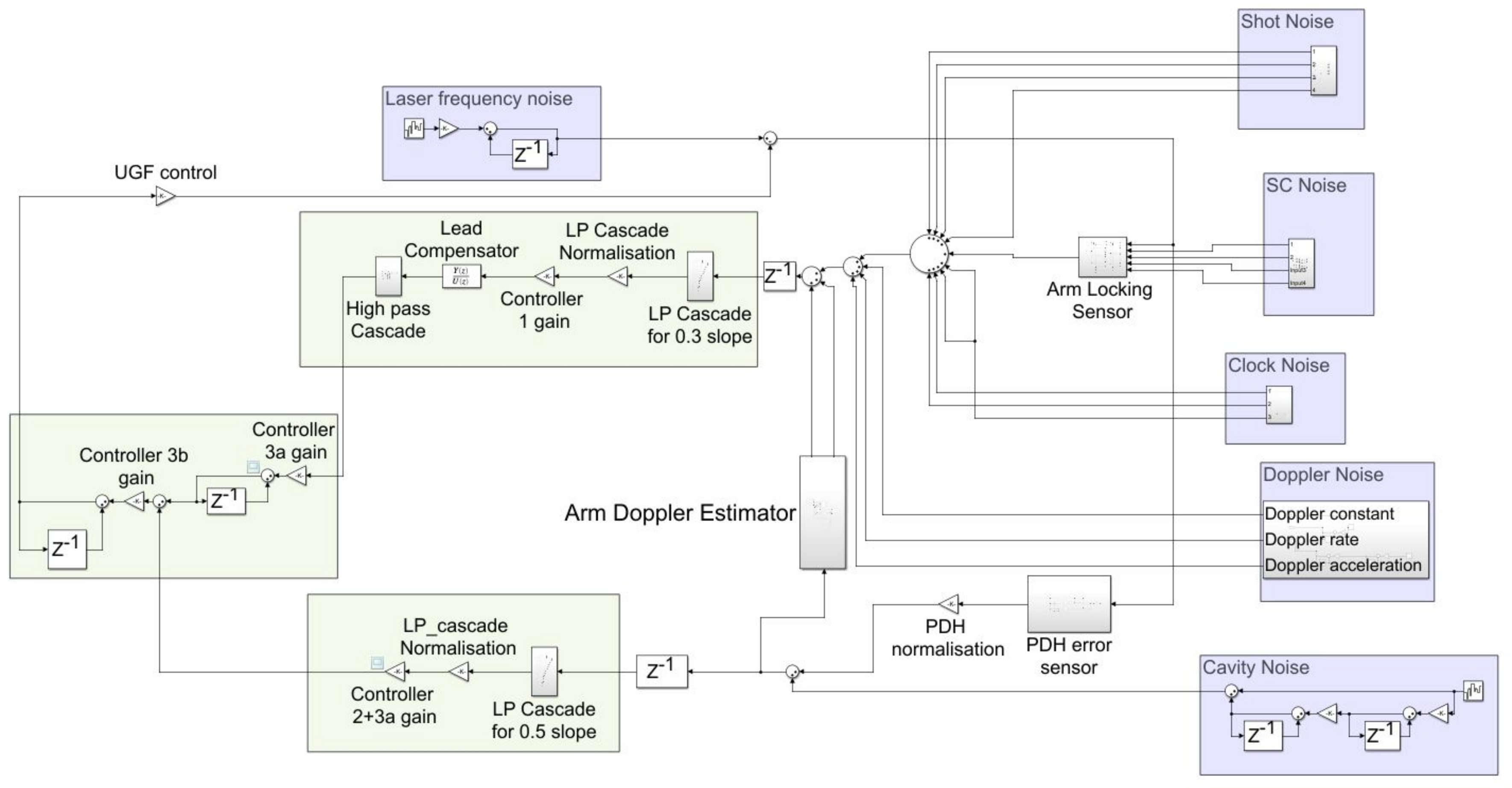




LISA Timeline (proposed for this combination)



