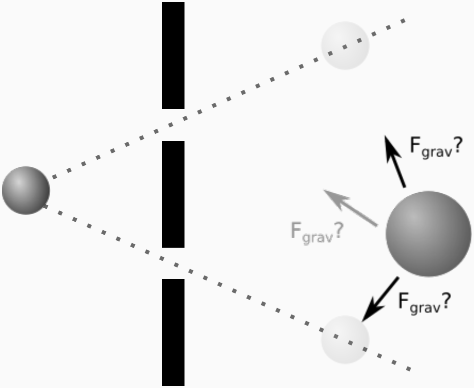
TESTING FUNDAMENTALLY SEMICLASSICAL GRAVITY

André Großardt

Friedrich Schiller University Jena, Germany

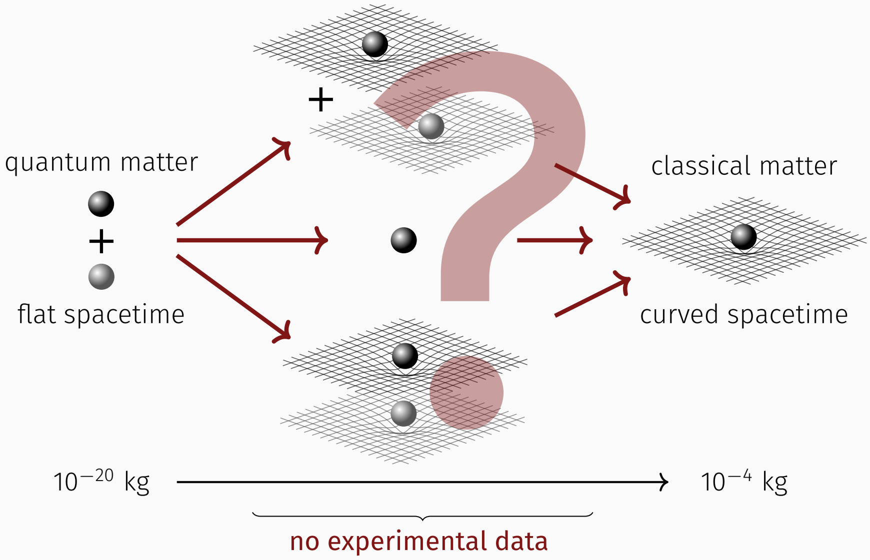
2nd EPS conference on gravitation: Measuring Gravity – July 2021



Does gravity need to be quantised?

Is curvature **of** spacetime a field living **on** spacetime?

FROM QUANTUM TO CLASSICAL MASS SCALES



Semiclassical gravity as a **fundamental** equation:

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = \frac{8\pi G}{c^4} \langle \hat{T}_{\mu\nu} \rangle$$

Weak-field **nonrelativistic** limit:

$\nabla^2 V = 4\pi G \, \langle \psi | \, \hat{\rho} \, | \psi \rangle$ with $\hat{\rho} = m \hat{\psi}^\dagger \hat{\psi}$

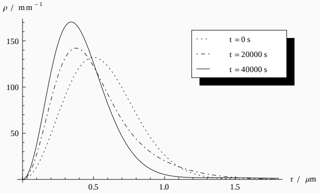
Results in the **Schrödinger-Newton equation** (here for one particle)

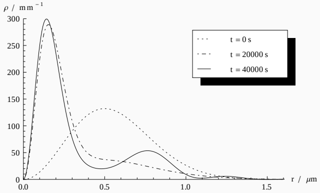
$$\mathrm{i}\hbar \, \dot{\psi}(t, \mathbf{r}) = \left(-\frac{\hbar^2}{2m} \nabla^2 - G m^2 \int \mathrm{d}^3 r' \, \frac{|\psi(t, \mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} \right) \psi(t, \mathbf{r})$$

$\Rightarrow$ **Nonlinear** Schrödinger equation

$\Rightarrow$ yields gravitational **self-interaction** of the wave function

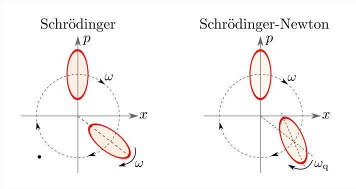
Free spreading of the wave function (interferometric tests):

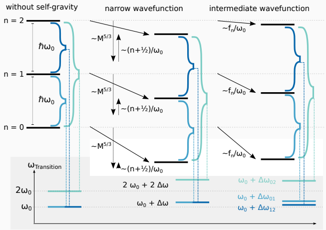




$\rho = 4\pi\, r^2\ |\psi|^2$ for masses of 7×10^9 u and 10^{10} u

Effects in optomechanical systems:

