Spectral separation of the stochastic background for LISA

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» Overview

Context: Stochastic background In LISA paper of BBH/BNS prediction

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Stochastic Background : Superposition of a large number of independent sources (unresolved sources):

* White dwarfs binaries in our galaxy Lamberts *et al.* simulation of the Waveform with galactic population of Double White Dwarf (DWD)
  $s(t) = \sum_{i=1}^{N} \sum_{P=+,\times} h_{P,i}(f_{orb,i}, M1_i, M2_i, X_i, Y_i, Z_i, t) \times F_P(\theta, \phi, t)\mathbf{D}(\theta, \phi, f)_P : \mathbf{e}_P$
  with a modulate waveform (firstly done with resolved sources to provide the modulation curve for the foreground Adams & Cornish)
* Binary Black Holes and Binary Neutron Stars from LIGO/Virgo Band
  * $\Omega_{GW} \simeq 1.8 \times 10^{-9} - 2.5 \times 10^{-9}$ at 25 Hz Chen *et al.* (2019)
  * $\Omega_{GW} \simeq 4.97 \times 10^{-9} - 2.58 \times 10^{-8}$ at 25 Hz Périgois *et al.* (2020)
* Cosmological sources (early universe)

» Context: Stochastic background In LISA paper of BBH/BNS prediction

Energy density spectrum:

Model of Energy density spectrum SGWB sources

$$\Omega_{GW}(f) = \frac{A_1 \left(\frac{f}{f_*}\right)^{\alpha_1}}{1 + A_2 \left(\frac{f}{f_*}\right)^{\alpha_2}} + \Omega_{Astro}\left(\frac{f}{f_*}\right)^{\alpha_{Astro}} + \Omega_{Cosmo}\left(\frac{f}{f_*}\right)^{\alpha_{Cosmo}}$$

with $\alpha = 0$ for the cosmological component, $\alpha = 2/3$ for the astrophysical component and low frequency DWD ($\Omega_{DWD,LF}(f) = \frac{A_1}{A_2}\left(\frac{f}{f_*}\right)^{\alpha_1 - \alpha_2}$).

Goal: Detecting a cosmological SGWB with LISA in the presence of an astrophysical background and Galactic foreground

MCMC: Markov chains Monte Carlo

» MCMC (Markov chain Monte Carlo)

Likelihood function, (d = data, θ = parameter)

likelihood

$$\mathcal{L}(\mathbf{d}|\theta) = -\frac{1}{2}\sum_{k=0}^{N}\left[\frac{d_A^2}{S_A+N_A} + \frac{d_E^2}{S_E+N_E} + \frac{d_T^2}{N_T} + \ln\left(8\pi^3(S_A+NA)(S_E+N_E)N_T\right)\right]$$

* posterior distribution $p(\theta|d) \propto p(\theta)\mathcal{L}(\mathbf{d}|\theta)$
* using log uniform and uniform prior $p(\theta) = \prod_i U(\theta_i, a_i, b_i)$
* Estimation 10 parameters $\theta = [N_{acc}, N_{pos}, \Omega_{astro}, \alpha_{astro}, \Omega_{cosmo}, \alpha_{cosmo}, A_1, \alpha_1, A_2, \alpha_2]$.

$\Longrightarrow$ using a Metropolis-Hasting sampler

Fisher Information Matrix

» Fisher Information Matrix

Fisher Information Matrix

$$F_{ab} = \frac{1}{2}\mathrm{Tr}\left(\mathcal{C}^{-1}\frac{\partial\mathcal{C}}{\partial\theta_a}\mathcal{C}^{-1}\frac{\partial\mathcal{C}}{\partial\theta_b}\right)$$
$$= \frac{1}{2}\sum_{I=A,E,T}\sum_{k=0}^{N}\frac{\frac{\partial S_I(f)+N_I(f)}{\partial\theta_a}\frac{\partial S_I(f)+N_I(f)}{\partial\theta_b}}{\left(S_I(f)+N_I(f)\right)^2}$$

Co-variance Matrix

$$\mathcal{C}(\theta, f) = \begin{pmatrix} S_A + N_A & 0 & 0 \\ 0 & S_E + N_E & 0 \\ 0 & 0 & N_T \end{pmatrix}$$

with N_I is the LISA noise of the channel $I = [A, E, T]$ and $S_I \propto \sum_\alpha \Omega_\alpha \left(\frac{f}{f_{ref}}\right)^\alpha$ the SGWB.

