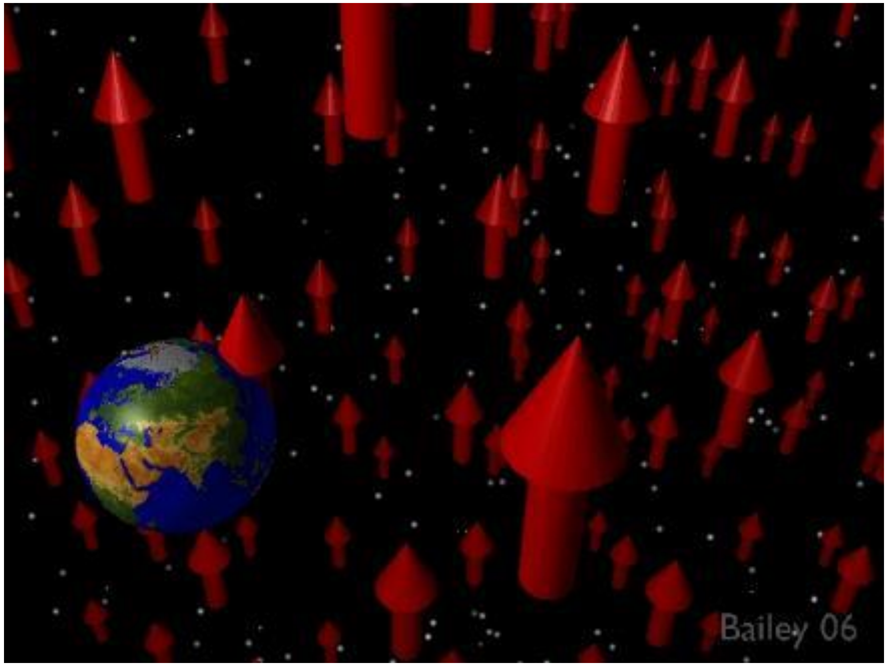
Testing Spacetime Symmetries with Gravitation



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Some work based on recent accepted paper: Ault, Bailey, Nilsson, PRD 2021

Second EPS conference on Gravitation

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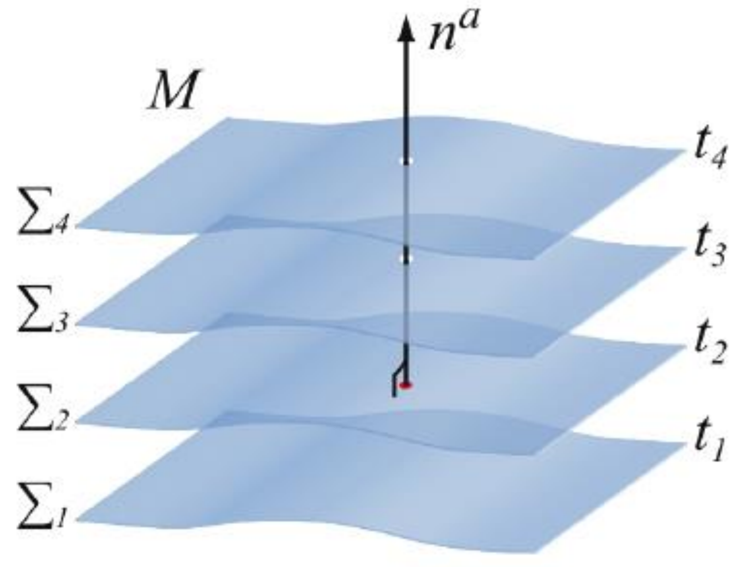
Outline

1) Spacetime and symmetries
2) SME gravity sector, 3 approaches
3) Recent tests
4) 3+1 formulation of mSME, features
5) Match to test models
6) Summary



Background on spacetime and symmetries

- Geometrical framework of General Relativity
  - pseudo-Riemann spacetime (no Torsion)
  - Metric $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- Symmetries:
  - **Local Lorentz symmetry** (~rotations, 6)