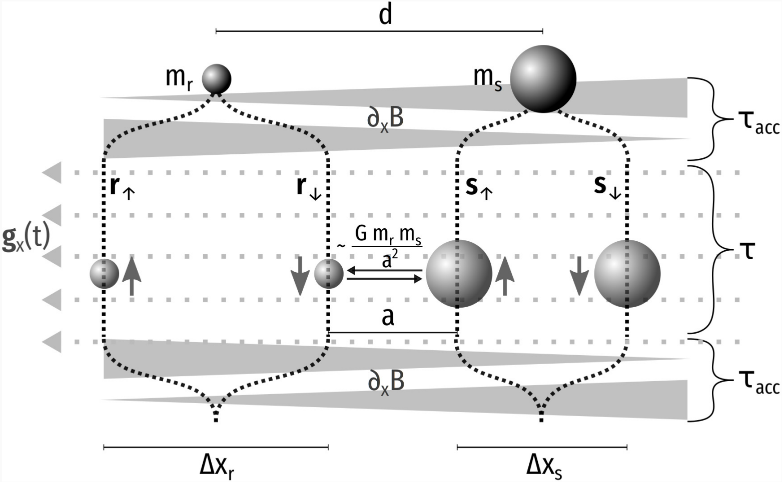




Yang et al. PRL 110 (2013) 170401

A.G. et al. PRD 93 (2016) 096003

Two adjacent Stern-Gerlach interferometers
Bose et al.: PRL 119 (2017) 240401



Gravitational phase shift

$$|\Psi\rangle = \frac{e^{i\varphi_{\uparrow\uparrow}}}{2}|\uparrow\uparrow\rangle + \frac{e^{i\varphi_{\uparrow\downarrow}}}{2}|\uparrow\downarrow\rangle + \frac{e^{i\varphi_{\downarrow\uparrow}}}{2}|\downarrow\uparrow\rangle + \frac{e^{i\varphi_{\downarrow\downarrow}}}{2}|\downarrow\downarrow\rangle$$

Simplest case: $a \ll \Delta x_{r,s}$, $\tau_{acc} \ll \tau \Rightarrow \varphi_{\downarrow\uparrow}$ **only contribution**

Semiclassical gravity: $|\Psi\rangle = \frac{1}{2}\left(|\uparrow\rangle + e^{i\varphi_{\downarrow}}|\downarrow\rangle\right) \otimes \left(e^{i\varphi_{\uparrow}}|\uparrow\rangle + |\downarrow\rangle\right)$

Entanglement witness (*Bose et al.*):

$$\mathcal{W} = \left|\left\langle \hat{\sigma}_x^{(r)} \otimes \hat{\sigma}_z^{(s)} \right\rangle + \left\langle \hat{\sigma}_y^{(r)} \otimes \hat{\sigma}_y^{(s)} \right\rangle\right|$$

$0 \leq \mathcal{W} \leq 2$ for quantised gravity but
$0 \leq \mathcal{W} \leq 1$ for semiclassical gravity.

EFFECT OF RELATIVE ACCELERATION

$$\frac{G\,M_{\text{source}}}{r^2} \sim \frac{(1\ \mu\text{m})^3}{(100\ \mu\text{m})^2} = \frac{(1\ \text{cm})^3}{(100\ \text{m})^2}$$

Phases with **external acceleration** (here for $|\uparrow\uparrow\rangle$):

$$\varphi_{\uparrow\uparrow} = \frac{Gm_r m_s}{\hbar}\int_0^t \frac{\mathrm{d}t'}{|\mathbf{r}_\uparrow(t') - \mathbf{s}_\uparrow(t')|} + \frac{1}{\hbar}\int_0^t \mathrm{d}t'\,\mathbf{g}(t')\cdot(m_r\mathbf{r}_\uparrow(t') + m_s\mathbf{s}_\uparrow(t'))$$

results in additional phase:

$$\varphi^{\text{ext}}_{r,s} = m_{r,s}\varphi^{\text{ext}}_0 \pm \frac{\mu_B \partial_x B\,\tau^2_{\text{acc}}}{4\hbar}\int_{\tau_{\text{acc}}}^{\tau+\tau_{\text{acc}}} \mathrm{d}t\,g_x(t)$$

White Gaussian noise: $\langle g_x(0)g_x(t)\rangle = \int \frac{\mathrm{d}\omega}{2\pi}S(\omega)\mathrm{e}^{-\mathrm{i}\omega t}$ and $S(\omega) \equiv S_0$

$\Rightarrow$ **mixed state** $\hat{\rho} = \int \mathrm{d}\varphi^{\text{ext}} P(\varphi^{\text{ext}})\,|\Psi(\varphi^{\text{ext}})\rangle\langle\Psi(\varphi^{\text{ext}})|$

We find the expectation values (assuming $m_r \approx m_s$)

$$\left\langle \sigma_x^{(r)} \otimes \sigma_z^{(s)} \right\rangle = \frac{e^{-\gamma}}{2} \left(\cos(\Delta\varphi + \delta\chi) - \cos(\delta\varphi - \delta\chi)\right)$$

$$\left\langle \sigma_y^{(r)} \otimes \sigma_y^{(s)} \right\rangle = \frac{1}{2} \left(\cos(\Delta\varphi - \delta\varphi) - e^{-4\gamma} \cos(2\delta\chi)\right)$$

with

$$\gamma = \frac{m_r \, m_s \, \Delta x^2 \, \tau \, S_0}{8\hbar^2}$$

$$\Delta\varphi \approx \frac{G m_r m_s \tau}{\hbar(d-\Delta x)} - \frac{G m_r m_s \tau}{\hbar d}, \quad \delta\varphi \approx \frac{G m_r m_s \tau}{\hbar(d+\Delta x)} - \frac{G m_r m_s \tau}{\hbar d}, \quad \delta\chi \approx \frac{G m_r m_s \tau \delta x}{\hbar d^2}$$

Implies for the entanglement witness: $\mathcal{W} \leq \frac{1}{2} + e^{-\gamma} + \frac{e^{-4\gamma}}{2}$

$$\mathcal{W} > 1 \quad \Leftrightarrow \quad \gamma \lesssim 0.75 \quad \Leftrightarrow \quad S_0 \lesssim \frac{6\hbar^2}{m_r m_s \tau \, \Delta x^2}$$