$\Longrightarrow \sqrt{F^{-1}_{aa}} = \sigma_a$

Results 6 parameter A-MCMC

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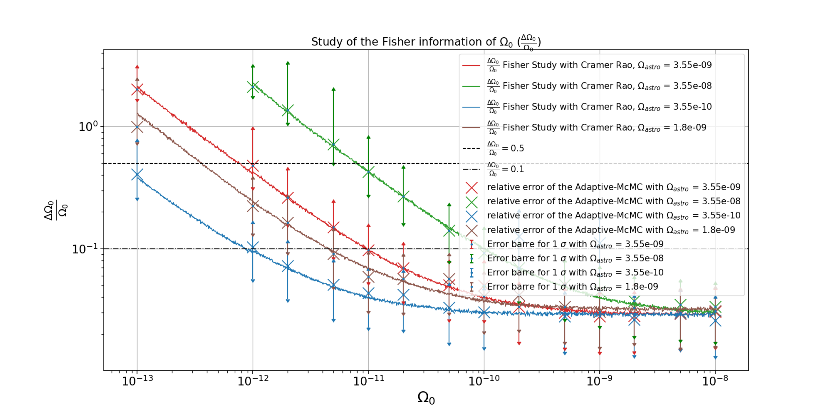

Uncertainty of the estimation of the Cosmological Amplitude from the Fisher Information study in line (with the Cramer-Rao calculation) and the parametric estimation from the A-MCMC in scatters for the channel A with the noise channel T. The upper horizontal dash line represent the error level 50%. In fact, above the line, the error is greater than 50%.

» Results 6 parameter A-MCMC

Prediction of the measurement limit of Cosmological Amplitude in 4 contexts of Isotropic astrophysical background In the LISA noise context (acceleration noise : $N_{Acc} = 9 \times 10^{-30}$ m^{2}s^{-4}Hz^{-1} and Optical Metrology System noise $N_{Opt} = 2.25 \times 10^{-22}$ m^{2}Hz^{-1}) of 4 years mission data measurement:

Limit for BBH/BNS + Cosmo + LISA noise

* $\Omega_{astro} = 3.55 \times 10^{-8}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.8 \times 10^{-12}$
* $\Omega_{astro} = 3.55 \times 10^{-9}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.8 \times 10^{-13}$
* $\Omega_{astro} = 1.8 \times 10^{-9}$ (25 Hz): $\Omega_{Cosmo,lim} = 3.6 \times 10^{-13}$
* $\Omega_{astro} = 3.55 \times 10^{-10}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.6 \times 10^{-14}$

$\Longrightarrow$ paper (PhysRevD.103.103529).

