Cosmic Explorer Suspension design

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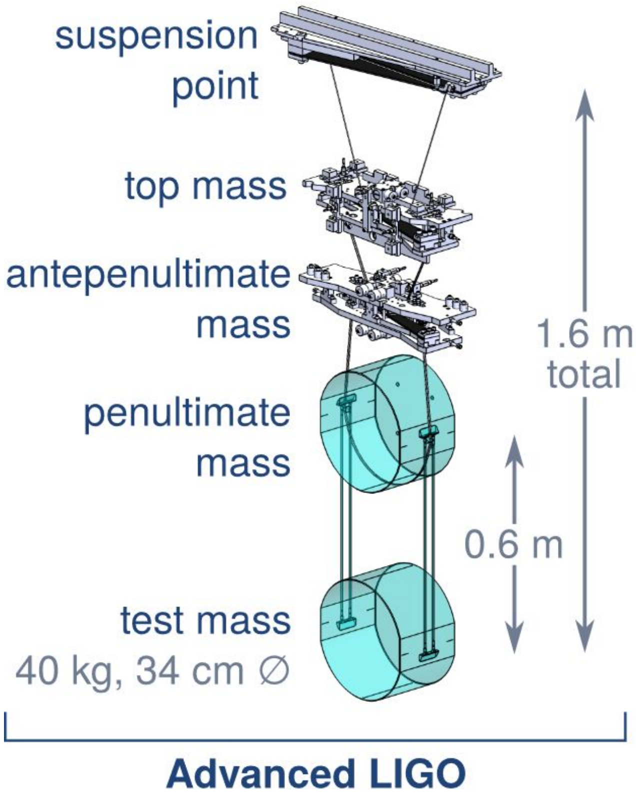
DCC: G2101051
CE DCC: G2100021

Background

aLIGO → Room temperature
1μm laser
Fused silica test mass+ fibers
40 kg test mass
~10^{-22} Hz$^{-½}$ @10Hz

CE1/CE2(1) → Room temperature
1μm laser
Fused silica TM + fibers
320 kg TM
~10^{-24} Hz$^{-½}$ @10Hz

CE2(2) → Cryogenic
2μm laser
Silicon TM + ribbons
320kg TM
~10^{-24} Hz$^{-½}$ @10Hz



???

Cosmic Explorer

From [1]

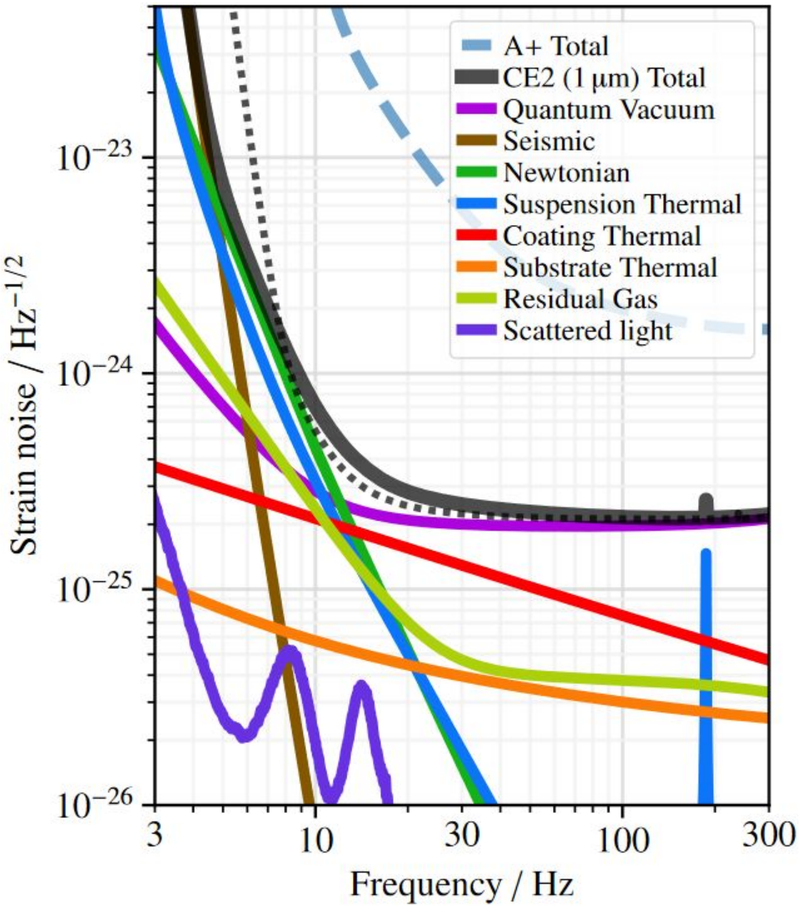
<u>Goal</u>: come up with a "realistic" CE suspension design

Outline

1. Requirements / What do we want
2. Silicon
3. Design approach
4. A few concepts
5. Conclusion

Requirements - CE2(1)

Courtesy of K. Kuns



Assume x100 better seismic isolation compare to aLIGO at 1Hz

Requirements:

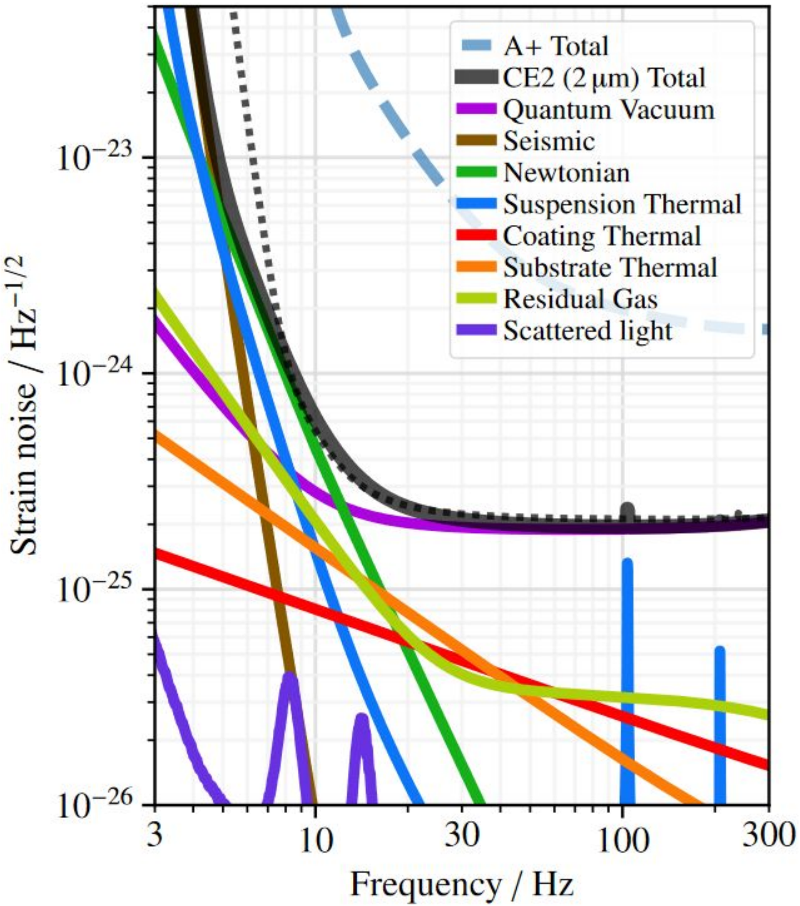
- Dia. 700mm, Mass: 320kg
- Suspension modes as low as possible. Goal: max ~3Hz

