Relativistic quantum theory and ab initio simulations of electroweak decay spectra in nuclei

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Introduction

I will present a new computational relativistic quantum-mechanical method for calculating decay rates in different scenarios

Introduction

I will analyze the following β^- decays

Astrophysical

$\Downarrow$

$$^{93}_{40}Zr \rightarrow ^{93}_{41}Nb + e^- + \bar{\nu}_e$$

$$^{176}_{71}Lu \rightarrow ^{176}_{72}Hf + e^- + \bar{\nu}_e$$

$$^{85}_{36}Kr \rightarrow ^{85}_{37}Rb + e^- + \bar{\nu}_e$$

Geological

$\Downarrow$

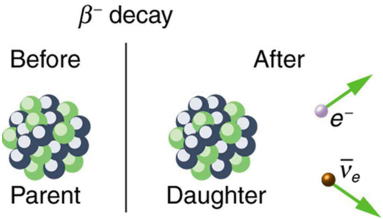
$$^{14}_{6}C \rightarrow ^{14}_{7}N + e^- + \bar{\nu}_e$$

$$^{10}_{4}Be \rightarrow ^{10}_{5}B + e^- + \bar{\nu}_e$$

$$^{87}_{37}Rb \rightarrow ^{87}_{38}Sr + e^- + \bar{\nu}_e$$

The β^- decay

The nuclear β^- decay is a transformation of an unstable nucleus ($^A_Z X$) into a nuclide ($^A_{Z+1} Y$) with the emission of an electron e^- (β^-particle) and an electron antineutrino $\bar{\nu}_e$



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the atomic number Z increases by one, while the mass number A is the same

Selection rules for β-decays

The β-decays can be classified into

Transitions	Angular momentum	Parity
Allowed transitions	$L' = \|j_i - j_f\| = 0, 1$	$\pi_i \cdot \pi_f = +1$
First non-unique forbidden transitions	$L' = \|j_i - j_f\| - 1 = 0, 1$	$\pi_i \cdot \pi_f = -1$
L^{th} non-unique forbidden transitions	$L' > 1$	$\pi_i \cdot \pi_f = (-)^{L'}$
$(L-1)^{th}$ unique forbidden transitions	$L' > 1$	$\pi_i \cdot \pi_f = (-)^{L'-1}$

Theory of beta decay

Fermi's theory of beta decay

$\Downarrow$

$$\mathcal{H}_\beta = G[\bar{\psi}_{f,p}(\vec{r})\gamma^\mu\psi_{i,n}(\vec{r})][\bar{\psi}_{f,e}(\vec{r})\gamma_\mu\psi_{i,\nu}(\vec{r})] + h.c.$$

Generalized Fermi's theory of beta decay

$\Downarrow$

$$\mathcal{H}_\beta = \frac{G_F}{\sqrt{2}}[\bar{\psi}_{f,p}(\vec{r})\gamma^\mu(1-x\gamma^5)\psi_{i,n}(\vec{r})\overline{]}[\psi_{f,e}(\vec{r})\gamma_\mu(1-\gamma^5)\psi_{i,\nu}(\vec{r})]+h.c.$$

Model[1]

Our approach is based on the calculation of the total Hamiltonian

$$H = H_{nucl} + H_{e-e} + H_{weak}$$

where

- H_{nucl} contains the interactions between nucleons in the initial and final nuclear states
- H_{e-e} is the electron-electron Coulomb correlation
- H_{weak} is the weak interaction Hamiltonian

[1] S. Taioli, D. Vescovi, M. Busso, S. Palmerini, S. Cristallo, A. Mengoni, S. Simonucci, 2022, arXiv:2109.14230v2 [astro-ph.SR].

