Linking 3D and 1D computational models to follow evolution of heated coronal plasma

J. Reid, A. W. Hood, P. J. Cargill, C. E. Parnell, and C. D. Johnston
School of Mathematics and Statistics, University of St Andrews, St Andrews, Fife, United Kingdom

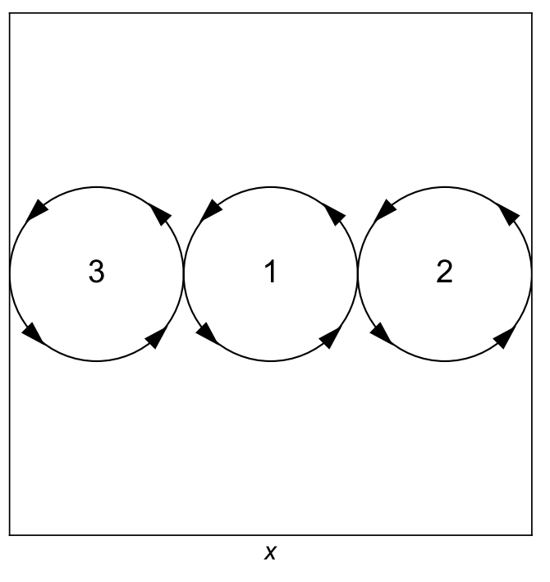
Abstract

A ‘proof of principle’ is presented, whereby the Ohmic and viscous heating determined by a three-dimensional (3D) MHD model of a coronal avalanche are used as the coronal heating input for field-aligned, one-dimensional (1D) hydrodynamic modelling.

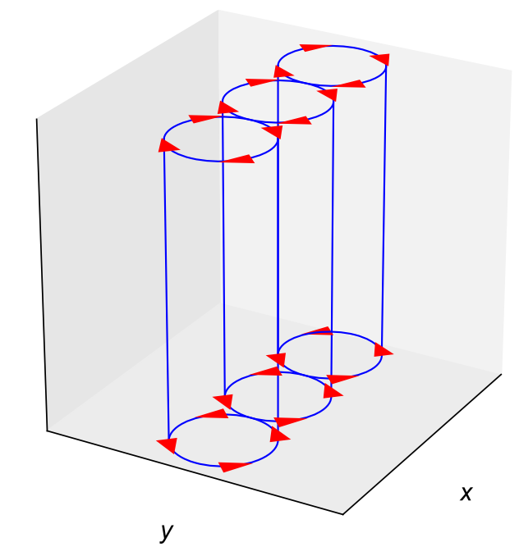
Three-dimensional MHD models cannot afford the computational resources to follow the magnetic field and the thermodynamic transport along field lines with realistic parameters. From a 3D MHD simulation, we extract the heating along single field lines and use these heating functions for 1D simulations that follow transport of energy. Proceeding from simple, ordered photospheric motions, this heating is spatially localized, dispersed, and impulsive, occurring in discrete, reconnection-facilitated bursts. MHD heating is shown to sustain coronal temperatures and densities, around $10^6\,\mathrm{K}$ and 10^{14}–$10^{15}\,\mathrm{m}^{-3}$ respectively, in a $90\,\mathrm{Mm}$ loop. Thermodynamic feedback on the plasma dynamics is limited, and the MHD evolution is largely robust to the field-aligned thermodynamic response. Advantages and drawbacks of the 3D and 1D models, within their respective spheres, are discussed and compared. Both models report similar temperature and density, but velocities diverge. Heating causes strongly asymmetric plasma flows, which differ significantly between 3D and 1D models and may have observable signatures. Velocities in the 1D model are comparable with 3D reconnection jets in the MHD model.

3D model: geometry and driving

- Our initial, 3D model (Reid et al. 2018) places three magnetic strands within a coronal loop, anchored at photospheric boundaries
- Slow rotational motions, in opposite directions, are applied at both footpoints of each strand
  - Motions are faster for the central strand than for the outer two
- Inducing twist, this creates flux tubes out of the initially uniform magnetic field, and stresses the field until critically unstable
  - We aim to cause a kink mode in the central strand, while the others remain stable



Vertical driving on base.



Rotation twisting strands.

