

# Jet and grooming algorithms at the future Electron-Ion-Collider

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Supported by the FELLINI project and the European Commission



# In this talk:

Asymmetric jet clustering in deep-inelastic scattering

[arXiv: 2006.10751](#)

In collaboration with:

M. Arratia, D. Neill, F. Ringer,  
N. Sato

Grooming algorithms for DIS: example 1-jettiness

[arXiv 2101.02708](#)

# Introduction to jets (Brief and basic)

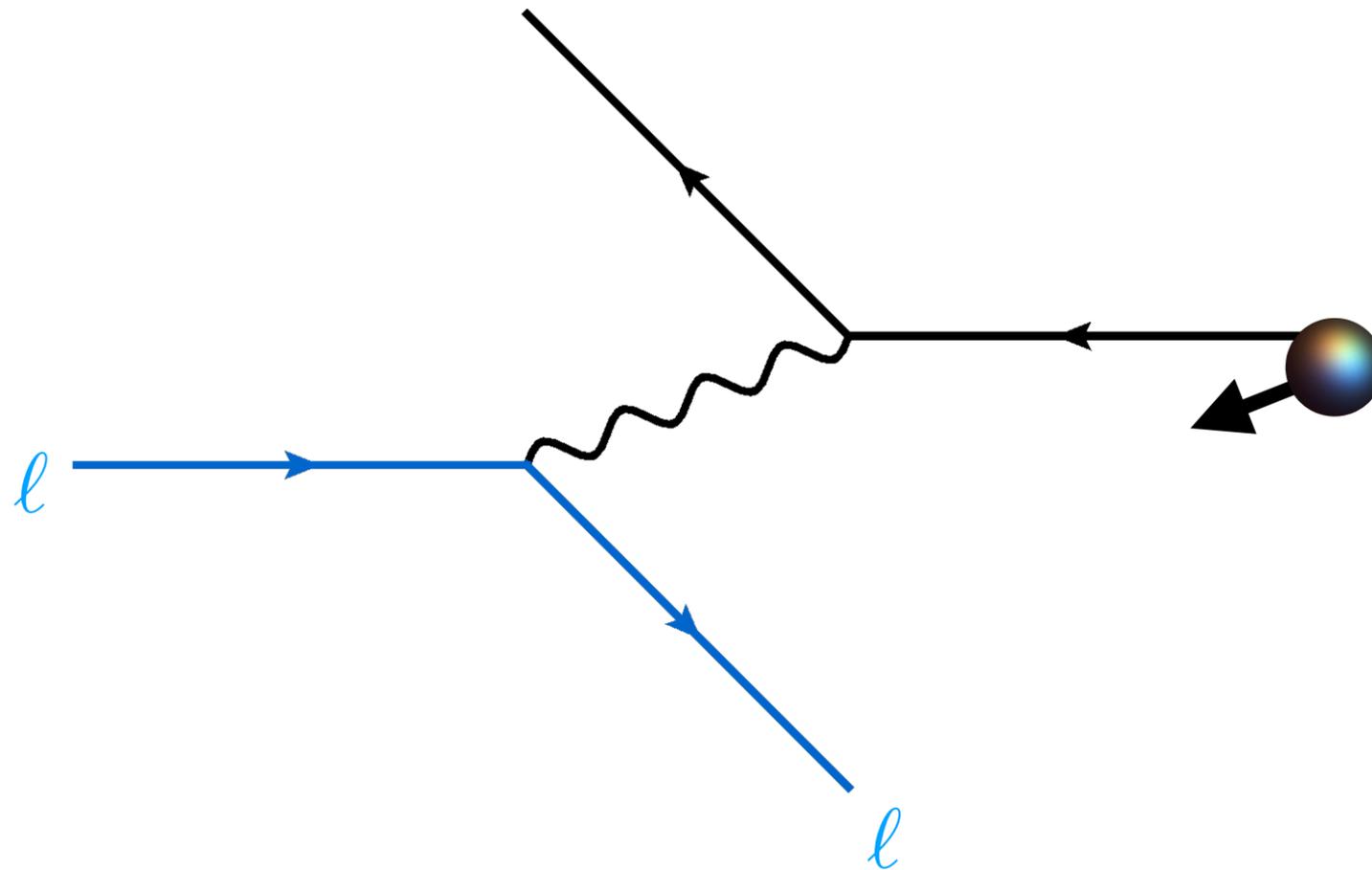
Jet algorithms (Part 1)

Event grooming and substructure (Part 2)

# Jets as proxies for partons

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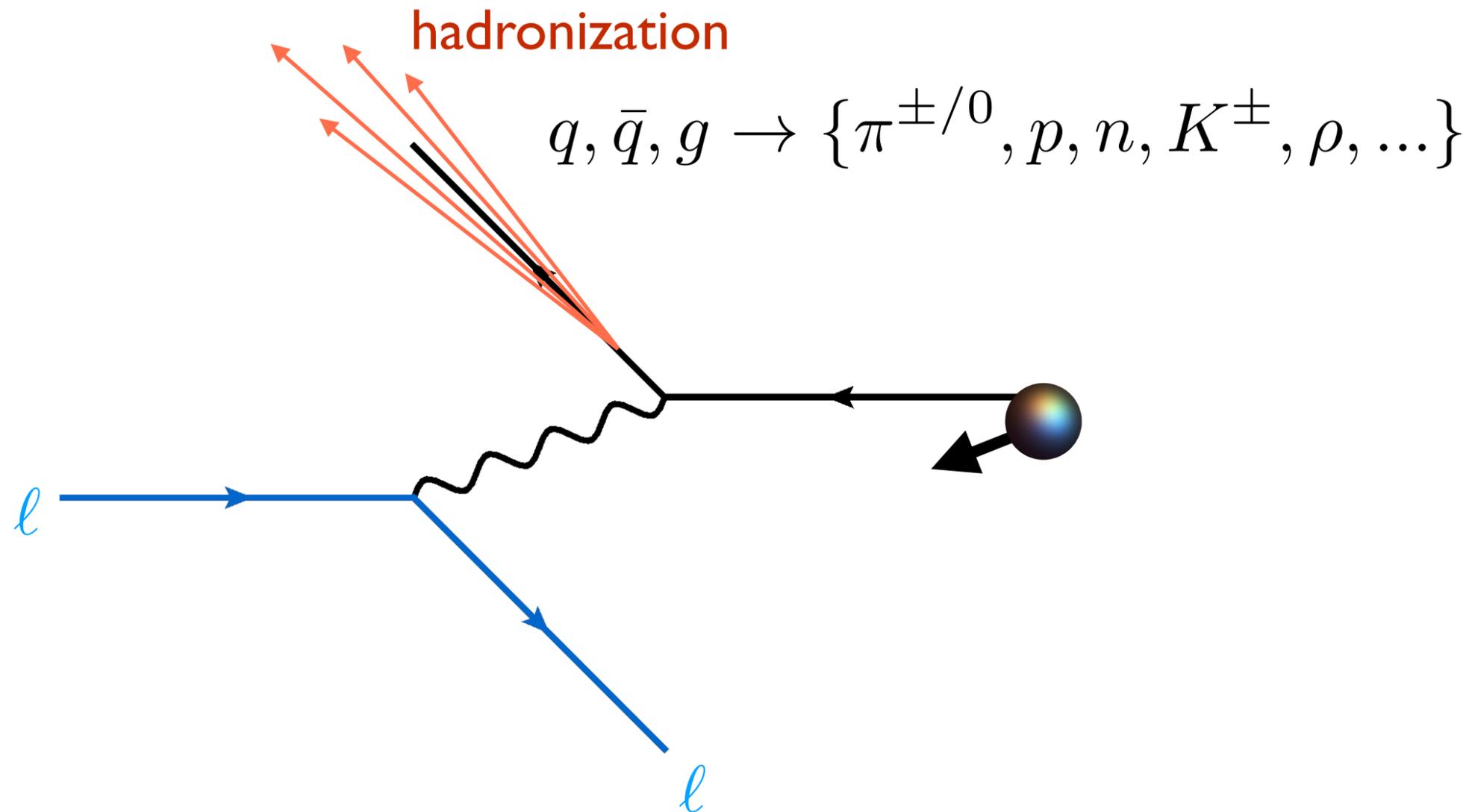
deep inelastic scattering



# Jets as proxies for partons

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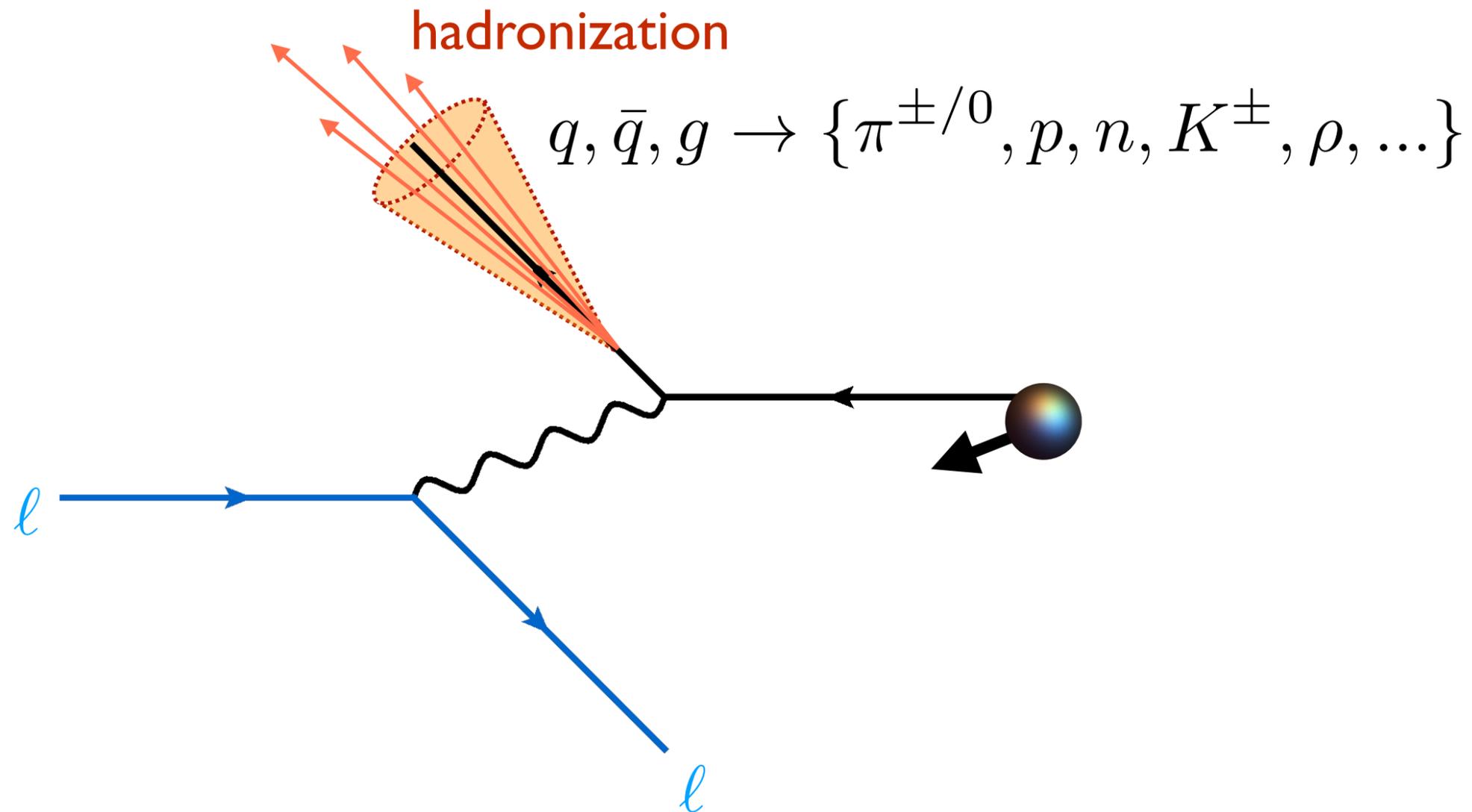
deep inelastic scattering