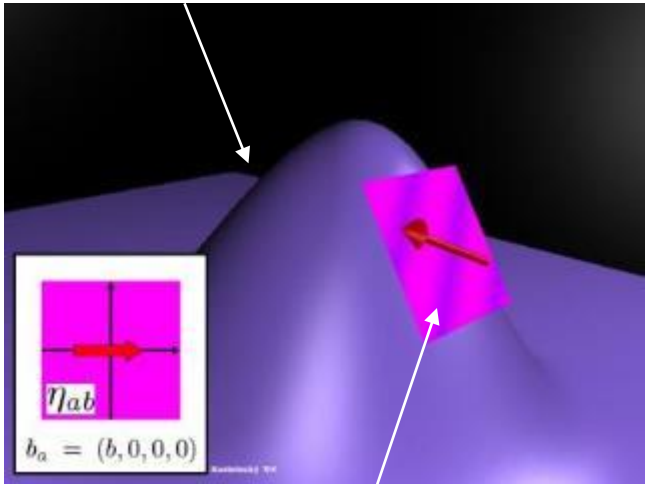
$$B'^a = \Lambda^a{}_b B^b$$

  - **Diffeomorphism** symmetry (P->Q, ~translations, 4)

$$B^\mu \longrightarrow B^\mu + (\partial_\nu \xi^\mu) B^\nu - \xi^\nu \partial_\nu B^\mu$$

(Also discrete C, P, T)

Curved spacetime manifold



Local Lorentz frame

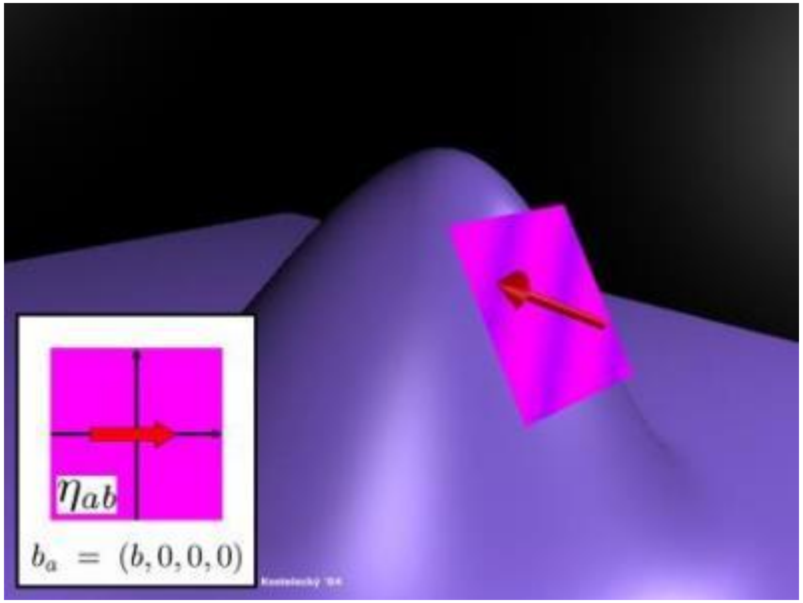
- Conservation laws associated with these symmetries

$$T^{\mu\nu} = T^{\nu\mu}$$

$$D_\mu T^{\mu\nu} = 0$$

Symmetry breaking

- Local Lorentz frame $b_a = (b, 0, 0, 0)$
- "Manifold" $b_\mu = e_\mu{}^a b_a$



Result 1: Lorentz breaking ↔ diffeomorphism breaking*

⬇

Coefficients control Lorentz **and** diffeomorphism breaking

(AK, Samuel, PRD 89, AK PRD 04, *AK and Li 2020)

Symmetry breaking

- **Explicit** Lorentz breaking – prescribed, nondynamical coefficients

$$b_\mu,\ c_{\mu\nu},\ g_{\lambda\mu\nu},\ \ldots,\ (k_F)_{\kappa\lambda\mu\nu},\ \ldots$$