Gravitational energy $\sim 1/a \gg$ Casimir-Polder energy $\sim 1/a^7$

$$\Rightarrow \quad a \gg \frac{1}{2\sqrt{\pi}} \left(\frac{3\alpha}{\rho} \sqrt{\frac{23\,\hbar c}{G}} \right)^{1/3}$$

With $G m_r m_s \tau \sim \hbar a$ and $\Delta x \gtrsim a$ for a detectable phase one finds the **noise limit**:

$$S_0 \lesssim \frac{8\,\hbar\,G}{3\pi\,a^3} \ll \frac{64\,\rho}{9\,\alpha} \sqrt{\frac{\pi \hbar G^3}{23\,c}}$$

depending only on material, with $\alpha \gtrsim 0.35$ and $\rho \lesssim 23$ g/cm^3

$$\Rightarrow \quad \sqrt{S_0} \ll 0.24 \text{ fm s}^{-2}/\sqrt{\text{Hz}}$$

For **unequal** $M \geq m$ with $\Delta x_r \approx \Delta x_s \approx \Delta x$ we find $\mathcal{W} > 1$ for $S_0 \lesssim \frac{\hbar^2}{M^2 \tau \Delta x^2}$
The closest approach is the radius of the larger particle $R \approx a \approx \Delta x$

$$\Rightarrow \quad \sqrt{S_0} \lesssim \frac{\hbar}{\rho R^4 \sqrt{\tau}}$$

Observable phase ($\Delta\varphi \sim \pi$): $GmM\tau \sim \hbar a \sim \hbar R$

$\tau <$ decoherence from gas collisions $\tau_{\text{dec}} \sim \sqrt{kTm_{\text{gas}}}/(PR^2)$:

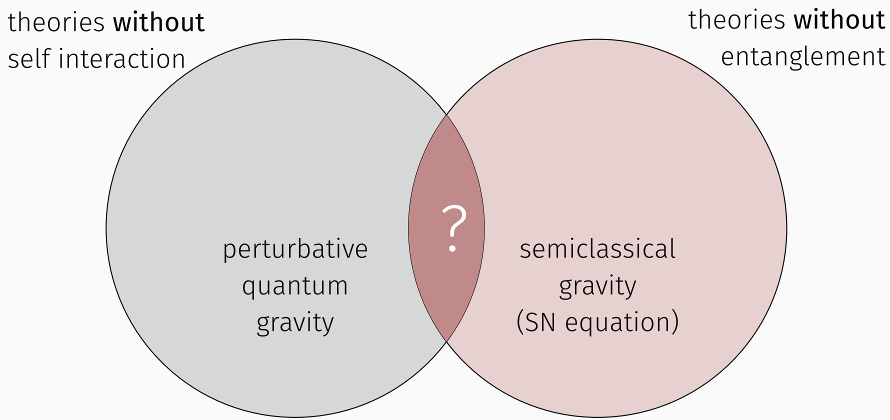
$$R^4\sqrt{\tau} \sim \frac{M^2\tau}{\rho^2 R^2 \sqrt{\tau}} > \frac{mM\tau}{\rho^2 R^2 \sqrt{\tau}} > \frac{\hbar}{G\rho^2 R \sqrt{\tau_{\text{dec}}}} \sim \frac{\hbar\sqrt{P}}{G\rho^2}(kTm_{\text{gas}})^{-1/4}$$

$$\Rightarrow \quad \sqrt{S_0} < \frac{8G\rho}{9\sqrt{\zeta(3/2)n_{\text{gas}}}} \left(\frac{m_{\text{gas}}}{3kT}\right)^{1/4}$$

	Bose et al.	interstellar
Material	diamond	osmium
Density	3.5 g/cm^3	23 g/cm^3
Medium (gas)	air	hydrogen
Temperature	150 mK	1 μK
Pressure	10^{-15} Pa	10^{-23} Pa
Particle density	500/cm^3	1/cm^3
$\sqrt{S_0} <$	1.4 pm s^{-2}/$\sqrt{\text{Hz}}$	2.1 nm s^{-2}/$\sqrt{\text{Hz}}$

- $\ll$ nano-g in drop tower experiments?
- active mitigation or precise tracking of $g_x(t)$?
- space? ($\sqrt{S_0} \sim 5$ fm s^{-2}/$\sqrt{\text{Hz}}$ in LISA Pathfinder)

What can different experiments teach us about gravity?



What ca

Gravity sourced by...

- weak measurements

  Kafri, Taylor, Milburn. New J. Phys. 16, 065020 (2014).

- objective collapse events

  Tilloy and Diósi. Phys. Rev. D 93, 024026 (2016).

theories **witho**
self interactio

without
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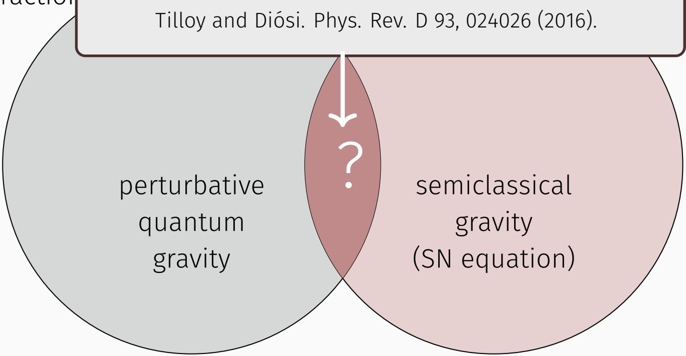
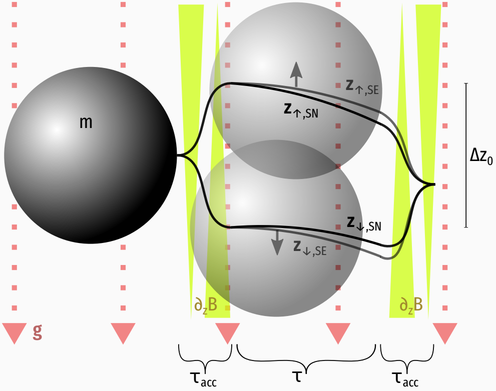


If gravitational spin-entanglement can be detected over 100 μm...

can a self-gravitational effect on spin be detected?