# Jets as proxies for partons

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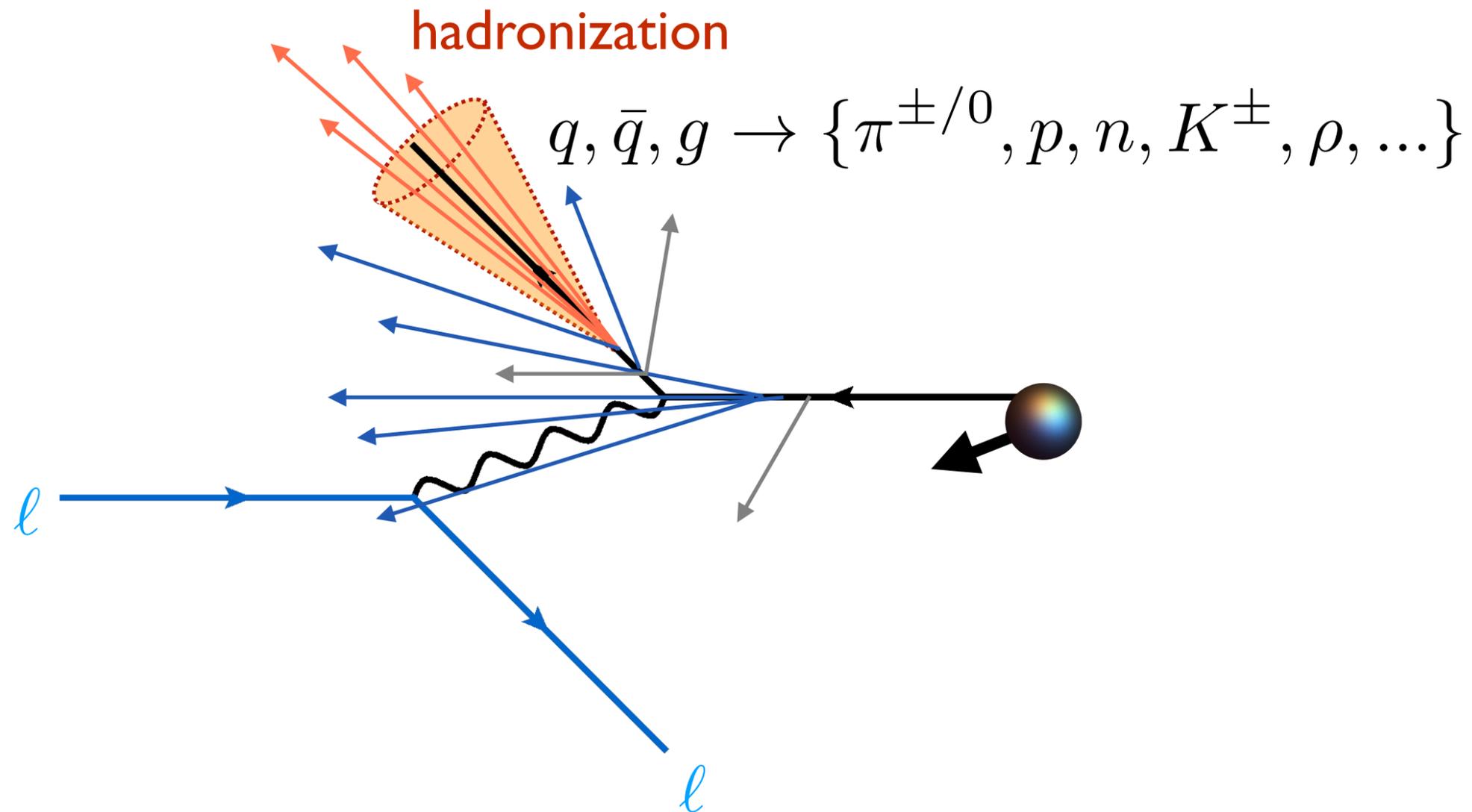
deep inelastic scattering



# Jets as proxies for partons

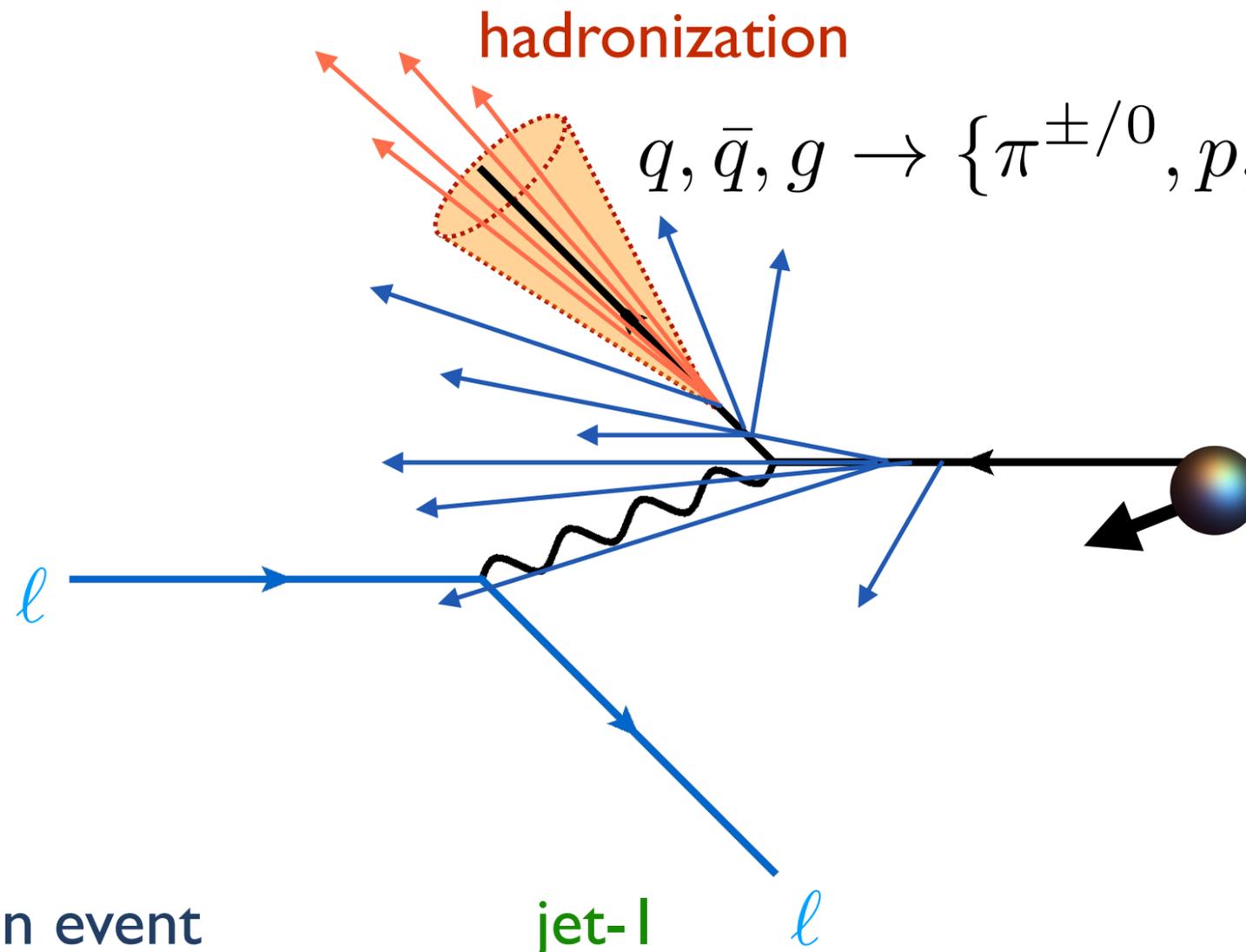
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deep inelastic scattering



# Jets as proxies for partons

deep inelastic scattering



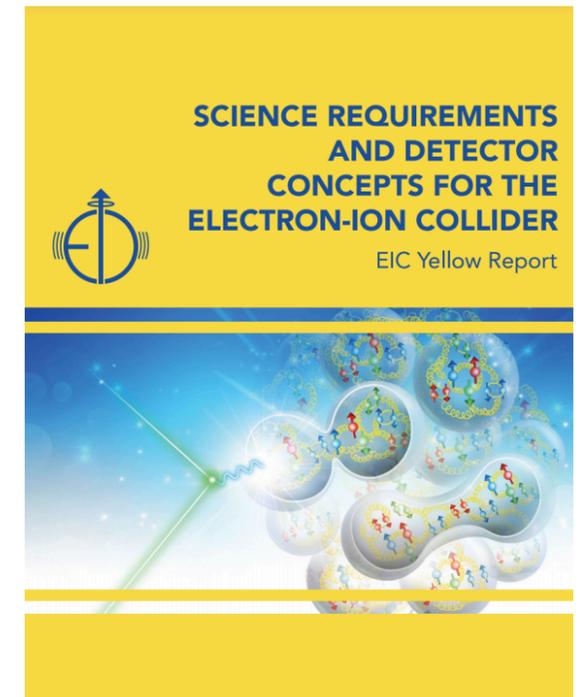
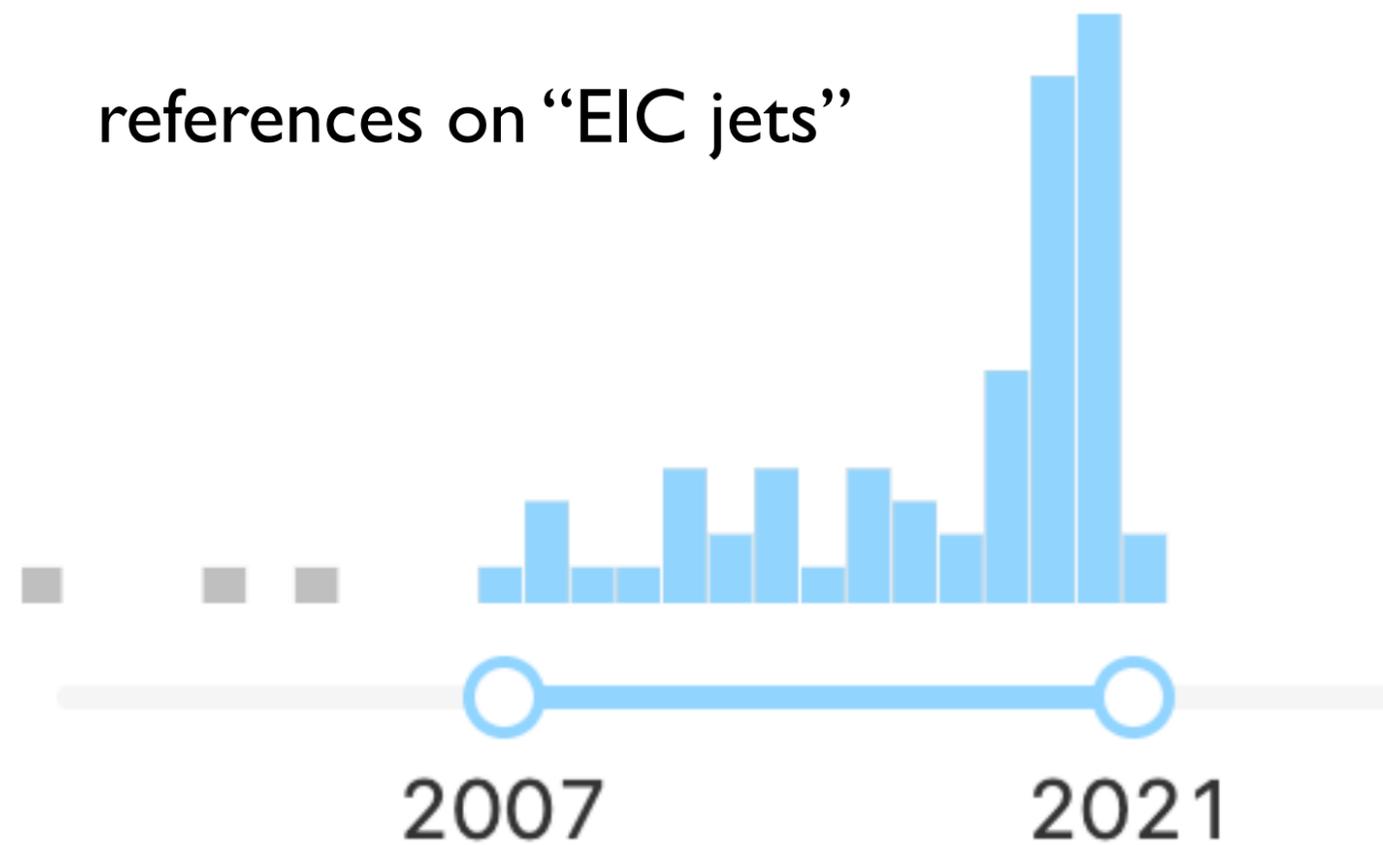
Jet algorithms:  
Algorithmic, reproducible,  
processes that **define** the jet  
IR-safe definitions of jets

All particles in event

$$p_1, p_2, p_3, \dots, p_n \rightarrow \{p_1, p_2, p_3, \dots\}_{\text{jet-1}}, p_i, p_{i+1}, \dots, \{p_k, p_{k+1}, \dots\}_{\text{jet-2}}$$

# Jets at Electron-Ion-Collider (EIC)

Source: [inspirehep.net](http://inspirehep.net)



<http://www.eicug.org/web/content/yellow-report-initiative>

# Application of jets at EIC

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**Extraction of strong-coupling:** multi-jet rates, event shapes, jet substructure,...