We want:

- Violin mode frequency ∝ stress in the fibers: stress as high as possible

Requirements - CE2(2)

Courtesy of K. Kuns



Assume x100 better seismic isolation compare to aLIGO at 1Hz

Requirements:

- Dia. 800mm, Mass: 320kg
- Suspension modes as low as possible. Goal: max ~3Hz

We want:

- Violin mode frequency ∝ stress in the ribbons: stress as high as possible
- Lowest spring constant in the laser beam direction to limit thermal noise: ~10:1 ratio between thickness and width of the ribbons

Silicon

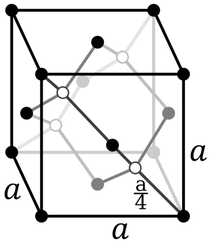
Silicon is a brittle, cubic crystalline solid.

Crystalline structure

1. Mechanical properties depend on crystal orientation → anisotropic (see next slide)

Diamond cubic crystal structure

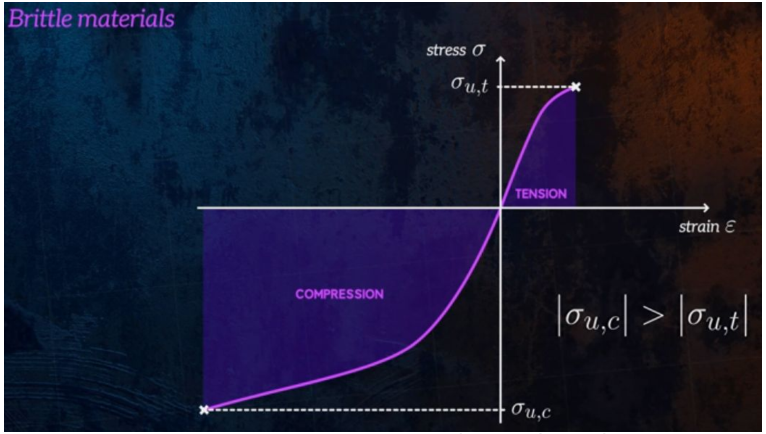
1. Uniform thermal expansion in all directions
2. Uniform change in elasticity in all directions [2]



Unit cell of silicon

Brittle material

1. Failure is considered to occur at fracture rather than yielding
2. Compressive strength larger than tensile strength



https://www.youtube.com/watch?v=xkbQnBAOFEg

Silicon - Young's modulus and Poisson's Ratio [3]

- Anisotropic material
- E varies between **130 and 188 GPa**
- E change due to thermal: ~-60ppm/°C [4-5]

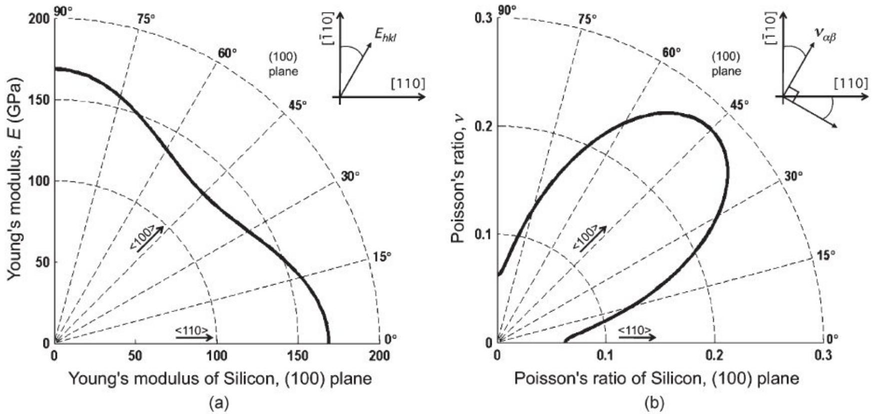
Thermal negligible for now

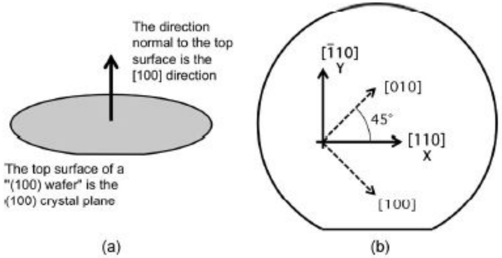
Assume isotropic behavior for now

E = 155 GPa

v = 0.27

(GWINC values)

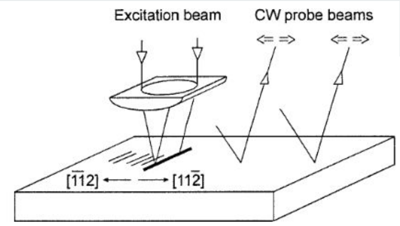




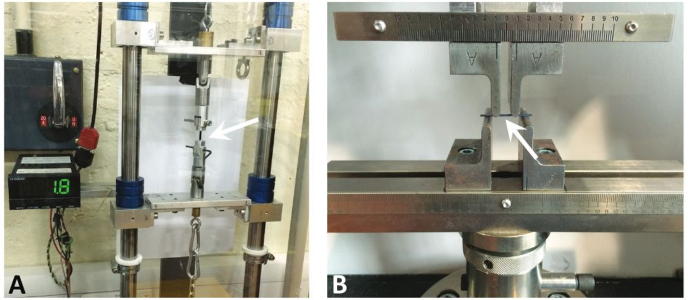
From [3]

Silicon - Tensile strength

- Calculated tensile strength: **22-47.3 GPa** [2,6-7]
- Measured tensile strength (SAW method):
  - **5-7 GPa** [8]
  - **0.8 GPa** [9]
  - **1.6 GPa** [10]
  - **>2.5 GPa** [11]
- Measured tensile strength (pull/point bending test):
  - **~200 MPa** (up to 450 MPa) [12]
  - **~350 MPa** (up to 700 MPa) [13]
  - **370 MPa** (up to 590 MPa) [14]
- **Fracture stress decreases with increasing cross section of the silicon sample.**
- Coated silicon? [15]



From [10]



From [14]

We chose: σ_{max} = 400 MPa (too optimistic?)