Orbital Modulation of the White dwarf binaries in our Galaxy

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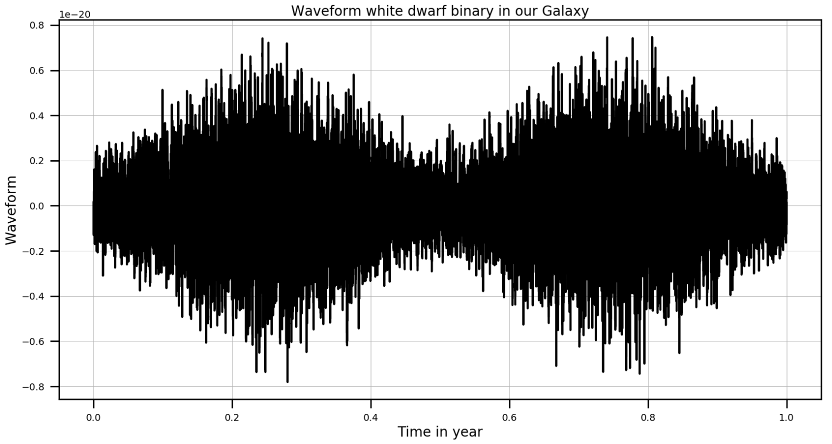
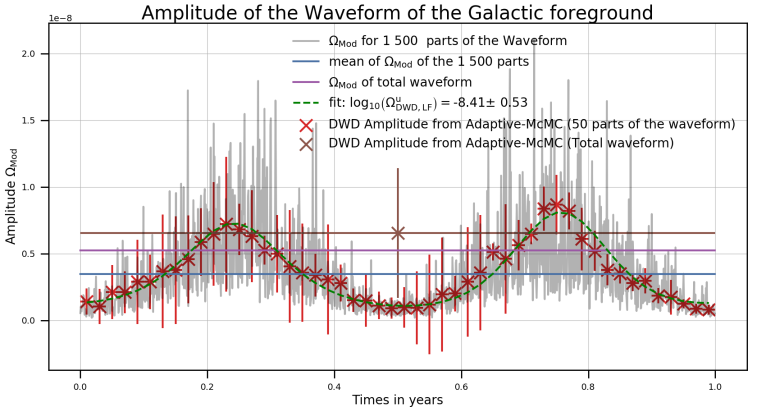


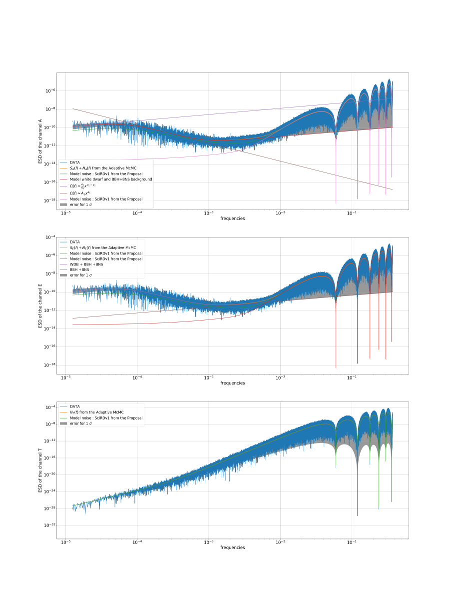
Total Gravitational signal of the white-Dwarf binaries seen by LISA

» Orbital Modulation of the White dwarf binaries in our Galaxy



Measurement of the orbital Modulation of the DWD amplitude. In **grey**: $\Omega_{Mod,i} = \frac{4\pi^2}{3H_0}\left(\frac{c}{2\pi L}\right)^2 A_i^2$. In **red** scatter : 50 A-MCMC of 8 parameters (BBH + WD + LISA noise) for small sections of the year. In green, fit on the 50 runs to estimate the modulation. Modulation model : $\Omega_{Mod,i} = \Omega^{u}_{DWD,LF}\left(F^2_{+,i} + F^2_{\times,i}\right)$.

» A-MCMC of 8 parameters (BBH + WD + LISA noise)



Input of the MCMC:

- $A_3 = 4.4 \times 10^{-12}$ ($f_* = \frac{2\pi L}{c} \simeq 0.019$ Hz) , $\alpha_3 = 2/3$
- data model:

$$S(f) = \frac{A_1 \left(\frac{f}{f_*}\right)^{\alpha_1}}{1 + A_2 \left(\frac{f}{f_*}\right)^{\alpha_2}} + A_3 \left(\frac{f}{f_*}\right)^{\alpha 3}$$

For $1 \ll A_2 \left(\frac{f}{f_*}\right)^{\alpha_2}$ (low frequency):

Energy spectral density of White dwarf binaries at low frequency

$$\Omega_{WD}(f) \simeq \frac{A_1}{A_2} \left(\frac{f}{f_*}\right)^{\alpha_1 - \alpha_2}$$

At low frequency, it can be approximated by a power law function, for a white dwarf binaries foreground the slope $\alpha = \alpha_1 - \alpha_2 = \frac{2}{3}$.