**Hadronic structure:** Jet-TMDs, in-jet fragmentation, multi-jet rates, diffractive processes,...

**Nuclear matter probes:** Jet substructure, jet rates and jet quenching,...

**Hadronization:** Jet substructure, in-jet fragmentation, jet energy loss,...

Introduction to jets (Brief and basic)

## Jet algorithms (Part 1)

Event grooming and substructure (Part 2)

# The kT-type algorithms

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1. Evaluate distance:

$$d_{ij} = \min[k_{Ti}^{2p}, k_{Tj}^{2p}] \frac{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}{R^2}$$

$$d_{iB} = k_{Ti}^{2p}$$

“particle”-“particle”

“particle”-“beam”

2. Merge “nearest”

3. Repeat

↓  
Selectors (e.g.  $p_T > 20$  GeV)

Final set of jets

Anti-kT:  $p = -1$

arXiv:0802.1189 (M. Cacciari, G. P. Salam and G. Soyez)

# Breit Frame kinematics

$$n^\mu = (1, 0, 0, +1) \quad \bar{n}^\mu = (1, 0, 0, -1)$$

Proton 

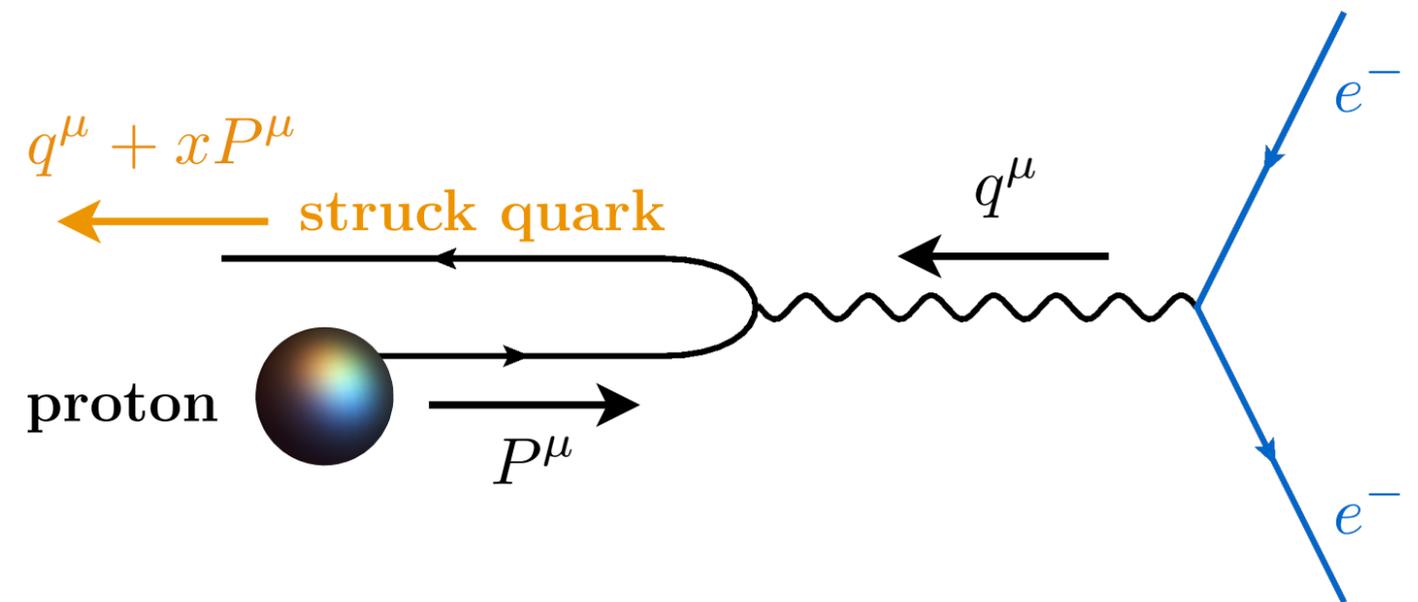
$$P^\mu = \frac{Q}{2x_B} n^\mu$$

Photon 

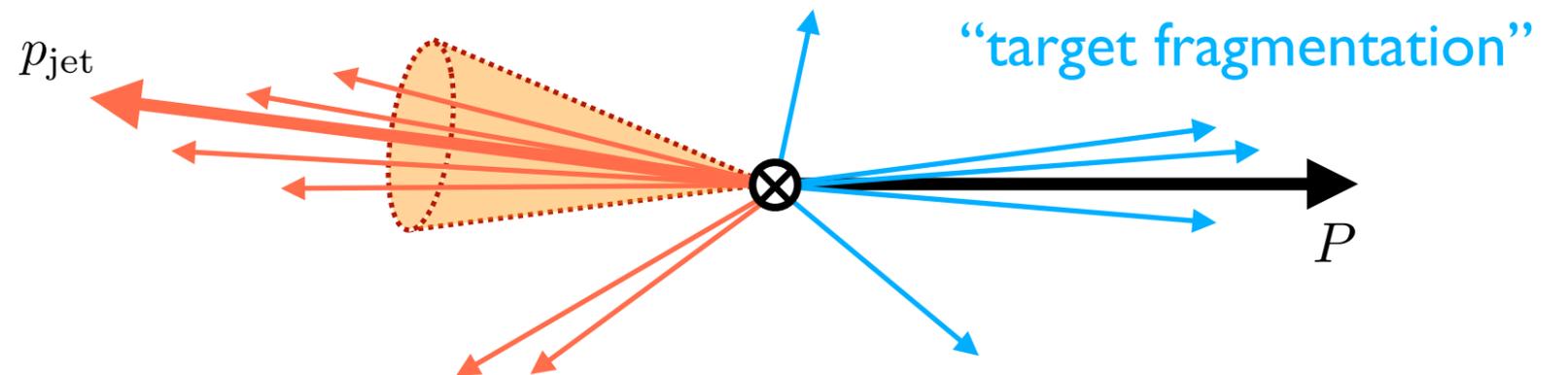
$$q^\mu = \frac{Q}{2} (n^\mu - \bar{n}^\mu)$$

Struck quark 

$$p^\mu = q^\mu + xP^\mu = \frac{Q}{2} \bar{n}^\mu$$

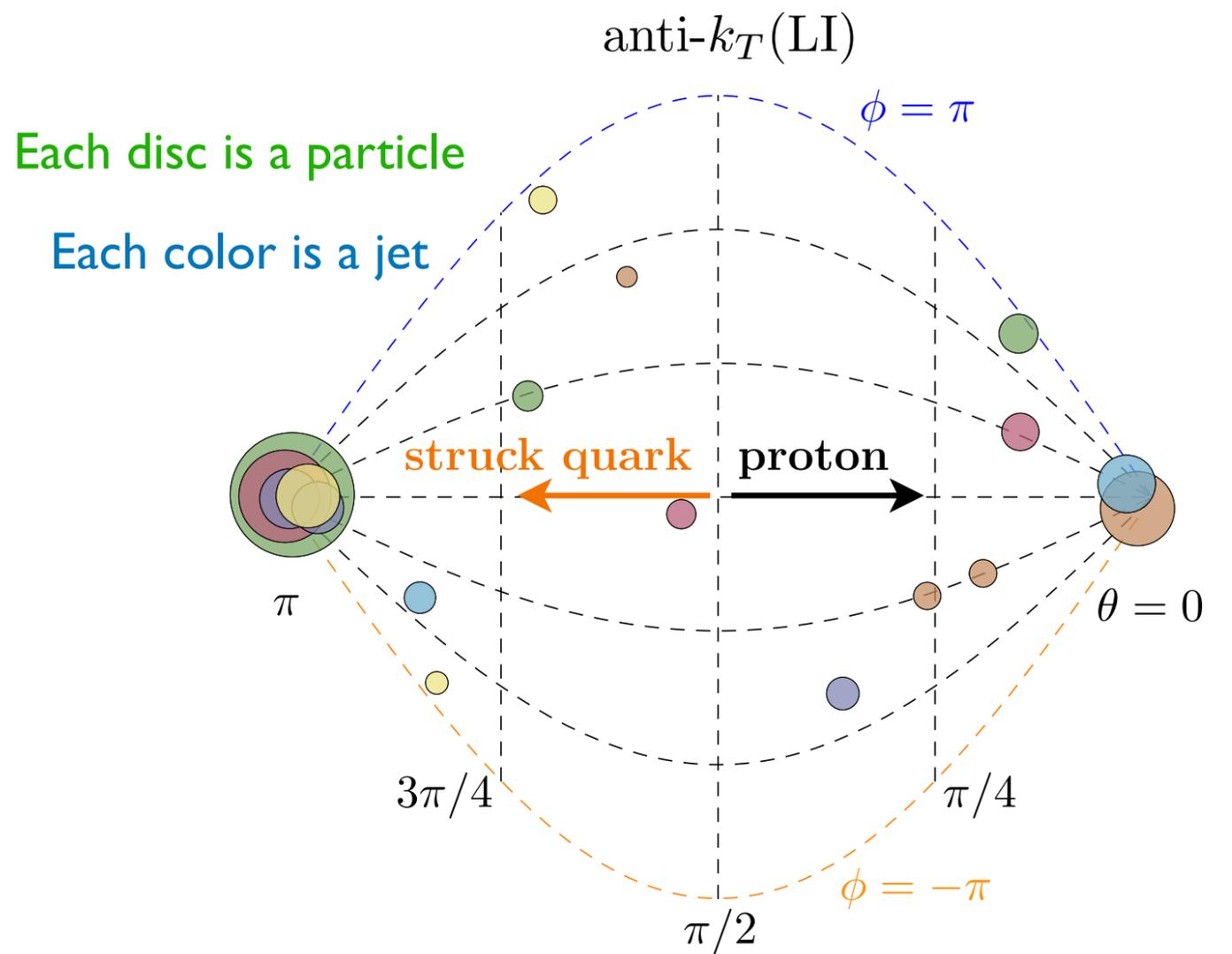
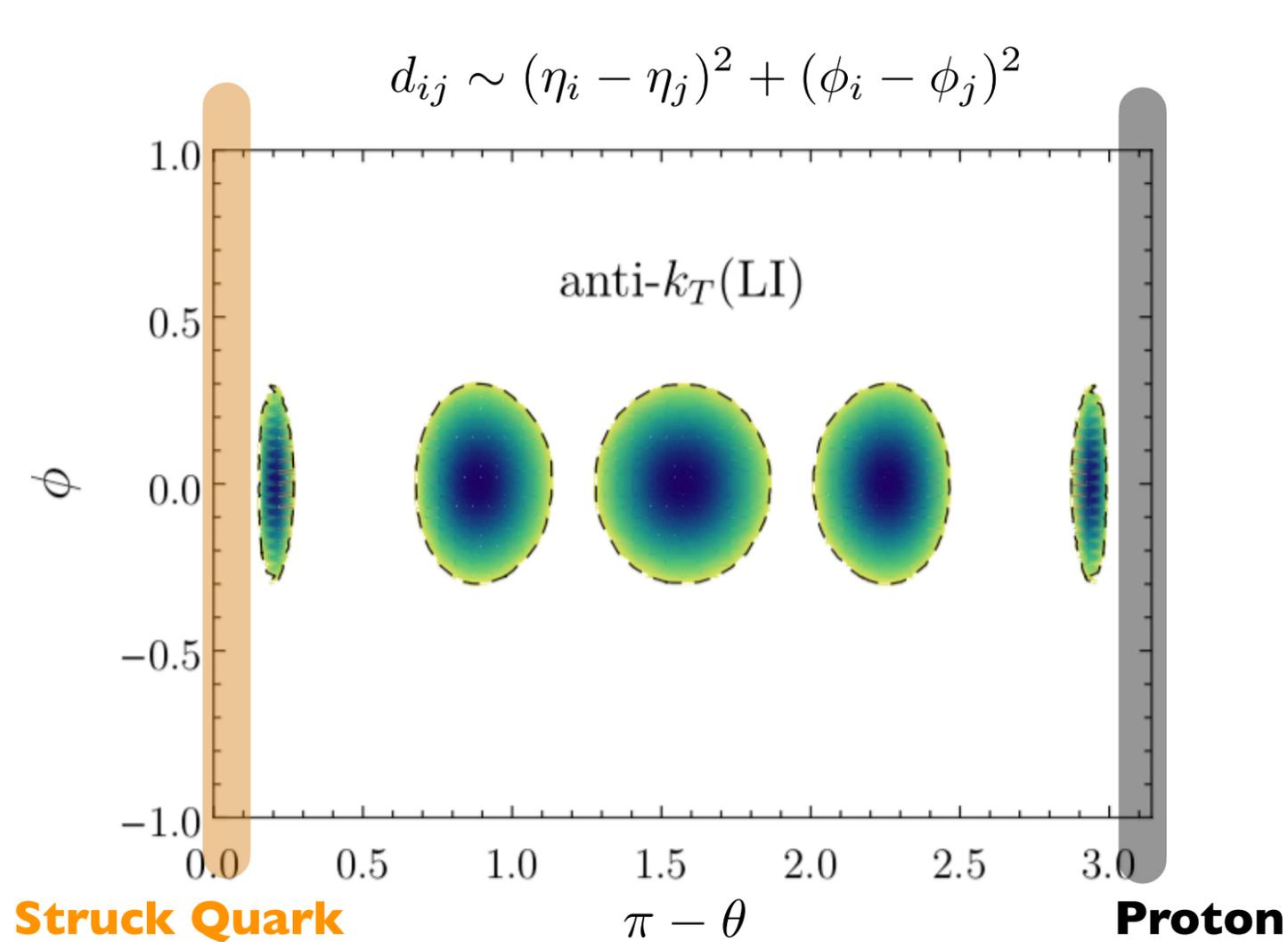


“current fragmentation”



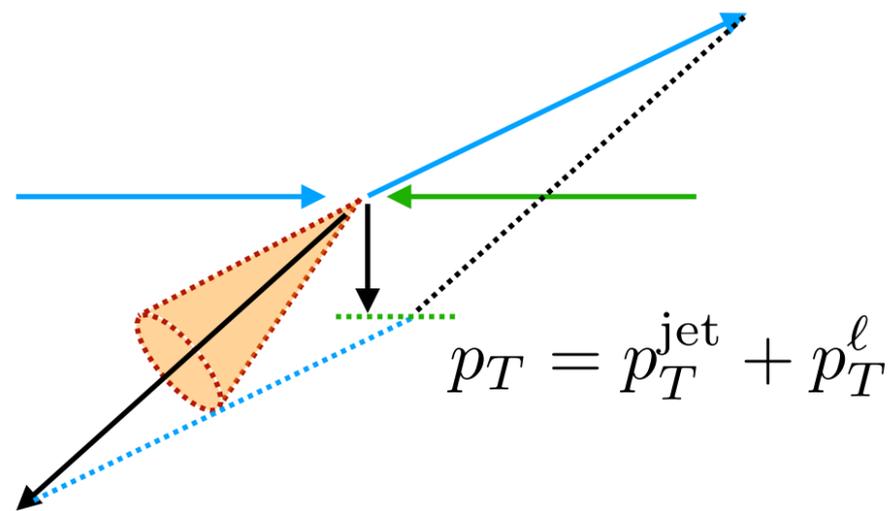
# Longitudinally invariant algorithms

However, in Breit Frame since they cluster in the  $\eta$ - $\phi$  space, they fail to capture jets in the very backward direction, where we are looking for one.



