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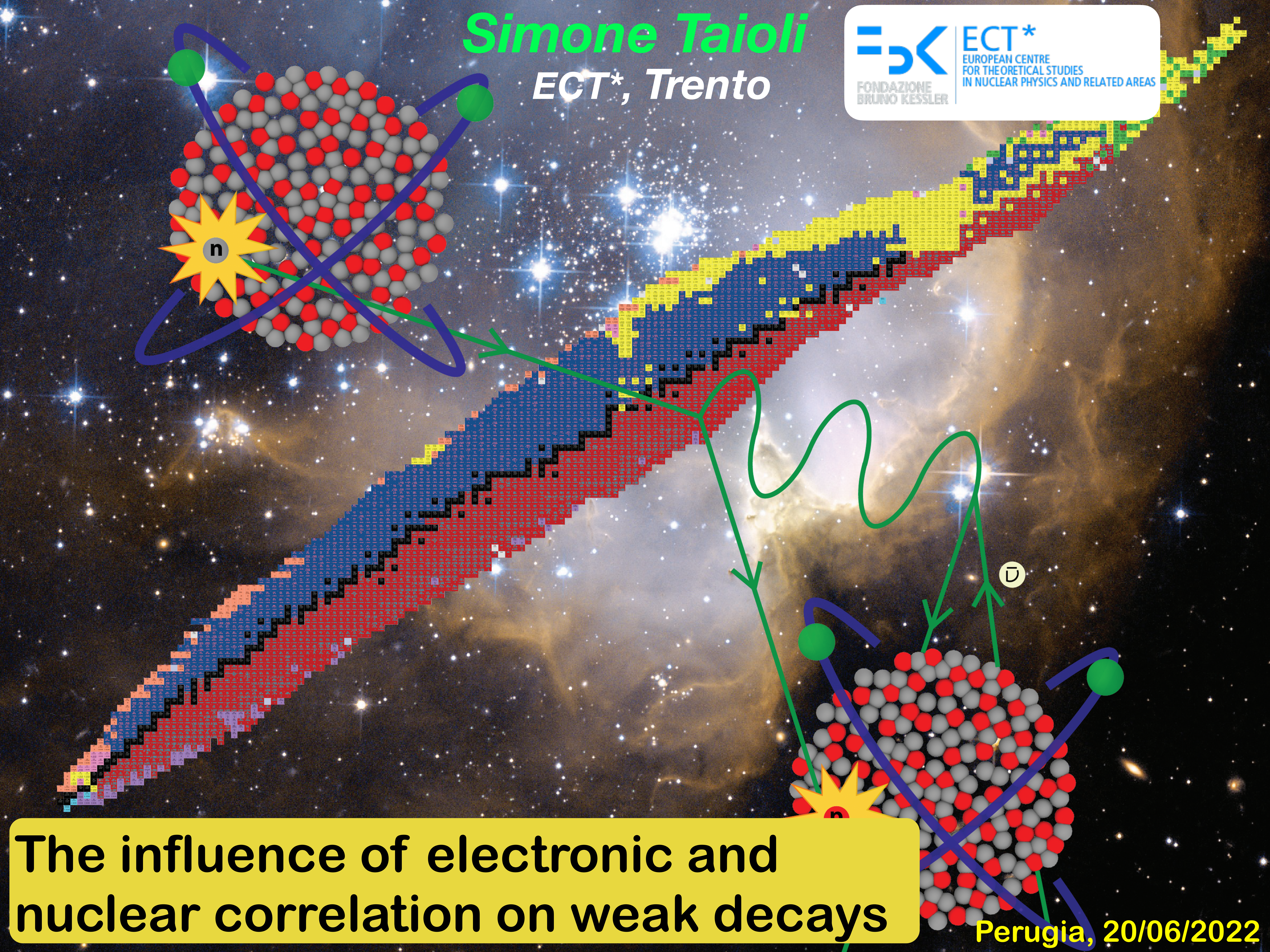


The influence of electronic and nuclear correlation on weak decays

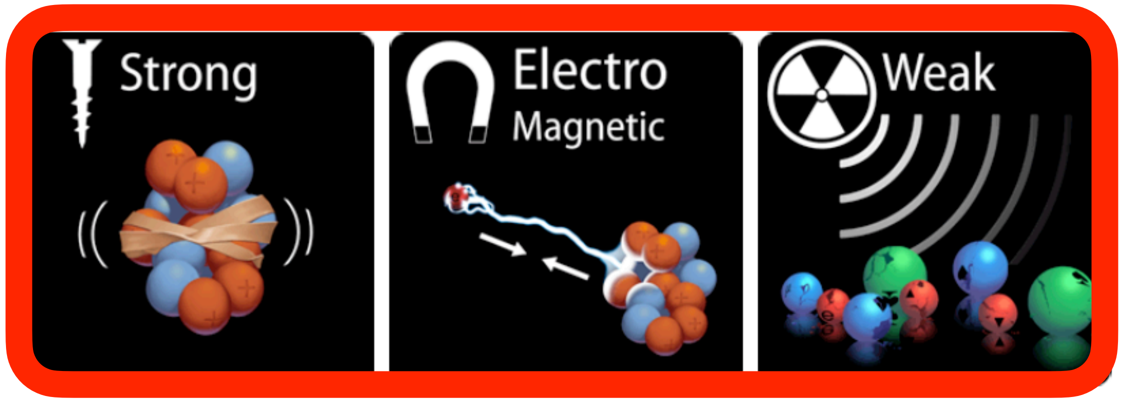
Perugia, 20/06/2022

Outline

an indicator of interaction frequency

10^{-2}
($t=10^{-15}$ s, 10^{-10} m)

1
(W $t=10^{-24}$ s, 10^{-15} m)



Coupl. const.$=10^{-7}$
($t>10^{-8}$ s, 10^{-18} m)

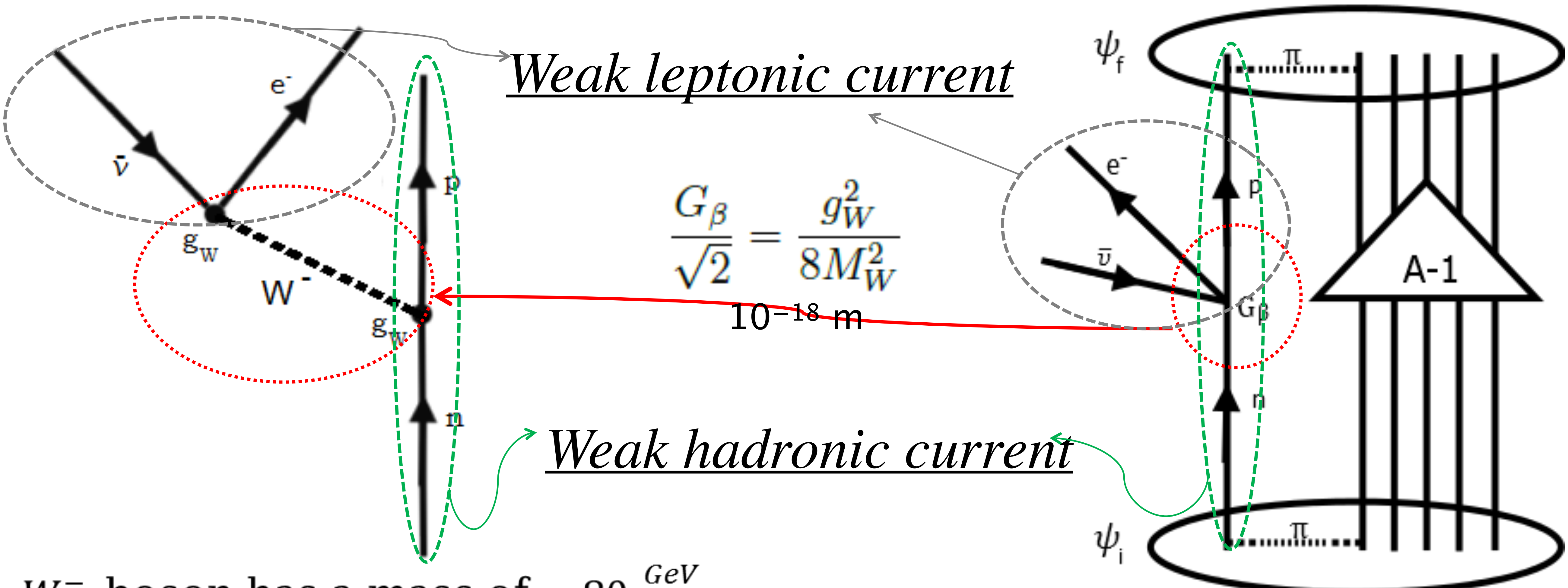
- **Introduction to our (relativistic) approach to β-decay (β-decay and EC are not only purely nuclear but also atomic processes)**

- **Application of our approach to a number of β-decay processes of light (^{7}Be) and heavy (^{134}Cs and ^{135}Cs) atoms in astrophysical scenarios**

- **Perspectives, future developments**

β-decay: tool basket, the nucleus

- **Standard Model of Particle physics: weak interaction is mediated by the emission or absorption of very massive bosons**



W^- boson has a mass of $\sim 80\ \frac{GeV}{c^2}$

⟶ *Uncertainty principle*: W^- can only live $\Delta t \leq \frac{\hbar}{M_W \cdot c^2}$

⟶ During this time it can travel at most $c\Delta t \leq \frac{\hbar}{M_W \cdot c} \sim 10^{-3}\ fm \ll 1\ fm$

Short range=Fermi contact interaction

(range of strong interaction)

We are deep in debt to ....

D. Mascali
(INFN-LNS)

S. Simonucci
(Unicam)

S. Palmerini
(UniPG)

T. Morresi
(Sorbonne)

M. Busso
(UniPG)

Maurizio's living legacy that endures

Let's celebrate the life and science of Prof. M. Busso!

From electronic structure to astrophysics

What I will present today is the very consequence of Maurizio's capability to think out-of-the box and imagine that we could apply our own tool box to tackle modern issues in astrophysics

Maurizio clearly believes that people speaking a different language and coming from a different background could try to improve our knowledge of the stellar processes, in particular to determine those nuclear and atomic inputs that affect many aspects of the overall stellar evolution.

I can hear Maurizio saying on several occasions that in order to understand the isotopic abundances and anomalies measured for example in presolar grains of stellar origin we must upgrade such nuclear inputs, which are poorly assessed

Maurizio’s living legacy that endures

Let’s celebrate the life and science of Prof. M. Busso!

So while our meeting with Maurizio here in Perugia about 10 years ago at one of these boring INFN meetings was random, the idea with which we came out of this meeting was crystal clear.

Maurizio encouraged and actually keeps encouraging us to pursue this goal, because he believes that this is one route to follow if one is to reconcile theory and observations.

I’ll be forever thankful for having introduced me to this extremely challenging field and for sharing to date his passion for this topic. Thank you Maurizio!

Maurizio's living legacy that endures

Following Maurizio's mandate we started by working on a few test cases whose decay may affect the overall stellar evolution

In particular we work on the assessment of the efficiency of the nuclear reaction rates of the nucleosynthesis processes that are in place during the evolution of massive stars, such as the e-capture of Be

Li, Be, and B are rare because they were poorly synthesized in primordial BBN. Elements heavier than beryllium could not be formed in the short period in which BBN occurred before being stopped by expansion and cooling (about 20 minutes)

The enrichment of Li in the Universe is still unexplained

We should rely on stellar nucleo-synthesis

Li is one of the primordial elements produced in Big Bang nucleosynthesis: very fragile

Contrary to these expectations, observations of the Sun and solar-like stars reveal that they undergo extensive Li-depleting processes (Li-dip) during central hydrogen burning as well as in late stages (RGB and AGB)

BBN predicts a ^{7}Li abundance > 3 than observed in metal-poor objects and in low metallicity MS stars

Galactic Cosmic Rays do not produce much ^{7}Li

For low mass star (below 2 - 3 $M_\odot$) Li is predicted to be destroyed in the early phases of evolution, preceding the MS

In interstellar medium Li content is higher than that expected by BBN

^{7}Li is produced in Novae and H-burning intermediate mass stars at the base of their envelope.

A small amount of ^{7}Li is produced in stars, but is thought to be burned during MS as fast as produced when convective processes can carry it to temperatures of a few millions K, where it undergoes p-captures.

The cosmic Li problem before BSM….

Thus, stellar burning depletes Li and its abundance is strongly influenced by several nuclear burning mechanisms as well as by the extension of the convective envelope.

- Production of ^{7}Li via EC (hereinafter "EC") of ^{7}Be

$$^7\mathrm{Be} + e^- \longrightarrow {}^7\mathrm{Li} + \nu_e$$

- production of ^{7}Li via antineutrino capture of ^{7}Be

$$^7\mathrm{Be} + \overline{\nu}_e \longrightarrow {}^7\mathrm{Li} + e^+$$

- production of ^{7}Be via positron capture of ^{7}Li

$$^7\mathrm{Li} + e^+ \longrightarrow {}^7\mathrm{Be} + \overline{\nu}_e$$

- production of ^{7}Be via neutrino capture of ^{7}Li

$$^7\mathrm{Li} + \nu_e \longrightarrow {}^7\mathrm{Be} + e^-$$

These reactions are interesting since previous simulations, which didn't account for electroweak processes at $10 < K_B T < 100$ KeV, gave a too high ^{7}Li abundance compared to observations

State-of-the-art of ^{7}Be beta-decay in the Sun

Theoretical predictions: about 20% of ^{7}Be in the center of the Sun might have a bound electron

Despite the main decay channel ^{7}Be in the Sun's interior is the free EC, bound e- can significantly change the decay probability

Bachall (1962): ^{7}Be lifetime in solar conditions for partially ionised atoms using DH screening for the e-e interaction shorter by 10%

Shaviv et al. (2001): ^{7}Be lifetime larger by ~ 20%–30% as compared to Bahcall due to fully ionized ^{7}Be is in the solar plasma

Quarati et al. (2009): ^{7}Be lifetime shorter by about 15%, using a modified DH screening potential.

Out-of-the-Sun conditions

Adelberger (2011): the most recent recommendations are based on the data extrapolation from Bahcall's, even though this procedure is rather insecure due to the very different ambient conditions.

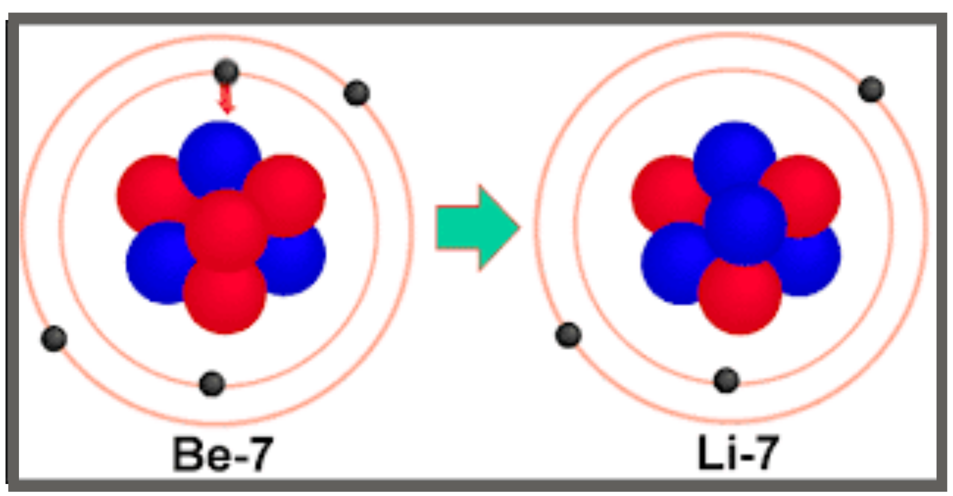
The situation is therefore quite unsatisfactory with a lot of scattered data and uncertainties affecting Li abundances in the Sun, MS, and RGB stars, such as the T and ρ dependence of the half-life.

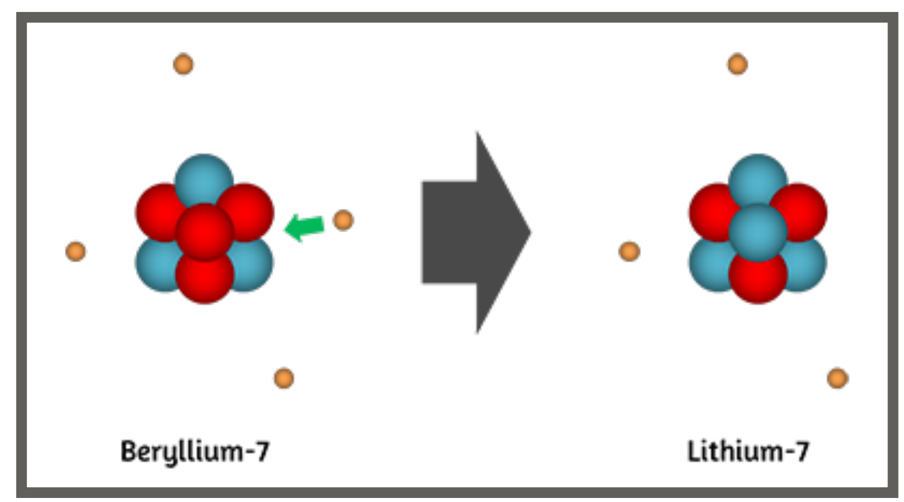
Open issues in Li depletion

Quantitative modeling is in particular hampered by a poor knowledge of how the Be decay rate changes in the rapidly varying conditions below the envelopes of Red Giants (T=1:80 MK, ϱ =1:5 o.o.m. lower than the Sun)

very different from those of our Sun (T_{core} = 10^7 K and $\varrho \sim 10^5$ kg/m^3)

In this conditions ^{7}Be is totally ionized, matter is a plasma that contains charged particles: positive ions, protons and electrons. Decay occurs by capture of an electron moving from both bound and energy continuum (or excited) states in the presence of the other charged particles screening its motion (Bahcall - 1962).





The lure and the lore of correlation

The classical Debye-Hückel approximation may not actually hold out of solar conditions (degenerate Fermi gas ???) and we do not know whether this introduces small or large deviations in the capture rate

Motivation of this work: provide the missing weak-interaction input data for Li nucleosynthesis calculations

$$^{7}_{4}\mathrm{Be} + e^{-} \rightarrow \,^{7}_{3}\mathrm{L}i + \nu_e$$

At ambient conditions ^{7}Be decays in 53 days into the ground state of ^{7}Li (3/2-) for 89.7% of cases, 10.3% it decays into the first excited state (1/2-)

Decay may occur by capture of an orbital e^{-} through the following process:

$$p + e^{-} \rightarrow n + \nu_e$$

change Z to Z-1 of an atom, keeping fixed A

$$W \propto \delta(\vec{r})$$

insensitive to extra-nuclear factors, such as chemical environment, ionization degree, pressure and temperature.

The driving force responsible for this decay is the weak interaction

One of the three β-decays mediated by the weak force.

Contrary to this simple view, there is evidence of changes in nuclear decay rates with these parameters. Why and how?

β-decay: standard approach

A typical nuclear β-decay process reads:

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}X^{'}_{N-1} + e^{-} + \bar{\nu}_{e} $$

Q-value: total energy released by the reaction ($m_\nu = 0$)

$$ Q_{\beta^-} = m_N(^AX) - m_N(^AX') \longrightarrow m_N(^AX) = m(^AX) - Zm_e + \sum_i^Z B_i $$

$$ Q_{\beta^-} = \{[m(^AX) - Z \cdot m_e] - [m(^AX^{'}) - (Z+1) \cdot m_e] - m_e\} \cdot c^2 + \{\sum_{i=1}^{Z} B_i - \sum_{i=1}^{Z+1} B_i\} $$

In the traditional theory of β-decay processes, spectra are typically calculated as product of three factors:

- **a phase-space factor to deal with the momentum sharing between the β-electron (p) and neutrino (q);**
- **a Fermi function F(Z,W) to take into account the static corrections due to the Coulomb field of the nucleus;**
- **a shape factor C(W) to include the coupling between nuclear and lepton dynamics.**

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Extra-nuclear factor

$$ Q_{\beta^-} = \{[m({}^{A}X) - Z \cdot m_e] - [m({}^{A}X') - (Z+1) \cdot m_e] - m_e\} \cdot c^2 + \{\sum_{i=1}^{Z} B_i - \sum_{i=1}^{Z+1} B_i\} $$

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$$ \frac{dN}{dW} \propto pWq^2 F(Z,W) C(W) $$

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$$ \nu = \pm Ze^2/\hbar v $$

$$ \frac{dN}{dW} \propto pWq^2 F(Z,W) C(W) \qquad F(Z,W) = \frac{2\pi\nu}{1 - \exp^{-2\pi\nu}} $$

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$$ \frac{dN}{dW} \propto pWq^2F(Z,W)C(W) \qquad C(W) = (2L'-1)!\sum_{k=1}^{L'} \lambda_k \frac{p^{2(k-1)}q^{2(L'-k)}}{(2k-1)![2(L'-k)+1]!} $$

- **a phase-space factor to deal with the momentum sharing between the β-electron (p) and neutrino (q);**
- **a Fermi function F(Z,W) to take into account the static corrections due to the Coulomb field of the nucleus;**
- **a shape factor C(W) to include the coupling between nuclear and lepton dynamics.** No sign of correlation!!!

β-decay: standard approach

$$\frac{dN}{dW} \propto pWq^2F(Z,W)C(W)$$

It works well to predict the lineshape allowed and forbidden unique transitions, at variance, nuclear structure effects cannot be neglected when dealing with forbidden non-unique transitions, and there is no such a simple relation for C(W)

One can treat first forbidden non-unique transitions as allowed if

$$2\xi = \frac{\alpha Z}{R_{nuc}} >> E_{max}$$

where E_{max} is the maximum escaping energy of the β-electron and α is the fine structure constant

Still a rigours treatment of these transitions including electronic and nuclear DOF is missing!!!

Our approach to beta-decay aims to solve these issues

How do we actually calculate e-capture rates?