Spin superposition state (initially **Gaussian** in space):

$$|\Psi(t)\rangle = \alpha\,|{\uparrow}\rangle \otimes \int \mathrm{d}^3 r\, \psi_\uparrow(t,\mathbf{r})\,|\,\mathbf{r}\rangle + \beta\,|{\downarrow}\rangle \otimes \int \mathrm{d}^3 r\, \psi_\downarrow(t,\mathbf{r})\,|\,\mathbf{r}\rangle$$

$$\hat{H} = \hat{I} \otimes \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{ext}} + |\alpha|^2\, U_\uparrow + |\beta|^2\, U_\downarrow\right) + \hat{\sigma}_z \otimes V_{\mathrm{acc}}$$

- $U_{\uparrow\downarrow}$ depend on $\psi_\uparrow$ **and** $\psi_\downarrow$ $\Rightarrow$ **nonseparable Schrödinger eq.**
- Can be made **linear** and **separable** by **perturbative approach**
- **Homogeneous** V_{ext}, V_{acc} only yield **phase + displacement**
- Both the **wave function width** and **split of classical trajectories** can be in **three regimes** defined by atomic scale ∼pm and particle size $\sim \mu$m

SPIN EXPECTATION VALUE (PRELIMINARY!)

$$\langle \hat{\sigma}_x \rangle = e^{-\gamma_{SN} - \gamma_g} \cos(\varphi_{\alpha\beta} + \varphi_g + \varphi_{SN})$$

- $\varphi_{\alpha\beta}$: initial relative phase between α and β
- $\varphi_g \sim \langle g \rangle$ COW phase, $\gamma_g \sim \mathrm{var}(g)$ acceleration noise decoherence
- φ_{SN} nonzero **only** for **unsymmetric** state $|\alpha|^2 \neq |\beta|^2$
- γ_{SN} loss of visibility due to reduced **overlap** of $\psi_\uparrow$ and $\psi_\downarrow$

	Bose et al.	**massive**
Particle size	1 μm	60 μm (Os.)
Separation	100 μm	3 pm
Wave function	0.1 pm ...100 nm	3 pm
$\gamma_g < 1$ **for** $\Delta g <$	10^{-17} m s^{-2}	10^{-13} m s^{-2}
γ_{SN} **(sym./unsym.)**	10^{-6} / $\mathcal{O}(1)$	$\mathcal{O}(10)/\mathcal{O}(1)$
Phase φ_{SN}	$-\mathcal{O}(1)$	$-\mathcal{O}(1)$

Thank You!

Questions?

ADDITIONAL SLIDES

PAGE-GEILKER “EXPERIMENT”

VOLUME 47, NUMBER 14 PHYSICAL REVIEW LETTERS 5 OCTOBER 1981

Indirect Evidence for Quantum Gravity

Don N. Page
and
C. D. Geilker

An experiment gave results inconsistent with the simplest alternative to quantum gravity, the semiclassical Einstein equations. This evidence supports (but does not prove) the hypothesis that a consistent theory of gravity coupled to quantized matter should also have the gravitational field quantized.

Quantum decision process: measuement of state $\frac{1}{\sqrt{2}}(|\,\psi\rangle + |\,\chi\rangle)$ used to place a macroscopic mass into opositions x_1 or x_2.

- **no collapse** interpretation:
  System is in state $\frac{1}{\sqrt{2}}(|\,\psi\rangle \otimes |\,x_1\rangle + |\,\chi\rangle \otimes |\,x_2\rangle)$ and gravitates towards $\bar{x} = \frac{x_1+x_2}{2}$ $\Rightarrow$ **not observed**
- **instantaneous collapse** (Copenhagen interpretation):
  violates $\nabla^\mu G_{\mu\nu} = 0$

$\Rightarrow$ **no obvious problem with non-instantaneous reduction**

SUPERLUMINAL SIGNALLING (I)

Claim: **any deterministic nonlinearity in the Schrödinger equation leads to the possibility to send faster than light signals** (Gisin, 1989)

- E.g. entangled spin-$\frac{1}{2}$ particles:

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A\,|\downarrow\rangle_B + |\downarrow\rangle_A\,|\uparrow\rangle_A\right) = \frac{1}{\sqrt{2}}\left(|+\rangle_A\,|+\rangle_B - |-\rangle_A\,|-\rangle_B\right)$$

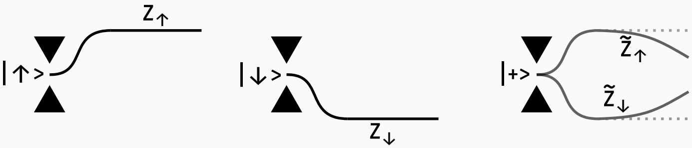
  where $|\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \pm |\downarrow\rangle\right)$ are the σ_x eigenstates