# Universal TMDs in the Breit frame

LAB frame

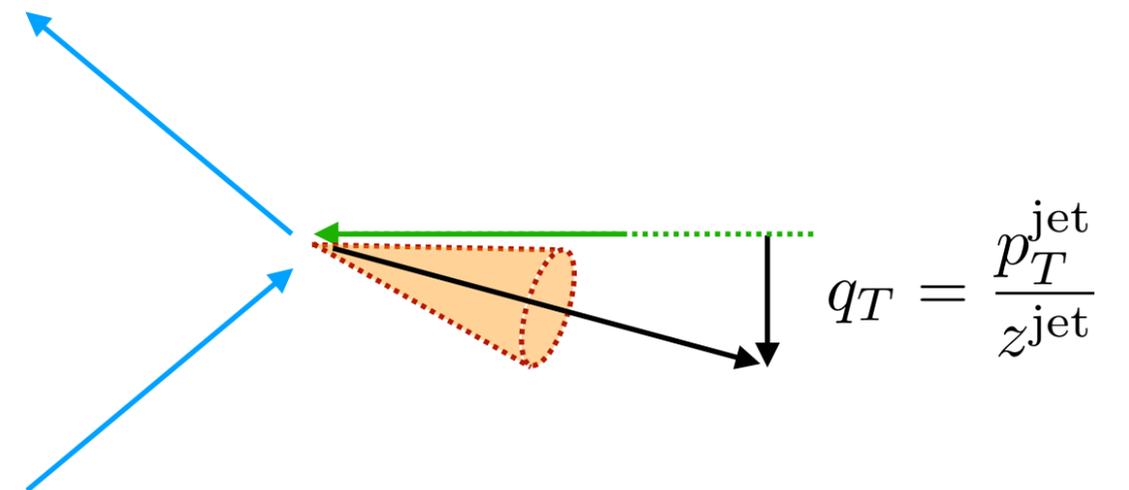


$$d\sigma \sim \left( f(x, b) S_{n_1 n_2}(b) \right) J(z, b)$$

**Not universal**

arXiv:1812.08077 (X. Liu, F. Ringer, W. Vogelsang, and F. Yuan)

Breit frame



$$d\sigma \sim \left( f(x, b) \sqrt{S_{n\bar{n}}(b)} \right) \left( D(z, b) \sqrt{S_{n\bar{n}}(b)} \right)$$

**Universal**

arXiv:1807.07573 (D. G.-Reyes, I. Scimemi, W. J. Waalewijn, L. Zoppi)

Also non-perturbative analysis for jet substructure observables (e.g. jet mass)  
is simplified in the Breit frame (work in progress)

# Alternative algorithms

kT-type (SI)

Spherically invariant

$$d_{ij} = \min(E_i^{2p}, E_j^{2p}) \frac{1 - \cos \theta_{ij}}{1 - \cos R}$$

$$d_{iB} = E_i^{2p}$$

Similar algorithm for DIS by:  
S. Catani, Y. L. Dokshitzer, and B. Webber, (1992)

Here we use:  $p = -1$

Centauro

Longitudinally invariant

$$d_{ij} = \min[z_i^{2p}, z_j^{2p}] (\Delta f_{ij}^2 + 2f_i f_j (1 - \cos \Delta \phi_{ij})) / R^2$$

$$d_{iB} = z_i^{2p}$$

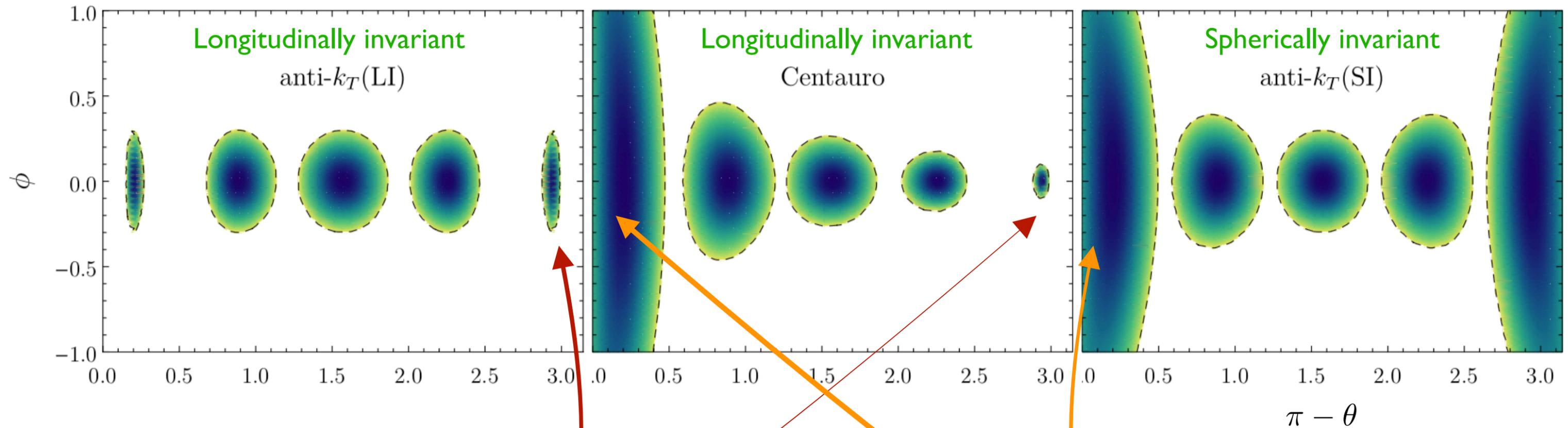
Asymmetric measure

$$f(x) = x + \mathcal{O}(x^2)$$

$$\bar{\eta}_i = -\frac{2Q}{\bar{n} \cdot q} \frac{p_i^\perp}{n \cdot p_i}$$

$$\bar{\eta}_i(\text{BF}) = 2p_i^\perp / p_i^+$$

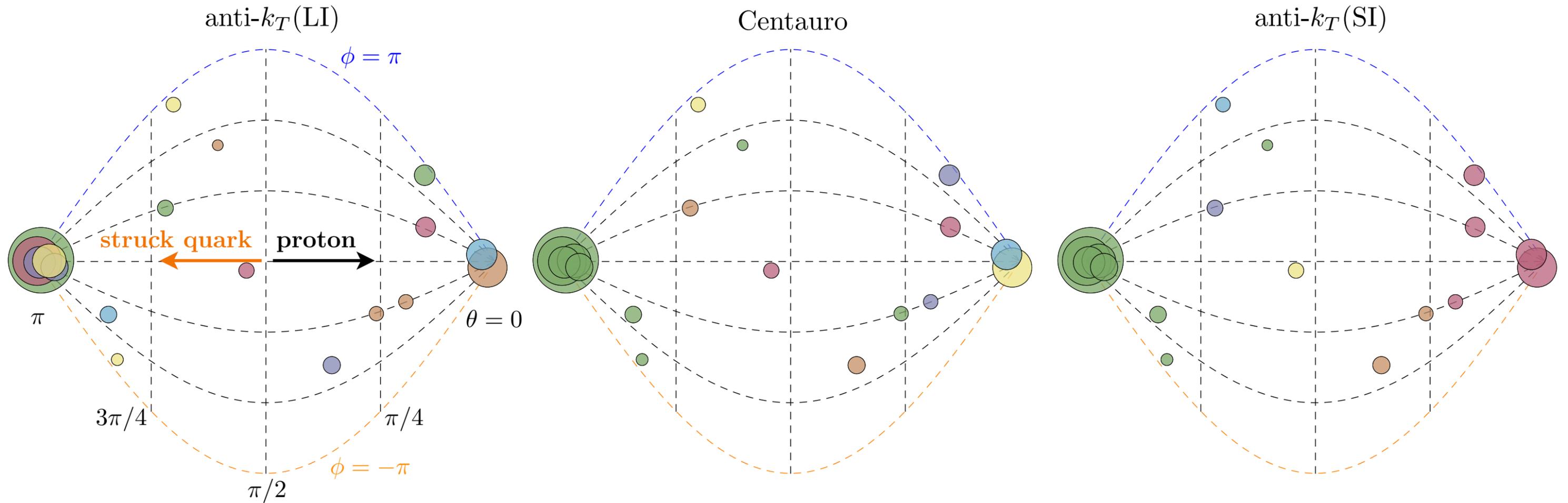
# Centauro: a hybrid algorithm



Jet area shrinks in the proton direction

The algorithm captures the beam axis along the struck quark direction

# Clustering



Longitudinally invariant  
Symmetric

Longitudinally invariant  
Asymmetric

Spherically invariant  
Symmetric

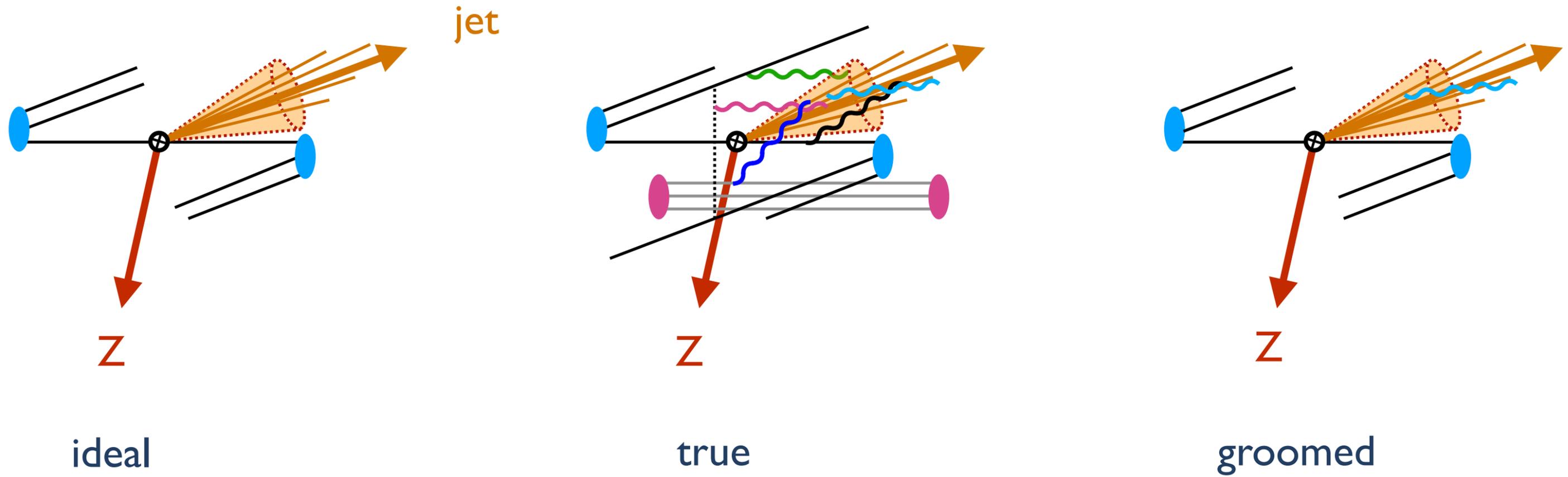
Introduction to jets (Brief and basic)

Jet algorithms (Part 1)