(current Voyager design [16]: σ_{max}=100 MPa)

Fused silica

Glass in amorphous (non-crystalline) form.

brittle

isotropic

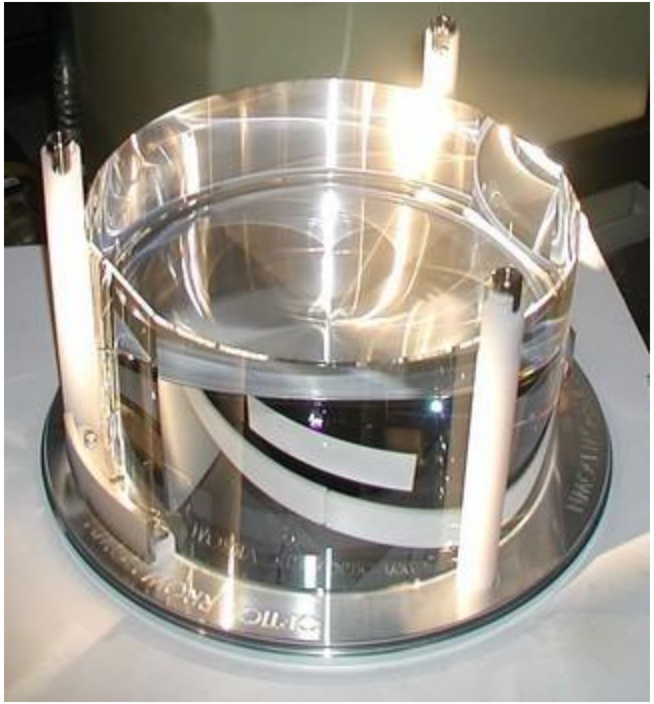
E = 72.7 GPa

(GWINC values)

ν = 0.167

σ_{max} = 800 MPa for blades (see next slide)

σ_{max} = 1.2 GPa for fibers [17]



https://www.ligo.caltech.edu/image/ligo20101010a

More conservative numbers for CE1/CE2a

Suspension parameters

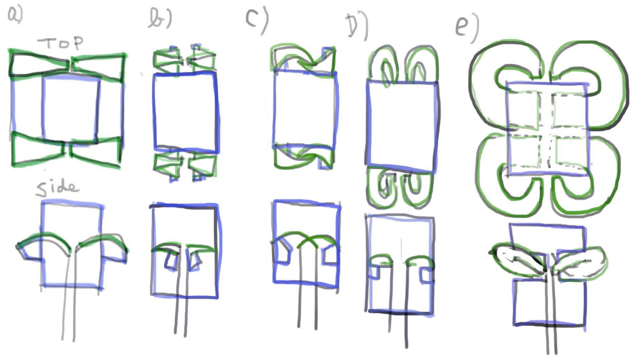
Max suspension mode ~3Hz:

- "Scale up" the upper stages (suspension point, top mass and antepenultimate mass), focus on lower stages
- Fiber/ribbon length: 2m
- **Blade springs between the penultimate mass and the test mass**

Component		Material	Dimensions (mm)	Comments
Test mass	CE1/CE2(1)	Fused Silica	700 dia. x 400	~320kg
	CE2(2)	Silicon	800 dia. x 300	
Fibers/Ribbons	CE1/CE2(1)	Fused Silica	1 dia. (body) x 2000	σ_{max} = 1.2 GPa
	CE2(2)	Silicon	2000 x 5 x 0.5	σ_{max} = 400 MPa 10:1 ratio between W and H

Blade model

- Find realistic dimensions for the blades
- Blade = simple cantilever beam with end mass
- 2 DOFs model (PUM+TM)
- Unknowns: Length (L), width (W) and thickness (H)
- H as small as possible



Courtesy of K. Arai

Requirement #1: $\sigma \leq \sigma_{max}$

Maximum stress in cantilever beam:

$$\sigma = \frac{FLH}{2I} = \frac{6FL}{WH^2} \qquad \Rightarrow H \geq \sqrt{\frac{6FL}{W\sigma_{max}}}$$

Requirement #2: $\omega \leq \omega_{bounce}$

First resonance frequency (blade weight negligible):

$$\omega^2 = \frac{k_2 m_1 + k_1 m_2 + k_2 m_2 + \sqrt{\left(k_2 m_1 + k_1 m_2 + k_2 m_2\right)^2 - 4 m_1 m_2 k_1 k_2}}{2 m_1 m_2}$$

$$\Rightarrow \quad H \leq L \sqrt[3]{\frac{2\beta\left(\beta - 2k_1 m_2\right)}{EW\left[\beta\left(m_1 + m_2 - 2m_1 m_2 k_1\right)\right]}}$$

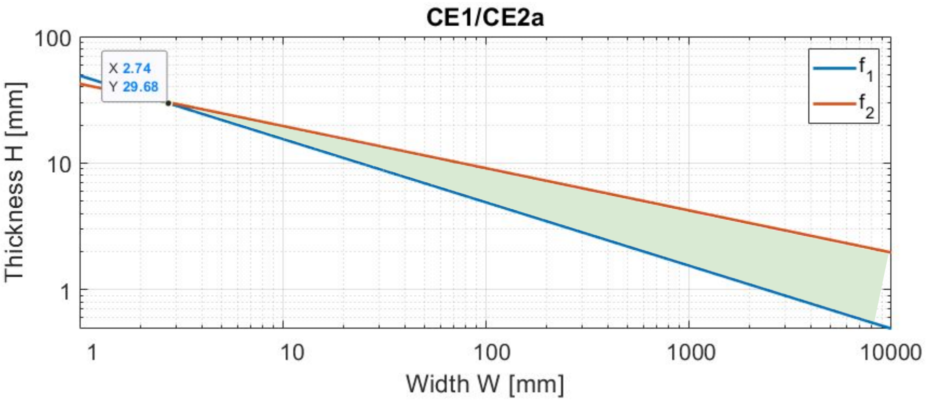
$$\text{with } \beta = 2 m_1 m_2 \omega^2$$