- Measuring in σ_z or σ_x basis results in same density matrix after tracing over possible outcomes $|\uparrow\rangle_B$ and $|\downarrow\rangle_B$ or $|+\rangle_B$ and $|-\rangle_B$:

$$\hat{\rho}_A = \frac{1}{2}\,|\uparrow\rangle\langle\uparrow| + \frac{1}{2}\,|\downarrow\rangle\langle\downarrow| = \frac{1}{2}\,|+\rangle\langle+| + \frac{1}{2}\,|-\rangle\langle-|$$

  equivalent mixtures (measurement at A independent of basis B) **remain equivalent** in a **linear** theory

SUPERLUMINAL SIGNALLING (II)



- **Semiclassical gravity**: assume spin of particle A becomes entangled with its position (e.g. in magnetic field gradient)

$$|\uparrow\rangle \;\to |\uparrow\rangle\otimes|z_\uparrow(t)\rangle\,, \qquad |\downarrow\rangle \;\to |\downarrow\rangle\otimes|z_\downarrow(t)\rangle$$

- **However** in **superposition** states $|\pm\rangle$

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \pm |\downarrow\rangle\right) \;\to\; \frac{1}{\sqrt{2}}\left(|\uparrow\rangle\otimes|\tilde{z}_\uparrow(t)\rangle \pm |\downarrow\rangle\otimes|\tilde{z}_\downarrow(t)\rangle\right)$$

with $\tilde{z}_{\uparrow\downarrow}(t) \approx z_{\uparrow\downarrow}(t) \pm \frac{G\,m}{2}\,\int_0^t dt'\int_0^{t'} dt''\,|z_\uparrow(t'') - z_\downarrow(t'')|^{-2}$

$\Rightarrow$ measurement outcomes at A **depend** on choice of basis at B

- Usual dogma: **stochastic** nonlinearity avoids superluminal signalling. Evolution of density matrix remains linear.

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] - \frac{\lambda}{2}\int d^3x \int d^3y\, \mathcal{G}(\mathbf{x}-\mathbf{y})\,[\hat{m}(\mathbf{x}), [\hat{m}(\mathbf{y}), \hat{\rho}]]$$

- Source of stochastic nonlinearity unknown $\Rightarrow$ **gravity?**
- **Diósi-Penrose**: $\mathcal{G}(\mathbf{x}) = G/\hbar\,|x|^{-1}$ $\Rightarrow$ collapse rate $\sim$ self-energy:

$$t_C \approx \frac{\hbar\, R_0}{G\, m^2}$$

- Needs to be regulated: **coarse graining** with length scale R_0
  $\Rightarrow$ instantaneous collapse only a good approximation above R_0

SUPERLUMINAL SIGNALLING (III)

Need to distinguish between $z_{\uparrow\downarrow}(t)$ and $\tilde{z}_{\uparrow\downarrow}(t) = z_{\uparrow\downarrow}(t) + \Delta z$

- Spatial resolution $\lambda \quad \Rightarrow \quad$ detection time $t_D \approx 2\Delta z \sqrt{\frac{\lambda}{G\,m}}$
- Uncertainty $\delta z\,\delta p_z \approx \hbar$ and $\lambda < \Delta z$ implies

$$\lambda > \delta z + \frac{t_D}{m}\,\delta p_z > \sqrt{\frac{\hbar\,t_D}{m}} > \left(\frac{\hbar^2\,\lambda^3}{G\,m^3}\right)^{1/4} \quad \Leftrightarrow \quad \lambda > \frac{\hbar^2}{G\,m^3}$$

- No collapse before separation:

$$t_C^2 \approx \frac{\hbar^2\,R_0^2}{G^2\,m^4} > t_D^2 > \frac{\lambda^3}{Gm} \qquad \Leftrightarrow \qquad \frac{\hbar^2}{G\,m^3} > \frac{\lambda^3}{R_0^2}$$

- Both conditions combined require $\lambda < R_0$

What is the **electric** field of a charged particle in a superposition?

$\Rightarrow$ field becomes entangled to particle state: $|\,\psi\rangle \otimes |\,E_\psi\rangle + |\,\chi\rangle \otimes |\,E_\chi\rangle$

- Evolves in time: $|\,\psi(t)\rangle \otimes |\,E_\psi(t)\rangle + |\,\chi(t)\rangle \otimes |\,E_\chi(t)\rangle$
- Test particle position gets entangled as well:
  $|\,\psi(t)\rangle \otimes |\,E_\psi(t)\rangle \otimes |\,x_{\mathrm{TP}}[E_\psi(t)]\rangle + |\,\chi(t)\rangle \otimes |\,E_\chi(t)\rangle \otimes |\,x_{\mathrm{TP}}[E_\chi(t)]\rangle$
- Interference terms at location x:
  $2P(t,x) = |\psi(t,x)|^2 + |\chi(t,x)|^2 + \psi^*(t,x)\chi(t,x) + \psi(t,x)\chi^*(t,x)$

$\Rightarrow$ no need for QED but accounts for "quantumness" of the field

QUANTISED GRAVITY AS SOLUTION?

In analogy to electrodynamics: $|\psi\rangle \otimes |G_\psi\rangle + |\chi\rangle \otimes |G_\chi\rangle$
$\Rightarrow$ **superposition of two spacetimes**

Incompatible with spacetime curvature:

- There is no well defined time translation operator in a superposition of spacetimes
- State $|\psi\rangle$ will evolve according to the Schrödinger equation in spacetime $|G_\psi\rangle$, whereas $|\chi\rangle$ evolves in spacetime $|G_\chi\rangle$
- How to **identify** points in **different** spacetimes? (e.g. for performing an interference experiment)

C.f. Penrose (1996)

The Schrödinger-Newton equation can be seen as...