Produces modified conservation laws

$$D_\mu T_e{}^\mu{}_\nu - J^x D_\nu k_x = 0$$

$$T_e{}^{\mu\nu} - T_e{}^{\nu\mu} = e^{\mu a} e^{\nu b} k_x (X_{[ab]})^x{}_y J^y$$

$$D_\mu G^{\mu\nu} = 0$$

➡ **Required for consistency with** Bianchi identities

Result 2: Explicit Lorentz/diffeo breaking is generically incompatible with **Riemann** geometry*

Result 3: Spontaneous symmetry breaking maintains conservation laws

e.g., vector field $\langle B^\mu \rangle = b^\mu$

*Can accomplish with solutions in some models

(AK PRD 04, Bluhm & AK PRD 05, 08, Bluhm et al PRD 15, Symmetry 16)

Standard-Model Extension gravity sector

- *Observer covariant* form through mass dimension 6

$$\mathcal{L} = \frac{\sqrt{-g}}{2\kappa}\Big[R + (k^{(4)})_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} + (k^{(5)})_{\alpha\beta\gamma\delta\kappa}D^{\kappa}R^{\alpha\beta\gamma\delta}$$
$$+\tfrac{1}{2}(k_1^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda}\{D^{\kappa},D^{\lambda}\}R^{\alpha\beta\gamma\delta} + (k_2^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}R^{\alpha\beta\gamma\delta}R^{\kappa\lambda\mu\nu}\Big] + \mathcal{L}'$$

- Quadratic action version (bulk of phenomenology work so far)

$$\mathcal{L}^{(4)} = \frac{1}{4\kappa}\bar{s}^{\mu\kappa}h^{\nu\lambda}\mathcal{G}_{\mu\nu\kappa\lambda}$$

$$\mathcal{L}^{(5)} = -\frac{1}{16\kappa}h_{\mu\nu}(q^{(5)})^{\mu\rho\alpha\nu\beta\sigma\gamma}\partial_{\beta}R_{\rho\alpha\sigma\gamma}$$

- Kost-Li extension

Component	Expression
$\mathcal{L}_{g0}$	$R - 2\Lambda$
$\mathcal{L}_g^{(2)}$	$\breve{k}^{(2)}$
$\mathcal{L}_g^{(3)}$	$(\breve{k}_{\Gamma}^{(3)})^{\mu}\Gamma^{\alpha}{}_{\mu\alpha}$
$\mathcal{L}_g^{(4)}$	$(\breve{k}_R^{(4)})^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$
$\mathcal{L}_g^{(5)}$	$(\breve{k}_D^{(5)})^{\alpha\beta\gamma\delta\kappa}D_{\kappa}R_{\alpha\beta\gamma\delta} + (\breve{k}_{\mathrm{CS},1}^{(5)})_{\kappa}\epsilon^{\kappa\lambda\mu\nu}\eta_{ac}\eta_{bd}(\omega_{\lambda}{}^{ab}\partial_{\mu}\omega_{\nu}{}^{cd} + \frac{2}{3}\omega_{\lambda}{}^{ab}\omega$ $+(\breve{k}_{\mathrm{CS},2}^{(5)})_{\kappa}\epsilon^{\kappa\lambda\mu\nu}\epsilon_{abcd}(\omega_{\lambda}{}^{ab}\partial_{\mu}\omega_{\nu}{}^{cd} + \frac{2}{3}\omega_{\lambda}{}^{ab}\omega_{\mu}{}^{ce}\omega_{\nu e}{}^{d})$

$$\breve{k}^{\mu} = k^{\mu} + k_{\nu}g^{\mu\nu} + k^{a}e^{\mu}{}_{a} + \ldots .$$