In particular we use the theory of scattering under two potentials in the center of mass, reducing the problem to a two-body scattering:

V = screened, short-range Coulomb potential
W = weak interaction coupling the Coulomb distorted initial state and the final decay channels

The cross section of the electron capture process can be written as:

$$\sigma_{i\to f} = \int \frac{d^3k}{(2\pi)^3}\frac{2\pi}{v}\left|\langle\psi^-_{f,\boldsymbol{k}}|W|\phi^+_{i,\boldsymbol{p}}\rangle + \langle\phi^-_{f,\boldsymbol{k}}|V|\phi_{i,\boldsymbol{p}}\rangle\right|^2 \delta\left(\frac{p^2}{2m_e}+E_i-E_f-ck\right)$$

$$= \int \frac{d^3k}{(2\pi)^3}\frac{2\pi}{v}\left|\langle\phi^-_{f,\boldsymbol{k}}|T_w|\phi^+_{i,\boldsymbol{p}}\rangle\right|^2 \delta\left(\frac{p^2}{2m_e}+E_i-E_f-ck\right)$$

=0 — Coulomb operator does not couple ini and fin channels

$\phi_{i,\boldsymbol{p}}$ **= free-plane wave**

$\phi^-_{f,\boldsymbol{k}}$ **= Coulomb perturbed out-state (ʋ emitted and target in final state f)**

$\psi^-_{f,\boldsymbol{k}}$ **= Coulomb and weak perturbed out-state (ʋ emitted and target in final state f)**

$\phi^+_{i,\boldsymbol{p}}$ **= Coulomb perturbed in-state (Coulomb distort + outgoing spherical)**

E_i, E_f = internal energies of the target ^{7}Be and of the final decay product
p = m_ev and k are relative e^- and neutrino momenta in the initial and final channels
v = electron velocity in the initial channel relative to ^{7}Be.

How do we actually calculate e-capture rates?

We can define the T-matrix of the weak interaction as:

$$\left\langle \psi^{-}_{f,\boldsymbol{k}} | W | \phi^{+}_{i,\boldsymbol{p}} \right\rangle = \left\langle \phi^{-}_{f,\boldsymbol{k}} | \mathrm{T}_W | \phi^{+}_{i,\boldsymbol{p}} \right\rangle$$

By multiplying the c.s. by the e$^-$ current one obtains the e-capture rate:

$$\Gamma_{i\to f} = \int 2\pi \frac{d^3k}{(2\pi)^3} \left| \left\langle \phi^{-}_{f,\boldsymbol{k}} | T_w | \phi^{+}_{i,\boldsymbol{p}} \right\rangle \right|^2 \delta\left(\frac{p^2}{2m_e} + E_i - E_f - ck \right) = \frac{\bar{k}^2}{\pi c} \left| t_{f,i} \langle i,0 | \phi^{+}_{i,\boldsymbol{p}}(0) \rangle \right|^2$$

$$= \frac{1}{\pi c^3} |t_{f,i}|^2 \langle i,0 | \phi^{+}_{i,\boldsymbol{p}} \rangle \left(\frac{p^2}{2m_e} + E_i - E_f \right)^2 \langle \phi^{+}_{i,\boldsymbol{p}} | i,0 \rangle$$

$|i,0><i,0|$

$T_W \propto \delta(\boldsymbol{r})$

$H_0 + V$

where $\bar{k} = \frac{1}{c}\left(\frac{p^2}{2m_e} + E_i - E_f \right)$ **and** $\langle \phi^{+}_{i,\boldsymbol{p}} | i,0 \rangle$ **is the electronic w.f. at the Be nucleus.**

Approximations made

1. 1st Be e.s. is found at 429.4 keV=5X10^9 K above the ground state
2. $T_W \propto \delta(\boldsymbol{r})$ = very short range contact interaction
3. $t_{f,i}$ are chosen equal to those measured on the Earth, neglect dependence on T and $p^2/2m_e$

IMPORTANT OUTCOME!

^{7}Be-e$^-$ can be modelled as a two-body scattering process at a given relative electron momentum p.

The rate is proportional to $\rho_e(0)$.

Be e-capture

$$ {}^{7}_{4}\mathrm{Be} + e^- \rightarrow {}^{7}_{3}\mathrm{L}i + \nu_e $$

At ambient conditions ^{7}Be decays in 53 days into the ground state of ^{7}Li (3/2-) for 89.7% of cases, 10.3% it decays into the first excited state (1/2-)

The energy of the Li excited state is 477.6 keV (~6X10^9 K) higher than GS

Be Q_0 and Q_1 the kinetic energies of the neutrinos escaping from ^{7}Li in its ground and first excited state

Q_0 = 861.815 keV ⟷ Q_1 = Q_0 - 477. 6 = 384. 2 keV

Since the kinetic energy is higher in the first case, the available phase space will be larger. We can roughly estimate that for T = 10^7 K:

$$ BR = 89.7/10.3 \times (Q_0 + kT)^2 / (Q_1 + kT)^2 / (Q_0^2/Q_1^2) = 8.684 $$

The percentage variation of BR due to an increase of the temperature by five orders of magnitude is thus only 0.3% → **neglect e.s. decay!!!**

1st Be e.s. is found at 429.08 keV=5X10^9 K above the ground state

Our framework

find a good theory to model for different T and ϱ the hot plasma composed by ^{7}Be atoms surrounded by N_p protons (hydrogen nuclei) and N_e electrons, as a degenerate (quantum) Fermi gas, taking into account accurately the electron-electron interaction!

How to calculate ρ_e?

State-of-the-art techniques are based on the the Debye-Hückel (DH) models of screening, valid only for solar conditions and when electrons are not degenerate (but in RBG they could).

Does DH approximation really stand???

Condition of the stellar material at high T

DEGENERACY CONDITIONS: CLASSICAL vs. QUANTUM

- The separation between identical particles is << λ_{DB}
- The density is >> N_q where N_q is the number of available quantum states

Solar core: T=15.6 X 10^6 K → ^{7}Be atoms are all ionized

(12000 K = 1 eV)!!!

De Broglie wavelength in the core of the Sun

$$l << \lambda_{DB} = h/p \simeq h/(3m_e kT)^{1/2} = 2.731 \times 10^{-11}\ m$$

Electronic density

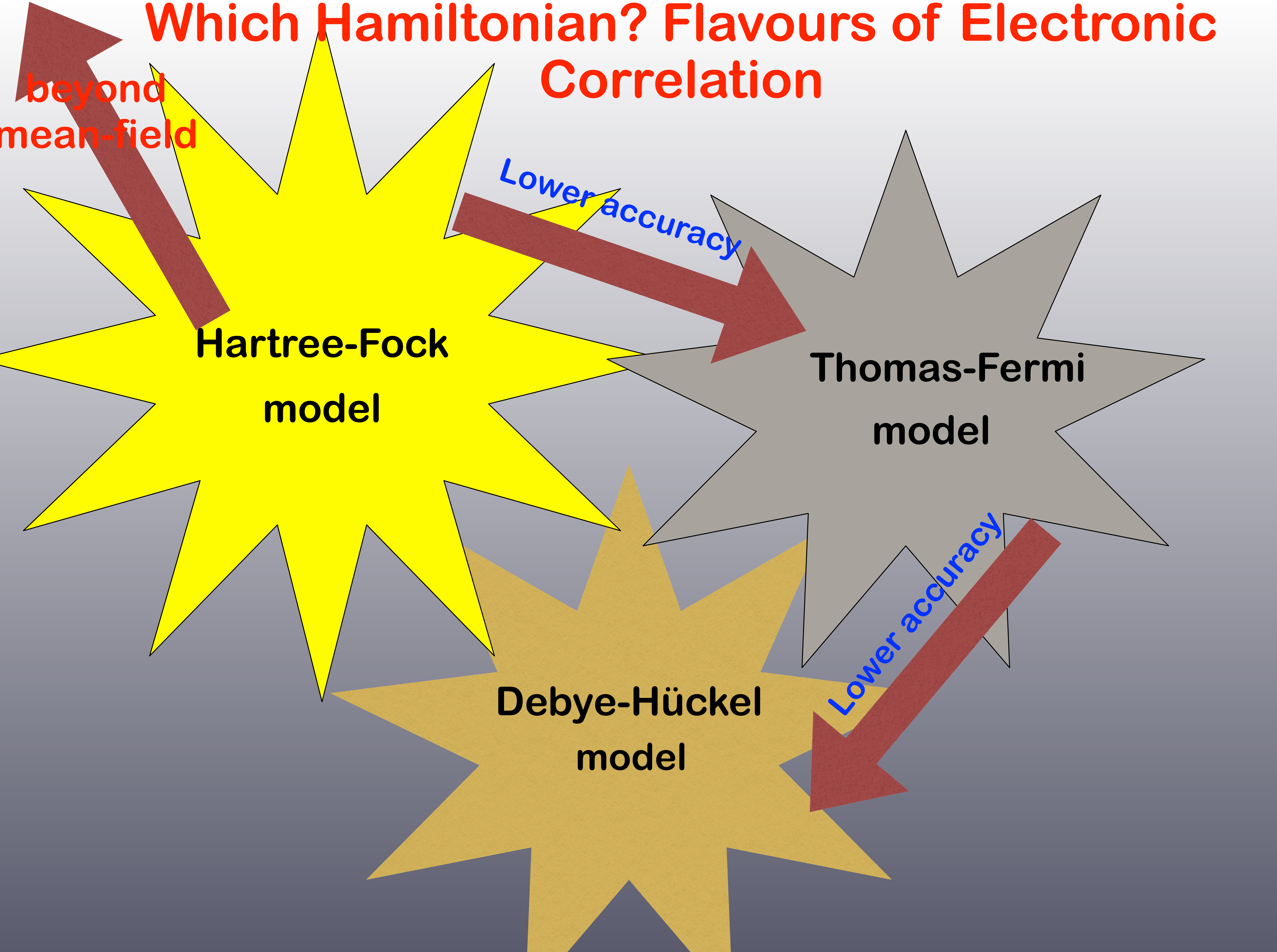
$$\rho_e >> n_{QNR} = (2\pi m_e kT/h^2)^{3/2} = 6.65 \times 10^{31}\ m^{-3}$$

To have degeneracy $T << h^2\rho^{2/3}/(2\pi mk) = 9.12 \times 10^6$ K

In the solar core the temperature is marginally too high for degeneracy of electrons, but decreasing R can set it in...

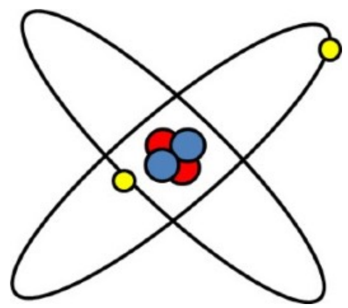
$T \propto 1/R$ and thus $n_{QNR} \propto T^{(3/2)} \propto R^{-3/2}$, which cannot keep the pace with $\rho_e \propto R^{-3}$ **Cold? Fermi gas can be degenerate even at millions of K.**

Which Hamiltonian? Flavours of Electronic Correlation



Hartree-Fock, TF, DH within BO approximation

There are 2 mechanisms to avoid each other: exchange and correlation, both lower the total energy and dress the e^--e^- bare interaction.



Many-body problem is replaced by many 1-body problem in which e^- are independent and feel an average potential $V_m(r)$

Correlation keeping the electrons apart is just among unsociable same spin electrons: Pauli exclusion principle

$$\Psi(r) = \psi_a(r_1)\psi_b(r_2) - \psi_a(r_2)\psi_b(r_1)$$

Thomas and Fermi (1920s) were the first to give an approximate expression of E as a function of the electronic density.

The kinetic, electronic exchange and correlations terms are taken from the theory of the uniform electron gas:

$$E[\rho] = T_{\mathrm{TF}}[\rho] + V_{\mathrm{ne}}[\rho] + J[\rho] + E_{\mathrm{x}}[\rho] + V_{\mathrm{nn}}$$

$$T_{\mathrm{TF}}[\rho] = C_{\mathrm{F}} \int_{\mathbb{R}^3} \rho^{5/3}(\mathbf{r})\, d^3\mathbf{r} \qquad E_{\mathrm{x}}[\rho] = -C_{\mathrm{x}} \int_{\mathbb{R}^3} \rho^{4/3}(\mathbf{r})\, d^3\mathbf{r}$$

Electronic density is far from uniform in a plasma

DH: Fermi-Dirac statistics to Boltzmann distribution linear in T

Some data...

Energy of the Isolated Beryllium Atom in Atomic Units and Spin-up Density at the Nucleus Obtained Through the HF and CI Calculations

	Energy	$\rho_{e\uparrow}(0)$
Hartree–Fock	−14.573	17.68521
Full-CI	−14.660	17.68060

Degenerate condition

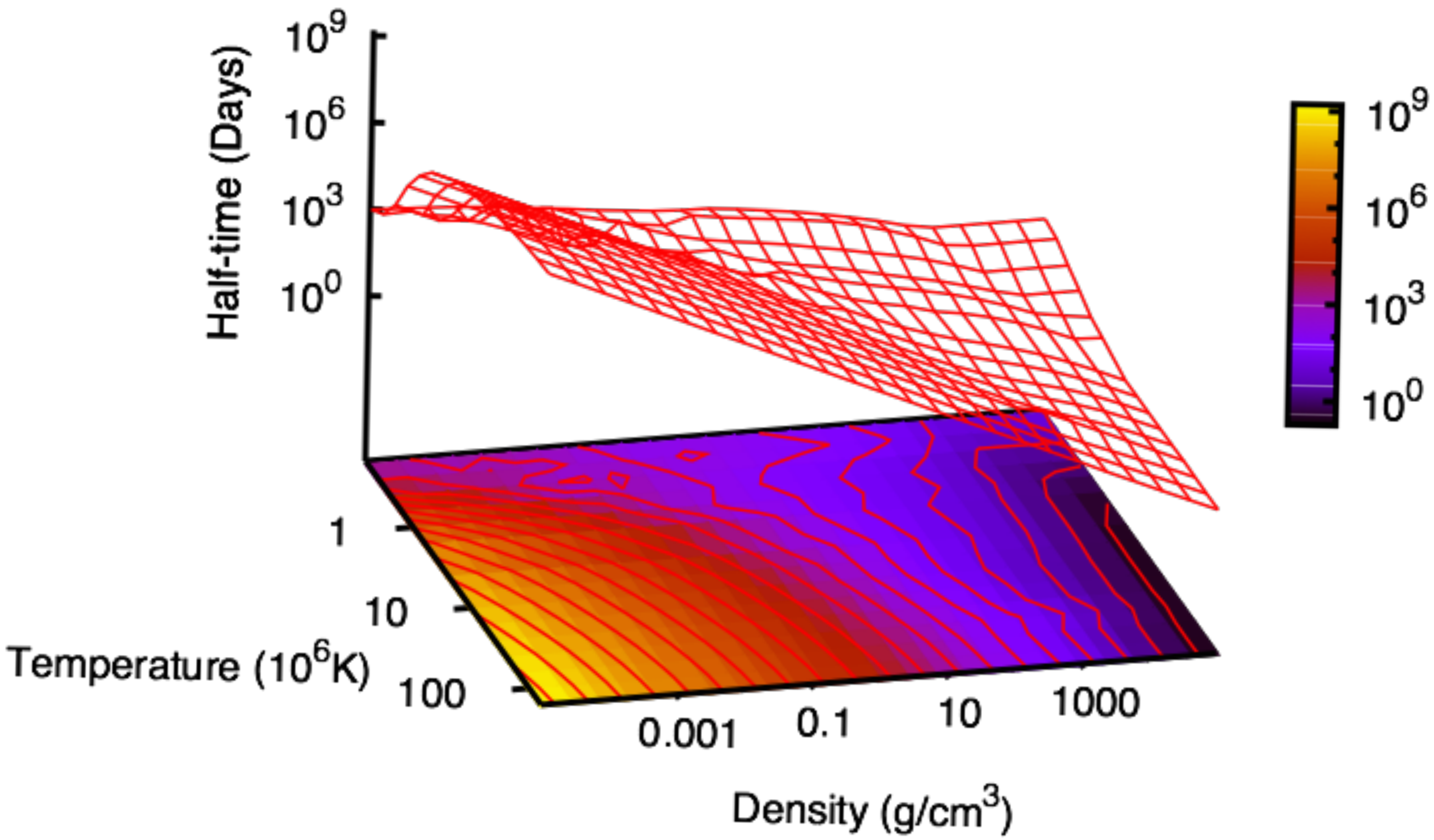
Solar condition

ρ (g cm^{-3})	T (10^6 K)	λ_{Debye} $a.u.$	$\lambda_{\mathrm{De\,Broglie}}$ $(e-p)$	$\rho_{\mathrm{HF}}(0)$ $a.u.$	$\rho_{\mathrm{TF}}(0)$	$\rho_B(0)$	$\rho_{\mathrm{DH}}(0)$
1000.	1.	0.038	1.409–0.0329	71.87 ÷ 71.97	68.99 ÷ 69.11	42.61 ÷ 42.74	47.46 ÷ 47.55
100.		0.119		33.52 ÷ 33.53	29.53 ÷ 29.55	4.027 ÷ 4.031	19.13 ÷ 19.14
10.		0.377		17.37 ÷ 17.37	13.83 ÷ 13.83	0.945 ÷ 0.945	13.33 ÷ 13.33
1.		1.193		7.839 ÷ 7.837	5.708 ÷ 5.707	0.184 ÷ 0.184	8.151 ÷ 8.149
0.1		3.771		1.940 ÷ 1.940	1.415 ÷ 1.415	0.044 ÷ 0.044	2.059 ÷ 2.058
0.01		11.93		0.278 ÷ 0.278	0.220 ÷ 0.220	0.0075 ÷ 0.0075	0.279 ÷ 0.279
0.001		37.71		0.0308 ÷ 0.0308	0.0264 ÷ 0.0264	0.0012 ÷ 0.0012	0.0303 ÷ 0.0303
1000.	10.	0.119	0.445–0.0103	122.43 ÷ 122.89	116.21 ÷ 116.68	51.77 ÷ 52.05	108.56 ÷ 109.01
100.		0.377		20.23 ÷ 20.27	19.53 ÷ 19.57	10.36 ÷ 10.39	19.54 ÷ 19.58
10.		1.193		2.578 ÷ 2.581	2.554 ÷ 2.558	2.515 ÷ 2.519	2.570 ÷ 2.573
1.		3.771		0.274 ÷ 0.275	0.274 ÷ 0.275	0.274 ÷ 0.274	0.274 ÷ 0.275
0.1		11.93		0.0281 ÷ 0.0282	0.0281 ÷ 0.0282	0.0281 ÷ 0.0282	0.0281 ÷ 0.0281
0.01		37.71		$(2.84 \div 2.84) \times 10^{-3}$	$(2.84 \div 2.84) \times 10^{-3}$	$(2.84 \div 2.84) \times 10^{-3}$	$(2.83 \div 2.83) \times 10^{-3}$
0.001		119.3		$(2.84 \div 2.84) \times 10^{-4}$	$(2.84 \div 2.84) \times 10^{-4}$	$(2.84 \div 2.84) \times 10^{-4}$	$(2.84 \div 2.84) \times 10^{-4}$
1000.	100.	0.377	0.141–0.0033	78.31 ÷ 80.39	78.24 ÷ 80.32	76.57 ÷ 78.64	78.22 ÷ 80.30
100.		1.193		9.051 ÷ 9.289	9.051 ÷ 9.288	9.031 ÷ 9.268	9.051 ÷ 9.288
10.		3.771		0.773 ÷ 0.787	0.773 ÷ 0.787	0.773 ÷ 0.787	0.773 ÷ 0.787
1.		11.93		0.0775 ÷ 0.0789	0.0775 ÷ 0.0789	0.0775 ÷ 0.0789	0.0775 ÷ 0.0789
0.1		37.71		$(7.75 \div 7.90) \times 10^{-3}$	$(7.75 \div 7.90) \times 10^{-3}$	$(7.75 \div 7.90) \times 10^{-3}$	$(7.75 \div 7.90) \times 10^{-3}$
0.01		119.3		$(7.75 \div 7.90) \times 10^{-4}$	$(7.75 \div 7.90) \times 10^{-4}$	$(7.75 \div 7.90) \times 10^{-4}$	$(7.75 \div 7.90) \times 10^{-4}$
0.001		377.1		$(7.75 \div 7.90) \times 10^{-5}$	$(7.75 \div 7.90) \times 10^{-5}$	$(7.75 \div 7.90) \times 10^{-5}$	$(7.75 \div 7.90) \times 10^{-5}$

^{7}Be half-life

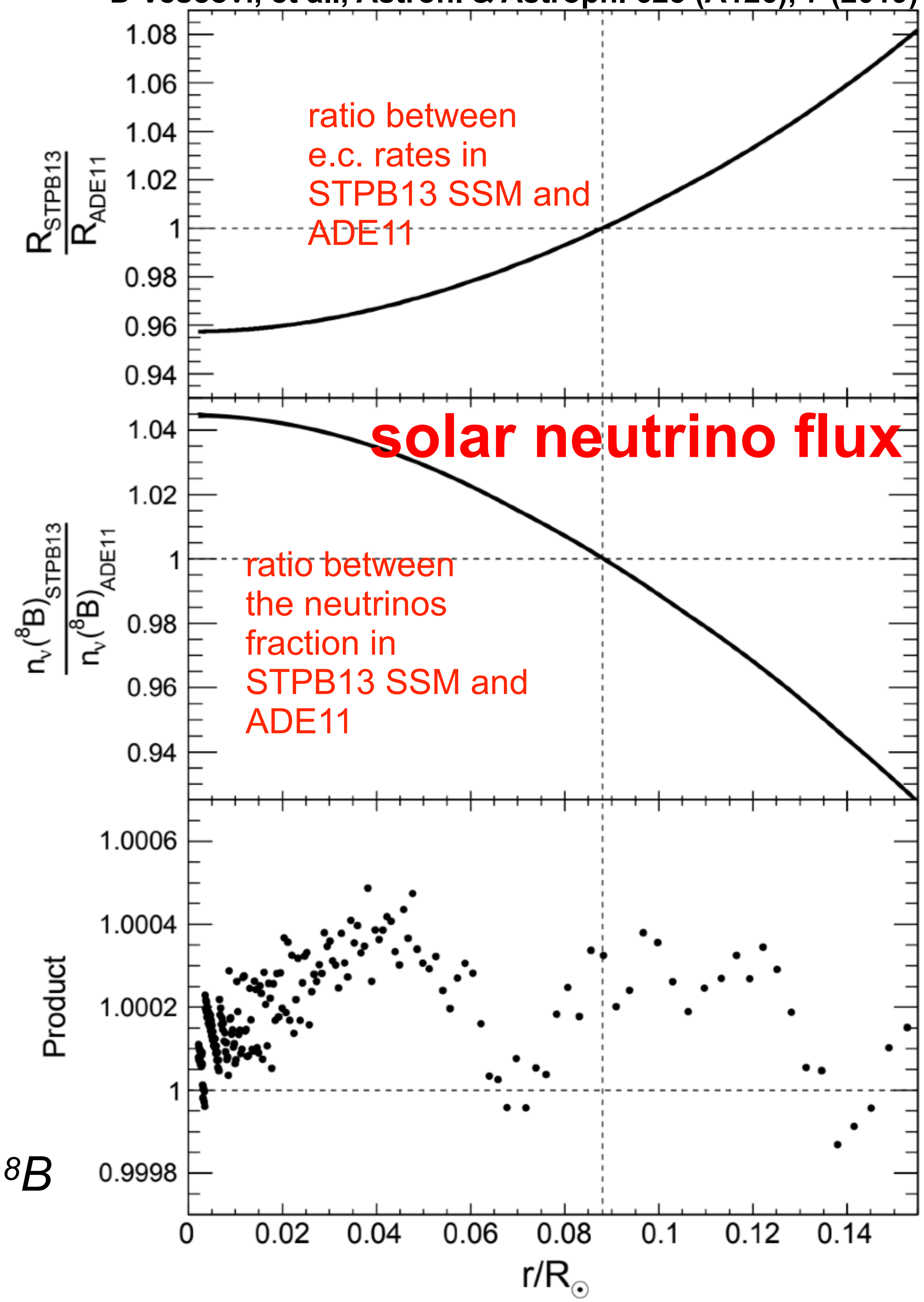
7Be and 8B neutrinos are produced in a hotter and narrower zone, ranging from the solar centre to about 0.15-0.2 R⊙

half-life (days)= 941.86881/$\varrho(0)$



S Simonucci, S Taioli, et al., The Astrophysical Journal 764 (2), 118

D Vescovi, et al., Astron. & Astroph. 623 (A126), 7 (2019)



A longer lifetime of ec ^{7}Be: more destruction via the ^{7}Be(p, γ)^{8}B changing the yield of the solar neutrino flux.