- Hartree approximation $\Psi_N = \psi \otimes \psi \otimes \cdots \otimes \psi$ for gravitational interaction potential (e.g. gravitating Bose-Einstein condensate)

$$V = \sum_{i \neq j} -\frac{Gm^2}{\left|\mathbf{x}_i - \mathbf{x}_j\right|}$$

- Nonrelativistic ($c \to \infty$) limit of the *classical* Einstein-Klein-Gordon (or Einstein-Dirac) equation

$$\varphi \sim \mathrm{e}^{imc^2t/\hbar} \left(\psi + \mathcal{O}(c^-2)\right)$$

- Supposed nonrelativistic approximation of a quantum field gravitating according to the semiclassical Einstein equations

Realistic systems for testing SN are **not** single particles:

$$i\hbar\dot{\Psi}_N(\mathbf{r}^N) = \left[-\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\Delta_{\mathbf{r}_i} + V_{\text{linear}}(\mathbf{r}^N) + V_G[\Psi_N(\mathbf{r}^N)]\right]\Psi_N(\mathbf{r}^N)$$

$$V_G[\Psi_N(\mathbf{r}^N)] = -G\sum_{i=1}^{N}\sum_{j=1}^{N} m_i m_j \int \frac{\left|\Psi_N(\mathbf{r}'^N)\right|^2}{\left|\mathbf{r}_i - \mathbf{r}'_j\right|}\,\mathrm{d}V'^N$$

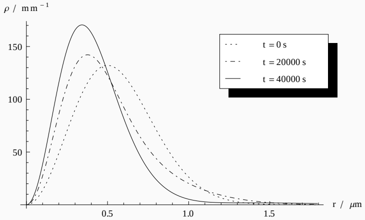
Centre of mass equation (**approx.**), separation $\Psi_N = \psi \otimes \chi_{N-1}$:

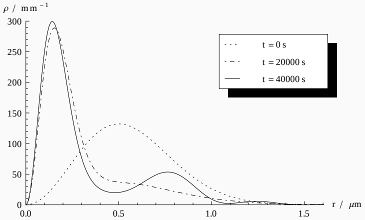
$$i\hbar\,\dot{\psi}(t,\mathbf{r}) = \left(-\frac{\hbar^2}{2M}\nabla^2 + V_{\text{lin.}}^{\text{ext.}} - G\int \mathrm{d}^3 r' \left|\psi(t,\mathbf{r}')\right|^2 I_\rho(\mathbf{r}-\mathbf{r}')\right)\psi(t,\mathbf{r})$$

$$I_\rho(\mathbf{d}) = \int \mathrm{d}^3x\,\mathrm{d}^3y\,\frac{\rho(\mathbf{x})\rho(\mathbf{y}-\mathbf{d})}{|\mathbf{x}-\mathbf{y}|} \quad \text{(where } \rho \text{ is given by } \left|\chi_{N-1}\right|^2\text{)}$$

wave function $\ll$ particle size $\quad\Rightarrow\quad \rho \approx \delta(\mathbf{r}_{cm}) \quad\Rightarrow\quad I_\rho(d) \approx 1/\left|d\right|$:

$$i\hbar\,\dot\psi(t,\mathbf{r}) = \left(-\frac{\hbar^2}{2m}\nabla^2 - Gm^2\int \mathrm{d}^3r'\,\frac{\left|\psi(t,\mathbf{r}')\right|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\psi(t,\mathbf{r})$$

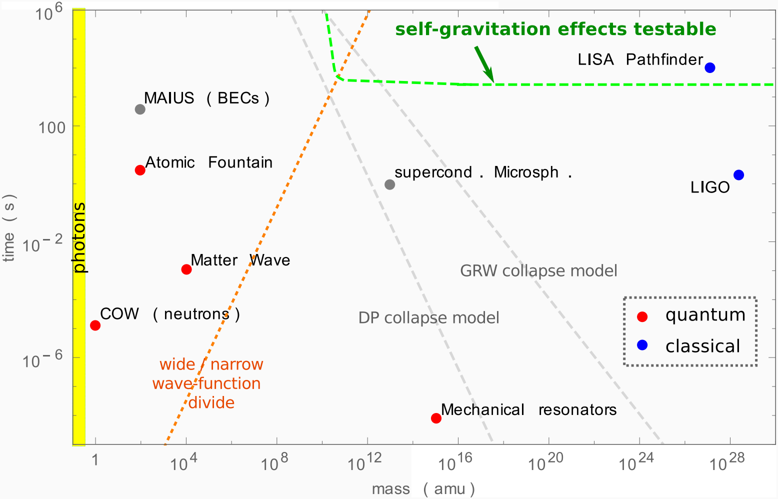




$\rho = 4\pi\, r^2\ |\psi|^2$ for masses of 7×10^9 u and 10^{10} u

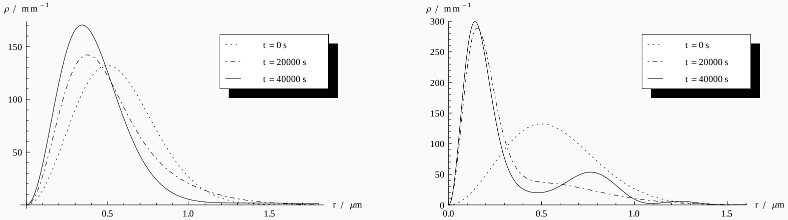
Problem: time scale (order of **hours**!)