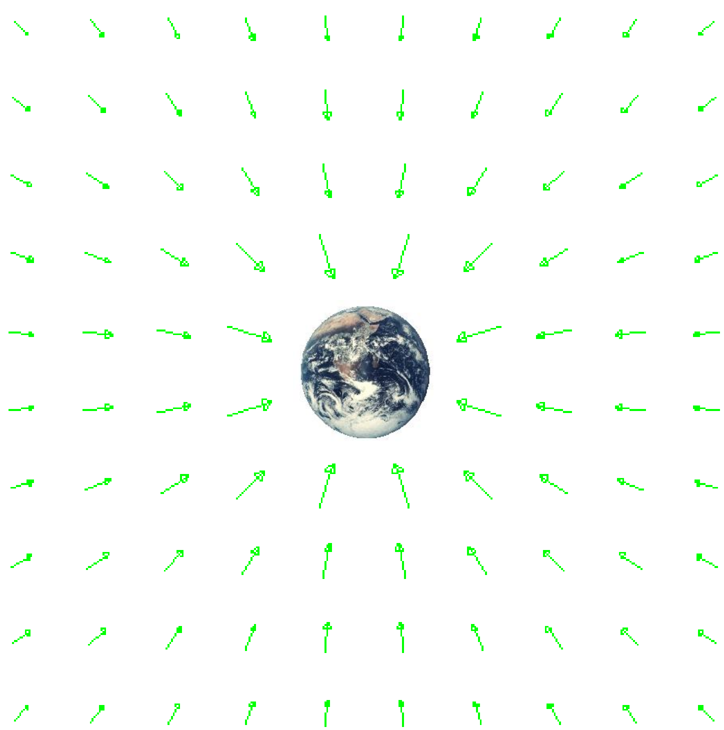
AK PRD 04, Bailey, AK, Xu PRD 15, weak field (Bailey, Mewes, Tasson, Xu +, various pubs 06-20), AK and Li 2020

Example: modifications to Newton's law of gravity

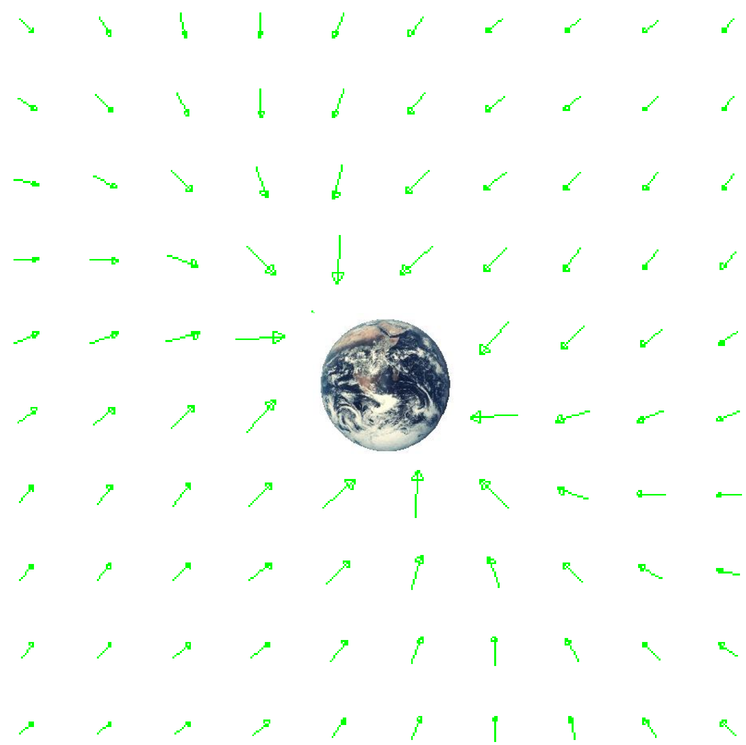
$$\vec{F} = \frac{-GMm}{r^2}\left[\hat{r} - \overline{s}\cdot\hat{r} + \frac{3}{2}(\hat{r}\cdot\overline{s}\cdot\hat{r})\hat{r}\right]$$