The new yield leads to a maximum difference in the efficiency of the ^{7}Be channel of about -4 % with respect to what is obtained with the previously adopted rate. This fact affects the production of neutrinos from ^{8}B, increasing the relative flux up to a maximum of 2.7%

Almost all OK for light nuclei!

However in AGB stars > $M_\odot$ the s-process represents the mechanism by which heavy nuclei from Sr to Bi are produced, and the nucleosynthesis path wanders along the valley of β-stability.

Still important issues are to be solved to reach a whole comprehension of s-process: several branching points along the s-process path require revisions of rates of both β-decays and n-capture reactions

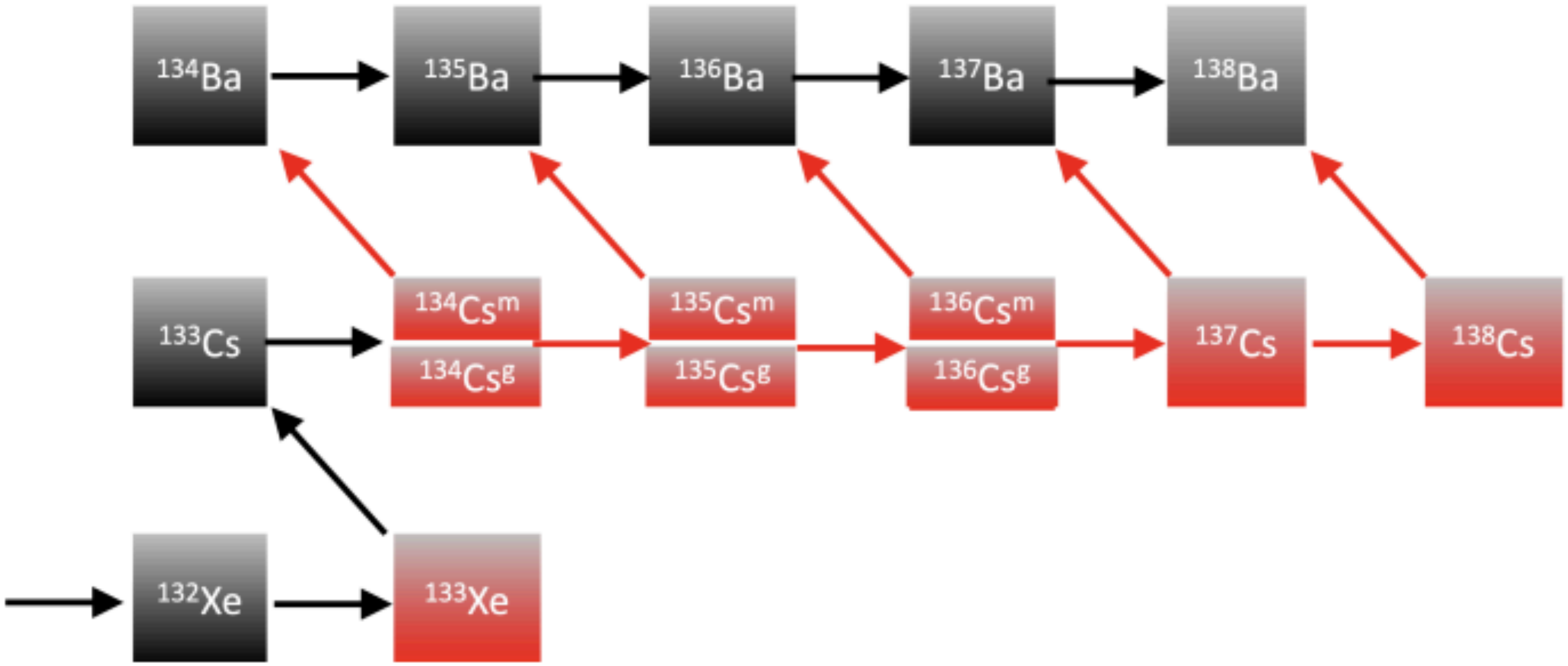
The region with A>140 has not been investigated yet, even if reproducing the solar distribution of the most heavy isotopes is still an issue for the galactic chemical evolution. Important pairs of cosmo-chronometers, such as Os-Re and Hf-Lu, belong to this region

The comparison between observations and theoretical yields computed using the most recent nuclear data in literature is poor: this can be due to inaccurate nuclear input data

Our analysis focused on the branching points close to ^{133}Cs, discussing cross sections and beta decays, e.g. giving a new preliminary estimate of the ^{134}Cs and ^{135}Cs half-lives.

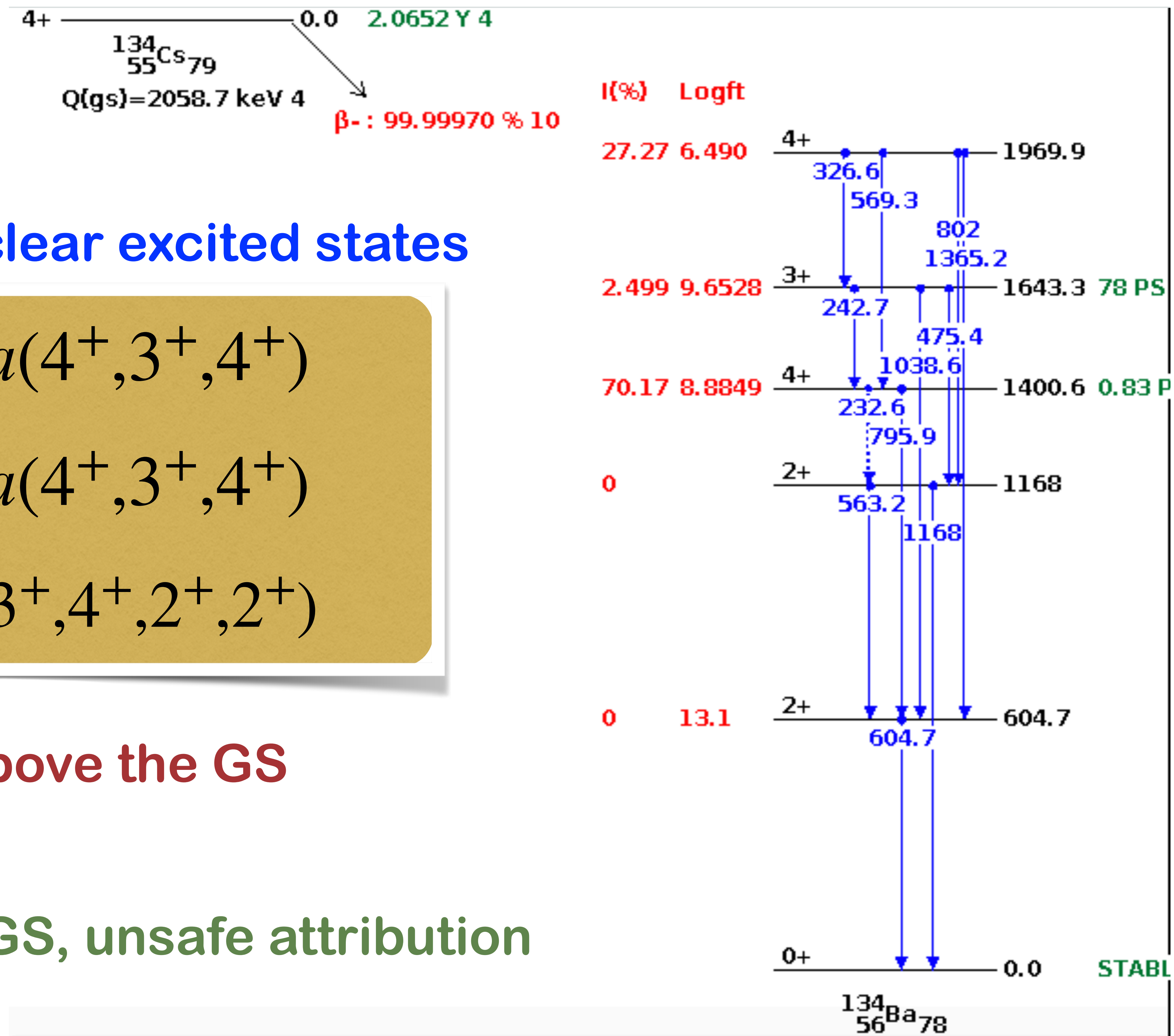
S Taioli, et al., The Astrophysical Journal, accepted and in press (2022)

The half-life for the radioactive ^{134}Cs and ^{135}Cs in astrophysical scenarios



- The abundance of Ba in AGB stars depends solely on slow (s) n-captures
- The s-process contribution to the element Ba starts from neutron captures on the stable isotope ^{133}Cs
- The flux proceeds through a branching point at the radioactive ^{134}Cs, where n-captures compete mainly with β–decay (laboratory half-life = 2 yr) to excited states of ^{134}Ba and, much less effectively, with electron captures to 134Xe (half-life = 6.8·105 yr)
- From ^{134}Cs, n-captures feed the longer-lived ^{135}Cs (half-life = 2.3E+6 y), and then ^{136}Cs (half-life = 13.16 d) and ^{137}Cs (half-life = 30.07 y), which are sites of branching points for the s-process path, but whose decay rates remain essentially unchanged for varying temperatures

$$^{134}_{55}Cs \rightarrow ^{134}_{56}Ba + e^{-} + \bar{\nu}$$



^{134}Cs short-lived nuclear excited states

$$Cs(4^+) \rightarrow Ba(4^+,3^+,4^+)$$

$$Cs(5^+) \rightarrow Ba(4^+,3^+,4^+)$$

$$Cs(3^+) \rightarrow Ba(4^+,3^+,4^+,2^+,2^+)$$

11 keV above the GS

60 keV above the GS, unsafe attribution

1 keV≈11.6 MK

How rates are typically assessed

Start from the allowed beta transition Fermi Formula

$$\frac{\mathrm{d}\lambda_\beta^0}{\mathrm{d}p} = g^2 \frac{|M_{if}^0|^2}{2\pi^3\hbar^7 c} p^2 q^2$$

$$\begin{aligned}
\lambda_\beta &= g^2 \frac{|M_{if}^L|^2}{2\pi^3\hbar^7 c} \int_0^{p_{\max}} \mathrm{d}p\, S^L(p,q) F^{\pm}(Z',p) p^2 q^2 \\
&= g^2 \frac{m_e^5 c^4 |M_{if}^L|^2}{2\pi^3\hbar^7} \left[\frac{1}{(m_e c)^5} \int_0^{p_{\max}} \mathrm{d}p\, S^L(p,q) F^{\pm}(Z',p) p^2 q^2 \right] \\
&\equiv g^2 \frac{m_e^5 c^4 |M_{if}^L|^2}{2\pi^3\hbar^7} f_L(Z',Q) \ ,
\end{aligned}$$

Include shape factor (accounting of “forbiddeness”) and Fermi function (accounting of Coulomb distortion of the e wf)

$$t_{1/2} = \log(2)/\lambda_\beta$$

Invert the eq.

Estimated by analogy to laboratory decays of nearby nuclei with similar transitions

$$f_L(Z',Q) t_{1/2} \equiv f t_{1/2} = \frac{\log_e(2) 2\pi^3\hbar^7}{g^2 m_e^5 c^4 |M_{if}^L|^2} \ . \quad \rightarrow \quad |M_{if}^L|$$

The ft’s can be quite large, and sometimes the “log ft” value is quoted. log(ft) can be measured, this is called systematics

Standard Model β-decay theory

β-decay rate is calculated by using Fermi's Golden Rule:

$$P_{i\to f} = 2\pi \int |\langle f|\hat{H}_\beta|i\rangle|^2 \rho(W_f)\delta(W_f - W_i)dW_f$$

Probability P per unit time that a system undergoes a transition from an initial state, *i*, to a number of final states, *f*, under the influence of a perturbation described by the Hamiltonian H_β

Weak Interaction Hamiltonian

$$\mathcal{H}_\beta = \frac{G_\beta}{\sqrt{2}}(\bar{\psi}_{f,p}(\mathbf{r})\gamma^\mu(1 - x\gamma^5)\hat{\psi}_{i,n}(\mathbf{r})) \cdot (\bar{\psi}_{f,e}(\mathbf{r})\gamma_\mu(1 - \gamma^5)\hat{\psi}_{i,\nu}(\mathbf{r})) + h.c.$$

Creates a proton

Destroys a neutron

Creates an electron

Destroys a neutrino (creates an antineutrino)

$\gamma^\mu \rightarrow$ Dirac gamma matrices

$G_\beta = G \cdot c_V \sim 1.13 \cdot 10^{-5} GeV^{-2}$; $x = \frac{c_A}{c_V} \sim 1.27$;

All the wavefunctions will be written as Dirac spinors

β-decay theory

Initial nuclear Fock-space state: $|\xi_n, j_n, \mu_n\rangle_A \equiv \hat{a}_n^\dagger |0\rangle_A$

Final nuclear Fock-space: $|\xi_p, j_p, \mu_p\rangle_A \equiv \hat{a}_p^\dagger |0\rangle_A$

Initial lepton Fock-space: $|0;\ 0\rangle_L$

Final lepton Fock-space: $|(n_B, \kappa_B, \mu_B + W_C^f, \kappa_C^f, \mu_C^f); W_\nu, \kappa_\nu, \mu_\nu\rangle_L \equiv (\hat{a}^+_{B,e} + \hat{a}^+_{C,e}) b_\nu^\dagger |0;0\rangle_L$

$j_{p,n,e}$ nuclear spin

$\mu_{p,n,e}$ projection along the quantization axis

$\xi_{p,n,e}$ quantum number characterizing the nuclear state

initial state: $|i\rangle \equiv |\xi_n, j_n, \mu_n\rangle_A \otimes |0;\ 0\rangle_L$

final state: $|f\rangle \equiv |\xi_p, j_p, \mu_p\rangle_A \otimes |W_C^f, \kappa_C^f, \mu_C^f;\ W_\nu, \kappa_\nu, \mu_\nu\rangle_L$

Field operators entering the Weak Interaction Hamiltonian

$$\hat{\psi}_n(\mathbf{r}) = \sum_{\xi_n, j_n, \mu_n} \langle \mathbf{r} | \xi_n, j_n, \mu_n \rangle\ \hat{a}_n + \text{antineutron creation term}$$

$$\hat{\psi}_e^+(\mathbf{r}) = \sum_{n'_B, \kappa'_B, \mu'_B} \langle n'_B, \kappa'_B, \mu'_B | \mathbf{r} \rangle\ \hat{a}^+_{B,e} + \int dW'_C \sum_{\kappa'_C, \mu'_C} \langle W'_C, \kappa'_C, \mu'_C | \mathbf{r} \rangle\ \hat{a}^+_{C,e} + \text{positron destruction term}$$

Inclusion of the antisymmetrization

$$\hat{\psi}_\nu(\mathbf{r}) = \int dW_\nu \sum_{\kappa_\nu, \mu_\nu} \left(\langle \mathbf{r} | W_\nu, \kappa_\nu, \mu_\nu \rangle\ \hat{a}_\nu + \langle \mathbf{r} | W_\nu, \kappa_\nu, \mu_\nu \rangle_-\ \hat{b}_\nu^\dagger \right)$$

$$\hat{\psi}_p^+(\mathbf{r}) = \sum_{\xi_p, j_p, \mu_p} \langle \xi_p, j_p, \mu_p | \mathbf{r} \rangle\ \hat{a}_p^+ + \text{antiproton destruction term}$$

→ In the standard approximation, one considers the particles entering the decay as non-interacting single particles

β-decay theory

Initial nuclear Fock-space state: $|\xi_n, j_n, \mu_n\rangle_A \equiv \hat{a}_n^\dagger|0\rangle_A$

Final nuclear Fock-space: $|\xi_p, j_p, \mu_p\rangle_A \equiv \hat{a}_p^\dagger|0\rangle_A$

Initial lepton Fock-space: $|0;\ 0\rangle_L$

Final lepton Fock-space: $|(n_B, \kappa_B, \mu_B + W_C^f, \kappa_C^f, \mu_C^f); W_\nu, \kappa_\nu, \mu_\nu\rangle_L \equiv (\hat{a}^+_{B,e} + \hat{a}^+_{C,e}) b_\nu^\dagger |0;0\rangle_L$

$j_{p,n,e}$ nuclear spin

$\mu_{p,n,e}$ projection along the quantization axis

$\xi_{p,n,e}$ quantum number characterizing the nuclear state

initial state: $|i\rangle \equiv |\xi_n, j_n, \mu_n\rangle_A \otimes |0;\ 0\rangle_L$

final state: $|f\rangle \equiv |\xi_p, j_p, \mu_p\rangle_A \otimes |W_C^f, \kappa_C^f, \mu_C^f;\ W_\nu, \kappa_\nu, \mu_\nu\rangle_L$

Field operators entering the Weak Interaction Hamiltonian

$$\hat{\psi}_n(\mathbf{r}) = \sum_{\xi_n, j_n, \mu_n} \langle \mathbf{r}|\xi_n, j_n, \mu_n\rangle\ \hat{a}_n + \text{antineutron creation term}$$

$$\hat{\psi}_e^+(\mathbf{r}) = \sum_{n'_B, \kappa'_B, \mu'_B} \langle n'_B, \kappa'_B, \mu'_B|\mathbf{r}\rangle\ \hat{a}^+_{B,e} + \int dW'_C \sum_{\kappa'_C, \mu'_C} \langle W'_C, \kappa'_C, \mu'_C|\mathbf{r}\rangle\ \hat{a}^+_{C,e} + \text{positron destruction term}$$

Inclusion of the antisymmetrization

$$\hat{\psi}_\nu(\mathbf{r}) = \int dW_\nu \sum_{\kappa_\nu, \mu_\nu} \left(\langle \mathbf{r}|W_\nu, \kappa_\nu, \mu_\nu\rangle\ \hat{a}_\nu + \langle \mathbf{r}|W_\nu, \kappa_\nu, \mu_\nu\rangle_-\ \hat{b}_\nu^\dagger \right)$$

$$\hat{\psi}_p^+(\mathbf{r}) = \sum_{\xi_p, j_p, \mu_p} \langle \xi_p, j_p, \mu_p|\mathbf{r}\rangle\ \hat{a}_p^+ + \text{antiproton destruction term}$$

→ In the standard approximation, one considers the particles entering the decay as non-interacting single particles

β-decay theory

To find the eigensolutions of the SM Hamiltonian for the β-decay we make a first "approximation": we assume that one can factorize this operator as the tensorial product of two non-interacting currents:

- **hadronic (nuclear);**
- **leptonic (electron + neutrino)**

Explicitly:

$$\langle f|\mathcal{H}_\beta|i\rangle = \frac{G_\beta}{\sqrt{2}} J^{H,\mu}_{i\rightarrow f}(\mathbf{r}) J^{L}_{i\rightarrow f,\mu}(\mathbf{r})$$

where:

e⁻ and ν can be considered uncoupled

$$J^{L}_{i\rightarrow f,\mu}(\mathbf{r}) = \psi^{+}_{f,e}(\mathbf{r})\gamma_0\gamma_\mu\left(1-\gamma^5\right)\psi_{i,\nu}(\mathbf{r})$$

n and p w.f. can be factorized provided that the nucleus is "hydrogenic", that is composed by a closed shell with only one single nucleon in one open shell embedded in the mean field generated by the closed shell

$$J^{H.\mu}_{i\rightarrow f}(\mathbf{r}) = \psi^{+}_{f,p}(\mathbf{r})\gamma_0\gamma^\mu\left(1-x\gamma^5\right)\psi_{i,n}(\mathbf{r})$$

$$J^{L}_{\mu}(r_h) = \begin{vmatrix} \langle\psi'_1|\phi_1\rangle & \langle\psi'_1|\phi_2\rangle & \cdots & \langle\psi'_1|\phi_N\rangle & Q_{L',q,1;\mu}(r_h) \\ \langle\psi'_2|\phi_1\rangle & \langle\psi'_2|\phi_2\rangle & \cdots & \langle\psi'_2|\phi_N\rangle & Q_{L',q,2;\mu}(r_h) \\ \vdots & & \ddots & & \vdots \\ \langle\psi'_N|\phi_1\rangle & \langle\psi'_N|\phi_2\rangle & \cdots & \langle\psi'_N|\phi_N\rangle & Q_{L',q,N;\mu}(r_h) \\ \langle\psi'_C|\phi_1\rangle & \langle\psi'_C|\phi_2\rangle & \cdots & \langle\psi'_C|\phi_N\rangle & Q_{L',q,C;\mu}(r_h) \end{vmatrix}$$

by combining the leptonic and the hadronic currents

$$J^{H,\mu}(r_h) = \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle\xi_p, j_p, \mu_p|\mathbf{r}_h\rangle\ \gamma^0\gamma^\mu(1-x\gamma^5)\langle\mathbf{r}_h|\xi_n, j_n, \mu_n\rangle \cdot r_h^2$$