Inhibition of free expansion of wave packets:



green line intuitively: free wave function would have increased by 25% but maintains its width due to self-gravity

In the wide wave function limit: one-particle SN equation



$\rho = 4\pi\, r^2\, |\psi|^2$ for masses of 7×10^9 u and 10^{10} u

- For a mass of $\sim 10^{10}$ u and a wave packet size of about 500 nm a significant deviation is visible after several *hours*
- **Scaling law:** with $\psi(t, \mathbf{x})$ for mass m, a solution for mass $\mu\, m$ is obtained as $\mu^{9/2}\psi(\mu^5 t, \mu^3 \mathbf{x}) \quad \Rightarrow \quad$ e. g. 10^{11} u at 0.5 nm would show an effect in *less than a second* **but** must remain in wide wave function regime (Os at 10^{10} u has 100 nm diameter)

Assumption: a **Gaussian** wave packet stays approximately **Gaussian**

The free spreading of a Gaussian wave packet and spherical particle can be approximated by a third order ODE for the width $u(t) = \langle r^2 \rangle(t)$:

$$\dddot{u}(t) = -3\omega_{\mathrm{SN}}^2 \, f(u(t)) \, \dot{u}(t)$$

with $\omega_{\mathrm{SN}} = \sqrt{Gm/R^3} \sim \sqrt{G\rho}$, initial conditions

$$u(0) = u_0 \,, \qquad \dot{u}(0) = 0 \,, \qquad \ddot{u}(0) = \frac{9\hbar^2}{2m^2 \, u_0} - \omega_{\mathrm{SN}}^2 \, g(u_0) \, u_0 \,,$$

and the functions (with u in units of R)

$$f(u) = \mathrm{erf}\left(\sqrt{\frac{3}{u}}\right) + \sqrt{\frac{u}{3\pi}} \left(u - \frac{7}{2} - \frac{324 - 162u - 35u^4 + 70u^5}{70\, u^4} \, \mathrm{e}^{-3/u} \right)$$

$$g(u) = \mathrm{erf}\left(\sqrt{\frac{3}{u}}\right) + \sqrt{\frac{u}{3\pi}} \left(\frac{2}{3}u - 3 + \frac{486 + 105u^3 - 70u^4}{105u^3} \, \mathrm{e}^{-3/u} \right)$$

$$u(t) \approx u_0 + \frac{1}{2}\,\ddot{u}(0)\,t^2$$

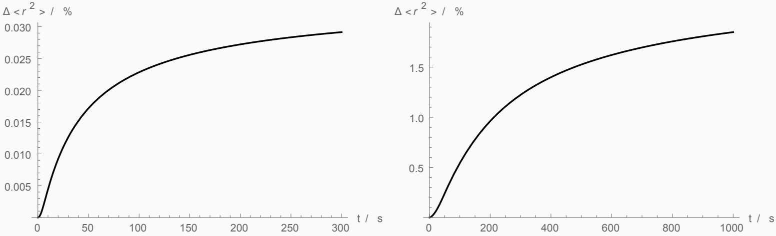
- *exact* without self-gravity term
- deviates from usual evolution by dependence on $g(u_0)$ in

$$\ddot{u}(0) = \frac{9\hbar^2}{2m^2\,u_0} - \omega_{\mathrm{SN}}^2\,g(u_0)\,u_0$$

- stationarity condition $\ddot{u}(0) = 0$ yields (pessimistic) estimate for the scales where self-gravity becomes important
- Assume **osmium** particle initially trapped with ω_0
  $\Rightarrow$ characteristic time scale $\tau = \omega_0^{-1}, \quad u_0 = 3\hbar\,\tau/m$
- $\ddot{u}(0) = 0$ determines characteristic (m, τ) graph
- limit $g(u) \to 1$ for $u \to 0$ yields $\tau(m) =$ const. for large m

For narrower wave functions (here $\mathcal{O}(10\ \mathrm{nm}) \lesssim$ particle size):
approximate ODE (assume: Gaussian wave packet remains Gaussian)

$$\frac{\mathrm{d}^3}{\mathrm{d}t^3}\langle r^2\rangle = -3\omega_{\mathrm{SN}}^2\, f(\langle r^2\rangle)\,\frac{\mathrm{d}}{\mathrm{d}t}\langle r^2\rangle$$



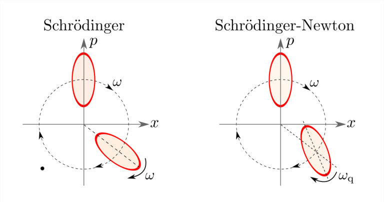
rel. deviation from standard Schrödinger evolution for $m = 10^9$ u and 10^{10} u

$\Rightarrow$ 1 % deviation after 200 s $\quad\rightarrow$ maybe in space?

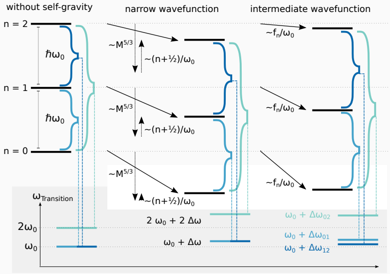
wave function $\gg$ particle size $\Rightarrow$ to $\mathcal{O}(|\mathbf{r}-\mathbf{r}'|^2)$, $I''(0) = m\omega^2 \mathbb{1}$:

$$i\hbar\,\dot\psi(t,\mathbf{r}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + \frac{m\omega^2}{2}\left((\mathbf{r}-\langle\mathbf{r}\rangle)^2 + \langle\mathbf{r}^2\rangle - \langle\mathbf{r}\rangle^2\right)\right)\psi(t,\mathbf{r})$$