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho\left(\nabla\cdot\mathbf{v}\right)$$
$$\rho\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = -\nabla P + \frac{1}{\mu_0}\left(\nabla\times\mathbf{B}\right)\times\mathbf{B} + \mathbf{F}_{\text{visc.}}$$
$$\rho\frac{\mathrm{D}\varepsilon}{\mathrm{D}t} = -P\left(\nabla\cdot\mathbf{v}\right) + \frac{j^2}{\sigma} + Q_{\text{visc.}}$$
$$\frac{\mathrm{D}\mathbf{B}}{\mathrm{D}t} = \left(\mathbf{B}\cdot\nabla\right)\mathbf{v} - \mathbf{B}\left(\nabla\cdot\mathbf{v}\right) - \nabla\times\left(\eta\nabla\times\mathbf{B}\right)$$

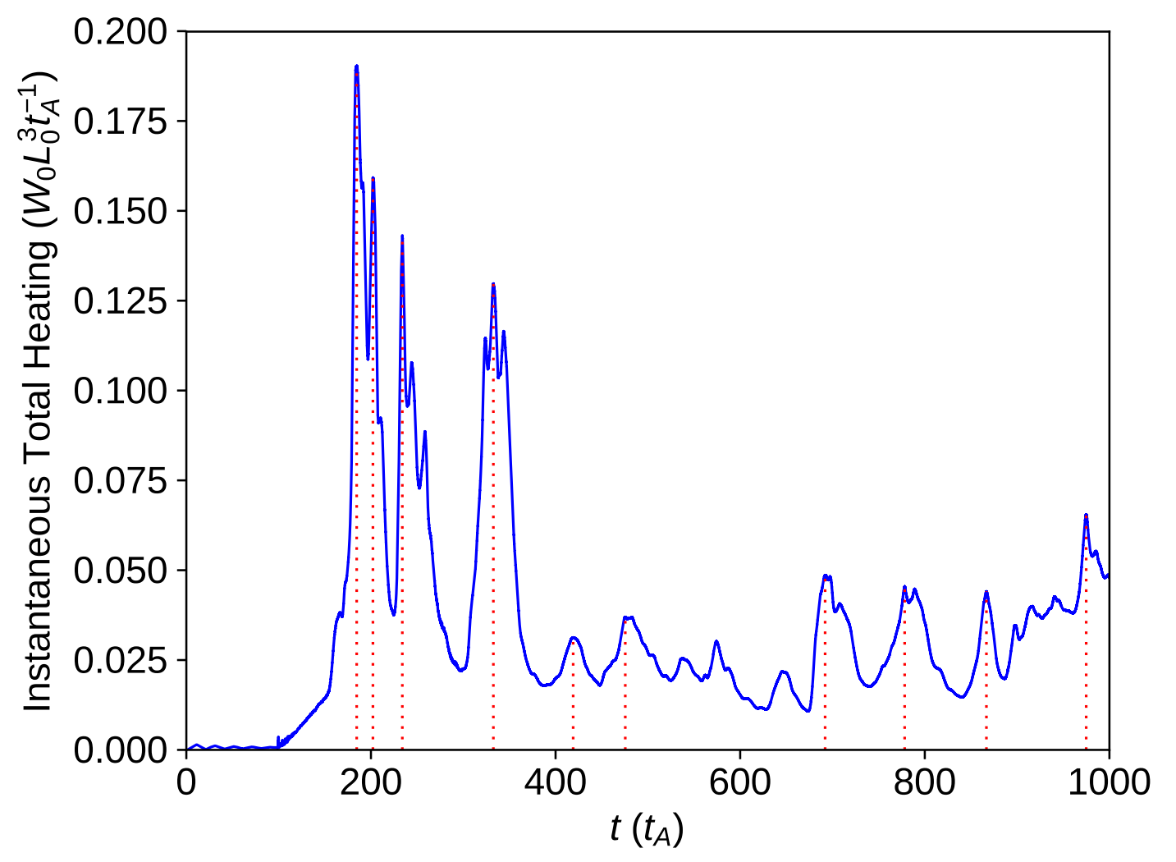
- We solve the 3D MHD equations in *Lare3d* (Arber et al. 2001)
- At this stage, we neglect gravity, radiative losses, and thermal conduction
  - Heat flux is too computationally difficult fully to resolve in 3D (Bradshaw & Cargill 2013)
- Numerical resolution: $512^2\times 2048$ cells, $\Delta z\sim\mathcal{O}\left(44\,\mathrm{km}\right)$
- Anomalous resistivity dissipates magnetic energy about particularly strong currents
- Shock viscosities and uniform, background viscosity dissipate energies in flows