**Event grooming and substructure (Part 2)**

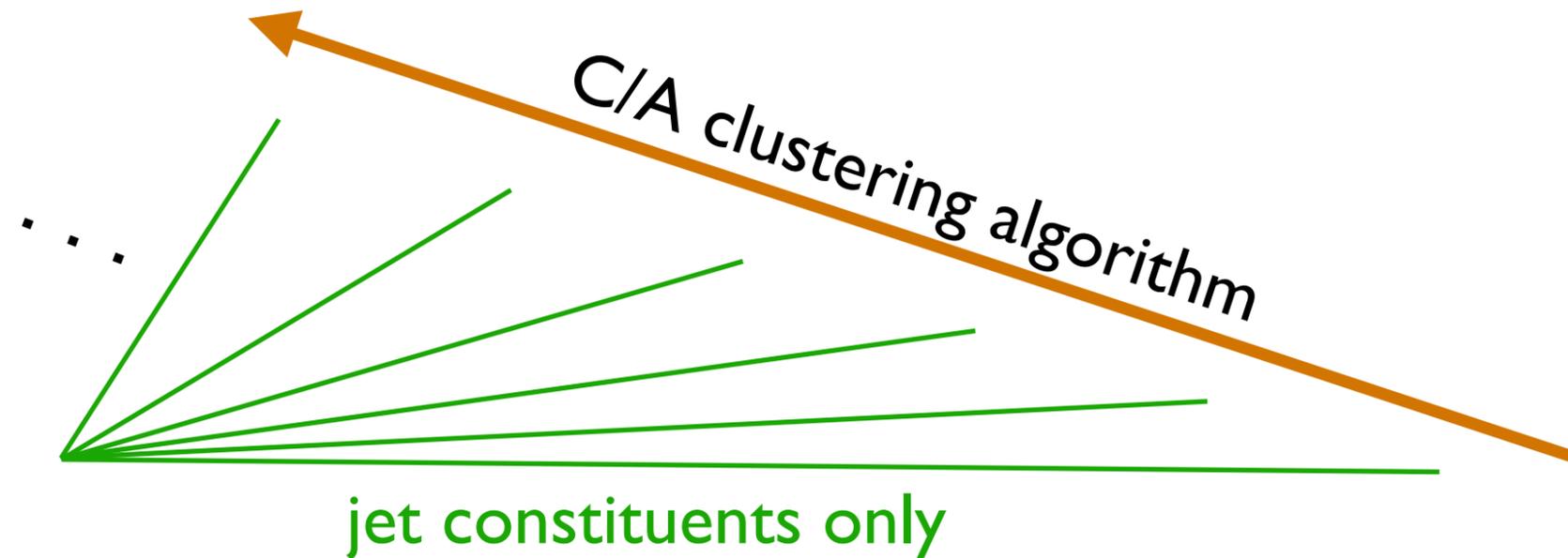
# Jet grooming

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# mMDT

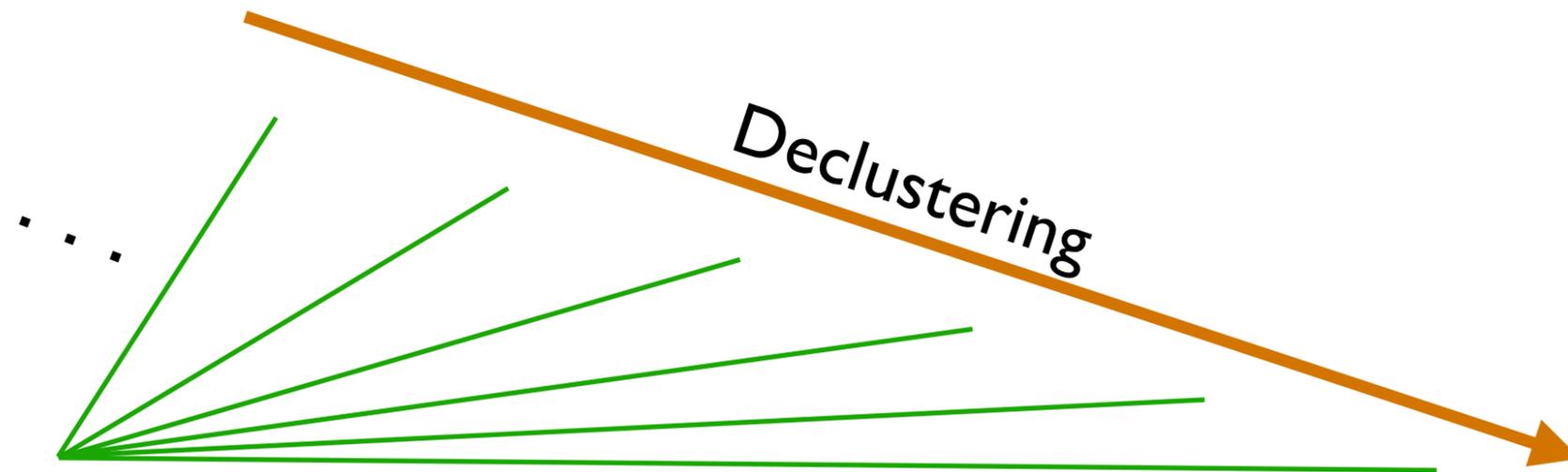
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- The algorithm is imposed only on the jet constituents
- Particles closer in angle get clustered first
- Record clustering history in each step (example coming soon)

# mMDT

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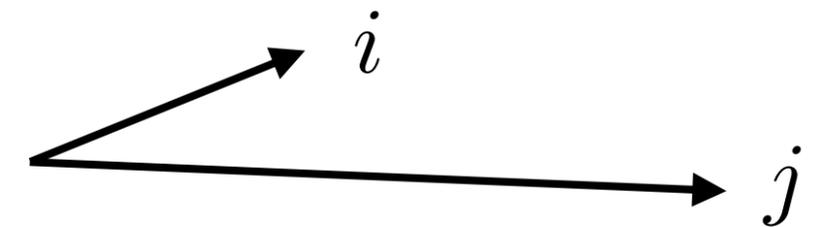
$$\frac{\min\{E_i, E_j\}}{E_i + E_j} > z_{\text{cut}}$$

True

Stop. Remaining particles  
consist groomed jet

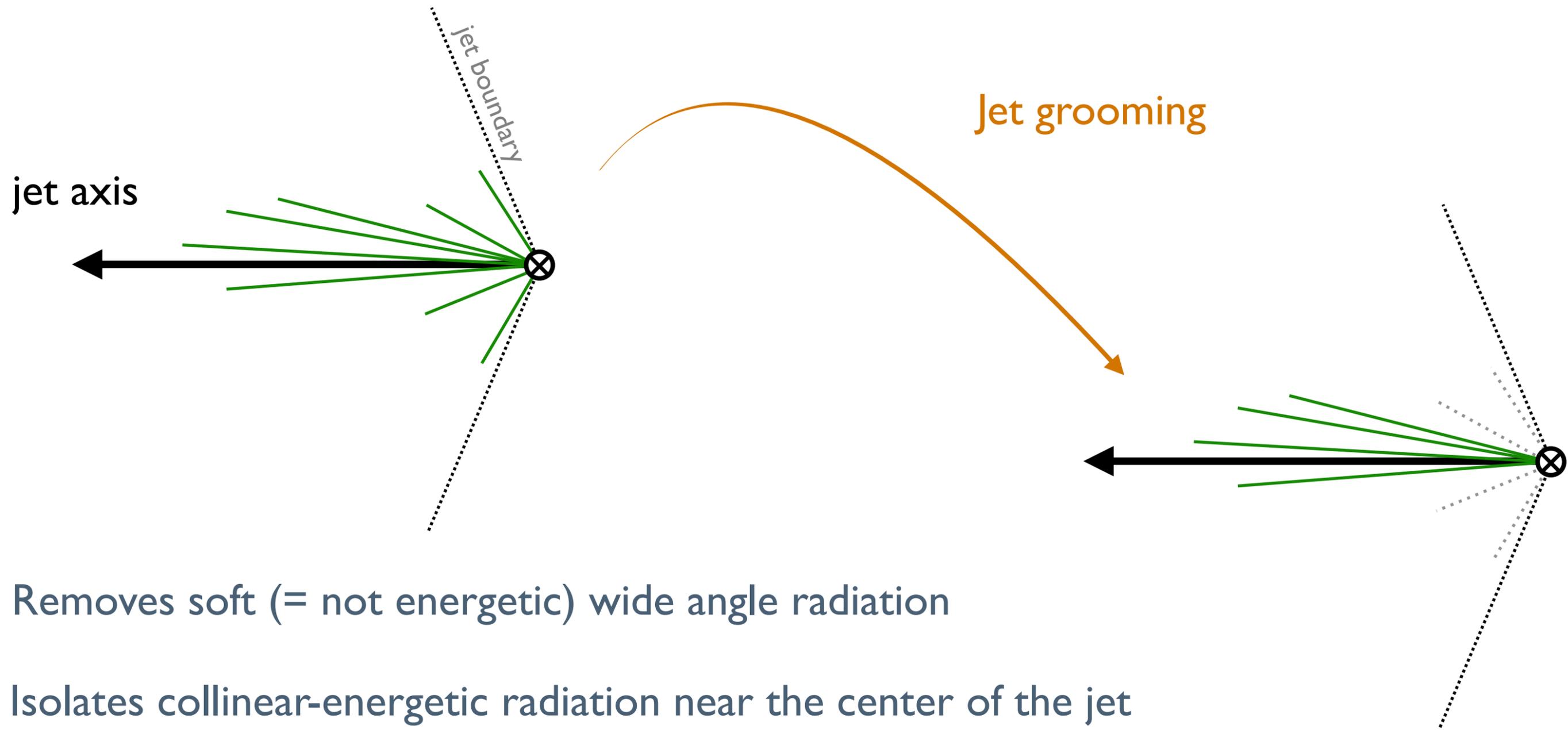
False

Drop the softer of the two  
branchings



# Jet grooming

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- Removes soft (= not energetic) wide angle radiation
- Isolates collinear-energetic radiation near the center of the jet  
insensitive to the cone/boundary
- Reduce contamination from underlying event

# The grooming algorithm in DIS events

UE will not be a problem in EIC ✖

No non-global effects ✔

Handle on soft radiation ✔

Dial on non-perturbative corrections ✔

Universal FS matrix elements ✔

Low particle multiplicity !