Differential decay rate (electron energy spectrum)

Adv. Theory Simul. 2018, 1800086

$$\frac{d\lambda}{dW_e^t} = \frac{\pi G_\beta^2}{(2j_n+1)(2J_B+1)} \sum_{\gamma'} \sum_{\mu_n,\mu_p} \sum_{\mu'_B,\mu_B} \sum_{\kappa'_C,\mu'_C} \sum_{\kappa_\nu,\mu_\nu}$$

$$\left| \sum_{L',q}(-)^q \begin{vmatrix} \langle\psi'_1|\phi_1\rangle & \langle\psi'_1|\phi_2\rangle & \cdots & \langle\psi'_1|\phi_N\rangle & M_{L',q,1}(W_\nu = Q - W_e^t) \\ \langle\psi'_2|\phi_1\rangle & \langle\psi'_2|\phi_2\rangle & \cdots & \langle\psi'_2|\phi_N\rangle & M_{L',q,2}(W_\nu = Q - W_e^t) \\ \vdots & & \ddots & & \vdots \\ \langle\psi'_N|\phi_1\rangle & \langle\psi'_N|\phi_2\rangle & \cdots & \langle\psi'_N|\phi_N\rangle & M_{L',q,N}(W_\nu = Q - W_e^t) \\ \langle\psi'_C|\phi_1\rangle & \langle\psi'_C|\phi_2\rangle & \cdots & \langle\psi'_C|\phi_N\rangle & M_{L',q,C}(W_\nu = Q - W_e^t) \end{vmatrix} \right|^2$$

➡ ***It gives the number of electrons per unit energy and per unit time***

Differential decay rate (electron energy spectrum)

$$\frac{d\lambda}{dW_e^t} = \frac{\pi G_\beta^2}{(2j_n+1)(2J_B+1)} \sum_{\gamma'} \sum_{\mu_n,\mu_p} \sum_{\mu'_B,\mu_B} \sum_{\kappa'_C,\mu'_C} \sum_{\kappa_\nu,\mu_\nu}$$

Adv. Theory Simul. 2018, 1800086

$$\left| \sum_{L',q} (-)^q \begin{vmatrix} \langle\psi'_1|\phi_1\rangle & \langle\psi'_1|\phi_2\rangle & \cdots & \langle\psi'_1|\phi_N\rangle & M_{L',q,1}(W_\nu = Q - W_e^t) \\ \langle\psi'_2|\phi_1\rangle & \langle\psi'_2|\phi_2\rangle & \cdots & \langle\psi'_2|\phi_N\rangle & M_{L',q,2}(W_\nu = Q - W_e^t) \\ \vdots & & \ddots & & \vdots \\ \langle\psi'_N|\phi_1\rangle & \langle\psi'_N|\phi_2\rangle & \cdots & \langle\psi'_N|\phi_N\rangle & M_{L',q,N}(W_\nu = Q - W_e^t) \\ \langle\psi'_C|\phi_1\rangle & \langle\psi'_C|\phi_2\rangle & \cdots & \langle\psi'_C|\phi_N\rangle & M_{L',q,C}(W_\nu = Q - W_e^t) \end{vmatrix} \right|^2$$

The final orbital ψ'_i depend on γ' that identifies the possible final (shake-up, shake-off, excited) states

$$M_{L',q,B} = \int \Big[\int d\Omega_h Y_{L',q}(\theta_h,\phi_h) \langle \xi_p, j_p, \mu_p | \mathbf{r}_h \rangle \; \gamma^0 \gamma^\mu (1 - x\gamma^5) \langle \mathbf{r}_h | \xi_n, j_n, \mu_n \rangle \cdot r_h^2 \cdot$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l) \langle n'_B, \kappa'_B, \mu'_B | \mathbf{r}_l \rangle \gamma^0 \gamma_\mu (1-\gamma^5) \langle \mathbf{r}_l | W_\nu, \kappa_\nu, \mu_\nu \rangle \delta(r_h - r_l) \Big] dr_h;$$

$$M_{L',q,C} = \int \Big[\int d\Omega_h Y_{L',q}(\theta_h,\phi_h) \langle \xi_p, j_p, \mu_p | \mathbf{r}_h \rangle \; \gamma^0 \gamma^\mu (1 - x\gamma^5) \langle \mathbf{r}_h | \xi_n, j_n, \mu_n \rangle \cdot r_h^2 \cdot$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l) \langle W'_C, \kappa'_C, \mu'_C | \mathbf{r}_l \rangle \gamma^0 \gamma_\mu (1-\gamma^5) \langle \mathbf{r}_l | W_\nu, \kappa_\nu, \mu_\nu \rangle \delta(r_h - r_l) \Big] dr_h;$$

Using L' = 0, $\langle\psi'_i|\phi_j\rangle = \delta_{ij}$,e- wfs at nuclear radius, and γ_0 one recovers standard beta-decay

Calculation of the leptonic wfs

DHF model

Dirac equation in a spherical potential

$$H\psi(\mathbf{r}) = (c\boldsymbol{\alpha}\cdot\mathbf{p} + \beta mc^2 + V(r))\psi(\mathbf{r}) = W\psi(\mathbf{r})$$

solutions are of the form: $\psi(\mathbf{r}) = \psi_{\kappa,\mu}(\mathbf{r}) = \begin{pmatrix} f_\kappa(r)\,\chi_{\kappa,\mu}(\Omega) \\ ig_\kappa(r)\,\chi_{-\kappa,\mu}(\Omega) \end{pmatrix}$

where $\chi_{\kappa,\mu}(\Omega) = \sum_{m_s=-\frac{1}{2}}^{\frac{1}{2}} \langle l\mu - m_s; \frac{1}{2} m_s | j\mu\rangle Y_{l,\mu-m_s}(\Omega)\phi_{m_s}$ are the spherical harmonics tensor

and calling $f_\kappa = \frac{u_\kappa}{r}$; $g_\kappa = \frac{v_\kappa}{r}$

u_κ and v_κ are solutions of

$$\begin{cases} \dfrac{\partial u_\kappa}{\partial r} = -\dfrac{\kappa}{r}u_\kappa + \dfrac{1}{c}(W - V(r) + mc^2)v_\kappa(r) = 0 \\ \dfrac{\partial v_\kappa}{\partial r} = \dfrac{\kappa}{r}v_\kappa - \dfrac{1}{c}(W - V(r) - mc^2)u_\kappa(r) = 0 \end{cases}$$

where

$$V(r) = -\frac{Z_f}{r} + \int \frac{\rho(r')}{r_>} d^3r' - V_{ex}(r)$$

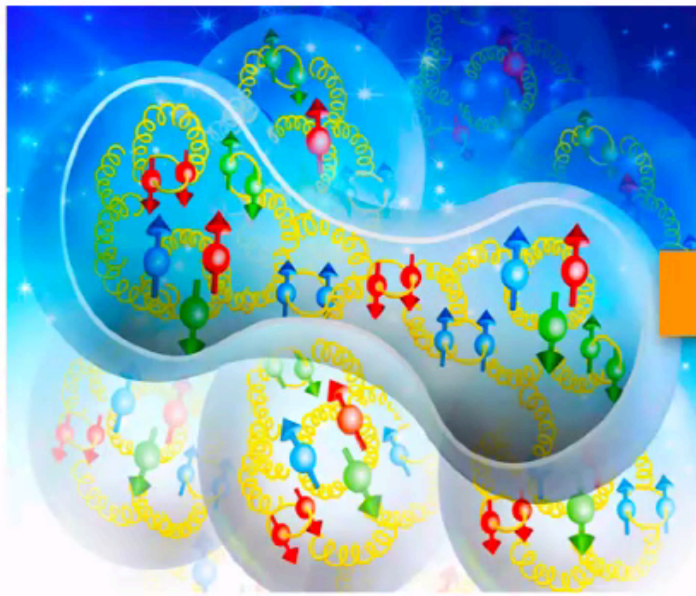
and we assume $V_{ex} = \frac{3}{2}\alpha_X\left[\frac{3}{\pi}\rho(r)\right]^{1/3}$ **which is local (TF or LDA)**

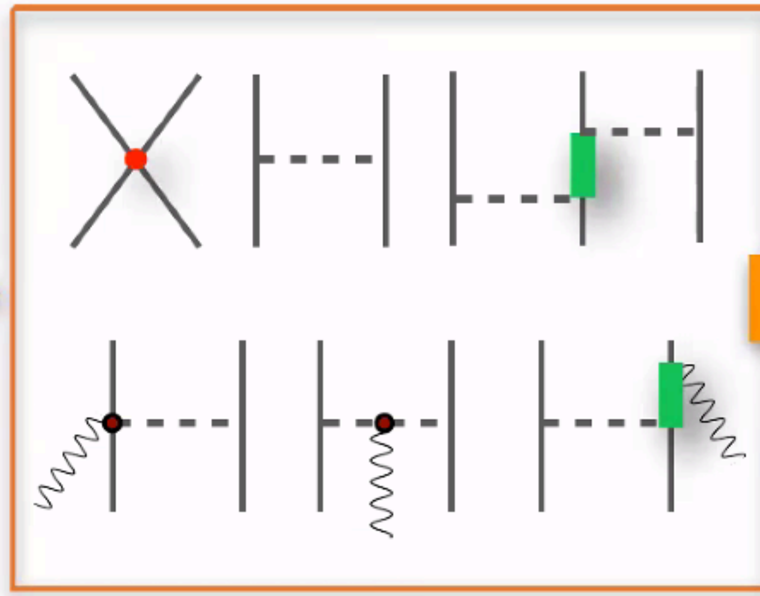
To numerically solve the DHF equations we use the collocation methods, which is a Runge-Kutta type integration method

THE NUCLEAR MANY-BODY PROBLEM

In the low-energy regime, quark and gluons are confined within hadrons and the relevant degrees of freedoms are protons, neutrons, and pions

Effective field theories are the link between QCD and nuclear observables.





Nucleons can treated as point-like particles interacting through the Hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

- **By far too computational expensive and far beyond our (current) capability**
- **The nucleon-nucleon interaction is modelled by a relativistic one-body Wood-Saxon potential**
- **Nuclear dynamic correlation is neglected**

Calculation of the hadronic wfs: DHF

By changing the interaction potential, the calculation of the hadronic wavefunctions within the nuclear matrix elements can be performed

Nuclear wfs simulations out of scope (WS model potential)

$$V_C(r) = -V_C\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1} \qquad V_C = V_0\left(1 \pm \chi\frac{N-Z}{A}\right)$$

Protons (+), Neutrons (−)

$$\tilde{V}_{SO}(r) = \tilde{V}_{SO}\left[1+\exp\left(\frac{r-R_{SO}}{a_{SO}}\right)\right]^{-1} \qquad \tilde{V}_{SO} = \lambda V_C$$

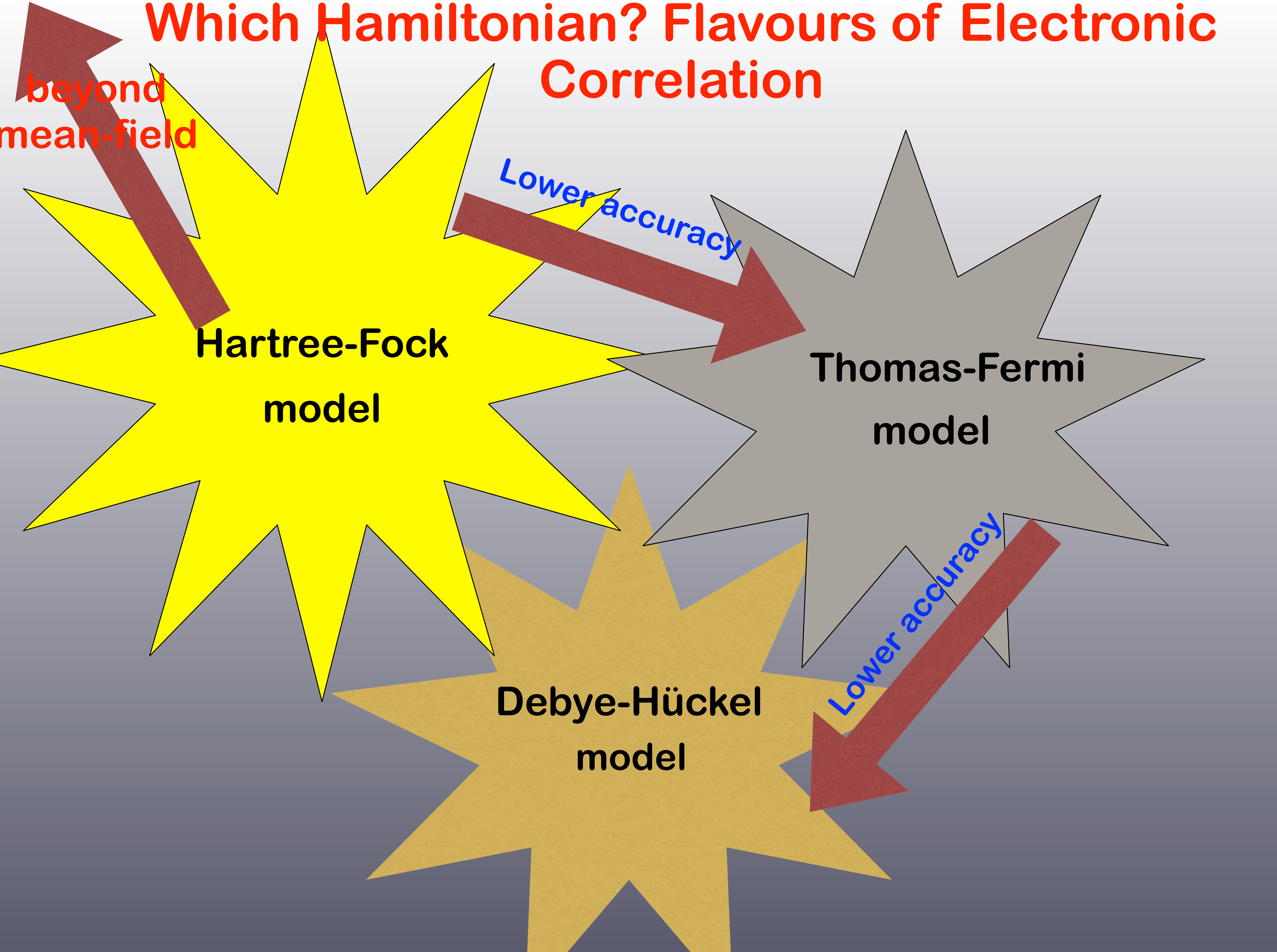
$R = R_0 A^{1/3}$ **and** $R_{SO} = R_{0,SO} A^{1/3}$ **= nuclear radius**

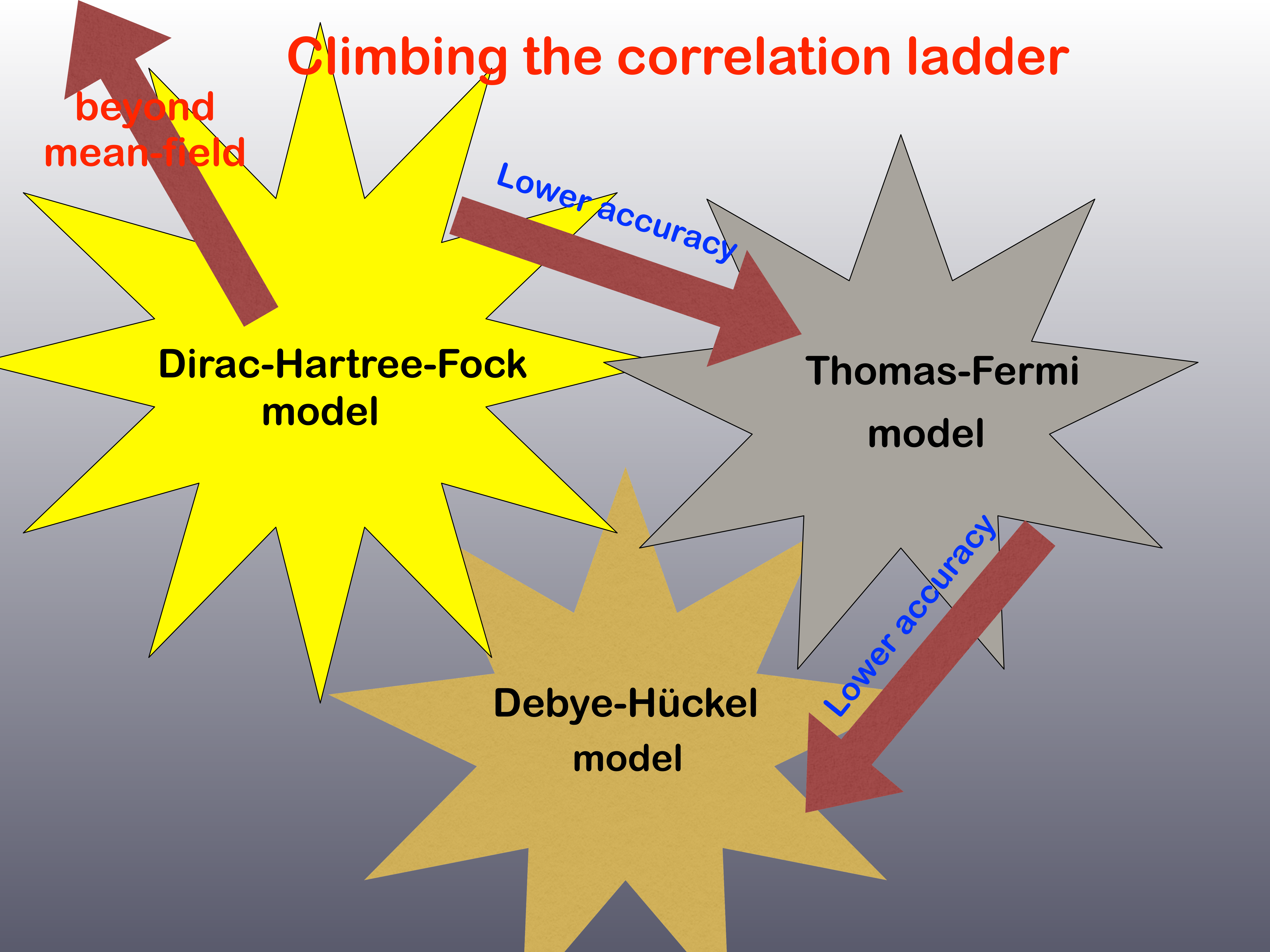
a **and** a_{SO} **= diffuseness**

$V_0, \chi, \lambda, a = a_{SO}, R_0, R_{0,SO}$ are parameters to be optimised according to experiments or ab-initio nuclear structure simulations

$$V_0 = 52.06\ MeV, \chi = 0.639, R_0 = 1.260\ fm, R_{0,SO} = 1.160\ fm, \lambda = 24.1, a = a_{SO} = 0.662\ fm$$

Which Hamiltonian? Flavours of Electronic Correlation





Climbing the correlation ladder
beyond
mean-field
Lower accuracy
Dirac-Hartree-Fock
model
Thomas-Fermi
model
Lower accuracy
Debye-Hückel
model

The beta-decay spectrum of $^{129}\mathrm{I}$

$$^{129}I \rightarrow \; 17374.6321 \; > \; 5486.6741$$
$$^{241}Pu \rightarrow \; 30566.4823 \; >> \; 763.6509$$

$$^{129}_{53}\mathrm{I}_{76} \rightarrow \, ^{129}_{54}\mathrm{Xe}^{*}_{75} + e^{-} + \bar{\nu}_e$$
$$J = 7/2^{+} \rightarrow J = 3/2^{+}$$

Second forbidden

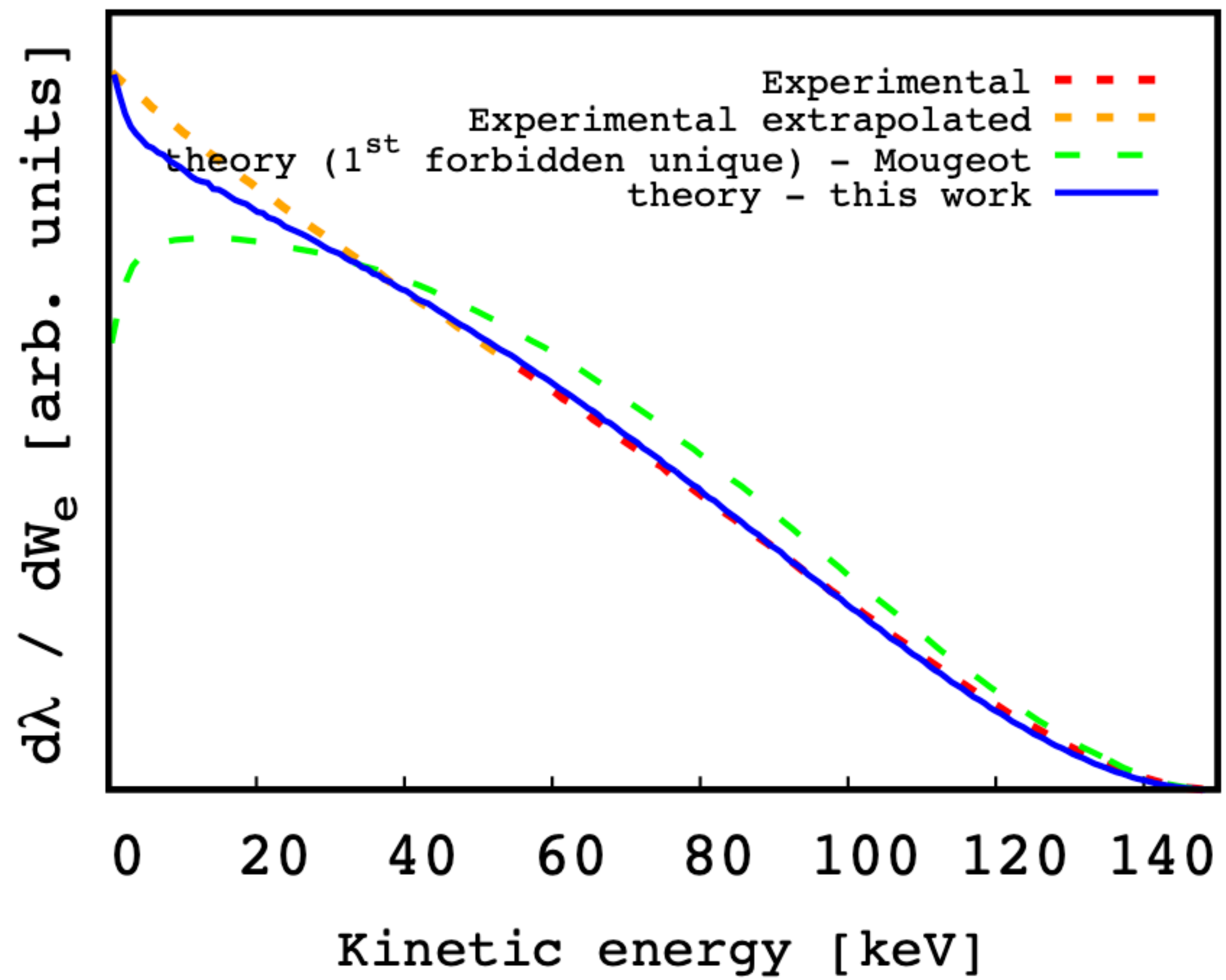
$^{129}_{53}I_{76}$ = odd-even

$^{129}_{54}\mathrm{Xe}_{75}$ = even-odd

β^{-} half-life = 1.57X10^{7} y (10.05)

Q-value = 149.4 keV
(ground state to excited state)

γ = 39.578 keV



Mean-field DHF + screening + exchange works just better than other approaches!!!!