Instability: from order to disorder

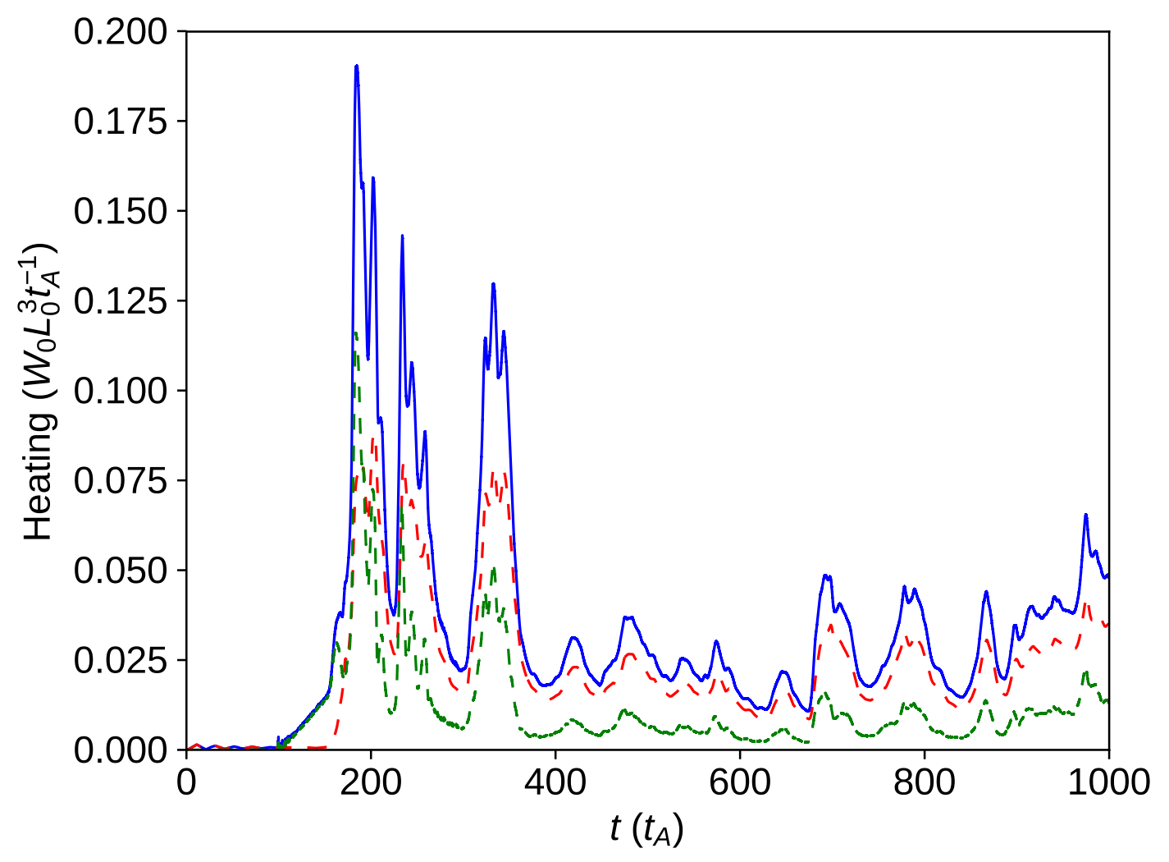
- Fastest-twisted central strand forms a strong helical current sheet and undergoes an ideal kink instability
- Reconnection occurs in this current sheet, beginning a process of wider disruption and heating
- Avalanche process engulfs outer strands
- Complex, interconnected magnetic fields continually produce heating out of the energy injected by constant driving on the boundary