# Breit Frame kinematics

Proton 

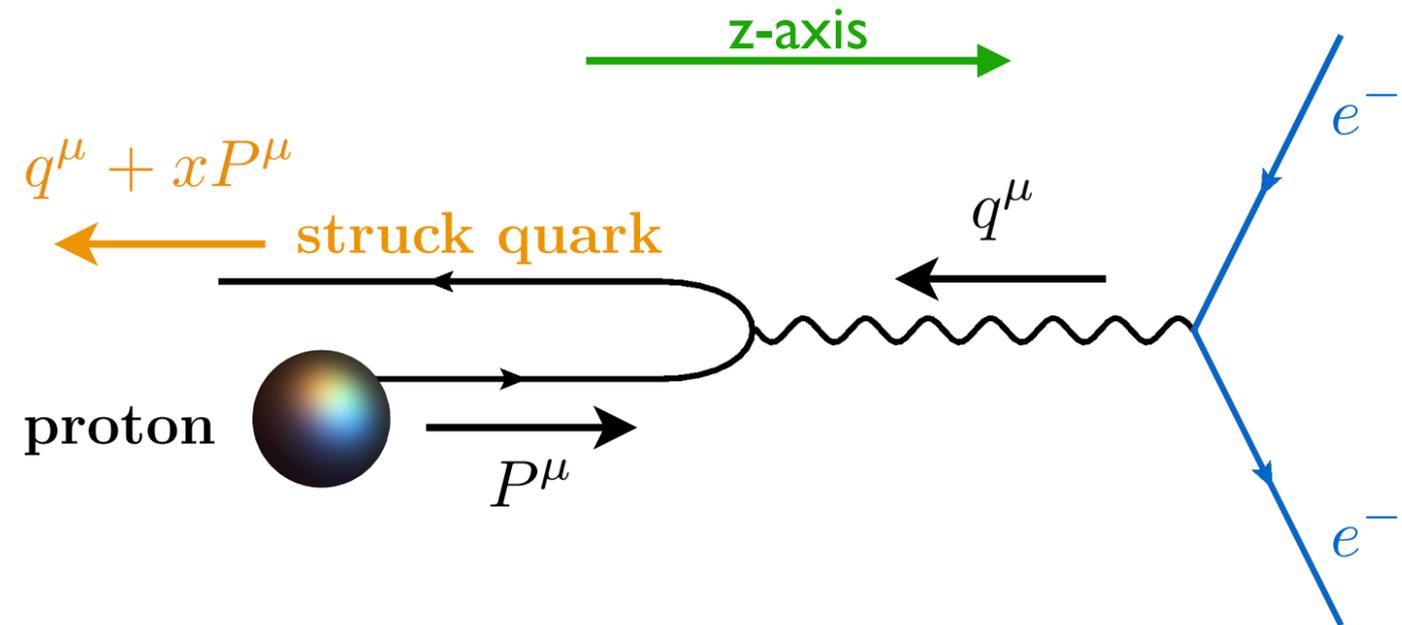
$$P^\mu = \frac{Q}{2x_B}(1, 0, 0, 1)$$

Photon 

$$q^\mu = (0, 0, 0, -Q)$$

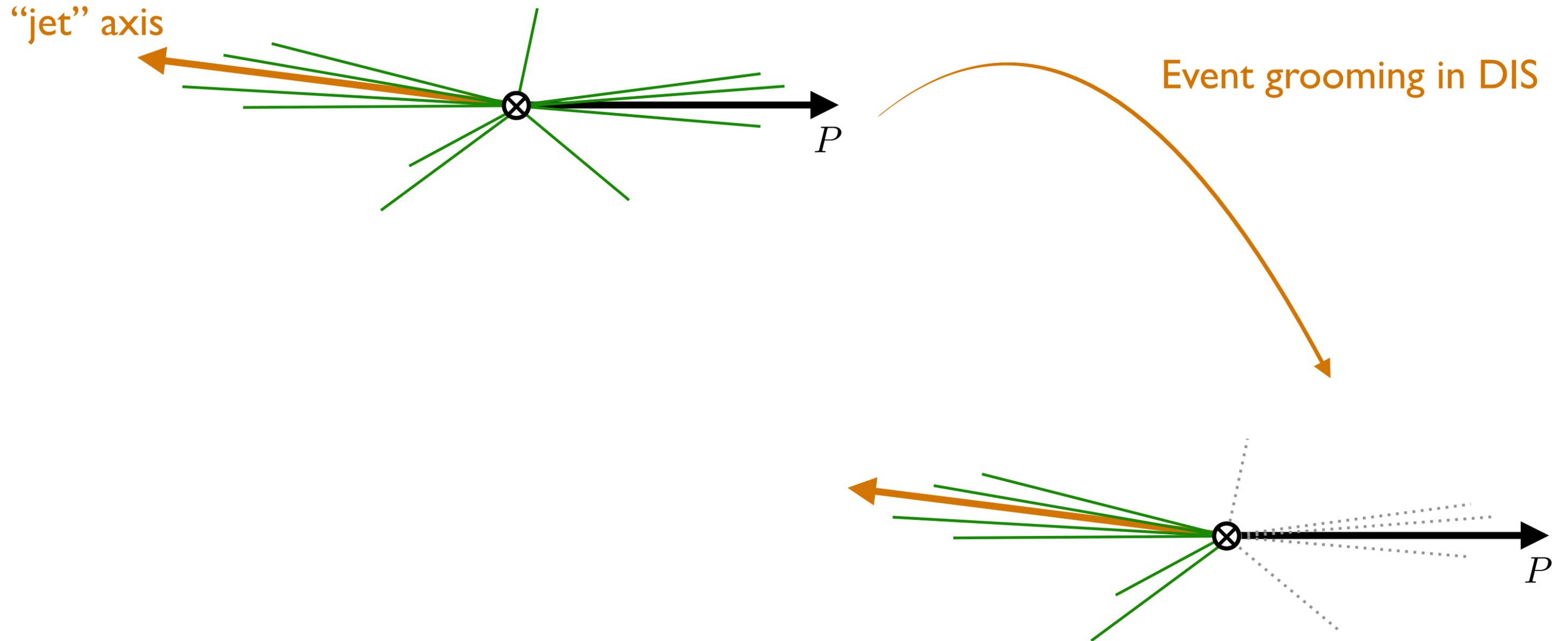
Struck quark 

$$p^\mu = \frac{Q}{2}(1, 0, 0, -1)$$



# Event grooming

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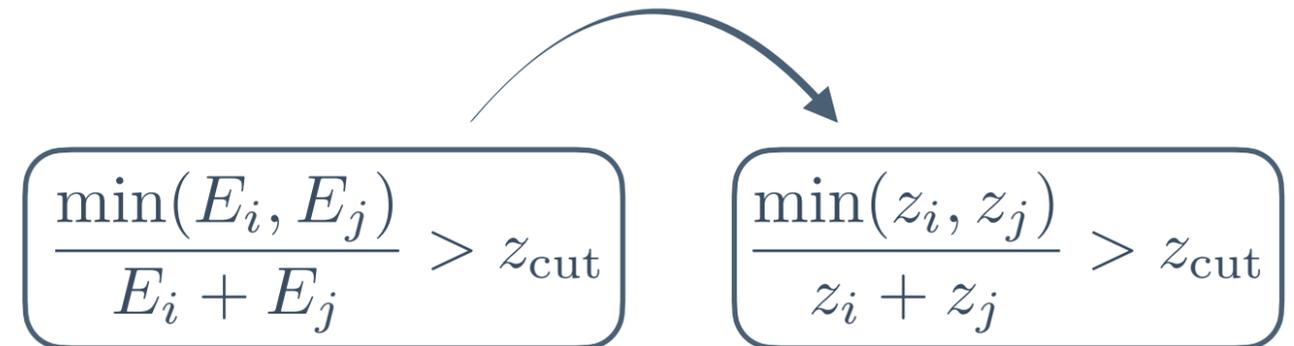
# The grooming algorithm

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modified MassDrop Tagger (mMDT)

Centauro clustering measure, (instead of C/A) \*

Modified grooming condition, ( $p^+$  instead of  $p^0$ )


$$\frac{\min(E_i, E_j)}{E_i + E_j} > z_{\text{cut}} \quad \frac{\min(z_i, z_j)}{z_i + z_j} > z_{\text{cut}}$$

Applied to the event, (instead of jets or hemispheres)

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\* A similar procedure can be imposed in the Laboratory frame using C/A (L.I.)

# The grooming algorithm

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## (Step 1/2) Clustering

1. For all pair of particles  $(i, j)$  calculate:  $d_{ij} = (\bar{\eta}_i - \bar{\eta}_j)^2 + 2\bar{\eta}_i\bar{\eta}_j(1 - \cos(\phi_i - \phi_j))$
2. Find the smallest measure and cluster particles  $i$  and  $j$  into a single branch
3. Repeat until all particles are clustered together

# The grooming algorithm

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(Step 2/2) DeClustering: Open the clustered tree in the reverse order.  
In each decluttering step there are two branches.

1. For these two branches test the condition:  $\frac{\min(z_i, z_j)}{z_i + z_j} > z_{\text{cut}}$   $z_i = \frac{P \cdot p_i}{P \cdot q}$
2. If the condition is false, drop the branch with smallest  $z_i$ ,  
and continue with the decluttering (point 1) of the branch largest  $z_i$ .
3. Repeat until the condition evaluate true

# Energy fraction (of particle $i$ )

Definition

$$z_i = \frac{P \cdot p_i}{P \cdot q}$$

Boundaries

$$z_i(\text{Breit frame}) = p_i^+ / Q$$

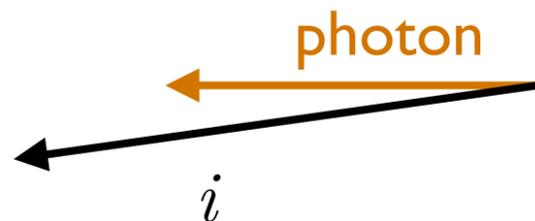
$$z_i \geq 0$$

$$z_i < 1$$

$$\sum_{i \in \text{event}} z_i = 1$$

Limits (geometric interpretation in the Breit frame)

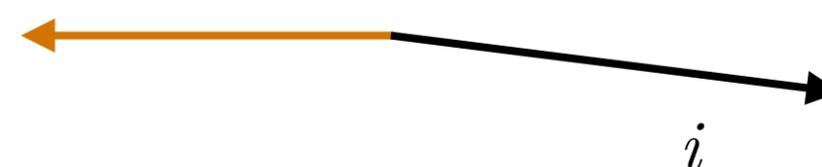
$$z_i \sim 1$$



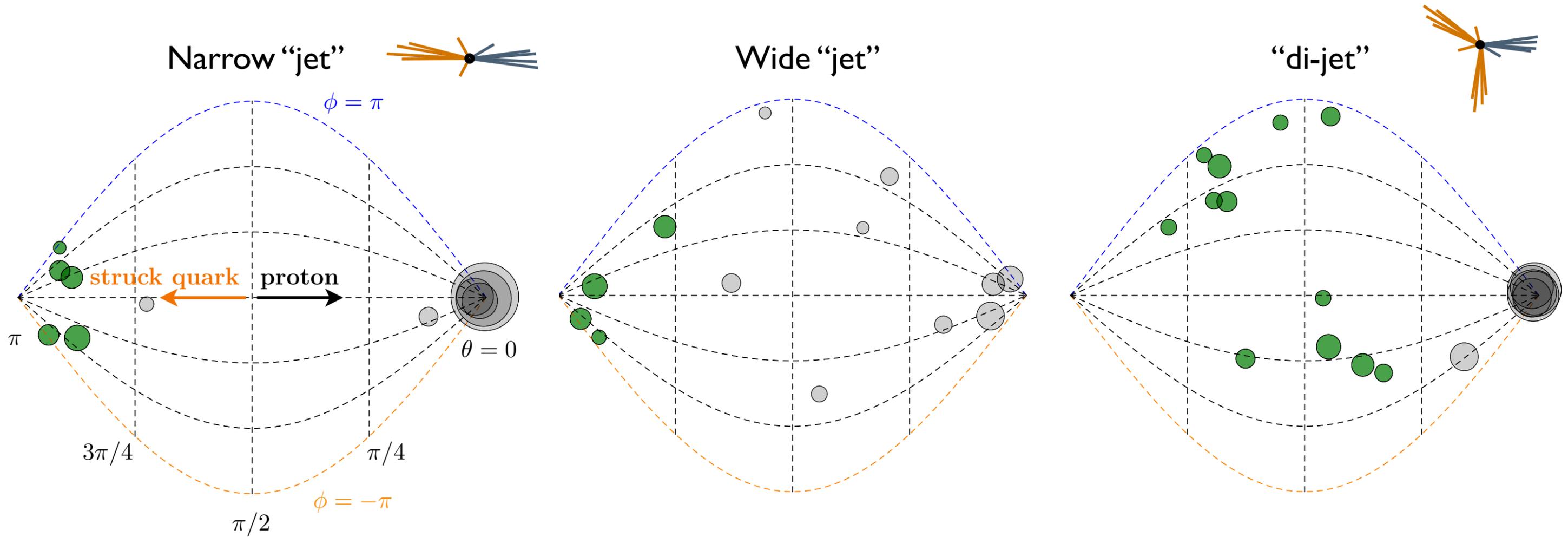
$$z_i \ll 1$$



$$z_i \ll 1$$



# The grooming algorithm



Each disc is a particle:

-  = Pass Grooming
-  = Fail Grooming

PYTHIA 8 :  $\sqrt{s} = 63 \text{ GeV}$

$e^- + p \rightarrow e^- + \text{jet} + X$

$Q > 10 \text{ GeV}$

# Applications for EIC

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## Event-shape observables in ep

Dynamics and interplay of perturbative QCD and hadronization  
Extracting QCD fundamental constants (strong coupling)  
Fragmentation of heavy quarks

## 3D-structure of the proton (complementary to semi-inclusive DIS)

Probe to initial state (proton/nuclear) TMDs (no TMD-FFs)  
Probe to final state TMDs and TMD evolution (no TMD-PDFs)

## Cold nuclear matter effects

Jet substructure  $\rightarrow$  Event substructure  $(z_g, \theta_g)$   
Clean access to modifications from propagation in nuclear medium

# Applications for EIC

---

Event-shape observables in ep  Discuss next (as an example 1-jettiness/“jet”-mass)

Dynamics and interplay of perturbative QCD and hadronization

Extracting QCD fundamental constants (strong coupling)

Fragmentation of heavy quarks

3D-structure of the proton (complementary to semi-inclusive DIS)

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Clean access to modifications from propagation in nuclear medium

# 1-jettiness in DIS

$$\tau_1 = \frac{2}{Q^2} \sum_{i \in \text{event}} \min(q_B \cdot p_i, q_J \cdot p_i)$$

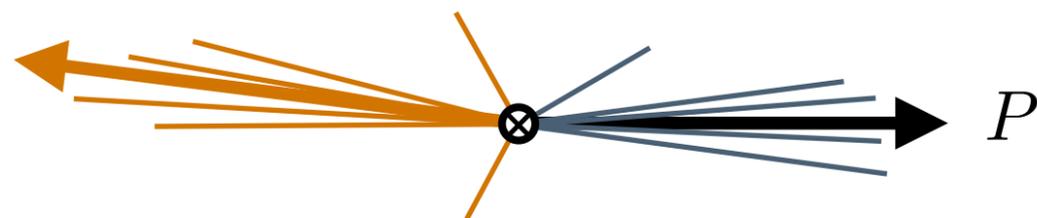
$$q_B^\mu = xP^\mu \quad q_J^\mu = \frac{Q}{2}(1, \hat{t}) \equiv \frac{Q}{2}n_J^\mu$$

arXiv: 1004.2489 (I.W. Stewart, F.J. Tackmann, and W.J. Waalewijn)

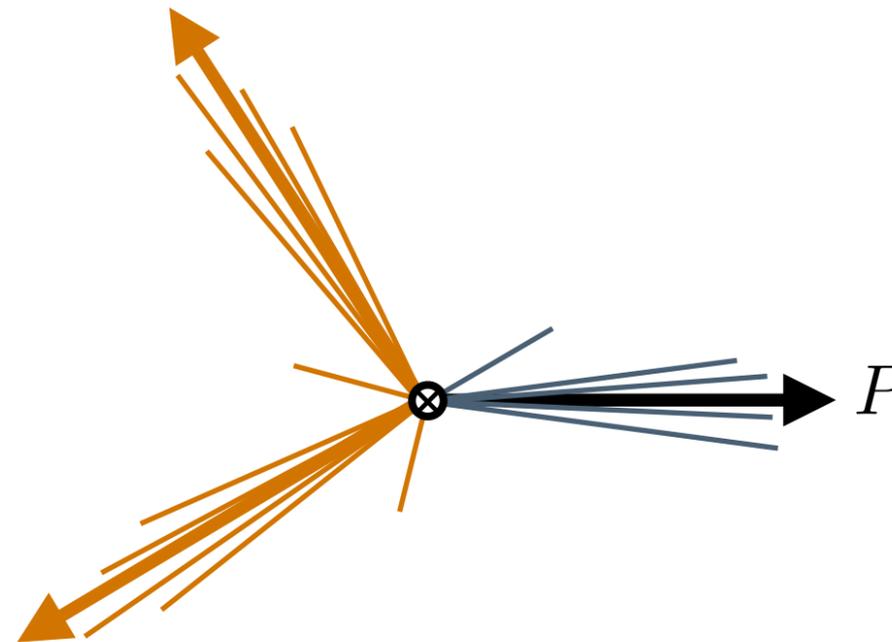
arXiv: 1204.5469 (Z.-B. Kang, S. Mantry, J.-W. Qiu)

arXiv: 1303.6952 (D. Kang, C. Lee, I.W. Stewart I)

Breit frame illustration:



$$\tau_1 \rightarrow 0$$



$$\tau_1 \sim 1$$

$\tau_1 \rightarrow 0 = 1 \text{ jet}$   
 $\tau_2 \rightarrow 0 = 2 \text{ jets}$   
 $\tau_3 \rightarrow 0 = 3 \text{ jets}$   
 $\vdots$

# Groomed 1-jettiness: definition

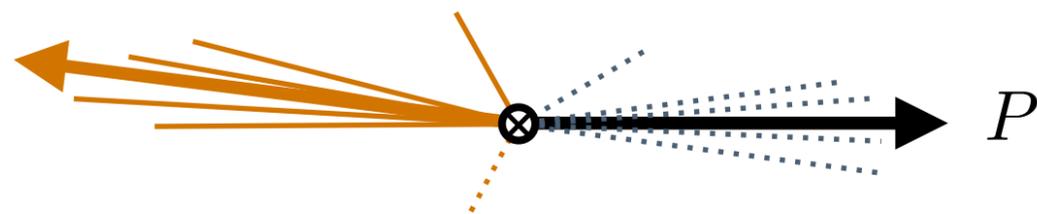
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$$\tau_1 = \frac{2}{Q^2} \sum_{i \in \text{gr. ent.}} \min(q_B \cdot p_i, q_J \cdot p_i)$$