The weak Hamiltonian

The weak Hamiltonian, which satisfies the Lorentz-invariance,

$$H_{weak} = \frac{G_F}{\sqrt{2}} H_\mu L^\mu + h.c.$$

is defined as the product of leptonic

$$L^\mu = \bar{u}_e \gamma^\mu (1 - \gamma^5) v_\nu$$

and hadronic currents

$$H_\mu = \bar{u}_p \gamma_\mu (1 - x\gamma^5) v_n$$

The leptonic part

- The leptonic current is factorized in the independent product of the electron and neutrino quantum field operators
  - Electrons interact via a mean-field, where the Fock term is substituted by the local density approximation (LDA) to the electron gas ($V_{ex} \propto \rho(r)^{1/3}$)
  - The electron wavefunction: Dirac-Hartree-Fock (DHF) equation in a central potential, whose numerical solution was calculated by using a modified Runge-Kutta method
  - The electron wavefunction is expressed in the field produced by both the nucleus and the surrounding electrons as a Slater determinant, to take into account the atomic exchange

$$\left[\sum_i (\vec{\alpha}_i \cdot \vec{p}_i + \beta_i mc^2 + V_i) + \sum_{i<j} (1 - \vec{\alpha}_i \cdot \vec{\alpha}_j) g_{ij}\right] \psi(\vec{r}_1, ..., \vec{r}_N) = E\psi(\vec{r}_1, ..., \vec{r}_N)$$

The leptonic part

- Electrons populate the energy levels according to a Fermi-Dirac (FD) distribution

$$n^i_{e^-} = \frac{1}{1 + e^{(\epsilon_i - \mu_{e^-})/K_B T}}$$

  where

  - the eigenvalues ϵ_i: self-consistent solution of the DHF equation for the leptons
  - the chemical potential μ_{e^-}: electrons behave as an ideal Fermi gas in a box with a relativistic energy-momentum dispersion

$$E^2 = m_e^2 c^4 + c^2 p^2.$$

- The neutrino wavefunction: free-particle Dirac equation

The hadronic part

- The hadronic current, reckoned by mean-field central potential, is separable into neutron and proton field operators
  - The decaying neutron acts as an independent particle correlated only geometrically to the *core* of the remaining nucleons
  - Protons and neutrons interact via a semi-empirical scalar and vector relativistic Wood-Saxon (WS) spherical symmetric potential
  - Protons are considered as non-relativistic particles

Transition probability

The main purpose is to compute the transition probability per unit time

$$N_{i\to f} = 2\pi Tr(\hat{\rho}_i H_{weak} P_f H_{weak})\delta(E_i - E_f) + h.c.$$

where $\hat{\rho}_i = p_i |i\rangle\langle i|$ is a statistical mixture of initial states $|i\rangle$ with

$$|i\rangle = |h_i\rangle \otimes |e_i\rangle$$

and $P_f = \sum_f |f\rangle\langle f|$ a mixture of final states $|f\rangle$ with

$$|f\rangle = |h_f\rangle \otimes |e_f\rangle \otimes |\bar{\nu}_f\rangle$$

Transition probability

The initial multi-nucleon states, the final multi-nucleon states and the initial multi-electron states are characterized by a discrete spectrum

- $|h_{(i,f)}\rangle \longrightarrow |J_{(i,f)}, M_{(i,f)}, T_{(i,f)}\rangle$
- $|e_i\rangle = |j_i, m_i; [n_1^b ... n_k^b]_{(i)}\rangle_{e-e}$

while the final multi-electron state is a continuum one

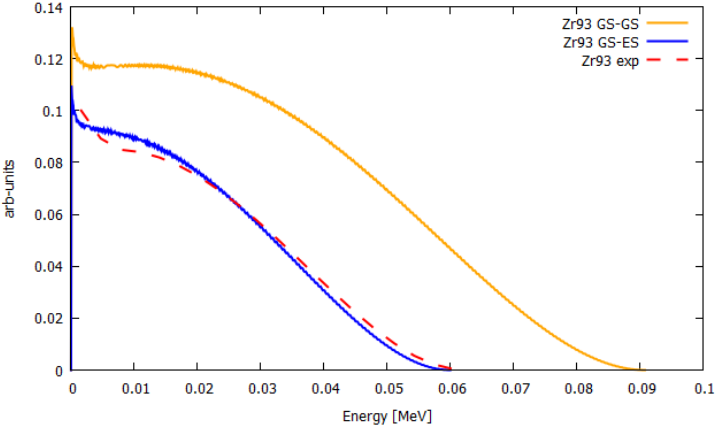
- $|e_f\rangle = \sum_j I_{j,f} \wedge |n_{j,f}\rangle$

the final anti-neutrino state $|\bar{\nu}_f\rangle$ is represented by a free plane wave