Assumptions in the nuclear simulations

- The decaying neutron in the Cs nucleus is found in the $2d_{3/2}$ shell and weak decays into a proton in the $1g_{7/2}$ shell of Ba. These states are geometrically coupled to the "core" of the other nucleons to recover the total J.

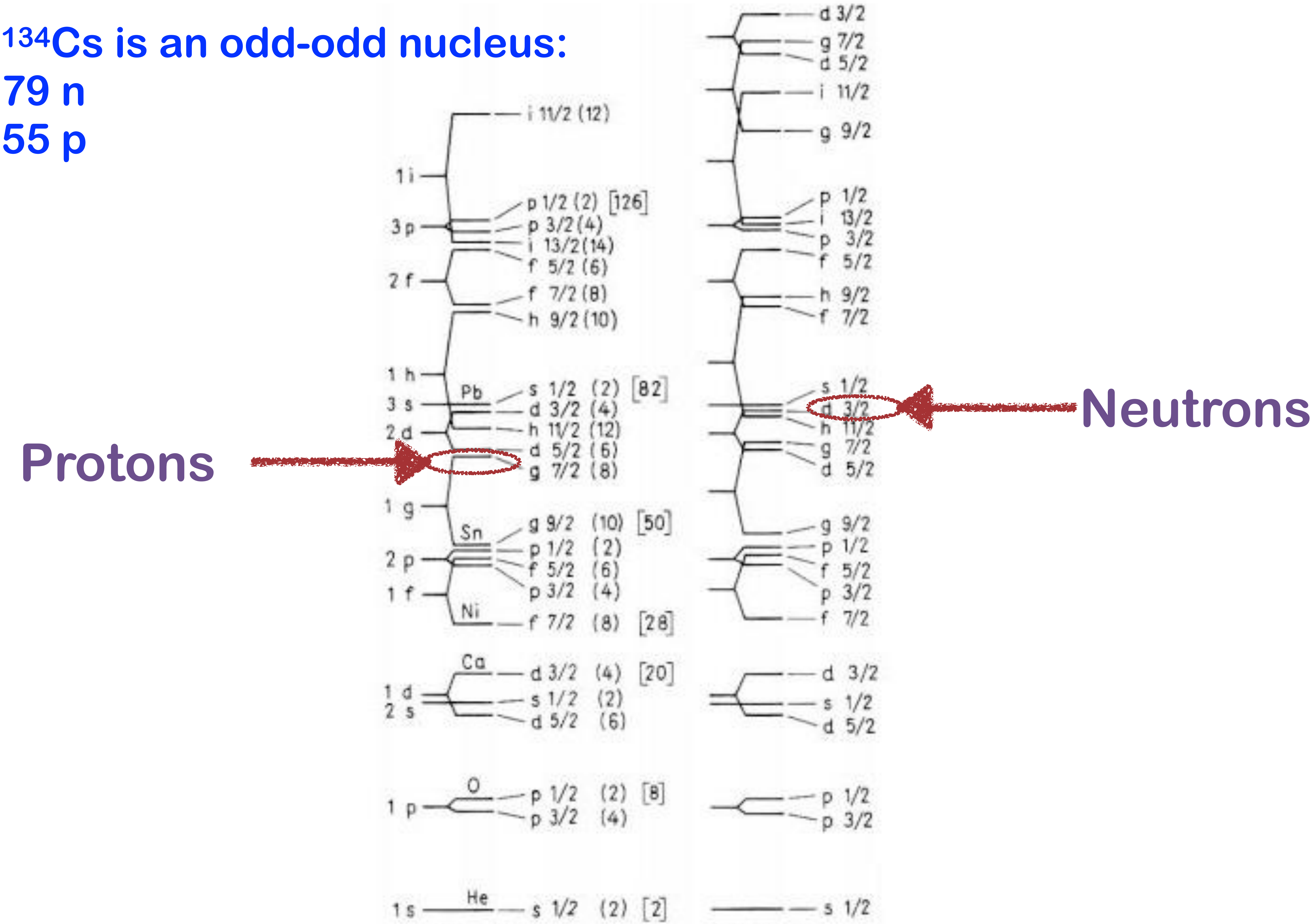
- In a true many-body approach, such as CI, the decaying neutron wave function is a superposition of several configuration of nearby energy.

- The population of nuclear states has been assumed to follow a Boltzmann probability distribution taking into account level degeneration [9(4^+), 11(5^+), and 7(3^+)]

- We renormalize the rate at all temperatures by a constant factor so as to recover the room temperature experimental log(ft) (mainly due to the accuracy of nuclear wavefunction calculations)

The nuclear shell model: practical view

^{134}Cs is an odd-odd nucleus:
79 n
55 p



Assumptions in the electronic structure calculations

- Chemical potential of e- and e+ as for an ideal Fermi gas using a relativistic energy-momentum dispersion $E^2 = c^2p^2 + m_e c^4$. A Fermi gas in thermal equilibrium is identified by temperature and chemical potential. Mean-field approximation for e- and e+.

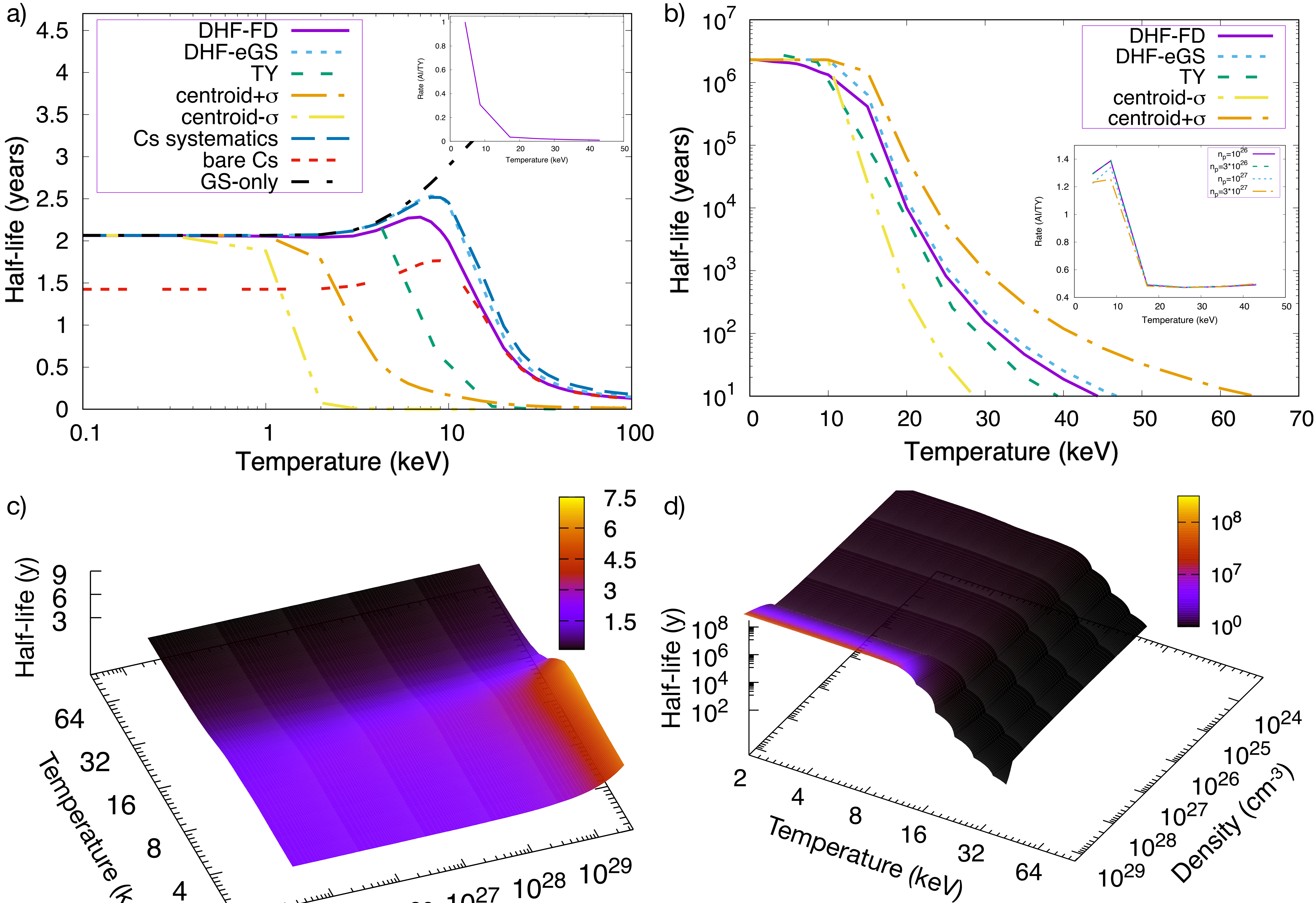
- Energy can be high enough to form e^+-e^- couples:

$$n_p = n_{e^-} - n_{e^+}$$

- The electronic levels of Cs (not re-optimized at each temperature) are populated according the Fermi-Dirac distribution

$n^i_{e^-} = \dfrac{1}{1 + e^{(\epsilon_i - \mu_{e^-})/(KT)}} = F(T, \mu)$, where the energies ϵ_i of the i-th level is obtained via the self-consistent solution of the DHF equation and the chemical potential from the implicit relation valid for a Fermi gas:

$$n_{e^-} = \int_0^\infty dp\ p^2/\pi^2 \times (F((c \times \sqrt{(}p^2 + c^2) - \mu_e)/kT) - F((c \times \sqrt{(}p^2 + c^2) + \mu_e)/kT))$$



ApJ: to appear soon

Take-home messages from rate calculations

The β decay rate of Cs is affected concurrently by two major factors:

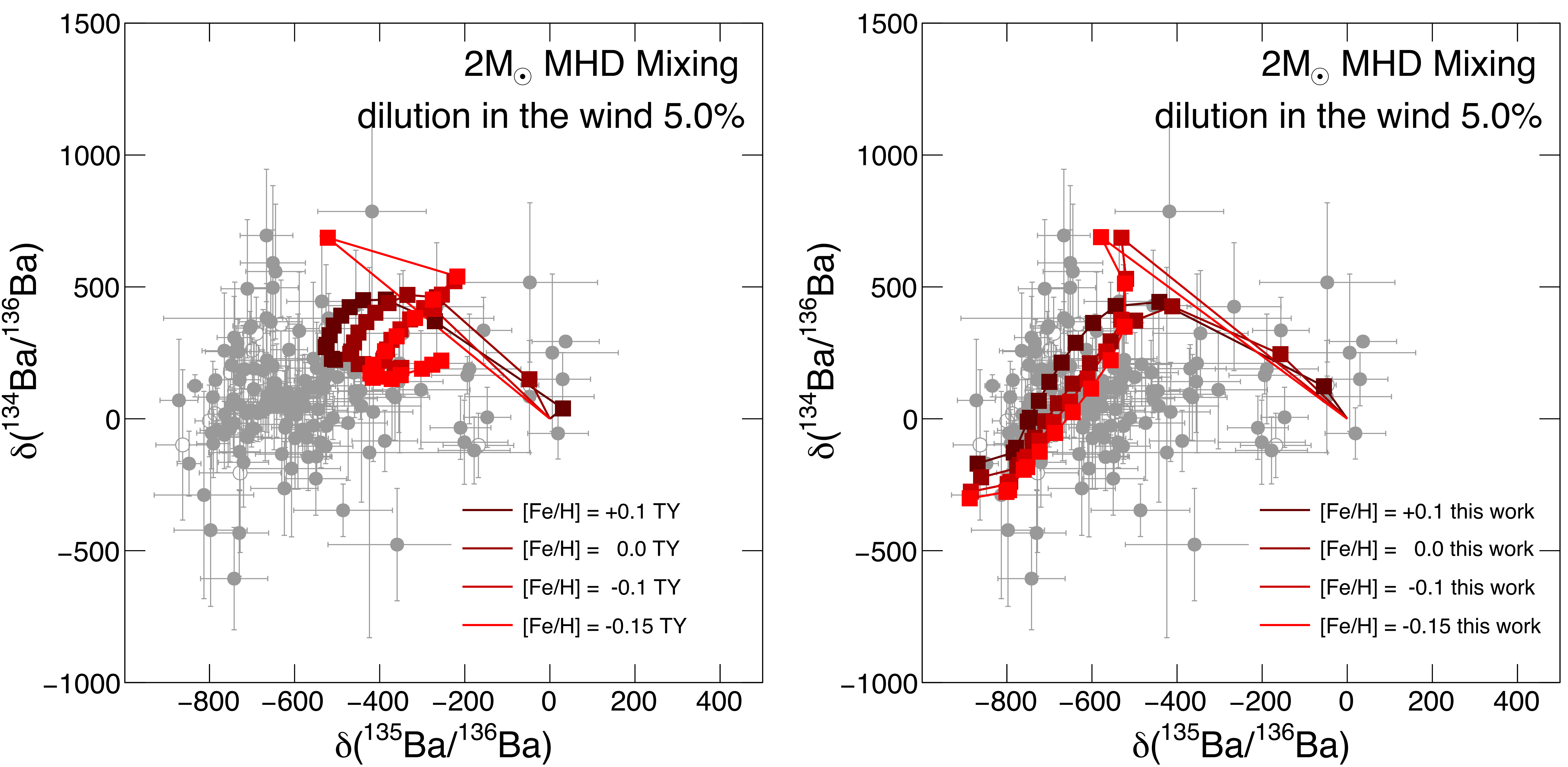
1. the presence of 3 nuclear excited states of Cs;
2. the electronic excitation, also up to a complete ionization

1. The nuclear excited state dynamics is the most relevant of the two, as it can increase the rate by a factor of 15 at 100 KeV (1 GK) to 23 at 1000 KeV with reference to room temperature conditions and by a factor of 3 at $T>10^8$ K for ^{134}Cs as compared to previous works based on systematics (basically due to populating the 60 keV fast-decaying ES).
2. The e^- temperature has the most pronounced impact on the rate in the range [0:15] keV (20% at 10 keV). Rate increases as electrons can be accommodated in empty bound orbitals. Despite being a quark-level process, the contribution of the electronic DOF to the rate is thus crucial.
3. Our half-life are consistently higher than TY and the rate increases ~ 3 times at 20 KeV (~ 230 MK), 6 times at 30 keV, 8 times at 40 keV (~ 464 MK) with respect to the GS decay only.

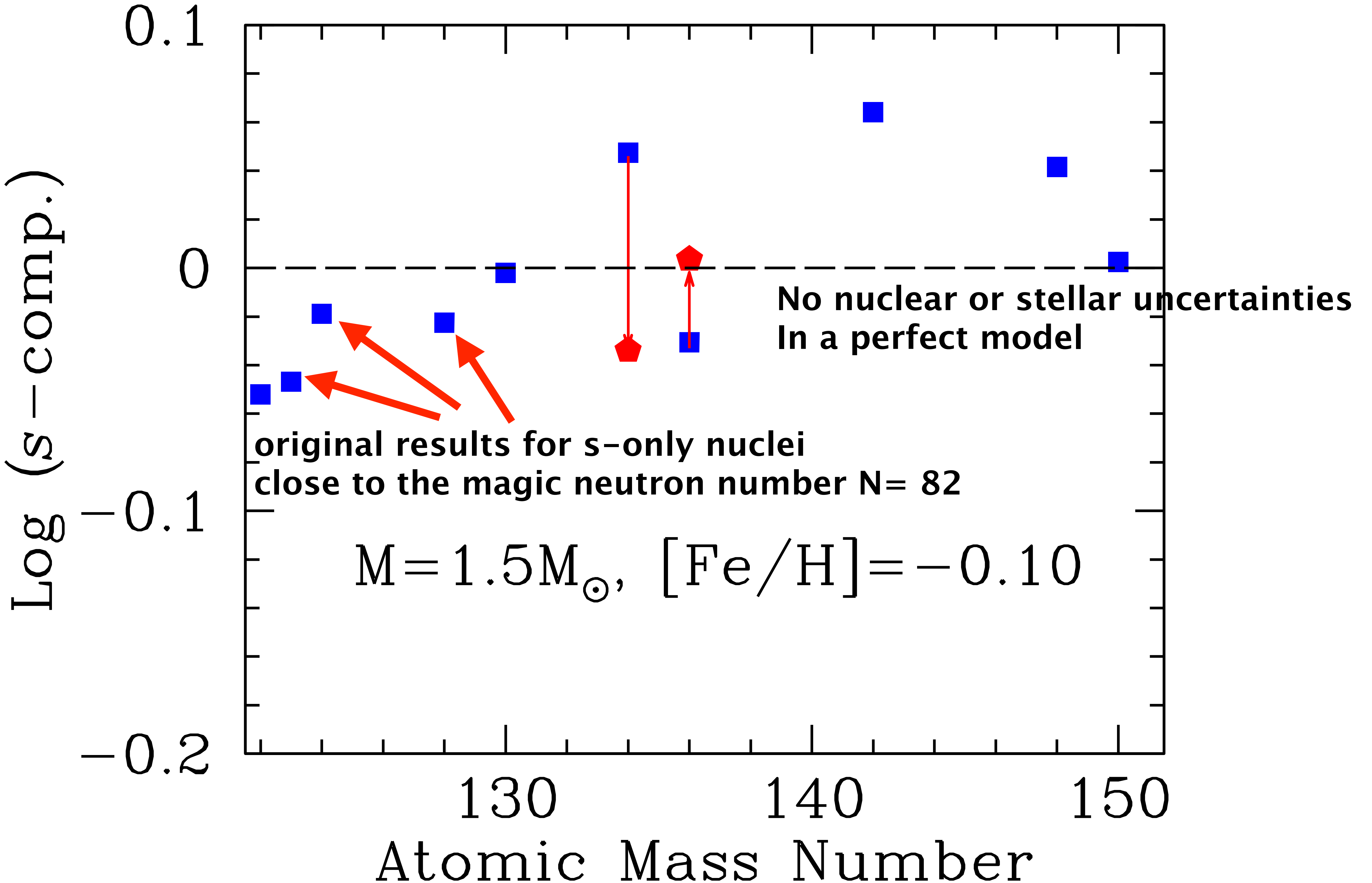
Major differences with state-of-the art methods

- We do not use semi-empirical approaches based on *log(ft)*
- We do not calculate *log(ft)* by e.g. using the nuclear shell model to obtain the stellar rate of ^{134}Cs within the standard approach to β-decay spectra.
- At variance, in our work we extend the theory and the computational methods by using a fully relativistic approach.
- We calculate directly the nuclear matrix elements that enter the hadronic current from first-principles. To do so, we adopt a mean-field approach, which can of course be systematically improved by using more correlated many-body techniques without modifying the backbone of our method.
- A second substantial difference relies on the treatment of the leptonic current, which is typically neglected or added via a semi-empirical Fermi function. We demonstrate that it may halve the half-life of ^{134}Cs around 10 keV. We include include both bound and continuum channels, the exchange interaction, the non-orthogonality between the parent and daughter electronic orbitals, as a function of plasma density, temperature and charge state distributions, reaching an unprecedented level of accuracy.