Heating: bursts and background

- Instantaneous heating is very impulsive and intermittent
- Above a largely steady ‘background’ are several strong ‘nanoflare’ events
- Heating is aperiodic
  - No preferred period, or ‘nanoflare timescale’
- Major heating events come with the destabilization of each strand and spread of the avalanche
  - Magnetic energy falls as it is dissipated Ohmically
  - Rapid outflows from reconnection show great kinetic energy
  - Fast flows are dissipated, leading to further heating
- Viscous heating greatly dominates over Ohmic



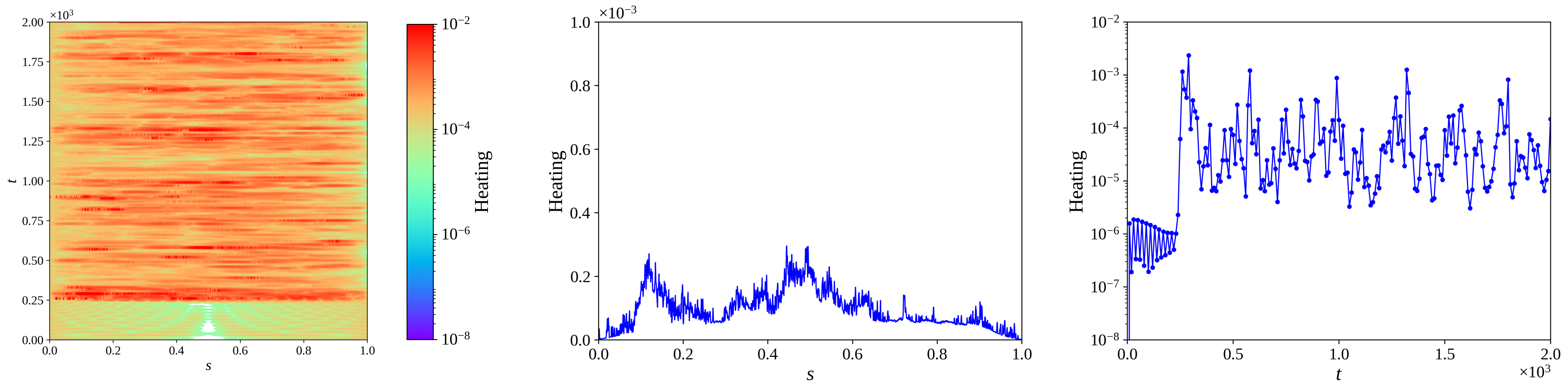
Total heating, with major events marked in red.



Total (blue), viscous (red), and Ohmic (green) heating.

Field-aligned heating

- Anisotropic thermal conduction in strong magnetic fields ⇒ thermodynamic behaviour is most important parallel to the magnetic field
- Large numbers of field lines are traced in the MHD model (Reid et al. 2020)
- From local viscous and Ohmic heating in MHD, we interpolate and determine heating along field lines as functions of space and time: $Q\left(s,t\right)$
- Field-aligned heating appears concentrated intensely in confined (largely Ohmic) pockets, amidst a general continuum of viscous damping



As a function of space and time. *Time-averaged.* *Spatially averaged.*

Heating along a sample field line.