Newtonian term

SME terms; 3x3 subblock of $\bar{s}^{\alpha\beta}$



GR



Exaggerated effect from $\bar{s}^{\alpha\beta}$

Gravity experimental/observational analysis with SME

- Atom interferometry/gravimeter tests (Müller et al PRL 08, Chung et al PRD 09, C.G. Shao et al PRD 18)
- gyroscope experiment (Bailey et al PRD 13)
- lunar laser ranging (Battat et al PRL 07, Bourgoin et al PRL 16, 17,...)
- pulsars (Shao PRL 14, PRD 14, Shao & Bailey PRD 18, 19)
- redshift/clock tests (Hohensee etal PRL 11, PRL 13,...)
- short-range gravity tests (Long PRD 15, Shao et al PRD 15, PRL 16, PRL 18)
- space-based WEP tests (Pihan-LeBars et al, PRL 2019)
- solar system ephemeris (Iorio CQG 12, Hees et al PRD 15)
- time delay/light-bending tests (SYRTE, Le Poncin-Lafitte et al PRD 16)
- torsion-pendulum tests (Schlamminger et al PRL 08)
- cosmic rays (Kostelecky and Tasson, PLB 15)
- gravitational waves (Kostelecky & Mewes, PLB 16, LIGO ApJL 17, Shao 2020)
- ring laser gyros (Tasson PRD 2019)
- cosmology (Bonder PRD 2017, Ault et al 2019), …

Example of overlap of different tests: "speed of gravity"

- Isotropic (simplified) version of the SME **gravity** sector:

$$\omega = \left(1 - \mathring{k}^{(4)}_{(I)}\right)|\boldsymbol{p}| \pm \mathring{k}^{(5)}_{(V)}\omega^2 - \mathring{k}^{(6)}_{(I)}\omega^3 \pm \mathring{k}^{(7)}_{(V)}\omega^4 - \mathring{k}^{(8)}_{(I)}\omega^5 \pm \ldots$$

- "Speed of gravity" is controlled by $\bar{s}^{(4)}_{00} = -\sqrt{16\pi}\ \mathring{k}^{(4)}_{(I)}$
- Independent constraints exist from other tests:

VLBI ~ 10^{-5}

GPB ~ 10^{-3}

Pulsars ~ 10^{-5}

Cosmic Rays (one sided) ~ 10^{-14}

->limit on speed of gravity from GW150914+ is poor compared to other tests!

GW170817 is different story… $\bar{s}^{(4)}_{00}$ less than $(-20 \text{ to } 5) \times 10^{-15}$

Le Poncin-Lafitte et al, PRD 16, Bailey et al PRD 13, Shao PRD 15, KT PLB 15, KM PLB 16, Abbot et al, Astro J L 17

State of the art for "sbar"

- Maximal sensitivity (which is ? FYI: **not a bound!**)

Coefficient	Sensitivity
$\bar{s}^{XY}$	10^{-11}
$\bar{s}^{XZ}$	10^{-11}
$\bar{s}^{YZ}$	10^{-11}
$\bar{s}^{XX} - \bar{s}^{YY}$	10^{-10}
$\bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}$	10^{-10}
$\bar{s}^{TT}$	10^{-5}
$\bar{s}^{TX}$	10^{-9}
$\bar{s}^{TY}$	10^{-8}
$\bar{s}^{TZ}$	10^{-8}

Lab/solar system limits from AI, LLR, ...

Mueller et al PRL 08, Bourgoin et al PRL 16, Kostelecky and Russell, Data Tables, 2021 version

State of the art for "sbar"

- Data Table experiment limits

Table D38. Gravity sector, d = 4 (part 1 of 3)