Model

- The temperature and density dependence
- The proton density $n_p = 10^{26}$
- The electronic and nuclear excited states (ES) population dynamics
- The non-orthogonality between the bound initial and final orbitals
- The presence of shake-up (excitation) and shake-off (ejection)

$${}^{93}_{40}Zr \rightarrow {}^{93}_{41}Nb + e^- + \bar{\nu}_e$$



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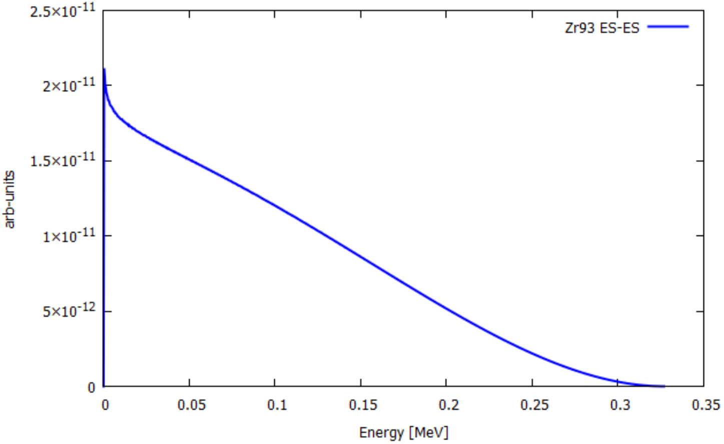

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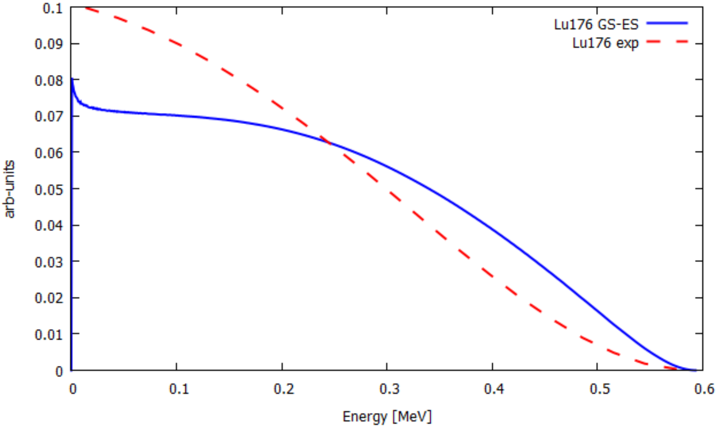

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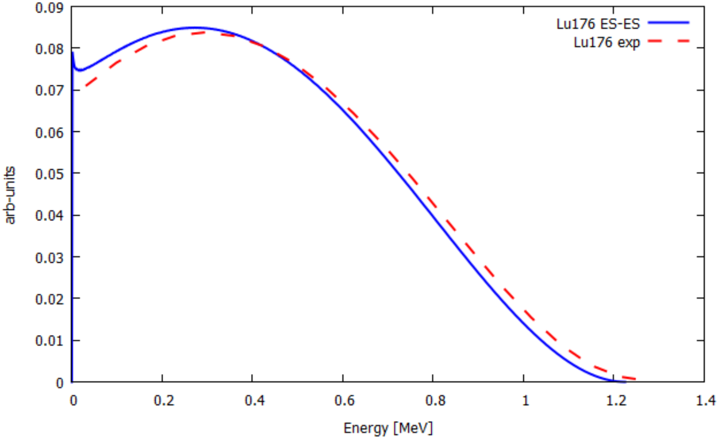