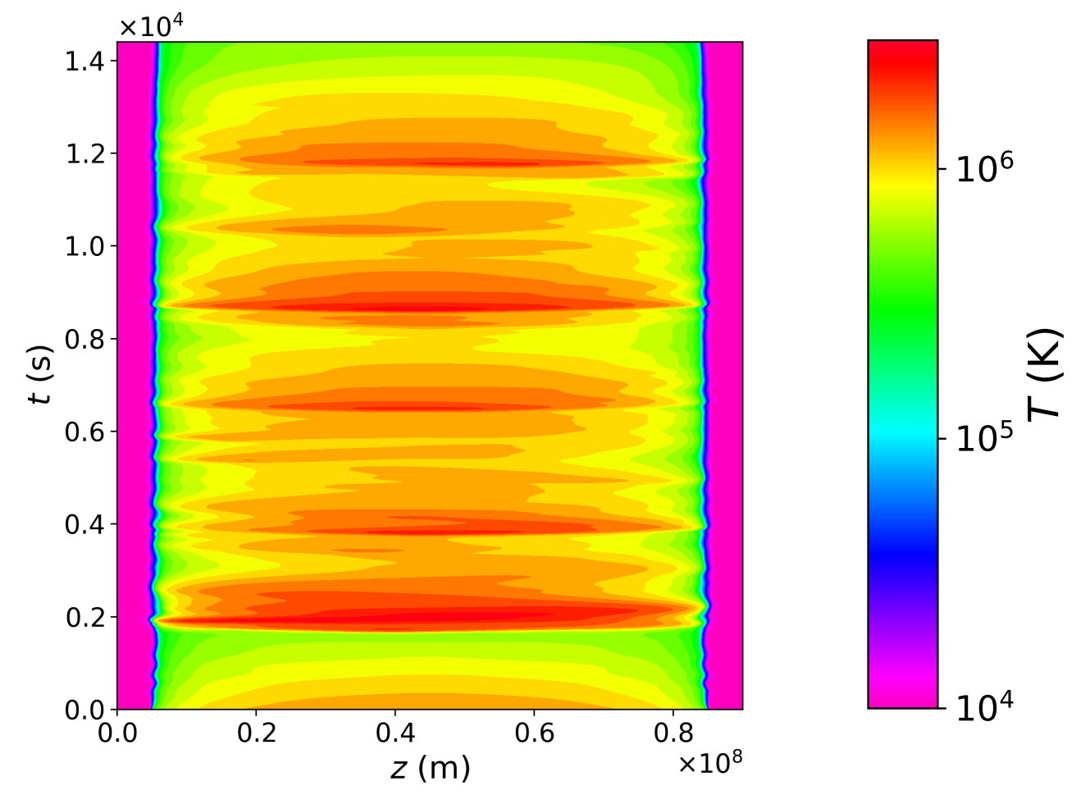
1D model: field-aligned in *Lare1d*

- Strong magnetic fields align thermal conduction with field lines ⇒ thermodynamic transport more easily resolved in 1D model, along a magnetic strand
- Field-aligned, 1D models include a ‘background’ heating term to maintain coronal temperature and density
- For this heating, we inject $Q\left(z,t\right)$ taken from field lines in MHD
- We solve the field-aligned MHD equations, now including gravity $g_{\|}\left(z\right)$, which varies along the field; radiation $-n^2\Lambda\left(T\right)$; our coronal heating function $Q\left(z,t\right)$; and thermal conduction $-\frac{\partial F_c}{\partial z}$.
- Thermal conduction treated with a new numerical technique, TRAC
  - Transition Region using Adaptive Conduction: Johnston & Bradshaw (2019) and Johnston et al. (2020)
  - Modifies thermal conduction term to resolve gradients, even with a far longer grid scale
  - Circumvents challenge on resolution and makes numerical simulations faster