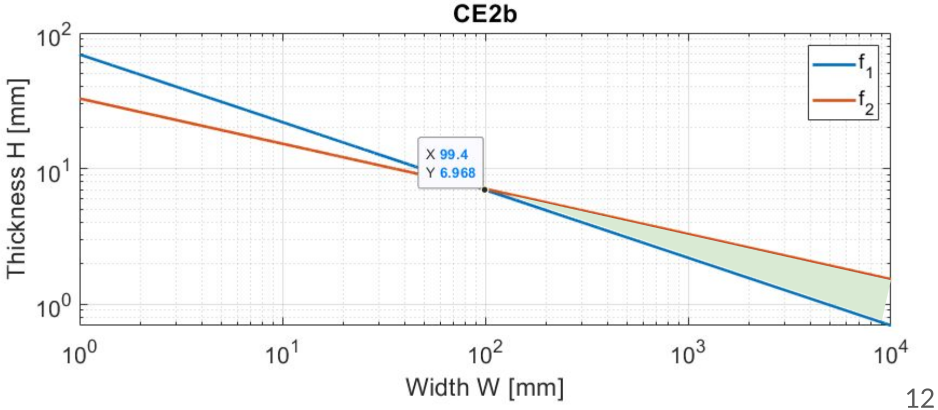
Blade model

- Two equations, three unknowns
- TM diameter up to 800mm. **Fix blade length at 410mm**

$$f_1(W) = \sqrt{\frac{6FL}{W\sigma_{max}}} \qquad f_2(W) = L\sqrt[3]{\frac{2\beta(\beta - 2k_1 m_2)}{EW[\beta(m_1 + m_2 - 2m_1 m_2 k_1)]}}$$

	CE1/CE2(1)	CE2(2)
Young's Modulus	72.7 GPa	155 GPa
TM weight	320 kg	
Bounce frequency	3Hz	
Maximum stress	800 MPa	400 MPa
PUM weight	400 kg	
Top blade stiffness (maraging steel)	4300N/m	
Length	**410 mm**	
Width	**135 mm**	
Thickness	**5.50 mm**	**7.25 mm**





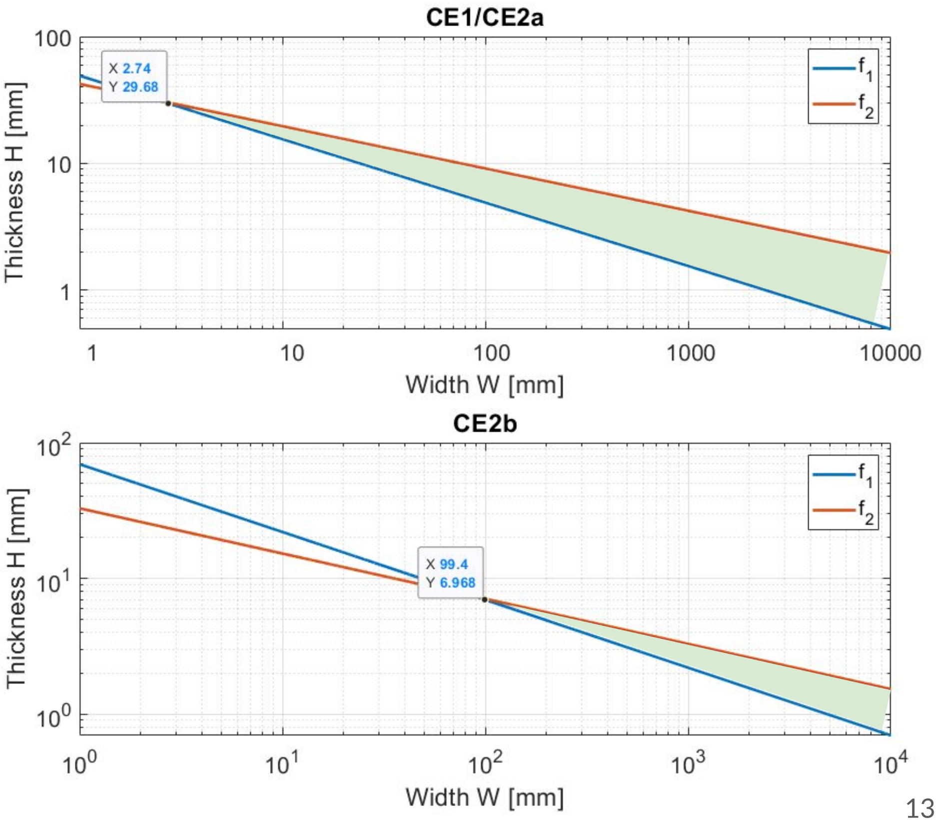
Blade model

- Two equations, three unknowns
- TM diameter up to 800mm. **Fix blade length at 410mm**

$$f_1(W) = \sqrt{\frac{6FL}{W\sigma_{max}}} \qquad f_2(W) = L\sqrt[3]{\frac{2\beta(\beta - 2k_1 m_2)}{EW\left[\beta\left(m_1 + m_2 - 2m_1 m_2 k_1\right)\right]}}$$

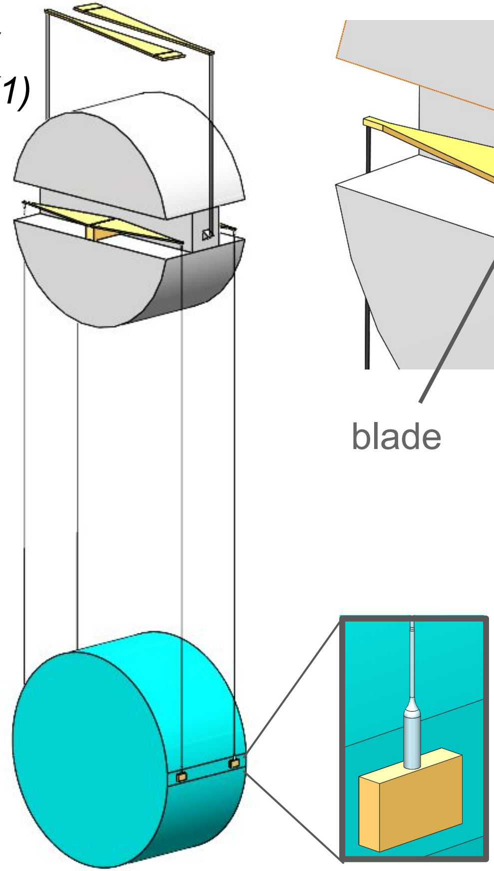
	CE1/CE2(1)	CE2(2)
Young's Modulus	72.7 GPa	155 GPa
TM weight	320 kg	
Bounce frequency	3Hz	
Length	410 mm	
Width	135 mm	
Thickness	5.50 mm	7.25 mm