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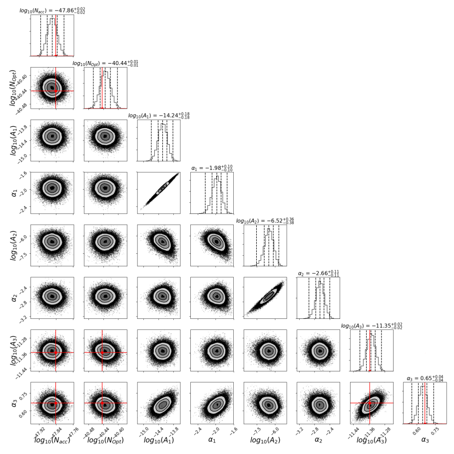

Ouput of the A-MCMC

* BBH/BNS:
  * $\Omega = 4.46 \times 10^{-12} \pm 1.2 \times 10^{-13}$
  * $\alpha = 0.65 \pm 0.04$
* WD low frequencies:
  * $\Omega = 1.94 \times 10^{-8} \pm 1.33 \times 10^{-8}$
  * $\alpha = 0.68 \pm 0.06$

10 parameters runs of A-MCMC (LISA noise + BBH/BNS + DWD + Cosmo)

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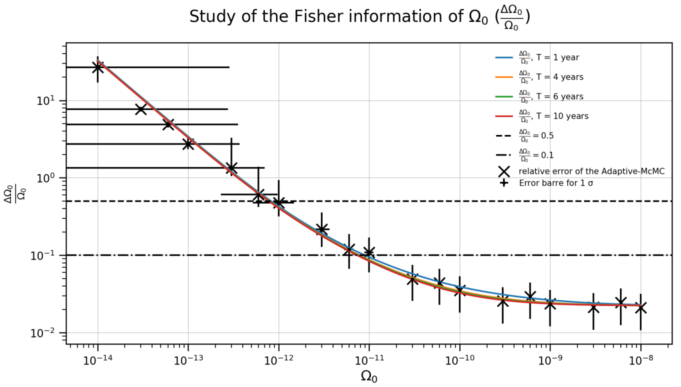


Uncertainty Estimation of the Cosmological Amplitude from the Fisher Information study in line (for 4 times duration) and the parametric estimation from the A-MCMC in scatters for the channel A with the noise channel T. The upper horizontal dash line represent the error level 50%. In fact, above the line, the error is greater than 50%.

» 10 parameters runs of A-MCMC (LISA noise + BBH/BNS + WD + Cosmo)

Times effect

We see no significant influence of the time duration (4,6 and 10 years) on the Cosmological measurement

Measurement limit

$\Omega_{Cosmo,lim} = 8 \times 10^{-13}$

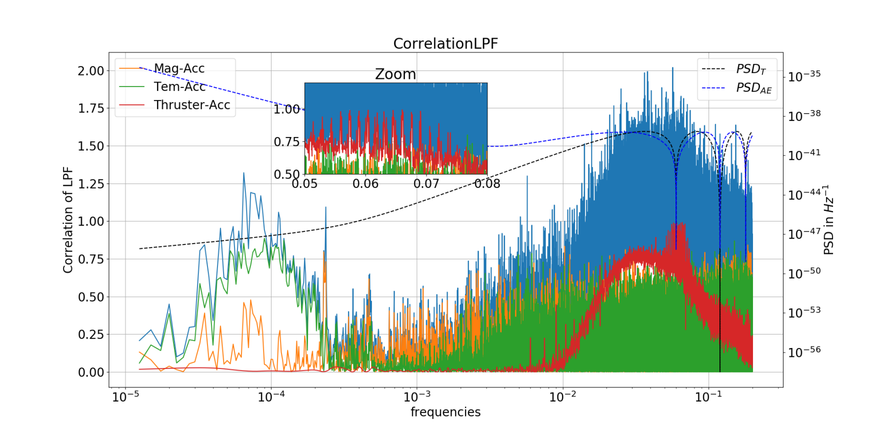
Limitation

BBH and BNS principal limitation for the Cosmological background

$\Longrightarrow$ paper in preprint (arXiv:2105.04283)