Combination	Result	System	Ref.
$\bar{s}^{(4)}_{00}$	$(-20 \text{ to } 5) \times 10^{-15}$	Gravitational waves	[254]
$\bar{s}^{(4)}_{10}$	$(-30 \text{ to } 7) \times 10^{-15}$	"	[254]
$\mathrm{Re}\,\bar{s}^{(4)}_{11}$	$(-2 \text{ to } 10) \times 10^{-15}$	"	[254]
$\mathrm{Im}\,\bar{s}^{(4)}_{11}$	$(-30 \text{ to } 7) \times 10^{-15}$	"	[254]
$\bar{s}^{(4)}_{20}$	$(-8 \text{ to } 40) \times 10^{-15}$	"	[254]
$\mathrm{Re}\,\bar{s}^{(4)}_{21}$	$(-2 \text{ to } 10) \times 10^{-15}$	"	[254]
$\mathrm{Im}\,\bar{s}^{(4)}_{21}$	$(-40 \text{ to } 8) \times 10^{-15}$	"	[254]
$\mathrm{Re}\,\bar{s}^{(4)}_{22}$	$(-10 \text{ to } 3) \times 10^{-15}$	"	[254]
$\mathrm{Im}\,\bar{s}^{(4)}_{22}$	$(-4 \text{ to } 20) \times 10^{-15}$	"	[254]
$\bar{s}^{(4)}_{00}$	$> -3 \times 10^{-14}$	Cosmic rays	[21]*
$\bar{s}^{(4)}_{10}$	$(-10 \text{ to } 7) \times 10^{-14}$	"	[21]*
$\mathrm{Re}\,\bar{s}^{(4)}_{11}$	$(-8 \text{ to } 8) \times 10^{-14}$	"	[21]*
$\mathrm{Im}\,\bar{s}^{(4)}_{11}$	$(-7 \text{ to } 9) \times 10^{-14}$	"	[21]*
$\bar{s}^{(4)}_{20}$	$(-7 \text{ to } 10) \times 10^{-14}$	"	[21]*
$\mathrm{Re}\,\bar{s}^{(4)}_{21}$	$(-7 \text{ to } 7) \times 10^{-14}$	"	[21]*
$\mathrm{Im}\,\bar{s}^{(4)}_{21}$	$(-5 \text{ to } 8) \times 10^{-14}$	"	[21]*
$\mathrm{Re}\,\bar{s}^{(4)}_{22}$	$(-6 \text{ to } 8) \times 10^{-14}$	"	[21]*
$\mathrm{Im}\,\bar{s}^{(4)}_{22}$	$(-7 \text{ to } 7) \times 10^{-14}$	"	[21]*

"maximal reach", or one-at-a-time

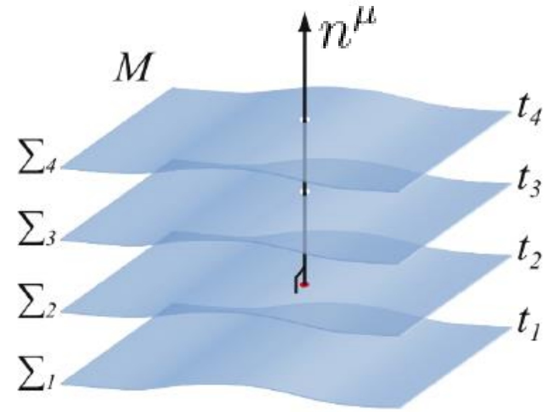
- Question: why not 10^{-15} in the maximal sensitivities?
- A: there is really on only one limit from BNS signal and Cosmic ray is a theorist limit*

Match to models of Lorentz violation

Model	Link to SME	Lorentz-violating fields	General Test Framework
PPN	Yes	None (α_1, α_2, w^j)	Yes, metric
SME, gravity sector	Yes	tensors, flavor dependent	Yes, EFT
Bumblebee	Yes	vector	No
Einstein-Aether	Partial	vector	No
Horava gravity	Yes	Two tensor	No
ATT model	Yes	Anti-symm. two-tensor	No
Cardinal	Yes	symm. two tensor	No
Massive gravity	Yes	two-tensor	No
CS gravity	Yes	scalar	No
GW modified dispersion	Yes	None (parameters)	Yes, Disp. Rel.
NC gravity	Yes	Theta^ab	No

many pubs: C. Will 70s, AK & Samuel 89, Jacobson & Mattingly 01, Jackiw & Pi 03, Carroll & Lim 04, Bluhm et al 08, Yunes 09, Seifert 09, Altschul et al 10, AK & Potting 05, Will, Yunes,... 12, Ciric et al PRD 2016, R Casana PRD 18, K and Li 2020, Bluhm et al 2016+