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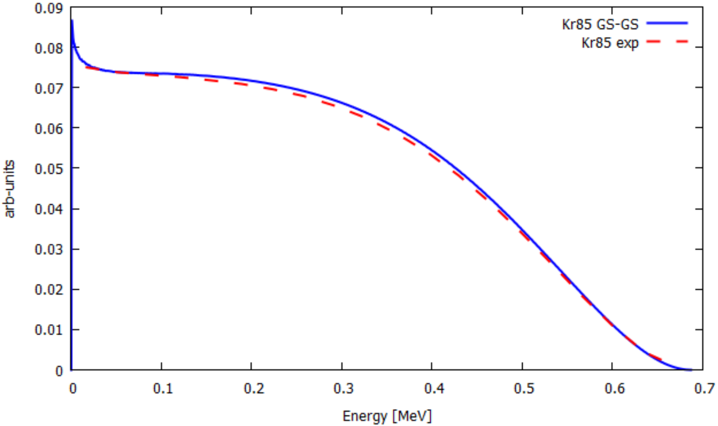

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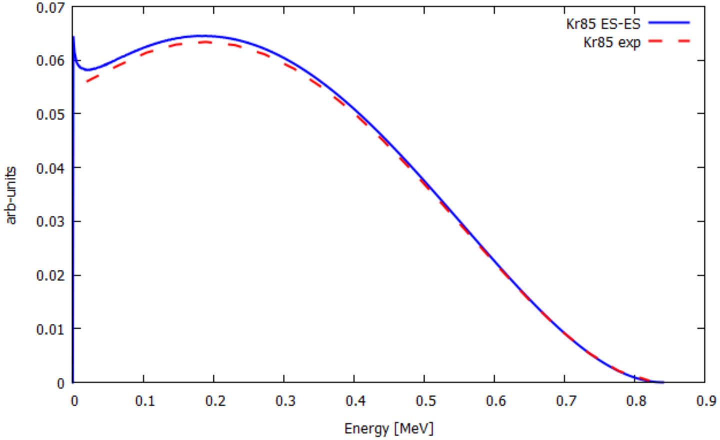

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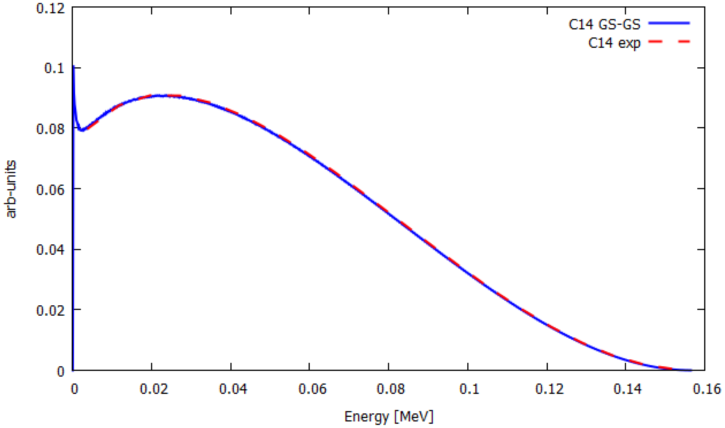

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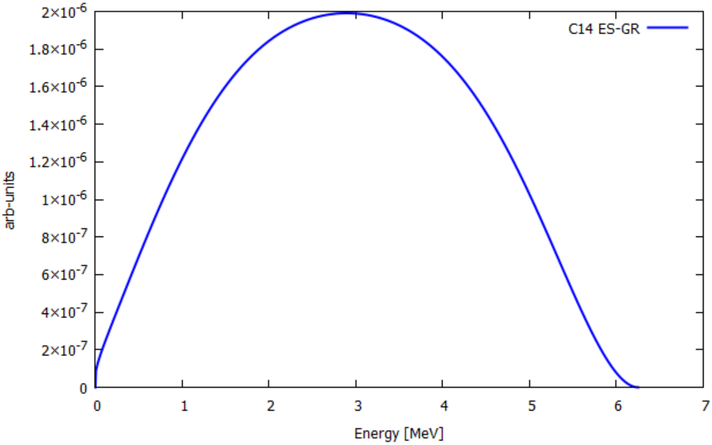