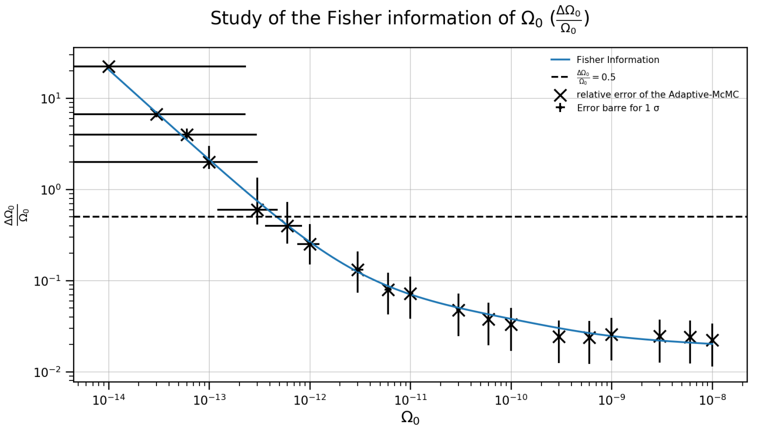
Correlation in LISA PathFinder (LPF) and LISA noise

» LPF Correlation Noise in LISA



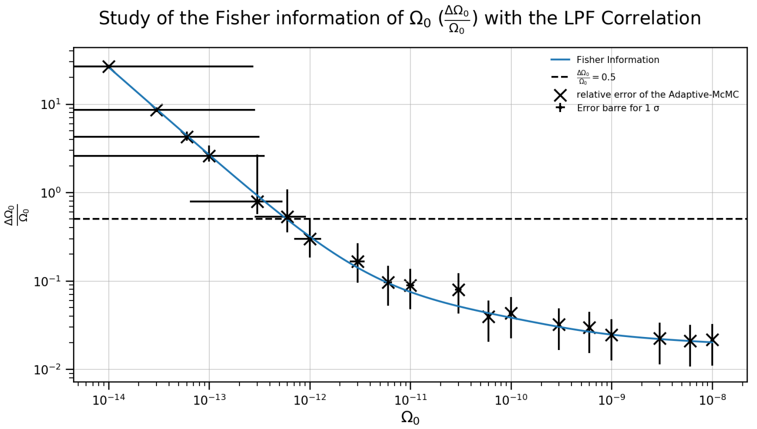
Correlation between differential Test-mass acceleration LPF L0 chains without glitch (de-glitch method on the spectral domain) and Magnetic Field, Temperature and μ-Thruster

» Spectral Separability : Uncertainty without LPF Correlation



Cosmological Amplitude Uncertainty Estimation, Fisher Information study in line and A-MCMC in scatters. The upper horizontal dash line represent the error level 50% (limitation criterion).

» Spectral Separability :Uncertainty with LPF Correlation



Cosmological Amplitude Uncertainty Estimation, Fisher Information study in line and A-MCMC in scatters. The upper horizontal dash line represent the error level 50% (limitation criterion).

Conclusion

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- We provide evidence that it is possible to measure for LISA the cosmological SGWB :

Measurement limit

$$\Omega_{Cosmo,lim} = 8 \times 10^{-14} - 8 \times 10^{-12}$$

Limitation

BBH and BNS principal limitation for the Cosmological background

- We use Lamberts *et. al.*, possibility to generate white dwarf waveform with other catalogues
- Interest to use estimating more complex backgrounds, like broken power laws, or spectrum with peaks. Our method can be easily expanded with more complex cosmological backgrounds

» Conclusion

- We provide evidence that it is possible to measure the cosmological SGWB with and without LPF correlation:

Measurement limit

$\Omega_{Cosmo,\ \mathrm{lim\ without\ LPF}} = 6.0\ \times 10^{-13}$
$\Omega_{Cosmo,\ \mathrm{lim\ with\ LPF}} = 4.7\ \times 10^{-13}$

- We measure the effect of the phase error due to LO light back-scattered in the channel A, E and T. High frequency effect, not a problem for stochastic, but potentially other searches.
- High flexibility of our code for other types of noise and correlated noise.

The End : Thank You !