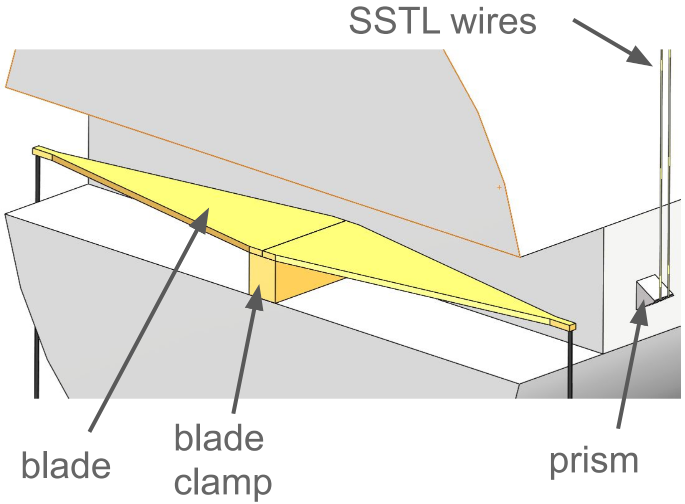
- Stringent dimension requirement with CE2(2)
- More freedom with CE1/CE2(1). Chose dimensions that seem reasonable (can be improved)



Concept #1

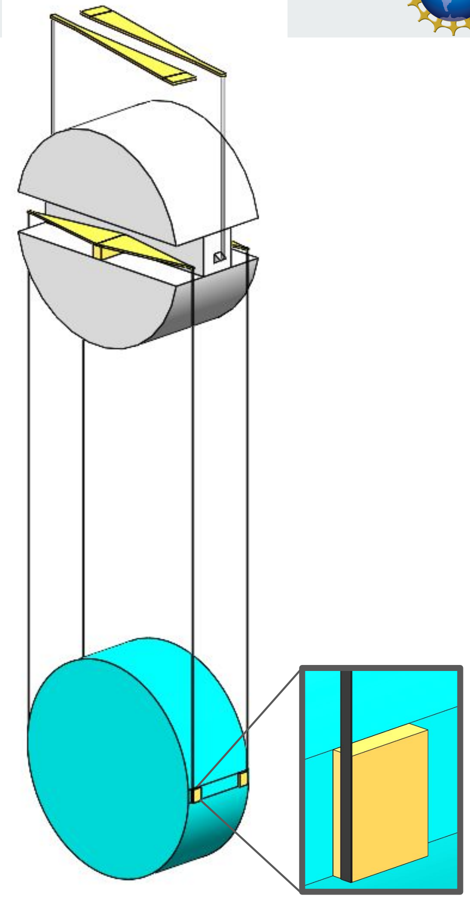
CE1/
CE2(1)



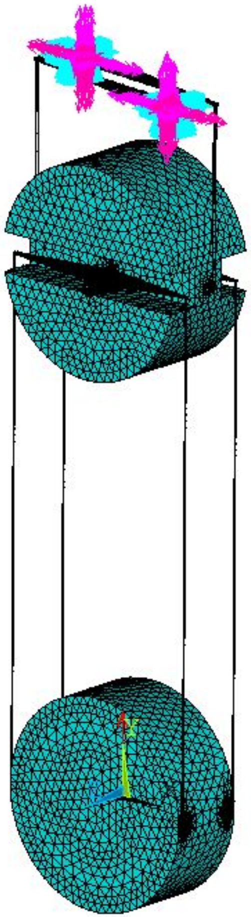


- CAD done in SolidWorks
- Blade width: 135 → 10 mm at the tip
- Assuming lossless bonds
- Flat clamps for now
- SSTL wires attached to prism

CE2(2)



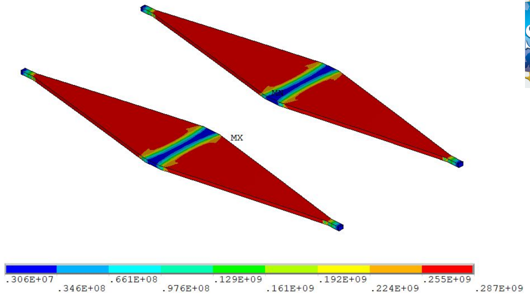
Concept #1



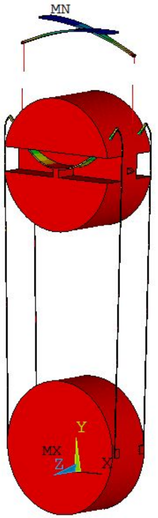
- Parasolid imported in ANSYS APDL
- Loads: SSTL wires clamped at the top, gravity

	CE1/CE2(1)	CE2(2)
Max stress in blade	687 MPa	287 MPa
Bounce freq.	1.36 Hz	3.46 Hz
First violin mode freq.	204Hz	99Hz

- Rough agreement between theory and FEA
- Blade's dimension validated to start exploring different concepts