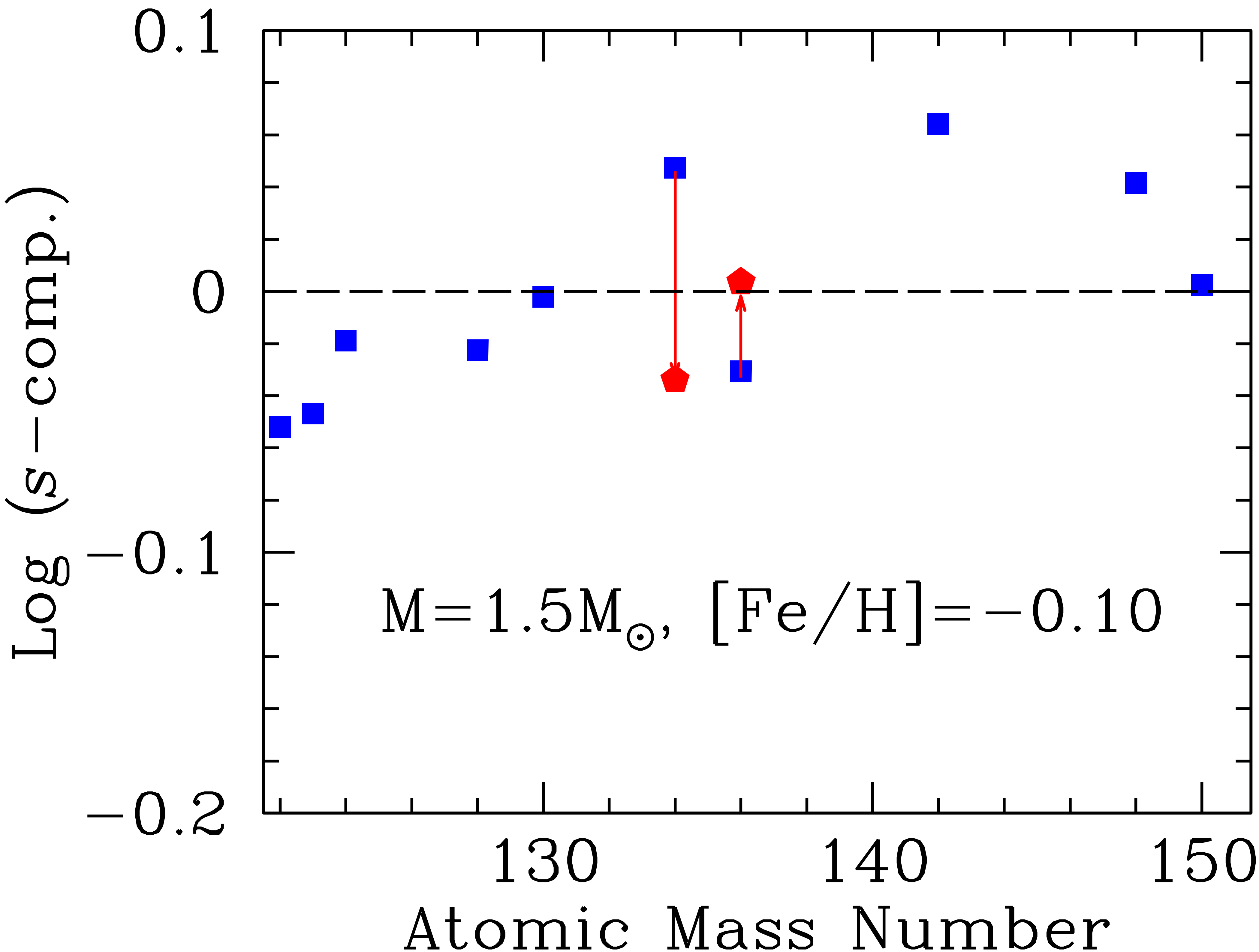
Left panel: isotopic ratios of ^{134}Ba and ^{135}Ba with respect to ^{136}Ba, displayed as part-per-mil deviations (indicated by the symbol δ), with decay rates from TY.
Right panel: the results of the same models, where only the decay rates for ^{134}Cs and ^{135}Cs are changed, using those of the present work. Computations are for 2 $M_\odot$ stars, where magneto-hydrodynamic processes induce the penetration of protons into He-rich layers, producing ^{13}C then releasing neutrons through $^{13}C(\alpha,n)^{16}O$. Abundances are computed in stellar winds, where magnetic blobs further add 5% of C-rich material in flare-like episodes. The symbol [Fe/H] indicates $Log(X_{Fe}/X_H)_{star}$ - $Log(^{13}C)_{sun}$



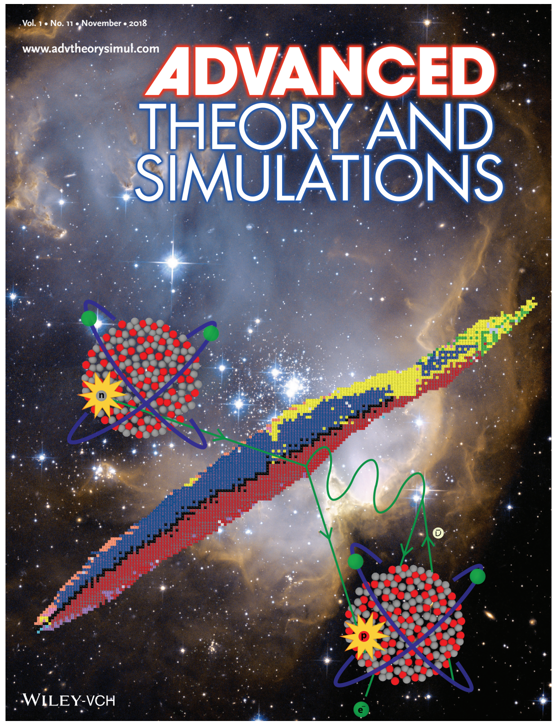
Percentage of s-process contributions (blue dots) as computed by M. Busso et al. ApJ 908, 55 (2021) for s-only nuclei near the magic neutron number N= 82.



Prof. M. Busso happy-go-lucky!



Conclusions



- A new method for calculating β- and e^--capture decay spectra in both light to heavy nuclei, which extends the standard approach in several ways

- It works also in astrophysical environment by including temperature, density and charge state distribution

- This method can be applied to any nuclear beta decay and include relativistic, many-body screening and post-collisional effects

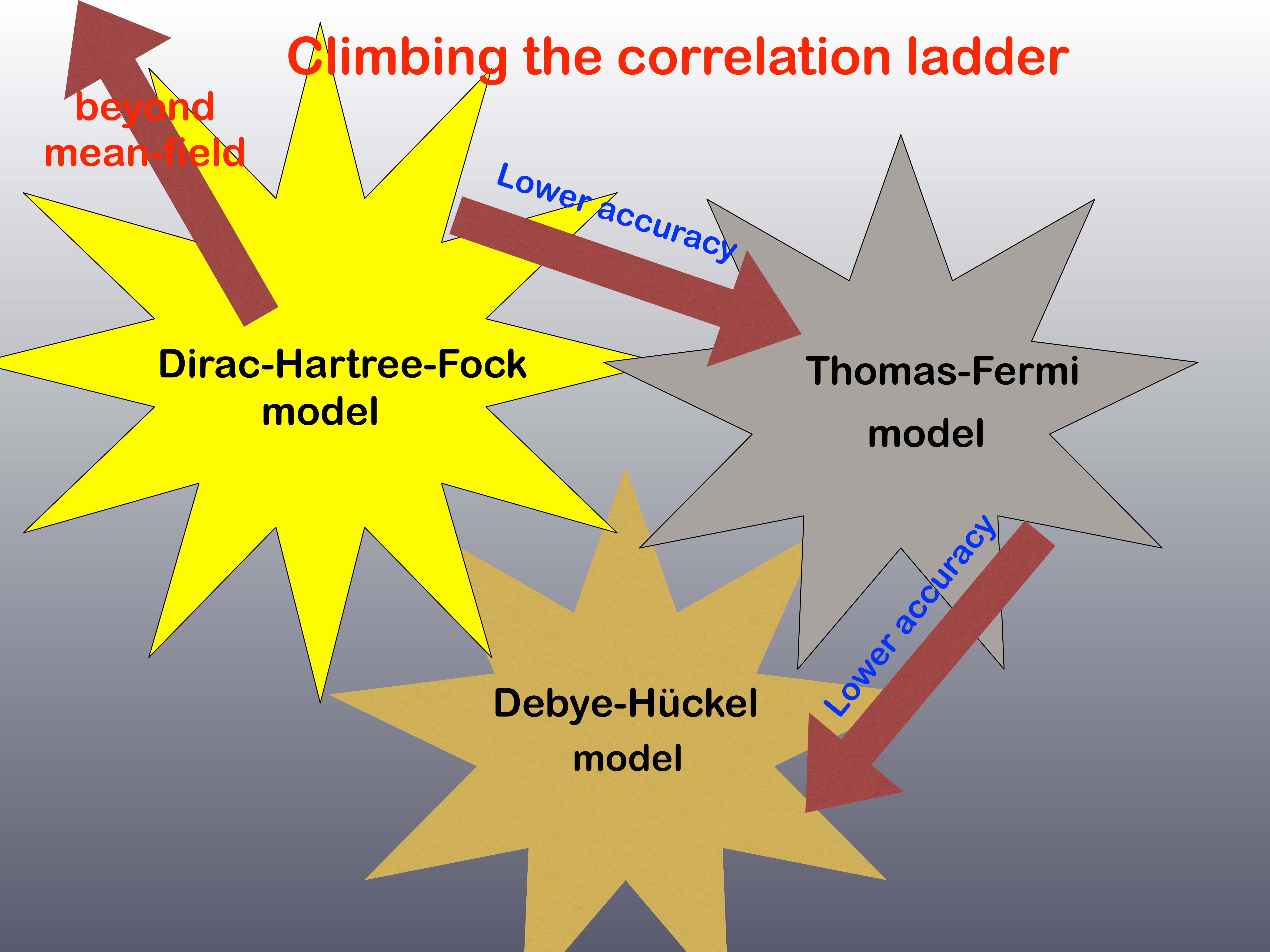
- Our approach is more accurate than state-of-the-art methods

Outlook

- Inclusion of nuclear dynamic correlation beyond mean-field approximation;

- Estimate of beta-decay rates of different elements (^{176}Lu, ^{94}Nb, any other suggestion from the Pandora collaboration AND you!)

Climbing the correlation ladder




That's all Folks!

EC on ^7Be

Thermal energy (keV)	$\Gamma_{tot}^{EC}(T)$ (s^{-1})	$t_{\frac{1}{2}}^{EC}$ (d)
10	$1.361402577510297 \times 10^{-17}$	589284680858.50464
20	$1.672038291185869 \times 10^{-16}$	47980580805.904541
30	$9.903842897623605 \times 10^{-13}$	8100428.2044962524
40	$1.116927353927415 \times 10^{-10}$	71826.845370664669
50	$2.10744362477365 84 \times 10^{-9}$	3806.7622496628555
60	$1.60706977580428 70 \times 10^{-8}$	499.20276984028214
70	$7.25222081269531 19 \times 10^{-8}$	110.62179491332135
80	$2.34704180223581 94 \times 10^{-7}$	34.181482521694548
90	$6.06512914503002 13 \times 10^{-7}$	13.227314113591401
100	$1.33515208016718 50 \times 10^{-6}$	6.0087063887706700

anti-neutrino capture on ^{7}Be

Thermal energy (keV)	$\Gamma^{\overline{\nu}}_{tot}(T)$ s^{-1})	$t^{\overline{\nu}}_{\frac{1}{2}}$ (d)
10	0	Infinity
20	0	Infinity
30	0	Infinity
40	0	Infinity
50	$3.904045341795658 \times 10^{-21}$	2054929216163012.8
60	$1.837750760978635 \times 10^{-18}$	4365410699005.1616
70	$1.418015915258997 \times 10^{-16}$	56575788379.749725
80	$3.910756562972360 \times 10^{-15}$	2051402766.9325206
90	$5.419568010960918 \times 10^{-14}$	148029083.09030658
100	$4.624080108779467 \times 10^{-13}$	17349476.318217535

p-capture on ^{7}Be

$$^{7}\mathrm{Be} + p \longrightarrow {}^{8}\mathrm{B}^{*} \longrightarrow {}^{8}\mathrm{Be} + \gamma + e^{+} + \nu$$

$$^{8}\mathrm{B}e \longrightarrow 2\ {}^{4}\mathrm{He}$$

Thermal energy (keV)	$t_{\frac{1}{2}}$ (s)
10	32074969039.417900
20	82519548.044600070
30	3866718.0085887648
40	519185.64739598491
50	119013.06295443069
60	37575.240054577698
70	14657.949390048834
80	6639.2646265137328
90	3359.1986918298267
100	1850.4832691326778

Calculation of the leptonic and hadronic wfs: DHF

$$\begin{pmatrix} mc^2 + W_V + W_S + \mathbf{A}_P \cdot \boldsymbol{\sigma} - E & -c\boldsymbol{\sigma} \cdot i\boldsymbol{\nabla} - \boldsymbol{\sigma} \cdot \mathbf{A} + W_{PS} \\ -c\boldsymbol{\sigma} \cdot i\boldsymbol{\nabla} - \boldsymbol{\sigma} \cdot \mathbf{A} + W_{PS} & -mc^2 + W_V + \mathbf{A}_P \cdot \boldsymbol{\sigma} - W_S - E \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_S \end{pmatrix} = 0$$

where

$$\begin{array}{ccll} W_S & - & \text{scalar} & \text{potential} \\ W_V & - & \text{vectorial} & \text{potential} \\ W_{PS} & - & \text{pseudoscalar} & \text{potential} \\ \mathbf{A}_P & - & \text{pseudo-vectorial} & \text{potential} \end{array}$$

For leptons:

$$W_S = 0$$

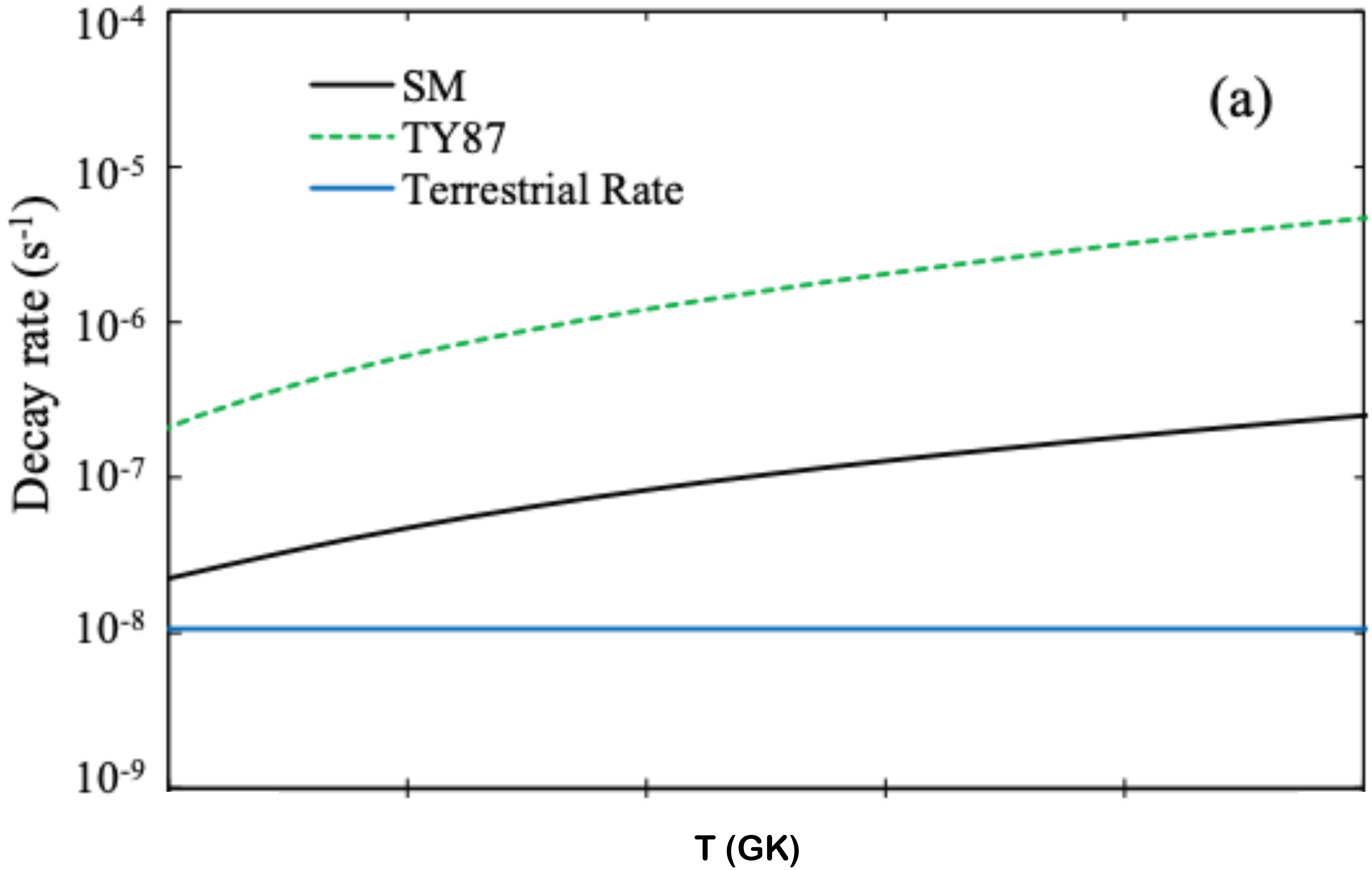
$$W_V = \text{Coulomb interaction}$$

$$A_P = 0$$

For hadrons:

$$\begin{array}{ccll} W_V + W_S & - & \text{Wood-Saxon} & \text{potential} \\ W_V - W_S & - & \text{spin-orbit} & \text{potential} \end{array}$$

$$\mathbf{A}_P \quad - \quad \text{magnetic field} = 0$$



^{134}Cs stellar β–decay rate of TY87 and of Li. et al. obtained with the shell model (Kuo-Ang Li et al. 2021 ApJL 919 L19)

The beta-decay spectrum of $^{63}Ni, ^{129}I, ^{241}Pu$

Allowed Gamow-Teller

$^{63}_{28}Ni_{35}$ **= even-odd**

$^{63}_{29}Cu_{34}$ **= odd-even**

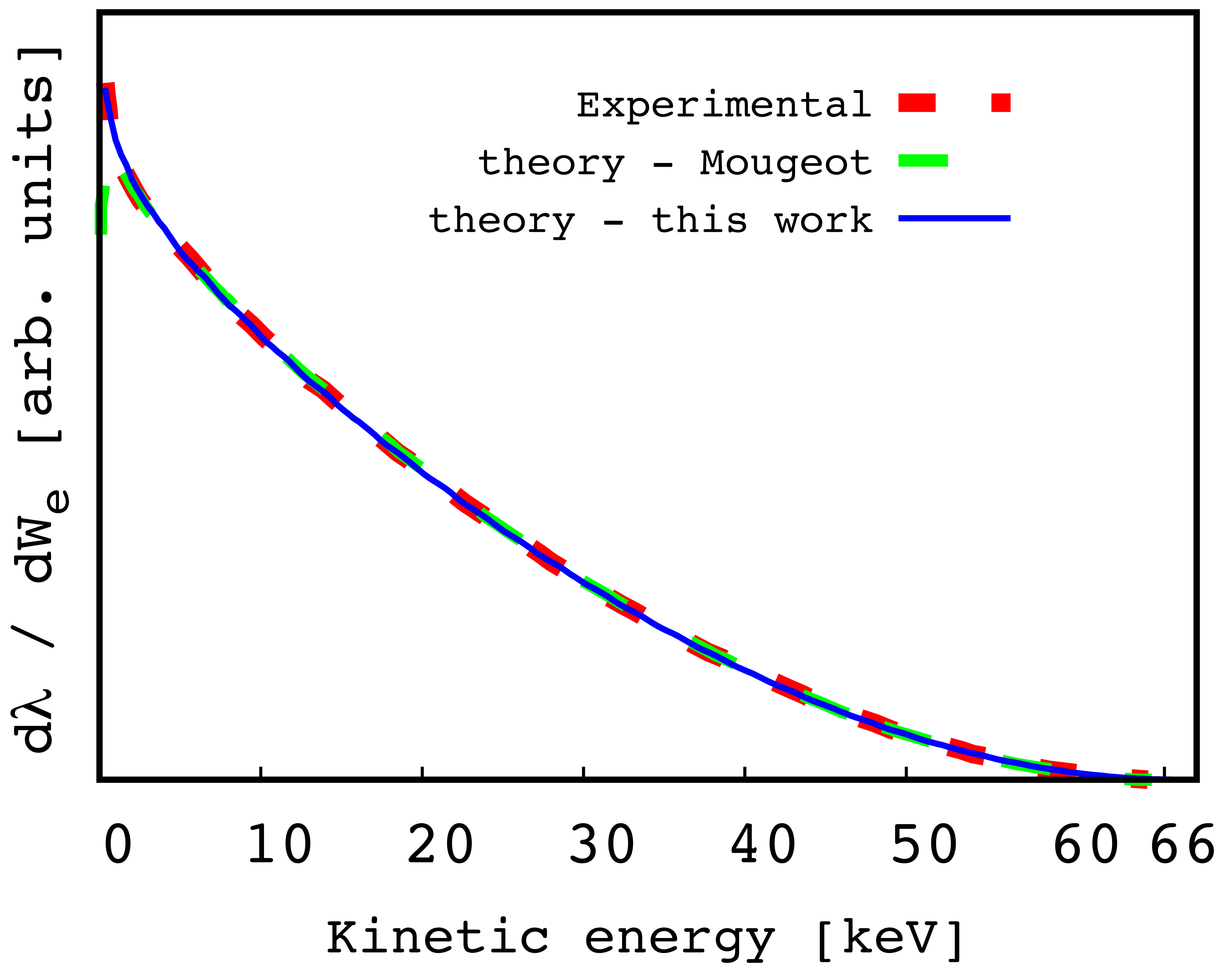
100% via β^- half-life = 101.2 y

Q-value = 66.945 keV
(ground state to ground state)

Mean-field DHF + screening + exchange works just fine as other approaches!!!!