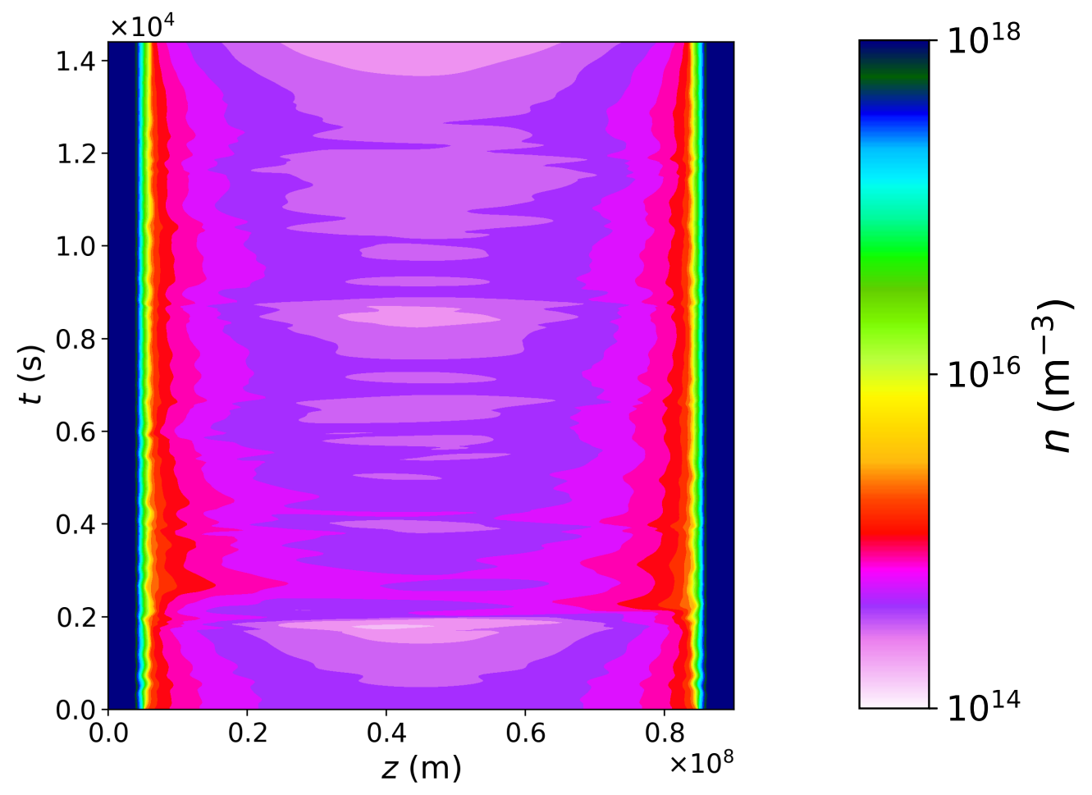
$$\frac{\partial\rho}{\partial t} + v\frac{\partial\rho}{\partial z} = -\rho\frac{\partial v}{\partial z}$$
$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial z} = -\frac{\partial P}{\partial z} - \rho g_{\|}\left(z\right) + \rho\nu\frac{\partial^2 v}{\partial z^2}$$
$$\rho\frac{\partial\varepsilon}{\partial t} + \rho v\frac{\partial\varepsilon}{\partial z} = -P\frac{\partial v}{\partial z} - \frac{\partial F_c}{\partial z} + Q\left(z,t\right) - n^2\Lambda\left(T\right) + \rho\nu\left(\frac{\partial v}{\partial z}\right)^2$$
$$P = \left(n_i + n_e\right)k_B T$$

Thermodynamic evolution along magnetic strands

- Initially, before the onset of major heating, the loop cools and drains, with temperature and density falling
- Strong heating comes with the major instabilities
- Bursts of heating locally raise temperature, which is quickly smoothed by conduction
- As heating is very time-dependent, there are cycles of heating and cooling, leading to evaporation and draining
- Transition region moves up and down in response to heating/cooling cycle



Evolution of temperature.



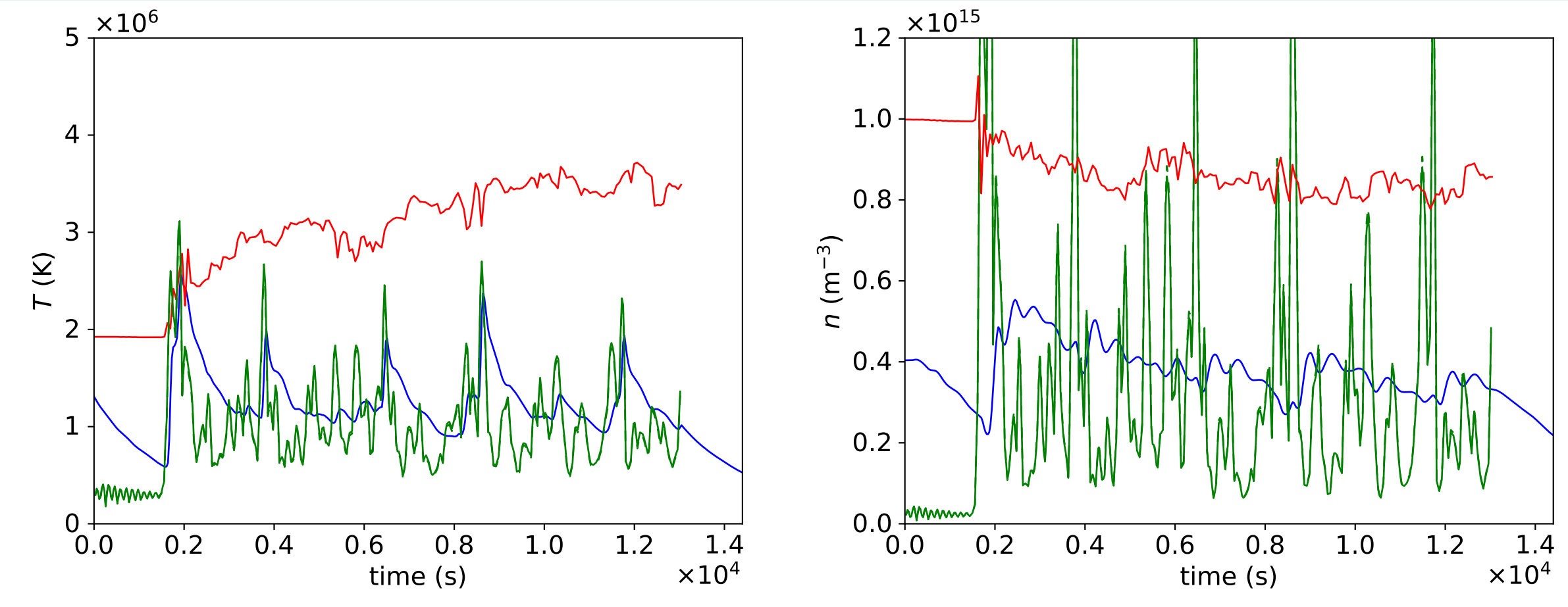
Evolution of density.