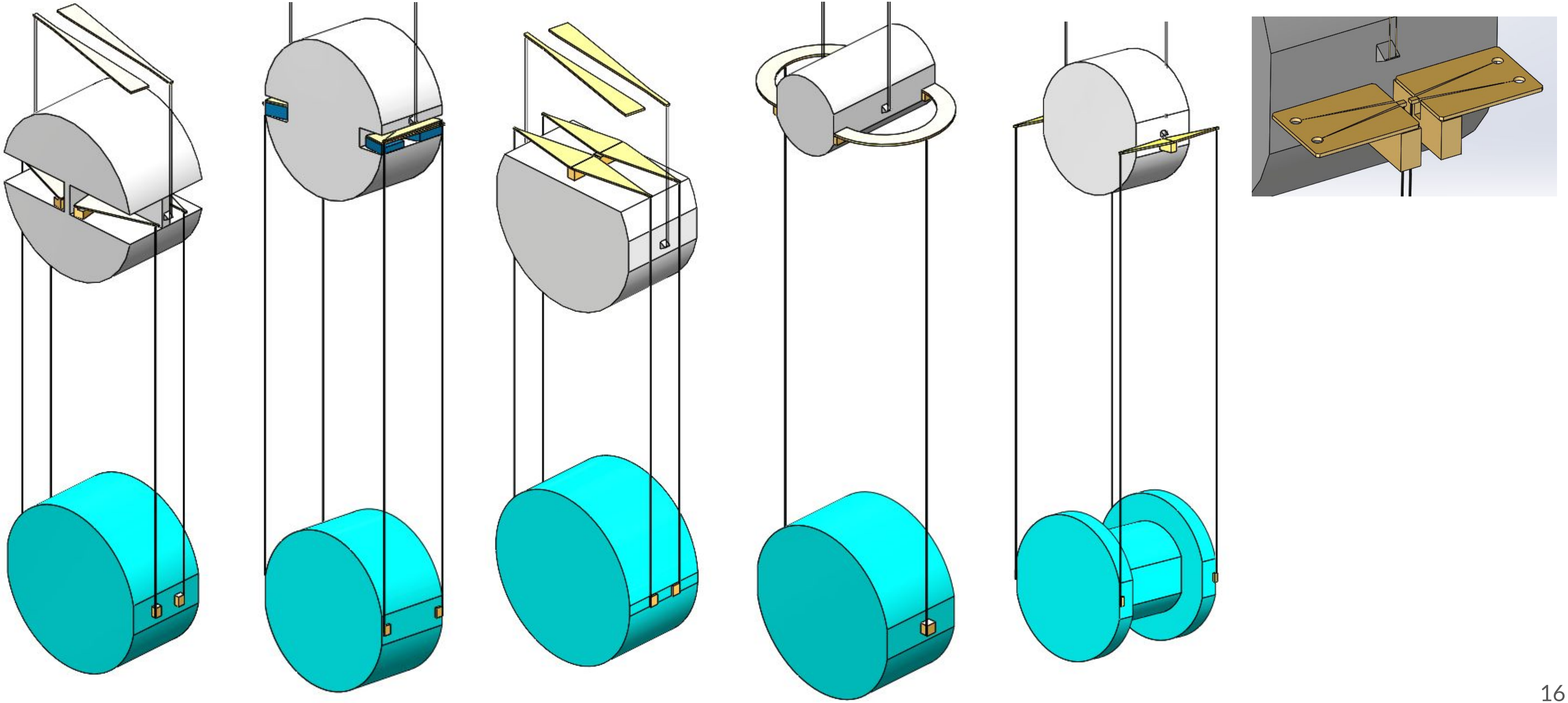
Von Mises stress



Displacement vector sum

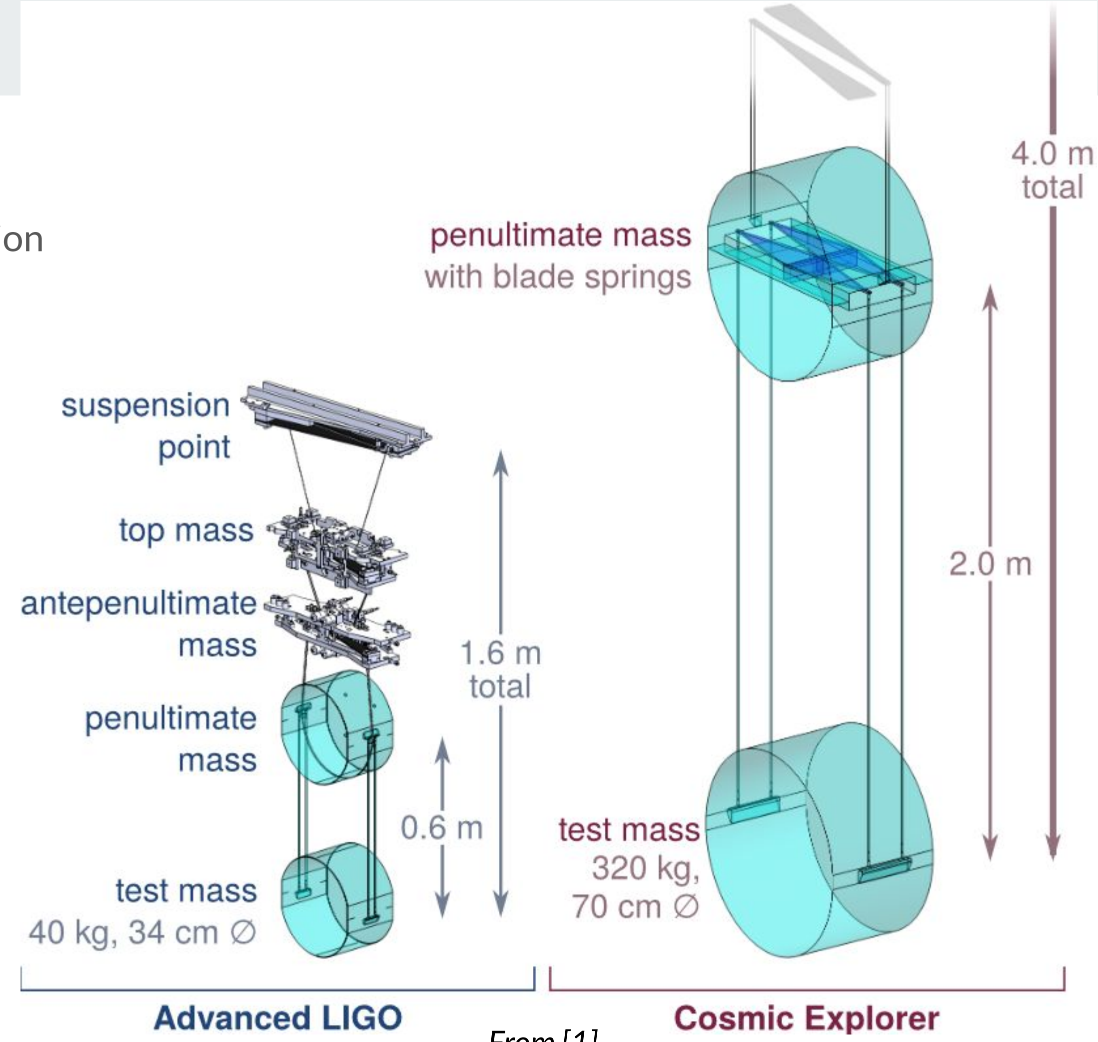
Concepts

- A lot of designs considered...



The one we like

- Two-parts PUM bonded together
- 4 spring blades in the middle section



From [1]

Conclusion

- Proposed design heavily inspired from the Advanced LIGO suspension
- This is just a concept! A lot of elements still need to be studied (upper stages, bonds, blade clamps, etc.)
- Using silicon put stringent requirements on the suspension design, but might be doable
- More ideas still to be explored (i.e. Euler springs [18] with MIT/UCLouvain)

Thank you

References

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[2] J. F. Nye, *Physical Properties of Crystals: Their Representation by Tensors and Matrices*. Oxford, U.K.: Oxford Univ. Press, 1985

[3] M. A. Hopcroft, W. D. Nix and T. W. Kenny, "What is the Young's Modulus of Silicon?," in Journal of Microelectromechanical Systems, vol. 19, no. 2, pp. 229-238, April 2010.

[4] Bourgeois, C., et al. "Design of resonators for the determination of the temperature coefficients of elastic constants of monocrystalline silicon." *Proceedings of International Frequency Control Symposium*. IEEE, 1997.