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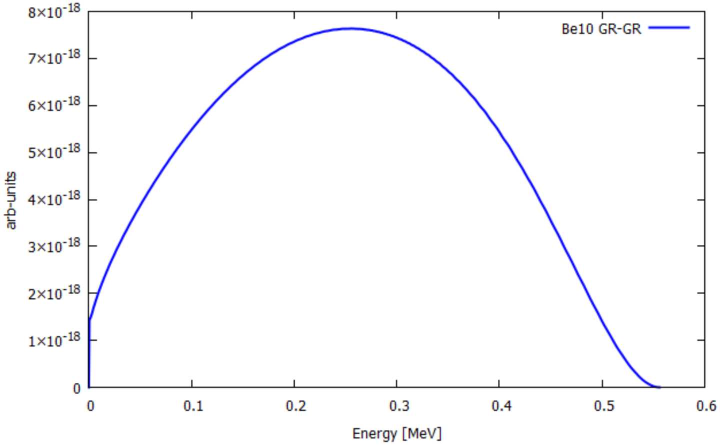

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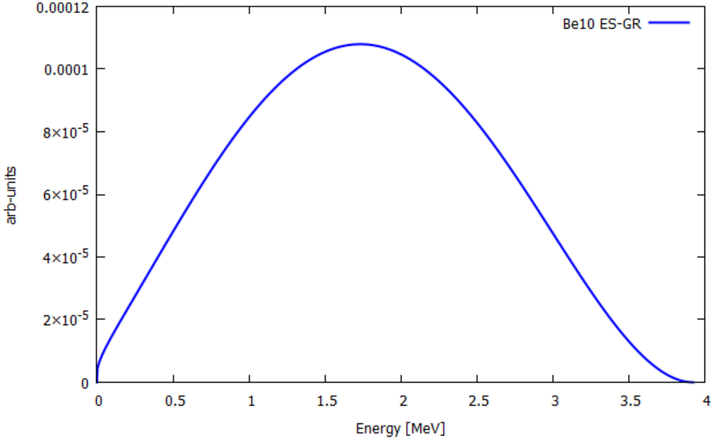

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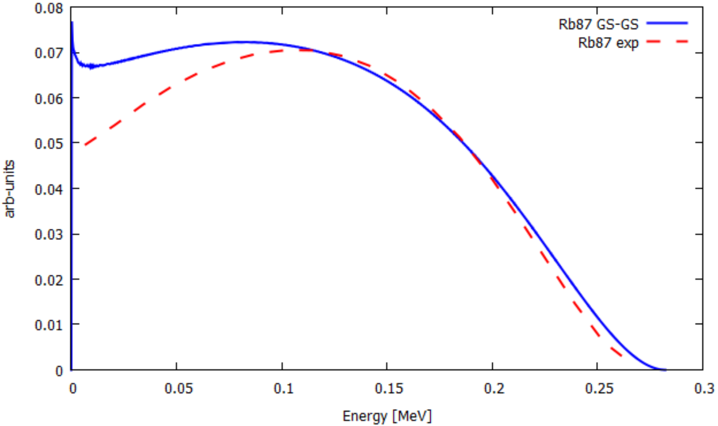

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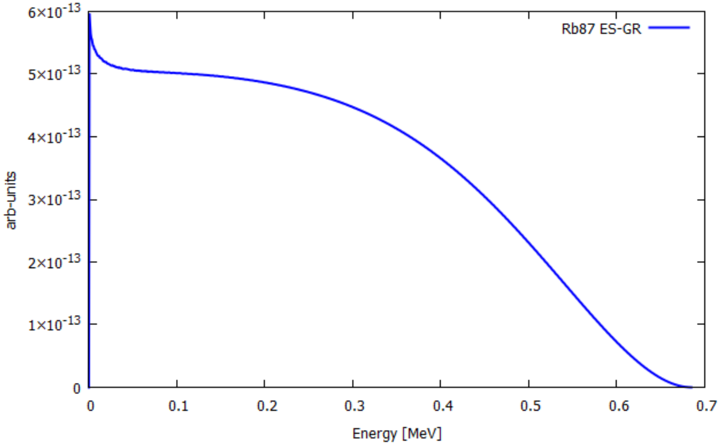