Comparing models: MHD and 1D

- Along field lines, the time-dependent heating function approximately maintains a certain ‘background’ level
- In thermodynamic simulation, this sustains coronal values of temperature and density:

$$Q\approx 8.59\times 10^{-6}\,\mathrm{J\,m^{-3}\,s^{-1}}\qquad T\approx 1.17\times 10^{6}\,\mathrm{K}\qquad n\approx 3.46\times 10^{14}\,\mathrm{m^{-3}}$$

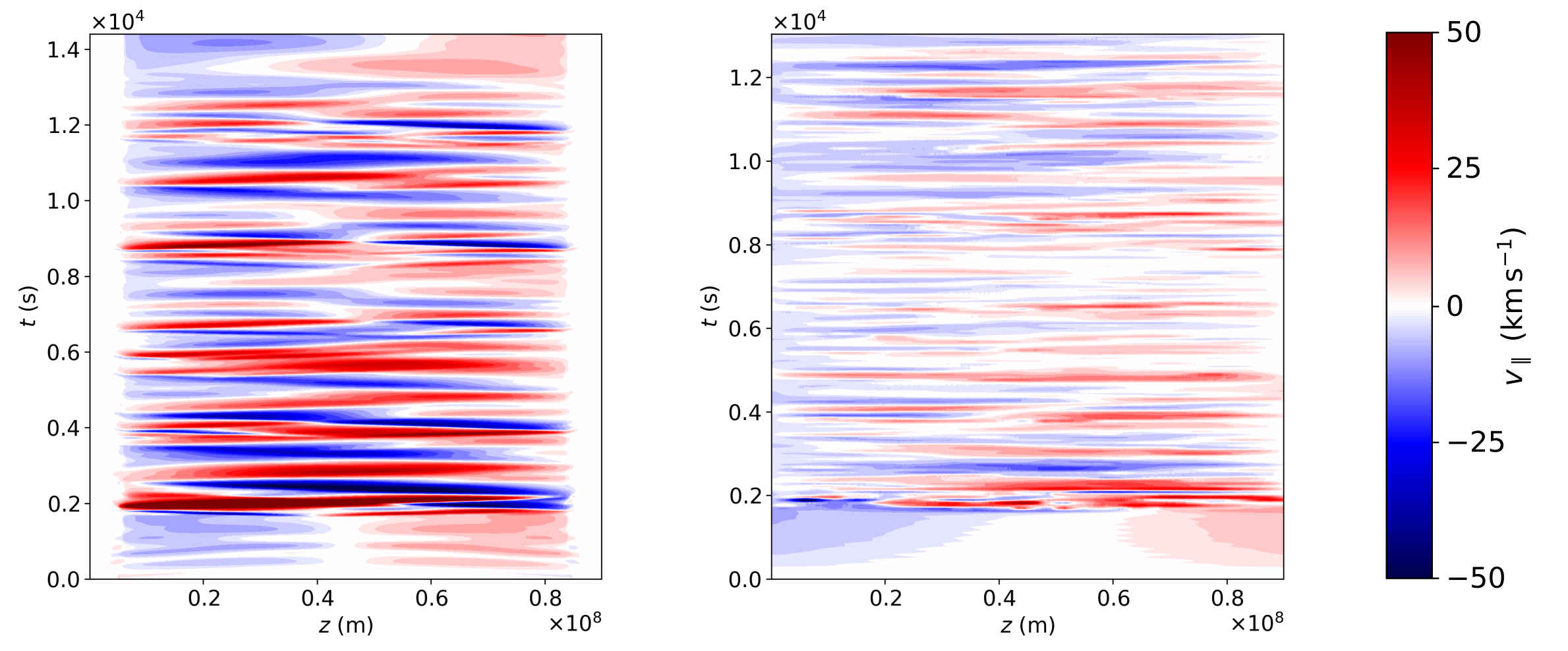
- Broadly, temperature and density change in-phase (although not always)
- 1D thermodynamic and 3D MHD models have similar temperatures and densities
  - MHD models faithful to dynamic evolution, notwithstanding neglecting thermal transport