[5] Lyon, K. G., et al. "Linear thermal expansion measurements on silicon from 6 to 340 K." *Journal of Applied Physics* 48.3 (1977): 865-868.

[6] Roundy, David, and Marvin L. Cohen. "Ideal strength of diamond, Si, and Ge." *Physical Review B* 64.21 (2001): 212103.

[7] Pokluda, J., et al. "Calculations of theoretical strength: State of the art and history." *Journal of computer-aided materials design* 11.1 (2004): 1-28.

[8] Kozhushko, V. V., A. M. Lomonosov, and P. Hess. "Intrinsic strength of silicon crystals in pure-and combined-mode fracture without precrack." *Physical review letters* 98.19 (2007): 195505.

[9] Lomonosov, A. M., and P. Hess. "Impulsive fracture of silicon by elastic surface pulses with shocks." *Physical review letters* 89.9 (2002): 095501.

[10] Lehmann, G., et al. "Impulsive fracture of fused quartz and silicon crystals by nonlinear surface acoustic waves." *Journal of applied physics* 94.5 (2003): 2907-2914.

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[11] Lomonosov, A. M., et al. "Laser-generated nonlinear surface wave pulses in silicon crystals." *Physical Review B* 69.3 (2004): 035314.

[12] Cumming, A. V., et al. "Silicon mirror suspensions for gravitational wave detectors." *Classical and Quantum Gravity* 31.2 (2013): 025017.

[13] Birney, Ross, et al. "Coatings and surface treatments for enhanced performance suspensions for future gravitational wave detectors." *Classical and Quantum Gravity* 34.23 (2017): 235012.

[14] Hu, S. M. "Critical stress in silicon brittle fracture, and effect of ion implantation and other surface treatments." *Journal of Applied Physics* 53.5 (1982): 3576-3580.

[15] Javvaji, Brahmanandam, et al. "Fracture Properties of Graphene‐Coated Silicon for Photovoltaics." *Advanced Theory and Simulations* 1.12 (2018): 1800097.

[16] Adhikari, Rana X., et al. "A cryogenic silicon interferometer for gravitational-wave detection." *Classical and Quantum Gravity* 37.16 (2020): 165003.

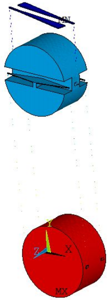
[17] Lee, Kyung-Ha, et al. "Improved fused silica fibres for the advanced LIGO monolithic suspensions." *Classical and Quantum Gravity* 36.18 (2019): 185018.

[18] Winterflood, John, Terran Barber, and D. G. Blair. "Using Euler buckling springs for vibration isolation." *Classical and Quantum Gravity* 19.7 (2002): 1639.

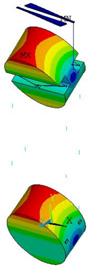
Extra-slides

CE1/CE2a

0.31 Hz

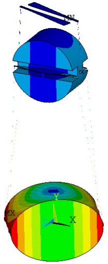


0.32 Hz

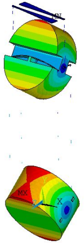


0.38 Hz

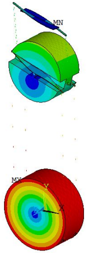
0.53 Hz



0.61 Hz

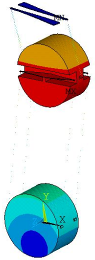


0.70 Hz

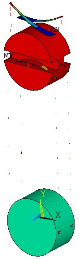


0.86 Hz

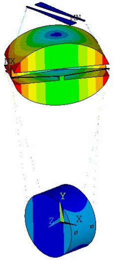
0.98 Hz



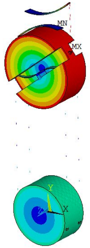
0.99 Hz



1.36 Hz

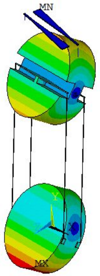


1.84 Hz

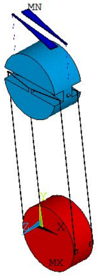


1.91 Hz

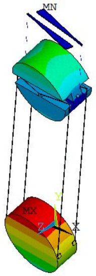
CE2b



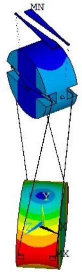
0.28 Hz



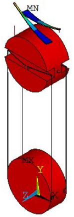
0.33 Hz



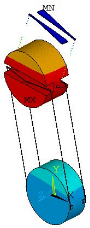
0.34 Hz



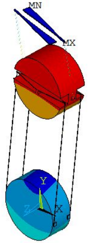
0.64 Hz



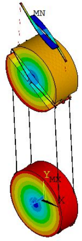
0.73 Hz



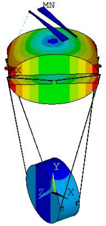
0.90 Hz



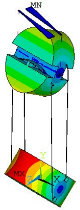
0.90 Hz



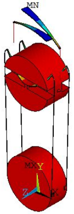
1.13 Hz



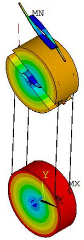
1.69 Hz



2.04 Hz



3.46 Hz



4.89 Hz

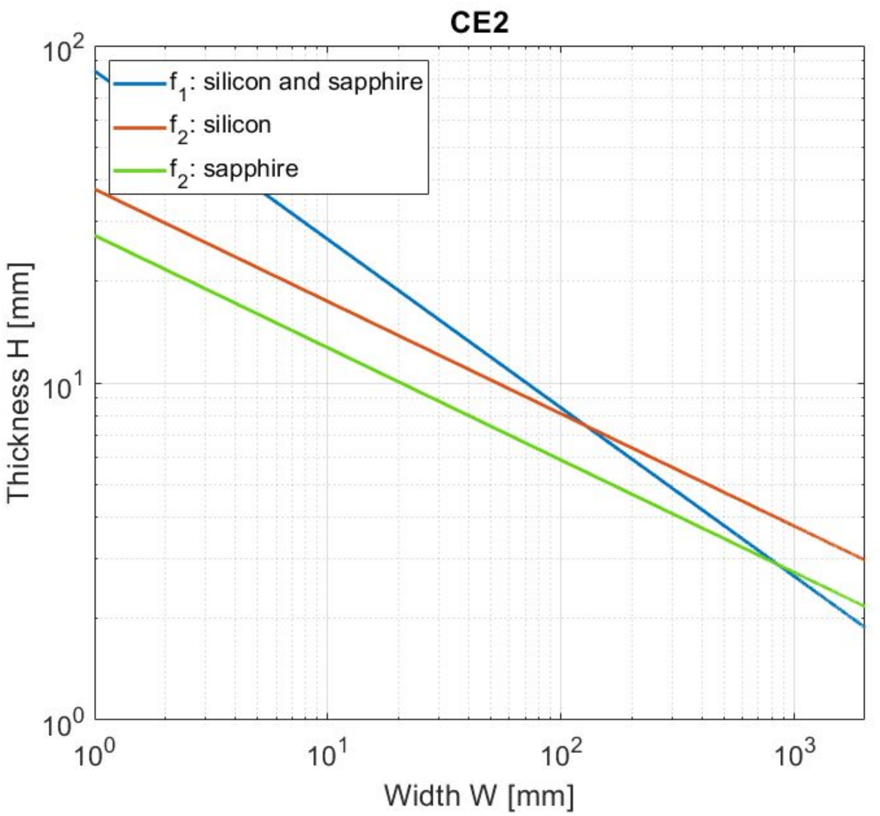
Sapphire?