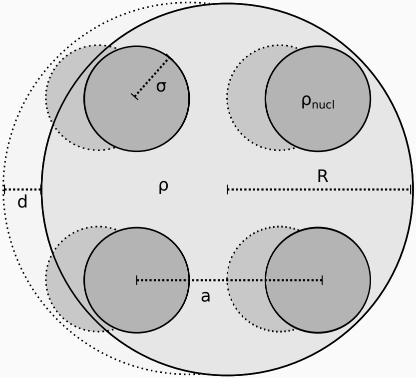
Effects in optomechanical systems:



Yang et al. PRL 110 (2013) 170401



A.G. et al. PRD 93 (2016) 096003



- the relevant radius is σ (localisation of the nuclei)
- effective mass density $\rho_{\mathrm{nucl}} \sim 10^3 \rho$
- $\omega_{\mathrm{SN}} = \sqrt{\frac{G m_{\mathrm{atom}}}{\sigma^3}} \sim \sqrt{G \rho_{\mathrm{nucl}}} \sim 1$ Hz for osmium

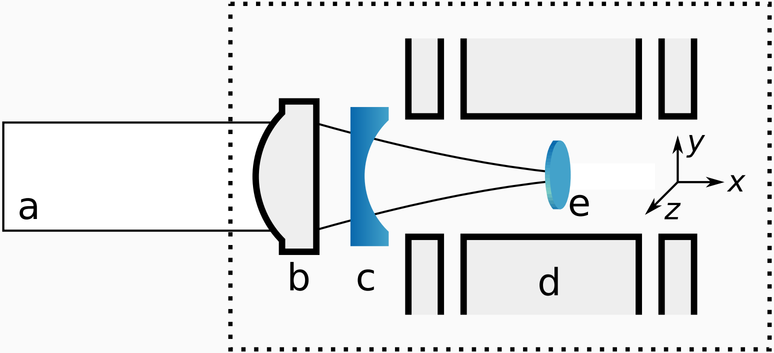
Need **ground state** cooling for:
mass $\sim 10^{15}$ u (μm sized) particle
trapped at $\mathcal{O}(10\,\mathrm{Hz})$

MATERIAL CHOICES

$$\omega_{SN} = \sqrt{\frac{G m_{atom}}{\sigma^3}}$$

Material	m_{atom} / u	ρ / g cm^{-3}	σ / pm	ω_{SN} / s^{-1}
Silicon	28.086	2.329	6.96	0.096
Tungsten	183.84	19.30	3.48	0.695
Osmium	190.23	22.57	2.77	0.996
Gold	196.97	19.32	4.66	0.464

Note: ω_{SN} enters **squared** in the evolution equation
$\Rightarrow$ osmium two orders of magnitude better than silicon


a
b
c
d
e
y
x
z

IS SEMICLASSICAL GRAVITY + DECOHERENCE ENOUGH?

- **Decoherence** yields classical mixtures **but no collapse**
- **Collapse models** describe nonlinear dynamics, and could be based on gravity **but with unclear relation to GR**
- **Semiclassical gravity** by itself **does not explain collapse**:
  - stationary states can be very distinct from likely position eigenstates
  - single particle SN dynamics has runaway probability
  - deterministic (no Born rule probabilities)

...but how about decoherence and semiclassical gravity combined?

✓ evolution into classical states from decoherence

✓ nonlinearity from gravity

? stochastic decoherence source? (dark matter? gravitational waves?)
$\rightarrow$ **can Born rule be derived from this?**

At the lowest order, gravity yields a phase $\varphi \sim \frac{G\, m_1\, m_2\, \Delta t}{\hbar\, \Delta x}$

- **Quantised gravity:**

$$(|L\rangle_1 + |R\rangle_1) \otimes (|L\rangle_2 + |R\rangle_2)$$
$$\rightarrow |L\rangle_1 \, |L\rangle_2 + |L\rangle_1 \, |R\rangle_2 + \mathbf{e}^{\mathrm{i}\varphi} \, |R\rangle_1 \, |L\rangle_2 + |R\rangle_1 \, |R\rangle_2$$

$$\mathcal{W} = \left| \left\langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \right\rangle + \left\langle \sigma_y^{(1)} \otimes \sigma_y^{(2)} \right\rangle \right| = \left| 1 + \mathrm{e}^{\mathrm{i}\varphi} \right| \leq 2$$

- **Schrödinger-Newton equation:**

$$(|L\rangle_1 + |R\rangle_1) \otimes (|L\rangle_1 + |R\rangle_1)$$
$$\rightarrow \mathbf{e}^{\mathrm{i}\varphi/2} \, |L\rangle_1 \, |L\rangle_2 + |L\rangle_1 \, |R\rangle_2 + \mathbf{e}^{\mathrm{i}\varphi} \, |R\rangle_1 \, |L\rangle_2 + \mathbf{e}^{\mathrm{i}\varphi/2} \, |R\rangle_1 \, |R\rangle_2$$
$$= (|L\rangle_1 + \mathbf{e}^{\mathrm{i}\varphi/2} \, |R\rangle_1) \otimes (\mathbf{e}^{\mathrm{i}\varphi/2} \, |L\rangle_2 + |R\rangle_2)$$

$$\mathcal{W} = \left| \left\langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \right\rangle + \left\langle \sigma_y^{(1)} \otimes \sigma_y^{(2)} \right\rangle \right| = \frac{1}{2} \left| 1 + \mathrm{e}^{\mathrm{i}\varphi} \right| \leq 1$$