Temperature. *Density.*

Coronal averages of temperature and density, in 1D thermodynamic (blue) and 3D MHD (red) simulations. Green shows values predicted by scaling laws from given heating.

- Velocity shows a cycle of heating and cooling, leading to upflows and downflows, with alternating signs of parallel velocity, $v_{\|}$
- Isolated bursts can cause upflows up one leg of the loop, and downflows along the other, in asymmetric velocity patterns
  - Assumption of symmetry about apex is not justified
- Parallel 1D flows are of the order of the perpendicular flows in 3D MHD (i.e. the reconnection jets)
  - Reconnection jets, a key observational signature of reconnection, in full, physical loops may be difficult to detect
  - Distinguishing orientation of flows along magnetic field is observationally challenging, but jets and evaporative flows may have detectably different temperatures



1D thermodynamic model. *3D MHD model.*

Parallel velocity in 1D and 3D models.

Conclusions and future work

- MHD avalanches give field-aligned heating that is predominantly viscous, punctuated by intense Ohmic bursts
  - temporally, heating is impulsive and time-dependent, but aperiodic: there is no preferred ‘nanoflare timescale’
  - spatially, heating is narrowly localized and dispersed, without obvious preference
- Thermodynamically, this heating maintains coronal conditions, with temperature and density similar to those in the 3D model
- Field-aligned models have strongly asymmetric velocity profiles, very different from those in 3D MHD simulations
  - 3D MHD and 1D thermodynamic models may predict very different observable signatures in velocity
- Future developments:
  - curved geometry and gravity in 3D model
  - in the long term, rigorous thermodynamic treatment (with TRAC) in 3D MHD model

Published in *Monthly Notices of the Royal Astronomical Society*.

References

Arber, T. D. et al. (2001). ‘A Staggered Grid, Lagrangian-Eulerian Remap Code for 3-D MHD Simulations’. *Journal of Computational Physics* **171**(1), pp. 151–181.
Bradshaw, S. J. and Cargill, P. J. (2013). ‘The Influence of Numerical Resolution on Coronal Density in Hydrodynamic Models of Impulsive Heating’. *The Astrophysical Journal* **770**, 12.
Johnston, C. D. and Bradshaw, S. J. (2019). ‘A Fast and Accurate Method to Capture the Solar Corona/Transition Region Enthalpy Exchange’. *The Astrophysical Journal Letters* **873**(2), L22.
Johnston, C. D. et al. (2020). ‘Modelling the solar transition region using an adaptive conduction method’. *Astronomy & Astrophysics* **635**, A168.
Reid, J. et al. (2018). ‘Coronal energy release by MHD avalanches: continuous driving’. *Astronomy & Astrophysics* **615**, A84.
Reid, J. et al. (2020). ‘Coronal energy release by MHD avalanches: Heating mechanisms’. *Astronomy & Astrophysics* **633**, A158.

`jr93@st-andrews.ac.uk`

`This work has been funded by the Carnegie Trust for the Universities of Scotland.`