$$f_1(W) = \sqrt{\frac{6FL}{W\sigma_{max}}}$$

$$f_2(W) = L\sqrt[3]{\frac{2\beta(\beta - 2k_1m_2)}{EW[\beta(m_1 + m_2 - 2m_1m_2k_1)]}}$$

Young Modulus	400 GPa
Tensile Strength	400 MPa

Worse than silicon!

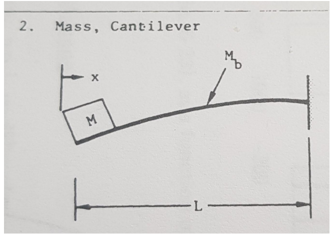


Maximum stress equation

$$I = \frac{WH^3}{12}$$

$$k = \frac{3EI}{L^3} = \frac{EWH^3}{4L^3}$$

$$\sigma = \frac{FLH}{2I} = \frac{6FL}{WH^2}$$



2DOF eigenvalues

$$-k_1x_1 + k_2(x_2 - x_1) = m_1\ddot{x_1}$$

$$-k_2(x_2 - x_1) = m_2\ddot{x_2}$$

$$-m_1\omega^2x_1 + x_1(k_1 + k_2) - k_2x_2 = 0$$

$$-m_2\omega^2x_2 + k_2x_2 - k_2x_1 = 0$$

$$\begin{bmatrix} -m_1\omega^2 + k_1 + k_2 & -k_2 \\ -k_2 & -m_2\omega^2 + k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Find eigenvalues:

$$det\begin{bmatrix} -m_1\omega^2 + k_1 + k_2 & -k_2 \\ -k_2 & -m_2\omega^2 + k_2 \end{bmatrix} = 0$$

$$(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2) - {k_2}^2 = 0$$

$$m_1m_2\omega^4 - m_1k_2\omega^2 - k_1m_2\omega^2 + k_1k_2 - k_2m_2\omega^2 = 0$$

$$m_1m_2\omega^4 + \omega^2(-k_2m_1 - k_1m_2 - k_2m_2) + k_1k_2 = 0$$

Let's define **x = ω²**

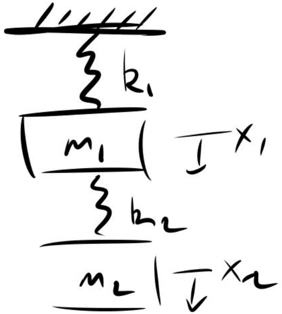
$$m_1m_2x^2 + x(-k_2m_1 - k_1m_2 - k_2m_2) + k_1k_2 = 0 \quad \text{(polynomial second order)}$$

$$\Delta = (-k_2m_1 - k_1m_2 - k_2m_2)^2 - 4m_1m_2k_1k_2$$

Roots:

$$x_1 = {\omega_1}^2 = \frac{k_2m_1 + k_1m_2 + k_2m_2 - \sqrt{(k_2m_1 + k_1m_2 + k_2m_2)^2 - 4m_1m_2k_1k_2}}{2m_1m_2}$$

$$x_2 = {\omega_2}^2 = \frac{k_2m_1 + k_1m_2 + k_2m_2 + \sqrt{(k_2m_1 + k_1m_2 + k_2m_2)^2 - 4m_1m_2k_1k_2}}{2m_1m_2}$$



Requirement

$\boldsymbol{\omega_2 \leq 2\pi f_b}$

Let's define $\alpha = k_2m_1 + k_1m_2 + k_2m_2$ and $\beta = 2m_1m_2(2\pi f_b)^2$:

$${\omega_2}^2 = \frac{\alpha + \sqrt{\alpha^2 - 4m_1m_2k_1k_2}}{2m_1m_2} \leq (2\pi f_b)^2$$

$$\sqrt{\alpha^2 - 4m_1m_2k_1k_2} \leq \beta - \alpha$$

$$\alpha^2 - 4m_1m_2k_1k_2 \leq [\beta - \alpha]^2 = \beta^2 + \alpha^2 - 2\alpha\beta$$

$$-4m_1m_2k_1k_2 \leq \beta^2 - 2\alpha\beta$$

$$-4m_1m_2k_1k_2 + 2\alpha\beta \leq \beta^2$$

$$-4m_1m_2k_1k_2 + 2\beta(k_2m_1 + k_1m_2 + k_2m_2) \leq \beta^2$$

$$k_2[-4m_1m_2k_1 + 2\beta m_1 + 2\beta m_2] \leq \beta^2 - 2\beta k_1m_2$$

2DOF eigenvalues

We know that

$$k_2 = \frac{3EI}{L^3} = \frac{EWH^3}{4L^3}$$

$$\frac{EWH^3}{4L^3}[-4m_1m_2k_1 + 2\beta m_1 + 2\beta m_2] \leq \beta^2 - 2\beta k_1 m_2$$

$$H^3[-4m_1m_2k_1 + 2\beta m_1 + 2\beta m_2] \leq (\beta^2 - 2\beta k_1 m_2)\frac{4L^3}{EW}$$

$$H^3[-2m_1m_2k_1 + \beta m_1 + \beta m_2] \leq (\beta^2 - 2\beta k_1 m_2)\frac{2L^3}{EW}$$

If $\beta(m_1 + m_2) - 2m_1m_2k_1 \geq 0$:

$$H \leq L\sqrt[3]{\frac{2\beta(\beta - 2k_1m_2)}{EW[\beta(m_1 + m_2) - 2m_1m_2k_1]}}$$

If $\beta(m_1 + m_2) - 2m_1m_2k_1 \leq 0$:

$$H \geq L\sqrt[3]{\frac{2\beta(\beta - 2k_1m_2)}{EW[\beta(m_1 + m_2) - 2m_1m_2k_1]}}$$