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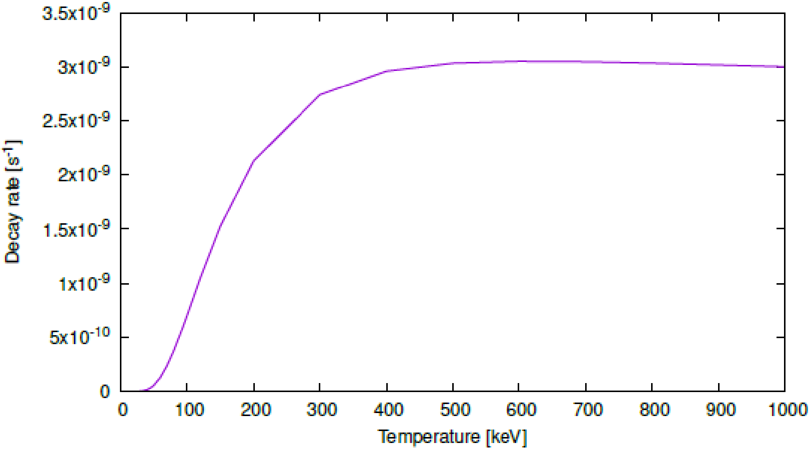

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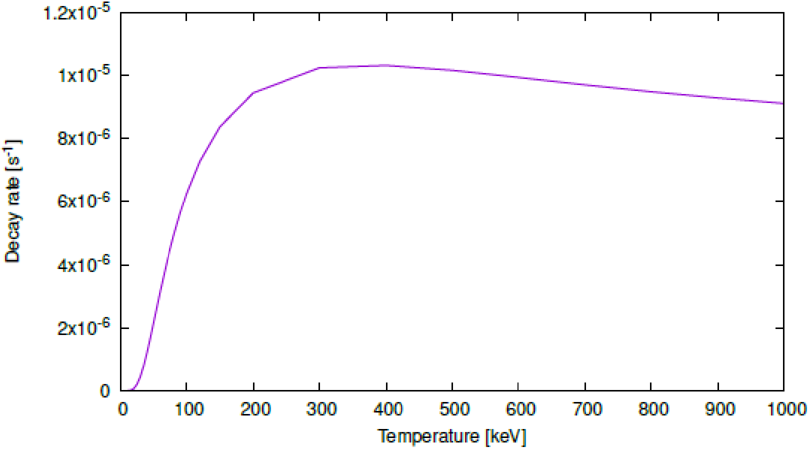

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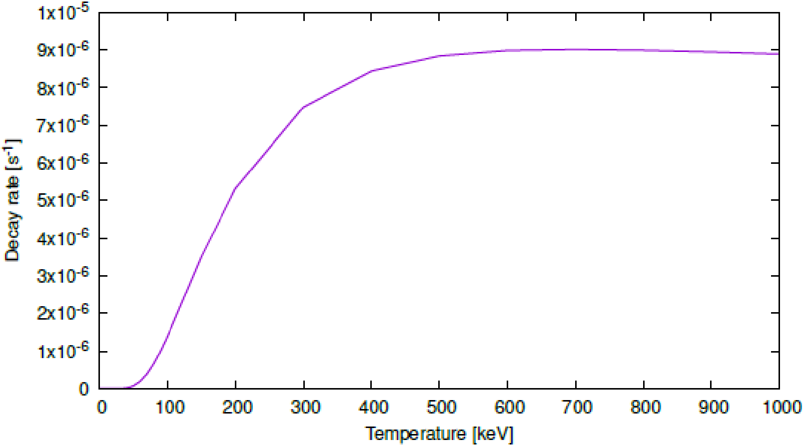

Conclusions

- In most cases the experimental and theoretical data coincide
- Temperature $\rightarrow$ decay rate
- Future goals? Implementation of the model $\rightarrow$ nuclear part

Thanks for the attention