$$^{63}_{28}\mathrm{Ni}_{35} \rightarrow {}^{63}_{29}\mathrm{Cu}_{34} + e^- + \bar{\nu}_e$$

$$J = 1/2^- \rightarrow J = 3/2^-$$



The beta-decay spectrum of $^{63}Ni, ^{129}I, ^{241}Pu$

$^{129}I \rightarrow \quad 17374.6321 \quad > \quad 5486.6741$

$^{241}Pu \rightarrow \quad 30566.4823 \quad >> \quad 763.6509$

$$^{241}_{94}\mathrm{Pu}_{147} \rightarrow {}^{241}_{95}\mathrm{Am}_{146} + e^- + \bar{\nu}_e$$

$$J = 5/2^+ \rightarrow J = 5/2^-$$

First forbidden

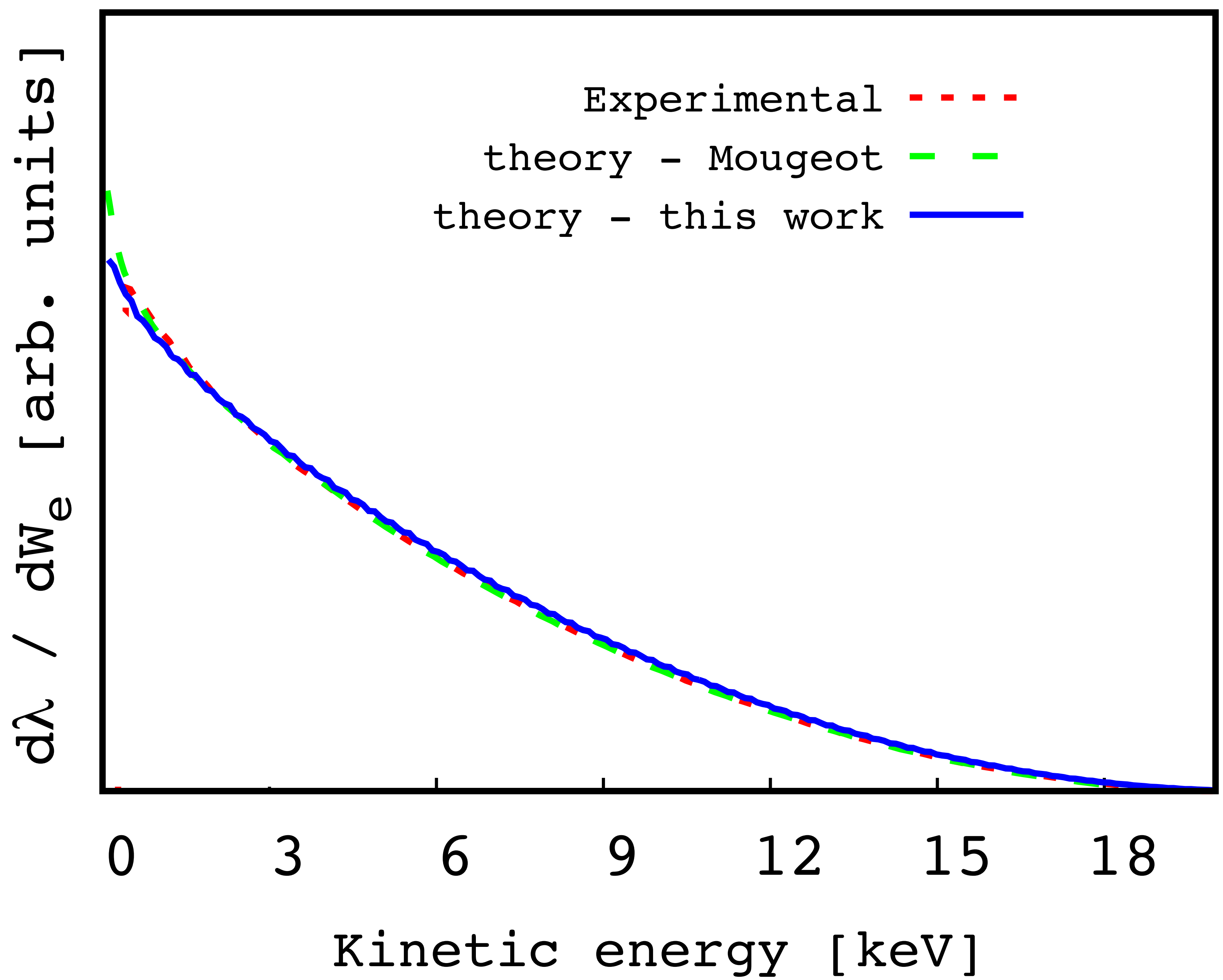
$^{241}_{94}\mathrm{Pu}_{147}$ **= even-odd**

$^{241}_{95}\mathrm{Am}_{146}$ **= odd-even**

β^- **half-life = 14.329 y (6.85)**

Q-value = 20.78 keV
(ground state to excited state)

γ **= 39.578 keV**



Mean-field DHF + screening + exchange works just fine as other approaches!!!!

The beta-decay spectrum of $^{210}_{83}\mathrm{Bi}_{127}$

$$^{210}_{83}\mathrm{Bi}_{127} \rightarrow {}^{210}_{84}\mathrm{Po}_{126} + e^- + \bar{\nu}_e$$

$$J = 1^- \rightarrow J = 0^+$$

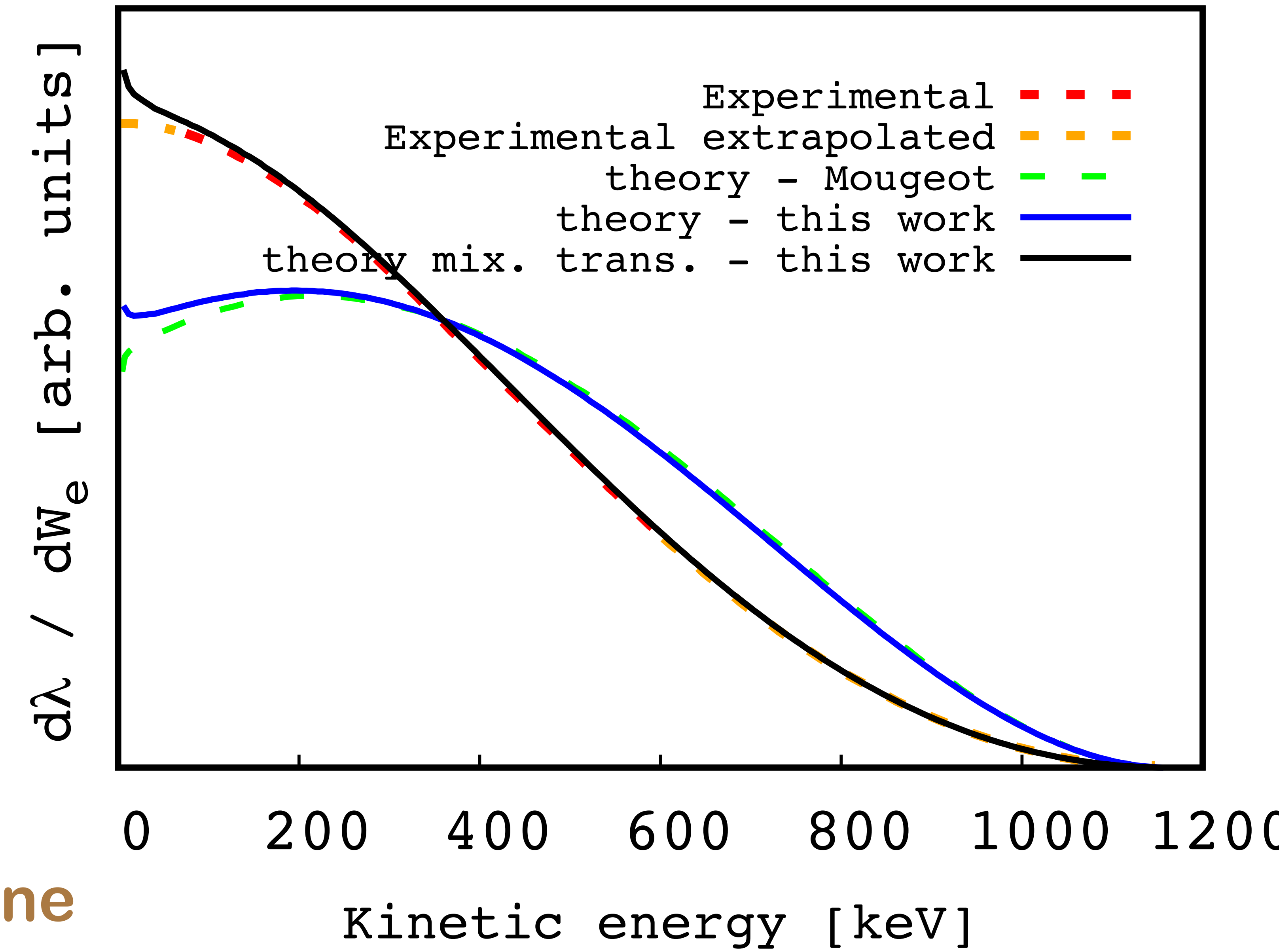
First forbidden non -unique

$^{210}_{83}\mathrm{Bi}_{127}$ **= odd-odd**

$^{210}_{84}\mathrm{Po}_{126}$ **= even-even**

β^- **half-life = 5.012 d**

Q-value = 1162.2 keV (ground state to excited state)



Mean-field DHF + screening + exchange does not work fine as standard approaches !!!!

Shake-up and shake-off modifies the decay by only 5%

The beta-decay spectrum of $^{36}_{17}\mathrm{Cl}_{19}$

$$^{36}_{17}\mathrm{Cl}_{19} \rightarrow\, ^{36}_{18}\mathrm{Ar}_{18} + e^- + \bar{\nu}_e$$

$$J = 2^+ \rightarrow J = 0^+$$

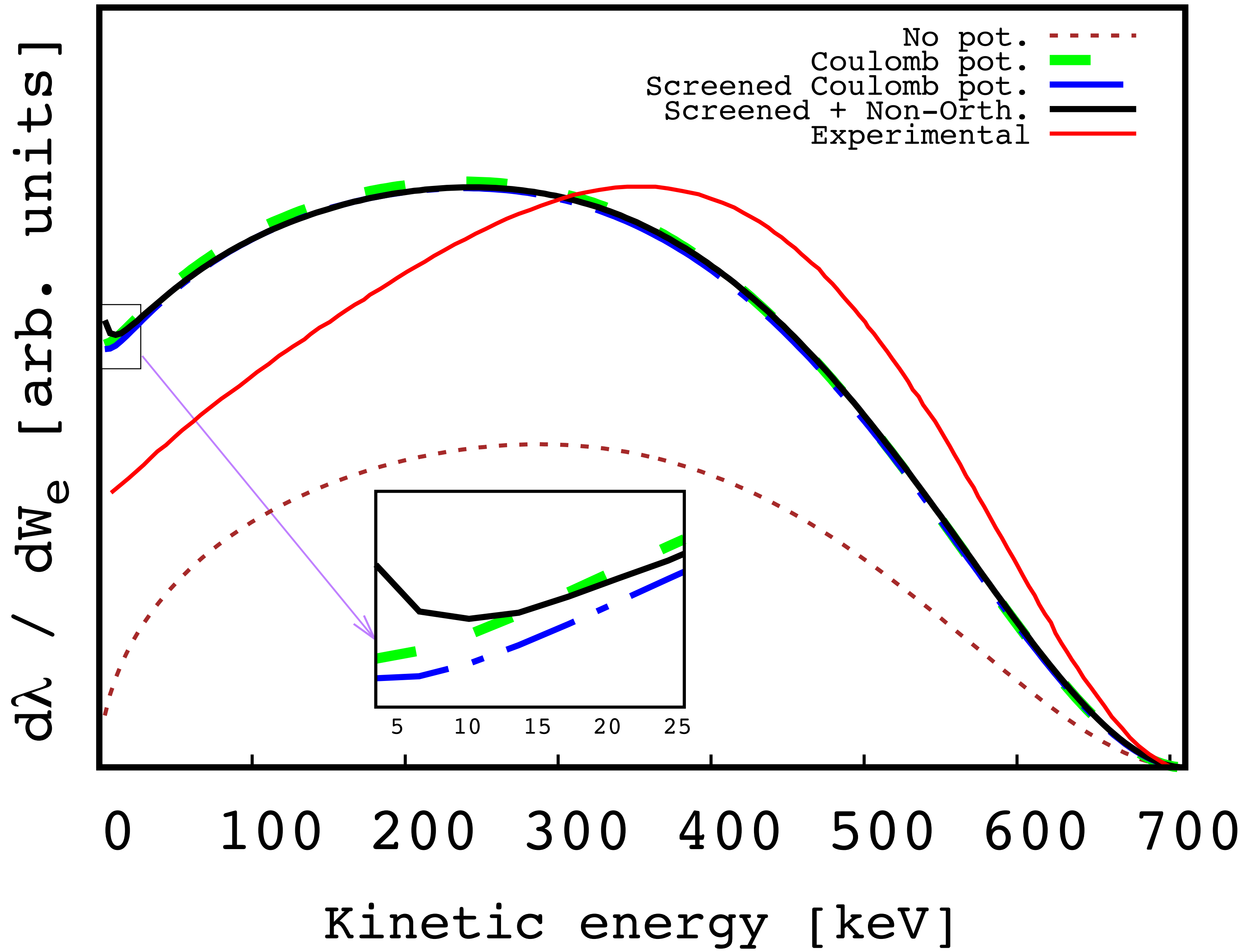
Second forbidden non-unique

$^{36}_{17}\mathrm{Cl}_{19}$ **= odd-odd**

$^{36}_{18}\mathrm{Ar}_{18}$ **= even-even**

β^- **half-life = 3.01X10^5 y**

Q-value = 709.547 keV
(ground state to excited state)



Mean-field DHF + screening (self-consistent DHF) + exchange (discrete-continuum interaction) does not work fine as standard approaches !!!!

Shake-up and shake-off modifies the decay by only 5%

Final-state nuclear many-body affects on beta-decay spectra of odd-odd nuclei?

Spectroscopic term	n	l	J	Number of states 2J+1	Number of nucleons in a shell	Magic Numbers
$1s_{1/2}$	1	0	1/2	2	2	2
$1p_{3/2}$	1	1	3/2	4	6	8
$1p_{1/2}$	1	1	1/2	2		
$1d_{5/2}$	1	2	5/2	6	12	20
$2s_{1/2}$	2	0	1/2	2		
$1d_{3/2}$	1	2	3/2	4		
$1f_{7/2}$	1	3	7/2	8	8	28
$2p_{3/2}$	2	1	3/2	4	22	50
$1f_{5/2}$	1	3	5/2	6		
$2p_{1/2}$	2	1	1/2	2		
$1g_{9/2}$	1	4	9/2	10		
$1g_{7/2}$	1	4	7/2	8	32	82
$2d_{5/2}$	2	2	5/2	6		
$2d_{3/2}$	2	2	3/2	4		
$3s_{1/2}$	3	0	1/2	2		
$1h_{11/2}$	1	5	11/2	12		
$1h_{9/2}$	1	5	9/2	10	44	126
$2f_{7/2}$	2	3	7/2	8		
$2f_{5/2}$	2	3	5/2	6		
$3p_{3/2}$	3	1	3/2	4		
$3p_{1/2}$	3	1	1/2	2		
$1i_{13/2}$	1	6	13/2	14		
$2g_{9/2}$	2	4	9/2	10	58	184
$3d_{5/2}$	3	2	5/2	6		
$1i_{11/2}$	1	6	11/2	12		
$2g_{7/2}$	2	4	7/2	8		
$4s_{1/2}$	4	0	1/2	2		
$2d_{3/2}$	2	2	3/2	4		
$1j_{15/2}$	1	7	15/2	16		

The experimentally determined final state of the $^{36}_{18}\mathrm{Ar}_{18}$ daughter nucleus is 0+. Within the nuclear shell model two protons and two neutrons all occupy the $1d_{3/2}$ single-particle state. By coupling the $1d_{3/2}$ n to p and to a $1d_{3/2}$ "core" to construct a 0+ final symmetry state, and by calculating the hadronic matrix element for this transition only, we obtain the lineshape reported as a blue curve in the previous figure. We could not yet find a good agreement between simulations and experimental data.

Adding "nuclear many-body effects" by mixing transitions to the $1d_{3/2}$ orbital with the $2s_{1/2}$ level, which is energetically close, we find good agreement with experiments

Assumptions in the nuclear simulations

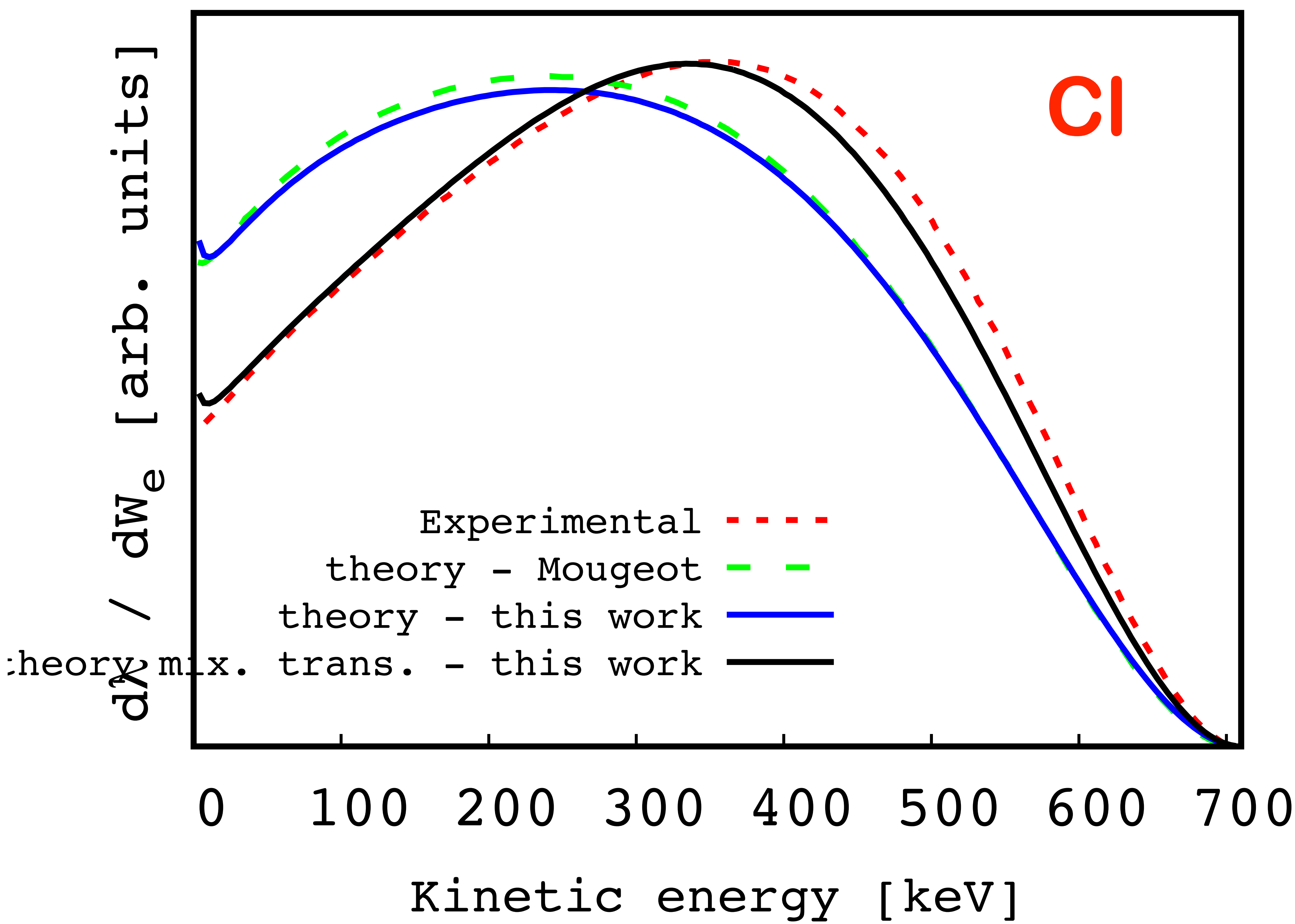
- The decaying neutron in the Cs nucleus is found in the $2d_{3/2}$ shell and weak decays into a proton in the $1g_{7/2}$ shell of Ba. This was deduced according to the nuclear shell model and can be a crude approximation particularly for the excited decays, where several states may participate in the decay. This state is geometrically coupled to the "core" of the other nucleons to recover the total J.

- In a many-body approach, such as CI, the decaying neutron wave function is a superposition of several configuration of nearby energy. In ^{134}Cs those are the $1h_{11/2}$ and $3s_{1/2}$ single-particle orbitals, respectively. However, this level of forbiddance is higher than the d to f owing to a bigger jump in ΔJ.

- The population of nuclear states has been assumed to follow a Boltzmann probability distribution, i.e. $\exp(-E/K_B T)$, where E is the energy of the nuclear level, T the temperature, and K_B the Boltzmann constant. We also took into account the degeneration of the three nuclear levels, which is 9(4^+), 11(5^+), and 7(3^+).