3+1 or AMD decomposition

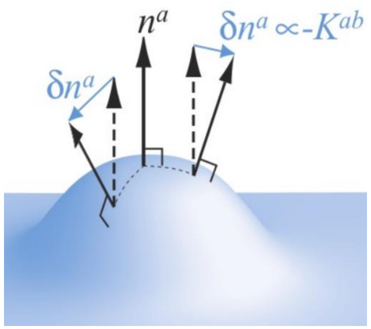


- Foliation of spacetime into spacelike hypersurface $t = t(x^{\mu\prime})$
- Normal vector $n^\mu = -\alpha g^{\mu\nu}\nabla_\nu t$
- Projector $\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$

- 4D curvature $R^\kappa{}_{\lambda\mu\nu}$ decomposed:

Extrinsic curvature: $K_{\mu\nu} = -\frac{1}{2}\mathcal{L}_{\mathbf{n}}\gamma_{\mu\nu}$

Lie derivative



components: $K_{ij} = -\frac{1}{2\alpha}(\partial_t\gamma_{ij} - \mathcal{D}_i\beta_j - \mathcal{D}_j\beta_i)$

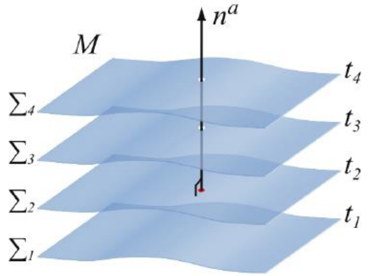
3D curvature (in the spacelike hypersurface):

$$R_{ij} = \partial_k\Gamma^k_{ij} - \partial_j\Gamma^k_{ik} + \Gamma^k_{ij}\Gamma^l_{kl} - \Gamma^k_{il}\Gamma^l_{jk}$$

- ADM metric $\alpha, \beta^j, \gamma_{ij}$

$$ds^2 = -(\alpha^2 - \beta^j\beta_j)dt^2 + 2\beta_j dt dx^j + \gamma_{ij}dx^i dx^j$$

ADM, Phys. Rev. 1959



3+1 General Relativity

Structure of the 3+1 split of Einstein equations

Constraints on hypersurface (4)

$$G^{0\mu} = \kappa T^{0\mu} \Rightarrow R + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$D_j(K^{ij} - \gamma^{ij}K) = 8\pi S^i$$

Evolution equations

$$G^{ij} = \kappa T^{ij} \Rightarrow \partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\begin{aligned}\partial_t K_{ij} &= \alpha(R_{ij} - 2K_{ik}K^k{}_j + KK_{ij}) - D_iD_j\alpha - 8\pi\alpha(S_{ij} - \tfrac{1}{2}\gamma_{ij}(S-\rho)) \\ &\quad + \beta^k\partial_k K_{ij} + K_{ik}\partial_j\beta^k + K_{kj}\partial_i\beta^k.\end{aligned}$$

Projections of stress-energy: $\rho = n_\mu n_\nu T^{\mu\nu}$, $S^i = -\gamma^{ij} n^\mu T_{\mu j}$, $S_{ij} = \gamma_{i\mu}\gamma_{j\nu}T^{\mu\nu}$

Variables: 10 $\alpha, \beta^j, \gamma_{ij}$

DOF counting: 4 constraints, 4 gauge conditions $\Rightarrow$ 2 DOF

$$g_{\mu\nu} \to g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu} = g_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

Equations from Baumgarte & Shapiro, *Numerical Relativity*

SME gravity sector, 3+1

$$\mathcal{L}_{\rm SME} = \frac{\sqrt{-g}}{2\kappa}(R - 2\Lambda + (k_R)_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}) + \mathcal{L}'$$