Simulink



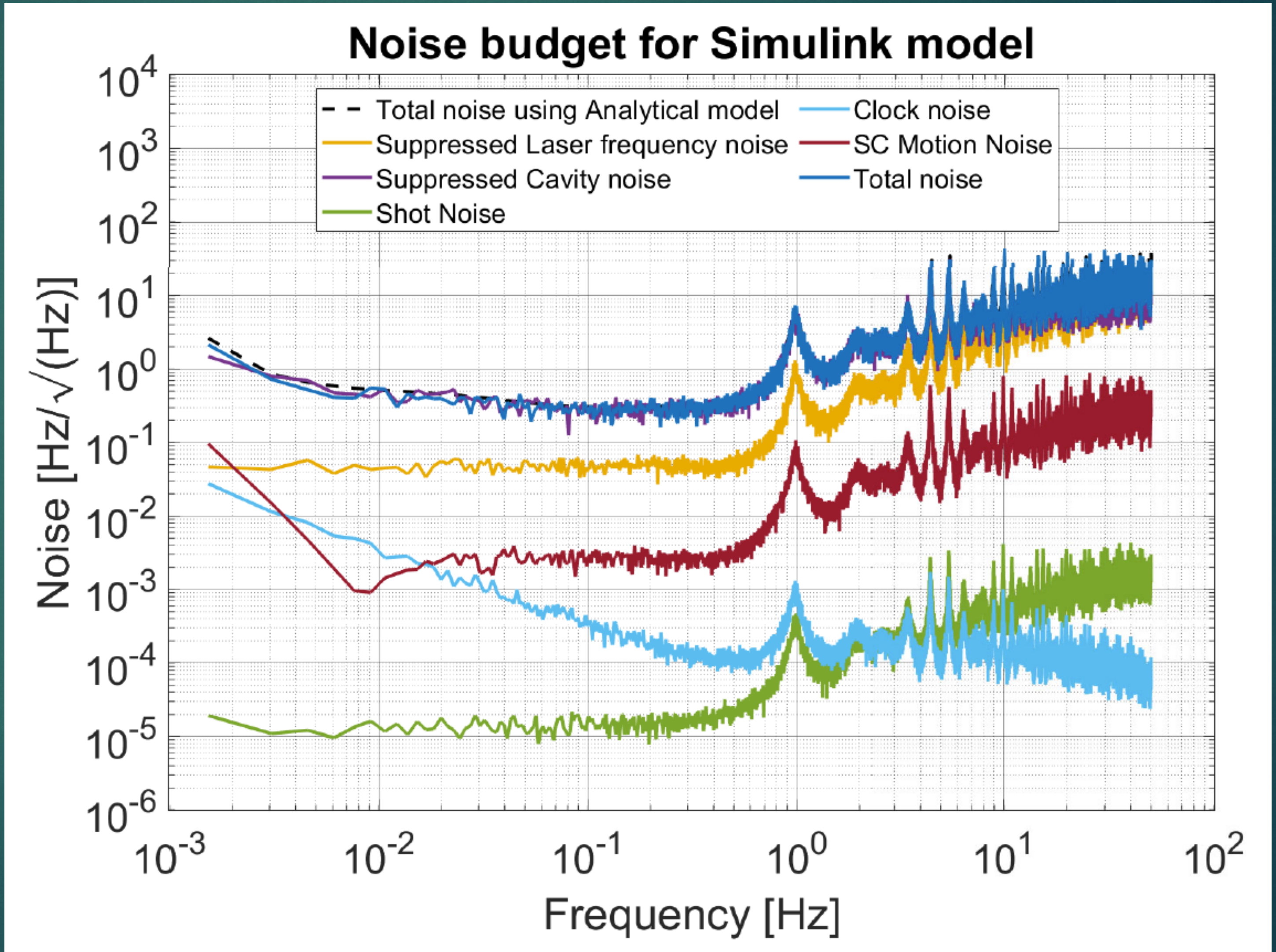
UGF is at 500 Hz, with the sampling frequency at 10 kHz.

Round trip time is 1s, with time difference of 0.1s.

A delay in the loop gives around 18 additional phase. (360*f/fs)

Noise budget using Simulink

Simulink results were verified with the analytical solutions



Future Work

- Explore complex sensors that could potentially
  - Reduce Doppler pulling
  - Improve noise budget.
- Experimental verification for the same

Thank you