Final-state nuclear many-body affects on beta-decay spectra of odd-odd nuclei?

$$1 \times J^{H,\mu}_{1d_{3/2}\to 1d_{3/2}} - 2.55 \times J^{H,\mu}_{1d_{3/2}\to 2s_{1/2}}$$



β-decay theory: total decay rate

$$\lambda_t = \frac{\pi G_\beta^2}{(2j_n+1)(2J_B+1)} \sum_{\gamma} \sum_{\mu_n,\mu_p} \sum_{\mu'_B,\mu'_C,\mu_B} \sum_{\mu_\nu} \int |I|^2 \rho(W_f)\delta(Q - T'_C - W_\nu)dW_\nu dW'_C$$

$\langle f|\mathcal{H}_\beta|i\rangle$

with electron energy $W_C^f = \sqrt{p^2c^2 + c^4} = c^2 + T_C^f$ and antineutrino energy $W_\nu = c \cdot p_\nu$

μ **and** γ' **runs over magnetic and principal quantum number and where**

$$I = \int\int \langle \xi_p, j_p, \mu_p|\ \hat{\psi}_p^+(\mathbf{r}_h)\gamma^0\gamma^\mu(1 - x\gamma^5)\hat{\psi}_n(\mathbf{r}_h)\ |\xi_n, j_n, \mu_n\rangle \cdot$$

$$\langle \bigwedge_{B,C} n'_B, \kappa'_B, \mu'_B, W'_C, \kappa'_C, \mu'_C; W_\nu, \kappa_\nu, \mu_\nu|\ \hat{\psi}_e^+(\mathbf{r}_l)\gamma^0\gamma_\mu(1 - \gamma^5)\hat{\psi}_\nu(\mathbf{r}_l)\ |\bigwedge_B n_B, \kappa_B, \mu_B; 0\rangle\ \delta(\mathbf{r}_h - \mathbf{r}_l)d\mathbf{r}_h d\mathbf{r}_l$$

expresses the point-like nature of the decay

$$\delta(\mathbf{r}_h - \mathbf{r}_l) = \sum_{L',q} \delta(r_h - r_l) \cdot r_l^{-2} \cdot Y_{L',q}(\theta_h, \phi_h) Y_{L',-q}(\theta_l, \phi_l) \cdot (-)^q$$

Spherical armonics

This notation is useful because it allows to split the matrix element into nuclear and lepton parts

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$$\langle \bigwedge_{B,C} n'_B, \kappa'_B, \mu'_B, W'_C, \kappa'_C, \mu'_C; W_\nu, \kappa_\nu, \mu_\nu | \, \hat{\psi}_e^+(\mathbf{r}_l)\gamma^0\gamma_\mu(1-\gamma^5)\hat{\psi}_\nu(\mathbf{r}_l) \, | \bigwedge_B n_B, \kappa_B, \mu_B; 0\rangle \, \delta(\mathbf{r}_h - \mathbf{r}_l) d\mathbf{r}_h d\mathbf{r}_l$$

expresses the point-like nature of the decay

$$\delta(\mathbf{r}_h - \mathbf{r}_l) = \sum_{L',q} \delta(r_h - r_l) \cdot r_l^{-2} \cdot Y_{L',q}(\theta_h, \phi_h) Y_{L',-q}(\theta_l, \phi_l) \cdot (-)^q$$

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$\langle f|\mathcal{H}_\beta|i\rangle$

with electron energy $W_C^f = \sqrt{p^2c^2 + c^4} = c^2 + T_C^f$ and antineutrino energy $W_\nu = c \cdot p_\nu$

μ and γ' runs over magnetic and principal quantum number and where

$$I = \int\int \langle \xi_p, j_p, \mu_p|\ \hat{\psi}_p^+(\mathbf{r}_h)\gamma^0\gamma^\mu(1 - x\gamma^5)\hat{\psi}_n(\mathbf{r}_h)\ |\xi_n, j_n, \mu_n\rangle \cdot$$

$$\langle W'_C, \kappa'_C, \mu'_C; W_\nu, \kappa_\nu, \mu_\nu|\ \hat{\psi}_e^+(\mathbf{r}_l)\gamma^0\gamma_\mu(1-\gamma^5)\hat{\psi}_\nu(\mathbf{r}_l)\ |0;0\rangle\ \delta(\mathbf{r}_h - \mathbf{r}_l)\ d\mathbf{r}_h\ d\mathbf{r}_l =$$

$$= \int J_{i\to f}^{H,\mu}(r_h) J_{i\to f,\mu}^{L}(r_h)\ dr_h$$

expresses the point-like nature of the decay

$$\delta(\mathbf{r}_h - \mathbf{r}_l) = \sum_{L',q} \delta(r_h - r_l) \cdot r_l^{-2} \cdot Y_{L',q}(\theta_h,\phi_h) Y_{L',-q}(\theta_l,\phi_l) \cdot (-)^q$$

Spherical armonics

<u>This notation is useful because it allows to split the matrix element into nuclear and lepton parts</u>

β-decay theory in central symmetry

Nuclear matrix element on a real space grid

$$J^{H,\mu}(r_h) = \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle \xi_p, j_p, \mu_p|\ \hat{\psi}_p^+(\mathbf{r}_h)\gamma^0\gamma^\mu(1-x\gamma^5)\hat{\psi}_n(\mathbf{r}_h)\ |\xi_n, j_n, \mu_n\rangle \cdot r_h^2 =$$

$$= \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle 0|\hat{a}_p\ \hat{\psi}_p^+(\mathbf{r}_h)\gamma^0\gamma^\mu(1-x\gamma^5)\hat{\psi}_n(\mathbf{r}_h)\ \hat{a}_n^+|0\rangle \cdot r_h^2$$

inserting the expressions for the field operators

$$J^{H,\mu}(r_h) = \sum_{\xi'_p, j'_p, \mu'_p}\ \sum_{\xi'_n, j'_n, \mu'_n} \langle 0|\hat{a}_p\ \hat{a}_{p'}^+\hat{a}_{n'}\ \hat{a}_n^+|0\rangle$$

$$\cdot \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle \xi'_p, j'_p, \mu'_p|\mathbf{r}_h\rangle\ \gamma^0\gamma^\mu(1-x\gamma^5)\langle \mathbf{r}_h|\xi'_n, j'_n, \mu'_n\rangle \cdot$$

and applying anti-commutation rules for creation/destruction Fock-space operators

$$\{\hat{a}_p, \hat{a}_n^\dagger\} = \{\hat{a}_p, \hat{a}_n\} = 0$$

$$\{\hat{a}_p, \hat{a}_{p'}^\dagger\} = \delta_{\xi_p,\xi'_p}\ \delta_{j_p,j'_p}\ \delta_{\mu_p,\mu'_p}$$

$$\{\hat{a}_n, \hat{a}_{n'}^\dagger\} = \delta_{\xi_n,\xi'_n}\ \delta_{j_n,j'_n}\ \delta_{\mu_n,\mu'_n}$$

Selection rules

one gets

$$J^{H,\mu}(r_h) = \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle \xi_p, j_p, \mu_p|\mathbf{r}_h\rangle\ \gamma^0\gamma^\mu(1-x\gamma^5)\langle \mathbf{r}_h|\xi_n, j_n, \mu_n\rangle \cdot r_h^2$$

β-decay theory: total decay rate

Lepton matrix element on a real space grid

$$J_\mu^L(r_h) = \int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle \bigwedge_{B,C} n'_B,\kappa'_B,\mu'_B,W'_C,\kappa'_C,\mu'_C;W_\nu,\kappa_\nu,\mu_\nu|$$

$$\hat{\psi}_e^+(\mathbf{r}_l)\gamma^0\gamma_\mu(1-\gamma^5)\hat{\psi}_\nu(\mathbf{r}_l)\ |\bigwedge_B n_B,\kappa_B,\mu_B;0\rangle\ \delta(r_h-r_l) =$$

$$= \int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle 0;0|\hat{a}'_{1,e}\ldots\ \hat{a}'_{N,e}\hat{a}'_{C,e}\hat{b}_\nu\ \hat{\psi}_e^+(\mathbf{r}_l)\gamma^0\gamma_\mu(1-\gamma^5)\hat{\psi}_\nu(\mathbf{r}_l)\hat{a}^+_{1,e}\ldots\ \hat{a}^+_{N,e}|0;0\rangle\delta(r_h-r_l)$$

inserting the expressions for the field operators

$$J_\mu^L(r_h) = \sum_{n'_B,\kappa'_B,\mu'_B}\int dW'_\nu \sum_{\kappa'_\nu,\mu'_\nu}\langle 0;0|\hat{a}'_{1,e}\ldots\ \hat{a}'_{N,e}\hat{a}'_{C,e}\hat{b}_\nu\hat{a}'^+_{B',e}\hat{b}^+_{\nu'}\hat{a}^+_{1,e}\ldots\ \hat{a}^+_{N,e}|0;0\rangle$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle n'_B,\kappa'_B,\mu'_B|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W'_\nu,\kappa'_\nu,\mu'_\nu\rangle\delta(r_h-r_l) +$$

$$+\int dW'_C \sum_{\kappa'_C,\mu'_C}\int dW'_\nu \sum_{\kappa'_\nu,\mu'_\nu}\langle 0;0|\hat{a}'_{1,e}\ldots\ \hat{a}'_{N,e}\hat{a}'_{C,e}\hat{b}_\nu\hat{a}'^+_{C',e}\hat{b}^+_{\nu'}\hat{a}^+_{1,e}\ldots\ \hat{a}^+_{N,e}|0;0\rangle$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle W'_C,\kappa'_C,\mu'_C|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W'_\nu,\kappa'_\nu,\mu'_\nu\rangle\delta(r_h-r_l)$$

β-decay theory: total decay rate

Lepton matrix element on a real space grid

and applying anti-commutation rules for creation/destruction Fock-space operators

$$\{\hat{a}'_{B,e}, \hat{a}'^{+}_{B',e}\} = \delta_{n_B,n'_B}\ \delta_{\kappa_B,\kappa'_B}\ \delta_{\mu_B,\mu'_B}$$

$$\{\hat{a}'_{C,e}, \hat{a}'^{+}_{C',e}\} = \delta(W_C - W'_C)\ \delta_{\kappa_C,\kappa'_C}\ \delta_{\mu_C,\mu'_C}$$

$$\{\hat{b}_{\nu}, \hat{b}^{+}_{\nu'}\} = \delta(W_\nu - W'_\nu)\ \delta_{\kappa_\nu,\kappa'_\nu}\ \delta_{\mu_\nu,\mu'_\nu}$$

$$\{\hat{a}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}^{+}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}_{C,e}, \hat{a}^{+}_{B,e}\} = 0$$

one gets

$$J^L_\mu(r_h) = \sum_{j=1}^{N} \prod_{B\neq j} (-)^j \langle 0;0|\hat{a}'_{B,e}\hat{a}'_{C,e}\hat{a}^{+}_{1,e}\ldots\hat{a}^{+}_{N,e}|0;0\rangle$$
$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle n'_B,\kappa'_B,\mu'_B|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h - r_l)\ +$$
$$+\ \langle 0;0|\hat{a}'_{1,e}\ldots\ \hat{a}'_{N,e}\hat{a}^{+}_{1,e}\ldots\hat{a}^{+}_{N,e}|0;0\rangle$$
$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle W'_C,\kappa'_C,\mu'_C|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h - r_l)$$

β-decay theory: total decay rate

Lepton matrix element on a real space grid

and applying anti-commutation rules for creation/destruction Fock-space operators

$$\{\hat{a}'_{B,e}, \hat{a}'^{+}_{B',e}\} = \delta_{n_B,n'_B}\ \delta_{\kappa_B,\kappa'_B}\ \delta_{\mu_B,\mu'_B}$$

$$\{\hat{a}'_{C,e}, \hat{a}'^{+}_{C',e}\} = \delta(W_C - W'_C)\ \delta_{\kappa_C,\kappa'_C}\ \delta_{\mu_C,\mu'_C}$$

$$\{\hat{b}_{\nu}, \hat{b}^{+}_{\nu'}\} = \delta(W_\nu - W'_\nu)\ \delta_{\kappa_\nu,\kappa'_\nu}\ \delta_{\mu_\nu,\mu'_\nu}$$

$$\{\hat{a}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}^{+}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}_{C,e}, \hat{a}^{+}_{B,e}\} = 0$$

one gets

$$J^L_\mu(r_h) = \sum_{j=1}^{N} \prod_{B\neq j} (-)^j \langle 0;0|\hat{a}'_{B,e}\hat{a}'_{C,e}\hat{a}^{+}_{1,e}\dots\hat{a}^{+}_{N,e}|0;0\rangle$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle n'_B,\kappa'_B,\mu'_B|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h - r_l)\ +$$

$$+\langle 0;0|\hat{a}'_{1,e}\dots\ \hat{a}'_{N,e}\hat{a}^{+}_{1,e}\dots\hat{a}^{+}_{N,e}|0;0\rangle$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle W'_C,\kappa'_C,\mu'_C|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h - r_l)$$

Standard beta-decay

$$Q_{L',q,C;\mu}(r_h)$$

β-decay theory: total decay rate

Lepton matrix element on a real space grid

and applying anti-commutation rules for creation/destruction Fock-space operators

$$\{\hat{a}'_{B,e}, \hat{a}'^{+}_{B',e}\} = \delta_{n_B,n'_B}\ \delta_{\kappa_B,\kappa'_B}\ \delta_{\mu_B,\mu'_B}$$

$$\{\hat{a}'_{C,e}, \hat{a}'^{+}_{C',e}\} = \delta(W_C - W'_C)\ \delta_{\kappa_C,\kappa'_C}\ \delta_{\mu_C,\mu'_C}$$

$$\{\hat{b}_{\nu}, \hat{b}^{+}_{\nu'}\} = \delta(W_\nu - W'_\nu)\ \delta_{\kappa_\nu,\kappa'_\nu}\ \delta_{\mu_\nu,\mu'_\nu}$$

$$\{\hat{a}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}^{+}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}_{C,e}, \hat{a}^{+}_{B,e}\} = 0$$

one gets

$$J^L_\mu(r_h) = \sum_{j=1}^{N} \prod_{B\neq j} (-)^j \langle 0;0|\hat{a}'_{B,e}\hat{a}'_{C,e}\hat{a}^{+}_{1,e}\ldots\hat{a}^{+}_{N,e}|0;0\rangle$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle n'_B,\kappa'_B,\mu'_B|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h - r_l) +$$

$$+\ \langle 0;0|\hat{a}'_{1,e}\ldots\ \hat{a}'_{N,e}\hat{a}^{+}_{1,e}\ldots\hat{a}^{+}_{N,e}|0;0\rangle$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle W'_C,\kappa'_C,\mu'_C|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h - r_l)$$

Inclusion of post-collisional effects: Fano's and Exchange interactions

β-decay theory: total decay rate

Lepton matrix element on a real space grid

and applying anti-commutation rules for creation/destruction Fock-space operators

$$\{\hat{a}'_{B,e}, \hat{a}'^{+}_{B',e}\} = \delta_{n_B,n'_B}\ \delta_{\kappa_B,\kappa'_B}\ \delta_{\mu_B,\mu'_B}$$

$$\{\hat{a}'_{C,e}, \hat{a}'^{+}_{C',e}\} = \delta(W_C - W'_C)\ \delta_{\kappa_C,\kappa'_C}\ \delta_{\mu_C,\mu'_C}$$

$$\{\hat{b}_{\nu}, \hat{b}^{+}_{\nu'}\} = \delta(W_\nu - W'_\nu)\ \delta_{\kappa_\nu,\kappa'_\nu}\ \delta_{\mu_\nu,\mu'_\nu}$$

$$\{\hat{a}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}^{+}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}_{C,e}, \hat{a}^{+}_{B,e}\} = 0$$

one gets

$$Q_{L',q,B;\mu}(r_h)$$

$$J^L_\mu(r_h) = \sum_{j=1}^{N} \prod_{B\neq j} (-)^j \langle 0;0|\hat{a}'_{B,e}\hat{a}'_{C,e}\hat{a}^{+}_{1,e}\ldots\hat{a}^{+}_{N,e}|0;0\rangle$$
$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle n'_B,\kappa'_B,\mu'_B|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h - r_l)\ +$$
$$+\ \langle 0;0|\hat{a}'_{1,e}\ldots\ \hat{a}'_{N,e}\hat{a}^{+}_{1,e}\ldots\hat{a}^{+}_{N,e}|0;0\rangle$$
$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle W'_C,\kappa'_C,\mu'_C|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h - r_l)$$

How do we actually calculate e-capture rates?

The e-capture rate for ^{7}Be is proportional to the electronic density at the nucleus!!!

Factors affecting this density, such as T (charge state distribution), ϱ, the level of ionization and the presence of other charged particles, screening the interaction, can appreciably modify the decay rate

Our model system of stellar plasma is a Fermi gas in the presence of neutralising particles, such as proton, helium, etc…

Calculation of the leptonic and hadronic wfs: DHF

The time independent Dirac Hamiltonian of a many particles system In the case of two different types of interactions, e.g. represented by scalar (g_S) and vector (g_V) potentials, the Dirac equation reads

$$\left\{ \sum_i \left(c\boldsymbol{\alpha}_i \cdot \mathbf{p}_i + \beta_i mc^2 + V_i \right) + \sum_{i<j} \left[\beta_i \beta_j g_{S,ij} + \left(1 - \boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j\right) g_{V,ij} \right] \right\} \psi\left(\mathbf{r}_1, \cdots \mathbf{r}_N\right) = E\psi\left(\mathbf{r}_1, \cdots \mathbf{r}_N\right)$$

which in second quantization can be written as follows:

$$H = \sum_{s_1 s_2} \int d\mathbf{r}\ \hat{\psi}^+_{s_1}(\mathbf{r}) \left[-ic\boldsymbol{\alpha}_{s_1 s_2} \cdot \boldsymbol{\nabla} + \beta_{s_1 s_2} mc^2 + \delta_{s_1 s_2} V(\mathbf{r}) \right] \hat{\psi}_{s_2}(\mathbf{r}) +$$
$$+ \frac{1}{2} \sum_{s_1 s_2 s'_1 s'_2} \int d\mathbf{r} d\mathbf{r}'\ \hat{\psi}^+_{s_1}(\mathbf{r}) \hat{\psi}^+_{s'_1}(\mathbf{r}') \left[\beta_{s_1 s_2} \beta_{s'_1 s'_2} g_S\left(\mathbf{r}, \mathbf{r}'\right) + \left(\delta_{s_1 s_2} \delta_{s'_1 s'_2} - \boldsymbol{\alpha}_{s_1 s_2} \cdot \boldsymbol{\alpha}'_{s'_1 s'_2} \right) g_V\left(\mathbf{r}, \mathbf{r}'\right) \right] \hat{\psi}_{s'_2}(\mathbf{r}') \hat{\psi}_{s_2}(\mathbf{r})$$

where s_1, s_2, s_1',s_2' index the bispinor two-components

To compute the electronic and hadronic current we use the HF approximation

$$\left\langle \hat{\psi}^+_{s_1}(\mathbf{r}) \hat{\psi}^+_{s'_1}(\mathbf{r}') \hat{\psi}_{s'_2}(\mathbf{r}') \hat{\psi}_{s_2}(\mathbf{r}) \right\rangle = \left\langle \hat{\psi}^+_{s_1}(\mathbf{r}) \hat{\psi}_{s_2}(\mathbf{r}) \right\rangle \left\langle \hat{\psi}^+_{s'_1}(\mathbf{r}') \hat{\psi}_{s'_2}(\mathbf{r}') \right\rangle - \left\langle \hat{\psi}^+_{s_1}(\mathbf{r}) \hat{\psi}_{s'_2}(\mathbf{r}') \right\rangle \left\langle \hat{\psi}^+_{s'_1}(\mathbf{r}') \hat{\psi}_{s_2}(\mathbf{r}) \right\rangle$$

Assumptions in the rate calculations

- In the neutral atom, while for the completely ionized atom (bare nucleus) the orbitals are optimized by considering only a bare Coulomb potential (Cs1s BE = 36.12 keV and 41 keV for neutral and completely ionized atom, respectively).

Table I. Comparison between ^{134}Cs rates obtained using our model and by TY [2] (units in $s^{-1} \times 10^{-8}$).

T_8 [a]	TY [b]	This work [b]
0.5 (4.31)	1.02	1.02
1 (8.62)	3.28	1.01
2 (17.23)	63.1	2.28
3 (25.85)	211.0	4.73
4 (34.47)	481.0	7.22
5 (43.09)	889.0	9.36

[a] $T_8 = 10^8$ K (corresponding values in keV in parentheses).
[b] $n_p = 10^{26}$ cm^{-3}

Table II. Comparison between ^{135}Cs rates obtained using our model and by TY [2] (units in s^{-1}).

T_8 [a]	TY [b]	This work [b]	TY [c]	This work [c]	TY [d]	This work [d]	TY [e]	This work [e]
0.5 (4.31)	8.12e-15	1.05e-14	7.90e-15	1.02e-14	7.92e-15	9.70e-15	7.39e-15	9.11e-15
1 (8.62)	1.04e-14	1.44e-14	8.78e-15	1.22e-14	8.04e-15	1.08e-14	7.81e-15	9.79e-15
2 (17.23)	6.91e-13	3.39e-13	6.65e-13	3.27e-13	6.09e-13	3.01e-13	5.52e-13	2.66e-13
3 (25.85)	8.64e-11	4.08e-11	8.55e-11	4.04e-11	8.24e-11	3.91e-11	7.74e-11	3.64e-11
4 (34.47)	9.77e-10	4.66e-10	9.65e-10	4.64e-10	9.52e-10	4.57e-10	9.17e-10	4.38e-10
5 (43.09)	4.18e-09	2.05e-09	4.15e-09	2.05e-09	4.08e-09	2.03e-09	3.96e-09	1.97e-09

[a] $T_8 = 10^8$ K (corresponding values in keV in parentheses).
[b] $n_p = 10^{26}$ cm^{-3}
[c] $n_p = 3 \times 10^{26}$ cm^{-3}
[d] $n_p = 10^{27}$ cm^{-3}
[e] $n_p = 3 \times 10^{27}$ cm^{-3}

Assumptions in the electronic structure calculations

- The chemical potential of electrons and positrons is calculated under the assumption to deal with an ideal Fermi gas in a box using a relativistic energy-momentum dispersion $E^2 = c^2p^2 + m_e c^4$. Protons are non-relativistic particles.

- The density of protons n_p (protons/cm³) and is equal to the density of electrons minus the density of positrons at that given temperature (energy can be high enough to form e⁺-e⁻ couples):

$$n_p = n_{e^-} - n_{e^+}$$

- The electronic levels of the Cs atom have not been re-optimized at each temperature. It is assumed that they are the same at any temperature, and we populate them according the Fermi-Dirac (FD) distribution $n^i_{e^-} = \dfrac{1}{1 + e^{(\epsilon_i - \mu_{e^-})/(KT)}} = F(T, \mu)$, where the energies ϵ_i of the i-th level is obtained via the self-consistent solution of the DHF equation and the chemical potential from the implicit relation valid for a Fermi gas:

$$n_{e^-} = \int_0^\infty dp\ p^2/\pi^2 \times (F((c \times \sqrt{(}p^2 + c^2) - \mu_e)/kT) - F((c \times \sqrt{(}p^2 + c^2) + \mu_e)/kT))$$

Important messages from rate calculations

The β decay rate of Cs is affected concurrently by two major factors:

1. **the presence of 3 nuclear excited states of Cs;**
2. **the electronic excitation, also up to a complete ionization**
3. **Our half-life are consistently higher than TY**

Nuclear DOF

The nuclear excited state dynamics is the most relevant of the two, as it can increase the rate by a factor of 15 at 100 KeV (1 GK) to 23 at 1000 KeV with reference to room temperature conditions and by a factor of 3 at $T>10^8$ K for ^{134}Cs as compared to previous works based on systematics.

This is basically due to populating fast-decaying nuclear excited states, in particular the 60 keV excited state of ^{134}Cs which delivers a rate ~ 80 times higher than the 4^+ GS decay. This number is obtained by comparing the decay rates from 4^+ and 3^+, as if they were the only occupied nuclear states from which the decay occurs.

Electronic DOF

At variance, in the range [0:15] keV the temperature of electrons has the most pronounced impact on the rate. Rate increases as electron can be accommodated also in empty bound orbitals. Despite being a quark-level process, the contribution of the electronic degrees of freedom to the rate is crucial.

Increasing temperature means both populating electronic excited states and changing the charge state. This may decrease the half-life even by 20% at 10 keV

To summarize some data: owing to the temperature acting on both nuclei and electrons we find an increase of the rate of about 3 times at 20 KeV (~ 230 MK), of 6 times at 30 keV, of 8 times at 40 keV (~ 464 MK) with respect to the GS decay only.