- Minimal SME, 3+1 form

$$\begin{aligned}\mathcal{L}_{k_R} = \frac{\sqrt{-g}}{2\kappa}\Big\{ & (k_R)_{\alpha\beta\gamma\delta}\Big[\mathcal{R}^{\alpha\beta\gamma\delta} + 2K^{\alpha\gamma}K^{\beta\delta} \\ & -12n^\alpha n^\gamma K^{\beta\epsilon}K^\delta{}_\epsilon + 4n^\alpha n^\gamma K^{\beta\delta}K^\epsilon{}_\epsilon + 8K^{\alpha\gamma}n^\beta a^\delta\Big] \\ & + 8K^{\epsilon\zeta}\mathcal{D}_\lambda\big((k_R)_{\alpha\beta\gamma\delta}\gamma^\alpha{}_\epsilon\gamma^{\beta\lambda}\gamma^\gamma{}_\zeta n^\delta\big) \\ & - 4a^\epsilon\mathcal{D}_\zeta\big((k_R)_{\alpha\beta\gamma\delta}\gamma^{\alpha\zeta}\gamma^\gamma{}_\epsilon n^\beta n^\delta\big) \\ & - 4K^{\epsilon\zeta}\mathcal{L}_{\mathbf{n}}\big((k_R)_{\alpha\beta\gamma\delta}\gamma^\alpha{}_\epsilon\gamma^\gamma{}_\zeta n^\beta n^\delta\big)\Big\}. \qquad (17)\end{aligned}$$

- Interesting feature for analysis: dynamical lapse, shift

$$\mathcal{L}_{k_R} \supset \frac{4\sqrt{-g}}{\kappa\alpha^2}K^{ij}n^\delta\big((k_R)_{i\beta j\delta}n^\beta\dot{\alpha} + (k_R)_{ilj\delta}\dot{\beta}^l\big)$$

-> No longer have constraint structure of GR, **IF** we assume explicit breaking of particle diffeomorphism symmetry

$$g_{\mu\nu} \to g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu} = g_{\mu\nu} + \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu$$

Ault, Bailey, and Nilsson, arXiv: 2009.00949, accepted in PRD

Match to Horava gravity

Renormalizable quantum gravity model that breaks diffeomorphism symmetry explicitly

$$\mathcal{L}_H = \alpha\sqrt{\gamma}(K_{ij}K^{ij} - \lambda K^2 + \xi\mathcal{R} + \eta a^i a_i + \dots)$$

Simplest isotropic "s" type SME lagrangian in 3+1 form

$$\mathcal{L}_2 = \alpha\sqrt{\gamma}\left[\mathcal{R}\left(1+\tfrac{1}{3}s\right) + \left(K^{ij}K_{ij} - K^2\right)\left(1 - s_{\bar{0}\bar{0}}\right)\right]$$

Insufficient, need second order terms (and two distinct sets of coefficients)

$$\begin{aligned}\mathcal{L}_{SME,Match} = \alpha\sqrt{\gamma}\Big[&\mathcal{R}\left(1+\tfrac{1}{3}s\right) + K^{ij}K_{ij} - K^2 \\ &+ a_5\tfrac{1}{2}(\nabla_\mu S^{\mu\lambda})(\nabla_\nu S^\nu{}_\lambda) \\ &+ a_{12}(S^{\mu\nu}\nabla_\mu S_{\nu\lambda})(S^{\kappa\rho}\nabla_\kappa S_\rho{}^\lambda)\Big]. \quad (\end{aligned}$$

Condition on coefficients $S_{\mu\nu} = n_\mu n_\nu$ Local Lorentz frame $S_{\bar{0}\bar{0}} = 1$

$$\Rightarrow \quad \begin{aligned}\mathcal{L}_{SME,Match} = \alpha\sqrt{\gamma}\Big[&\mathcal{R}\left(1+\tfrac{1}{3}s\right) + K^{ij}K_{ij} \\ &- K^2\left(1+\tfrac{1}{2}a_5\right) + \left(a_{12} + \tfrac{1}{2}a_5\right)a^i a_i\Big]\end{aligned}$$

P Horava, PRD 2009, D. Blas et al, PRL 2010 +, Ault, Bailey, and Nilsson, arXiv: 2009.00949, accepted in PRD

Summary

1) Three formulations of gravitational SME: covariant, quadratic, Kost-Li extension
2) Constraints on mSME are tight, some constraints exist on nmSME from GW, pulsars, short-range gravity
3) 3+1 SME gravity sector study, match to Horava gravity and quadratic SME
4) Rich phenomenology possible with new terms in the SME expansion!