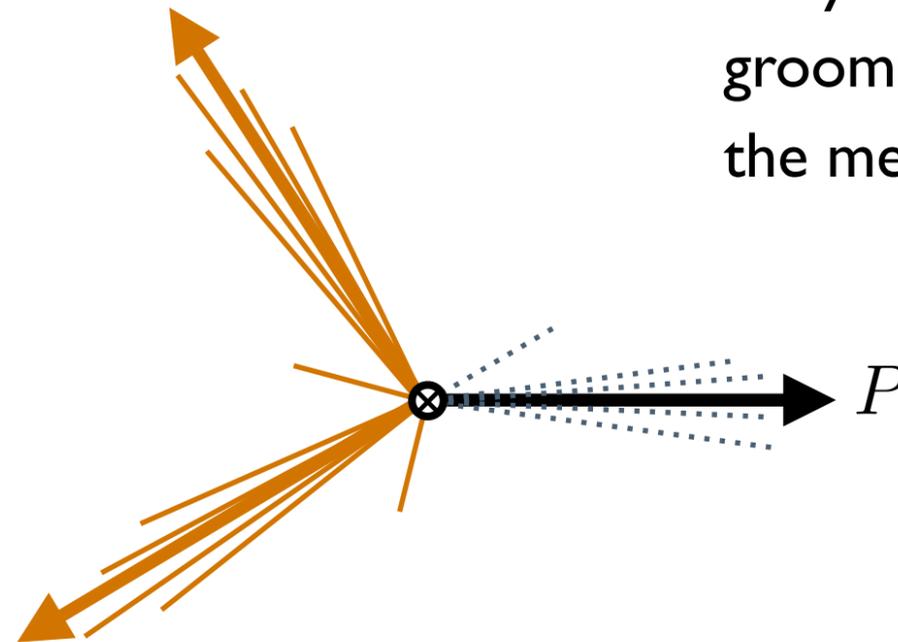
$$q_B^\mu = x P^\mu$$

$$q_J^\mu = \frac{Q}{2} (1, \hat{t}) \equiv \frac{Q}{2} n_J^\mu$$

Breit frame illustration:



$\tau_1 \rightarrow 0$



$\tau_1 \sim 1$

Only radiation that passed grooming will contribute to the measurement.

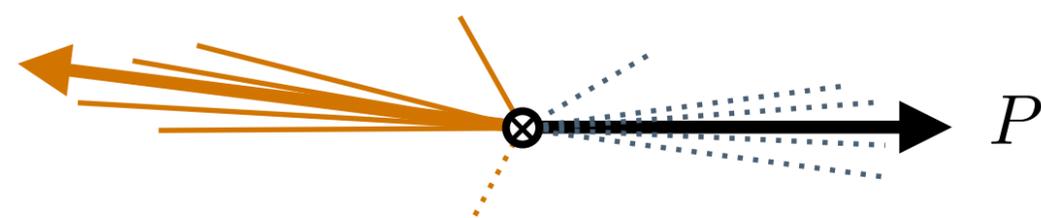
# Groomed invariant-mass: definition

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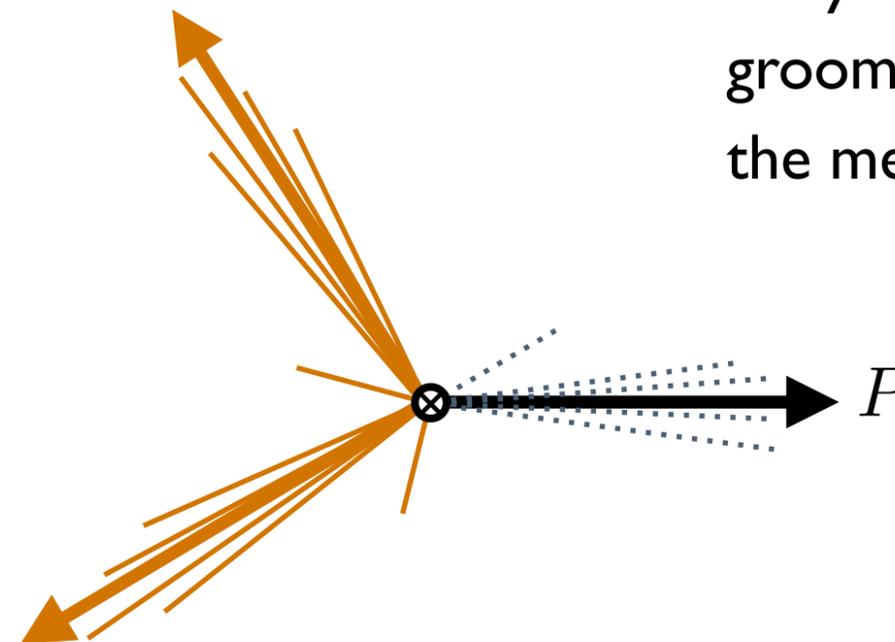
$$m_{\text{gr.}}^2 = \left( \sum_{i \in \text{gr. ent.}} p_i^\mu \right)^2$$

No hemispheres, no boundary conditions  
no jet definitions.

Breit frame illustration:



$$m_{\text{gr.}}^2 / Q^2 \rightarrow 0$$



$$m_{\text{gr.}}^2 / Q^2 \sim 1$$

Only radiation that passed  
grooming will contribute to  
the measurement.

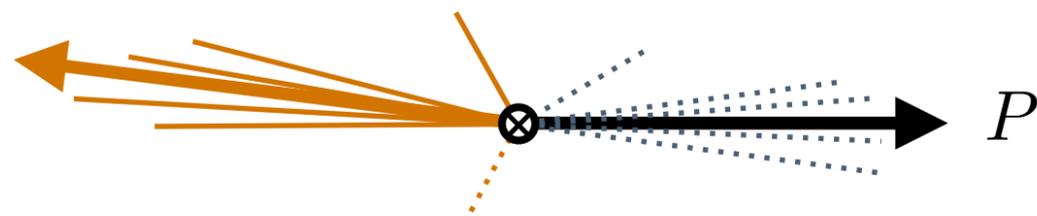
# Groomed invariant-mass: definition

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Breit frame illustration:



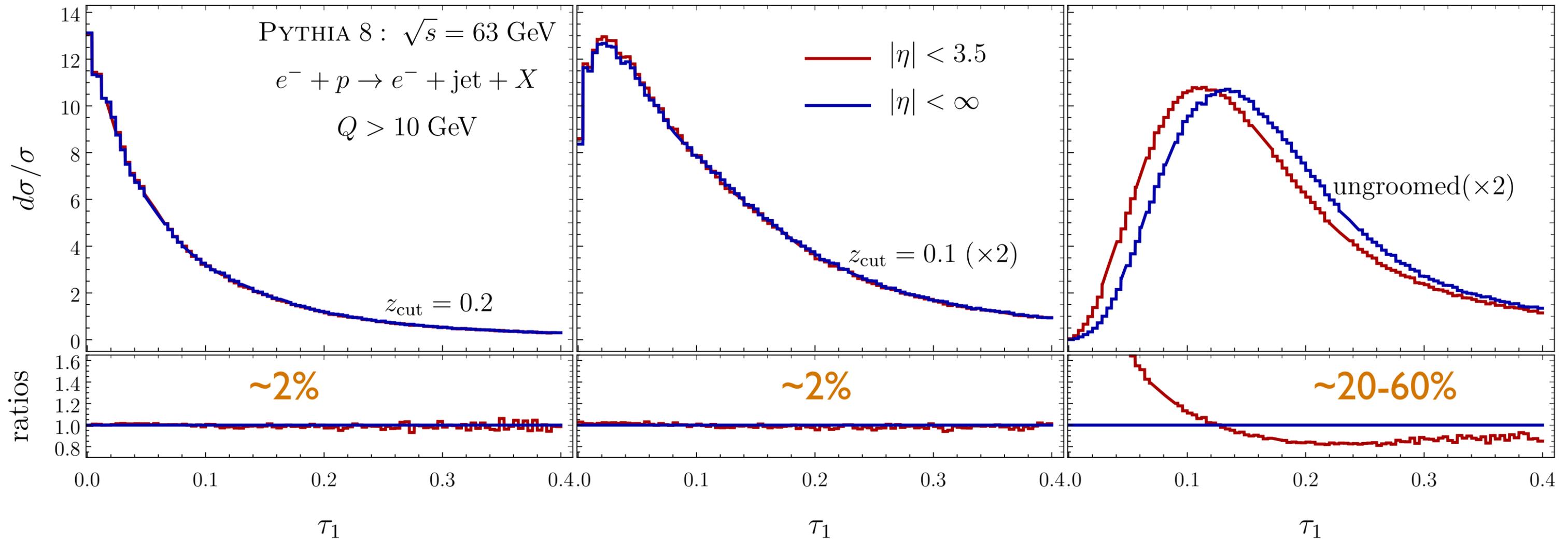
$$m_{\text{gr.}}^2 / Q^2 \rightarrow 0$$

In the back-to-back limit the two observables are related:

$$m_{\text{gr.}}^2 = Q^2 \tau_1 + \mathcal{O}(\tau_1^2)$$

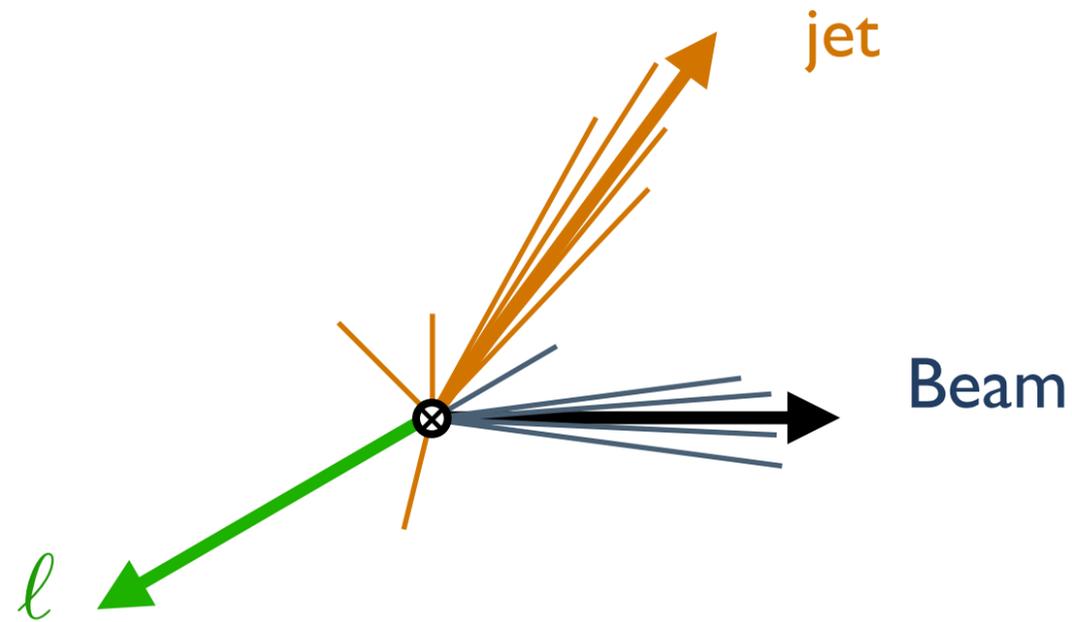
# Groomed 1-jettiness: Simulations

The groomed observable  
is mostly insensitive to the rapidity  
cutoff imposed in the laboratory frame



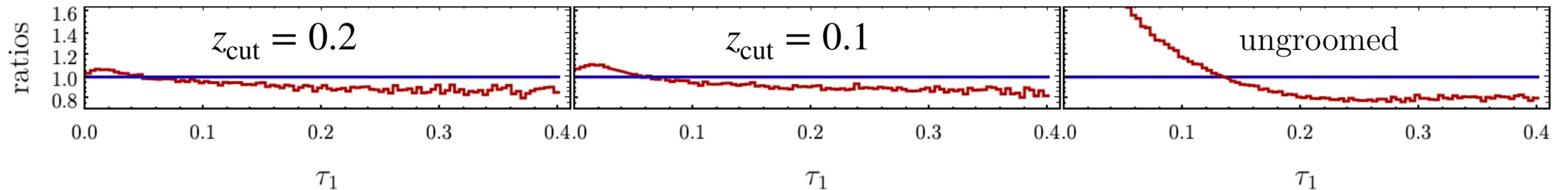
# Groomed 1-jettiness: Simulations

Laboratory frame illustration:



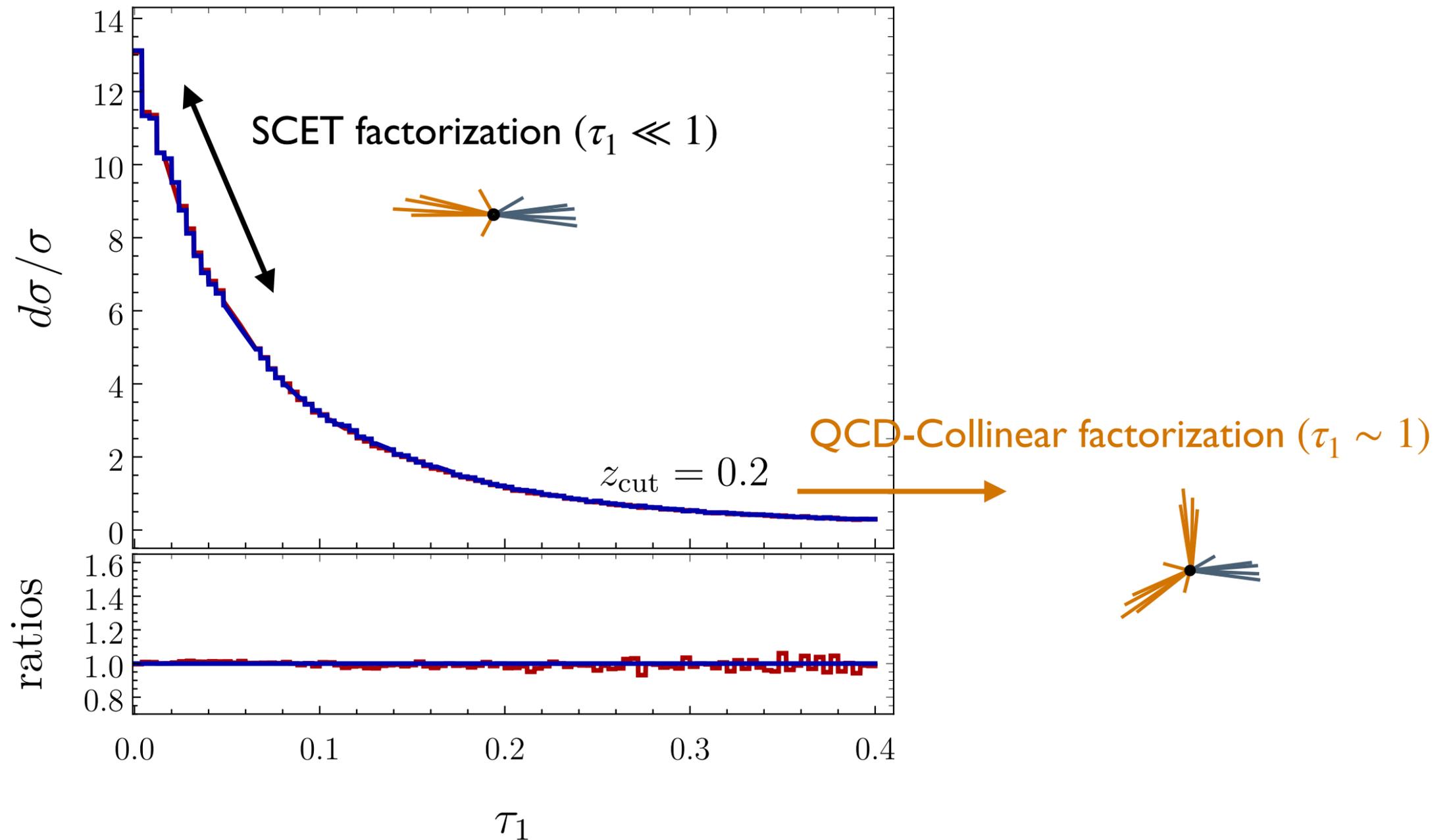
minimum particle  $p_T$  threshold shows larger effect but still much smaller compared to the ungroomed case.

— No cutoff  
—  $p_T > 200$  MeV



# Groomed 1-jettiness: Factorization

SCET = Soft-Collinear Effective Theory



# Groomed 1-jettiness: Factorization

$$\tau_1 \ll z_{\text{cut}} \ll 1$$

For invariant mass:  $\tau_1 \rightarrow m_{\text{gr.}}^2/Q^2$

$$\frac{d\sigma}{dx dQ^2 d\tau_1} = \sigma_0(x, Q) H(Q, \mu) S(Q z_{\text{cut}}, \mu) \sum_f \mathcal{B}(x, Q^2 z_{\text{cut}}, \mu) \int_{\gamma-i\infty}^{\gamma+i\infty} du \frac{\exp(u\tau_1)}{2\pi i} J\left(\frac{Q^2}{u}, \mu\right) \mathcal{C}\left(\frac{Q^2}{u z_{\text{cut}}}, \mu^2\right)$$

## Hard

Matching between  
QCD and SCET

## Soft

Uniform soft  
dynamics.  
Dim-reg: scaleless  
up-to clustering  
effects

$$S = 1 + \mathcal{O}(\alpha_s^2)$$

## “Beam”

Incoming hadron  
matrix element  
Matched onto the  
PDF for:  
 $Q\sqrt{z_{\text{cut}}} \gg \Lambda_{\text{QCD}}$

## Jet

Jet-mass  
jet function  
Final-state collinear  
matrix element

## Collinear-soft

The same as for  
SoftDrop (jet mass).

Mode scaling:

$$p_{cs}^\mu \sim Q z_{\text{cut}} \left( \frac{\tau_1}{z_{\text{cut}}}, 1, \sqrt{\frac{\tau_1}{z_{\text{cut}}}} \right)$$

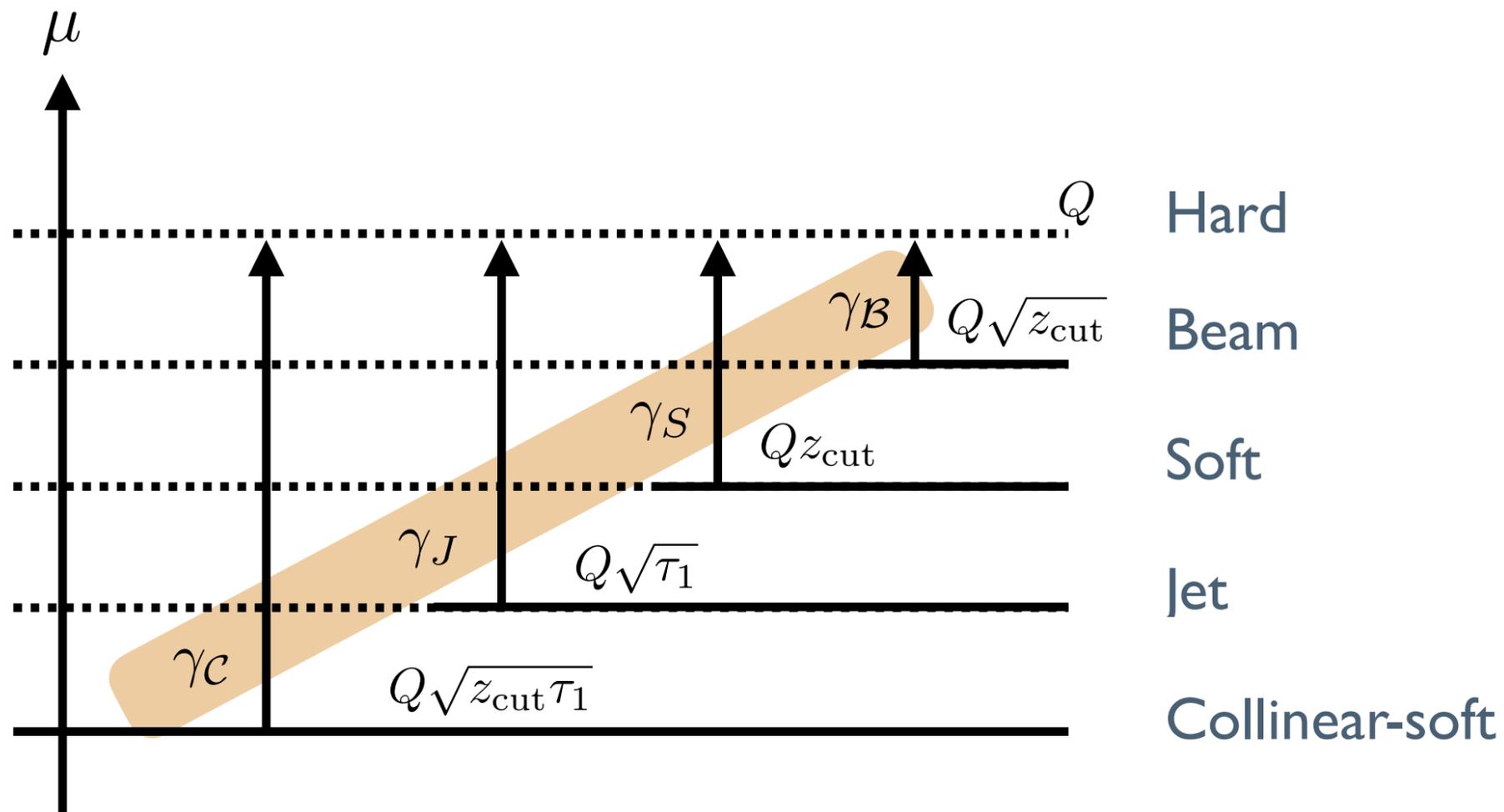
# Groomed 1-jettiness: Resummation

$$\frac{d}{d \ln \mu} F(\mu) = \gamma_F(\mu) F(\mu)$$

From RG invariance of cross-section:

$$\gamma_H + \gamma_S + \gamma_B + \gamma_J + \gamma_C = 0$$

Non trivial check of factorization  
order-by-order in PT



# The normalized distribution for $[x, Q]$ bins

## (1) re-scaled observable

$$e \equiv m_{\text{gr}}^2$$

$$e \equiv Q^2 \tau_1$$

## (2) normalized cross section

$$\frac{d\sigma^{\text{norm.}}}{de}(e, e_{\text{min}}, e_{\text{max}}) \equiv \left[ \int_{e_{\text{min}}}^{e_{\text{max}}} de \int_{x, Q} \frac{d\sigma}{dx dQ^2 de}(e) \right]^{-1} \times \int_{x, Q} \frac{d\sigma}{dx dQ^2 de}(e)$$

## (3) in terms of factorization

$$\frac{d\sigma^{\text{norm.}}}{de}(e, e_{\text{min}}, e_{\text{max}}) = \mathcal{N}(e_{\text{min}}, e_{\text{max}}, \mu) \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{d\tilde{u}}{2\pi i} \exp(\tilde{u}e) J\left(\frac{1}{\tilde{u}}, \mu^2\right) \mathcal{C}\left(\frac{z_{\text{cut}}}{\tilde{u}}, \mu^2\right)$$

$$\mathcal{N}^{-1}(e_{\text{min}}, e_{\text{max}}, \mu) = \int_{e_{\text{min}}}^{e_{\text{max}}} de \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{d\tilde{u}}{2\pi i} \exp(\tilde{u}e) J\left(\frac{1}{\tilde{u}}, \mu^2\right) \mathcal{C}\left(\frac{z_{\text{cut}}}{\tilde{u}}, \mu^2\right)$$

**Warning:**  $e_{\text{min}}$  and  $e_{\text{max}}$  must remain within the region of validity of factorization!

# NLL versus Pythia @ $\sqrt{s} = 318$ GeV

Softest scale becomes non-perturbative:

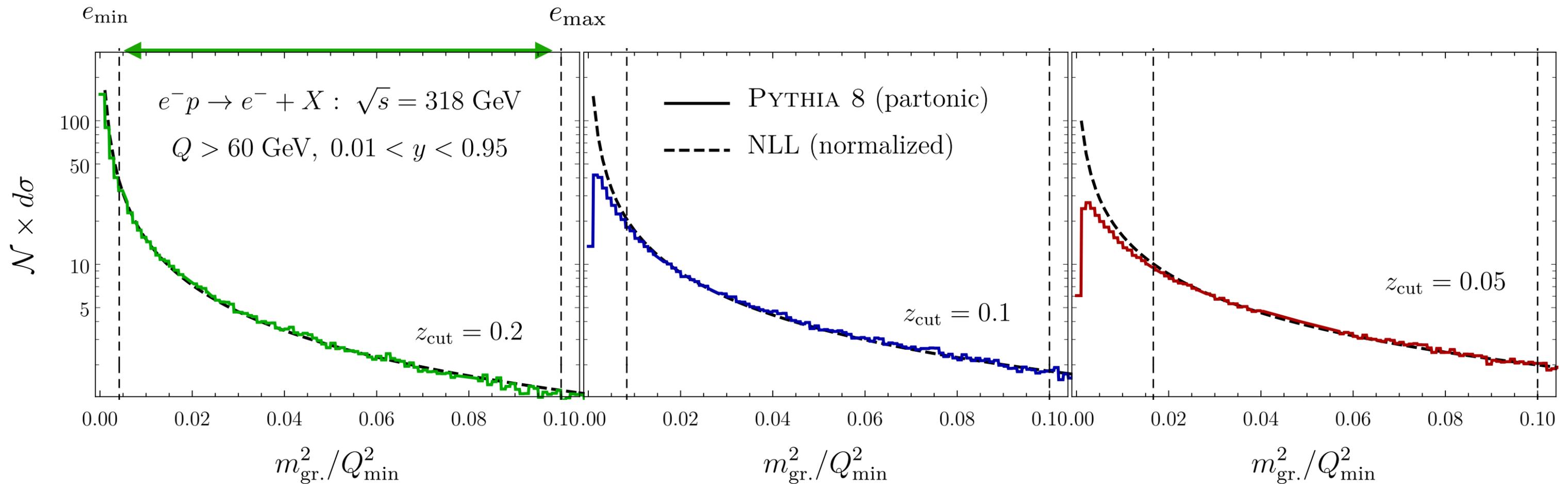
$$e_{\min} = \frac{\Lambda_{\text{NP}}^2}{z_{\text{cut}}}$$

Power corrections to factorization:

$$e_{\max} = 0.1$$

The effective region shrinks quickly with decreasing  $z_{\text{cut}}$ .

This will only get worse at EIC expected energies where  $Q \sim 10$  GeV



# NLL versus Pythia @ $\sqrt{s} = 140$ GeV

Softest scale becomes non-perturbative:

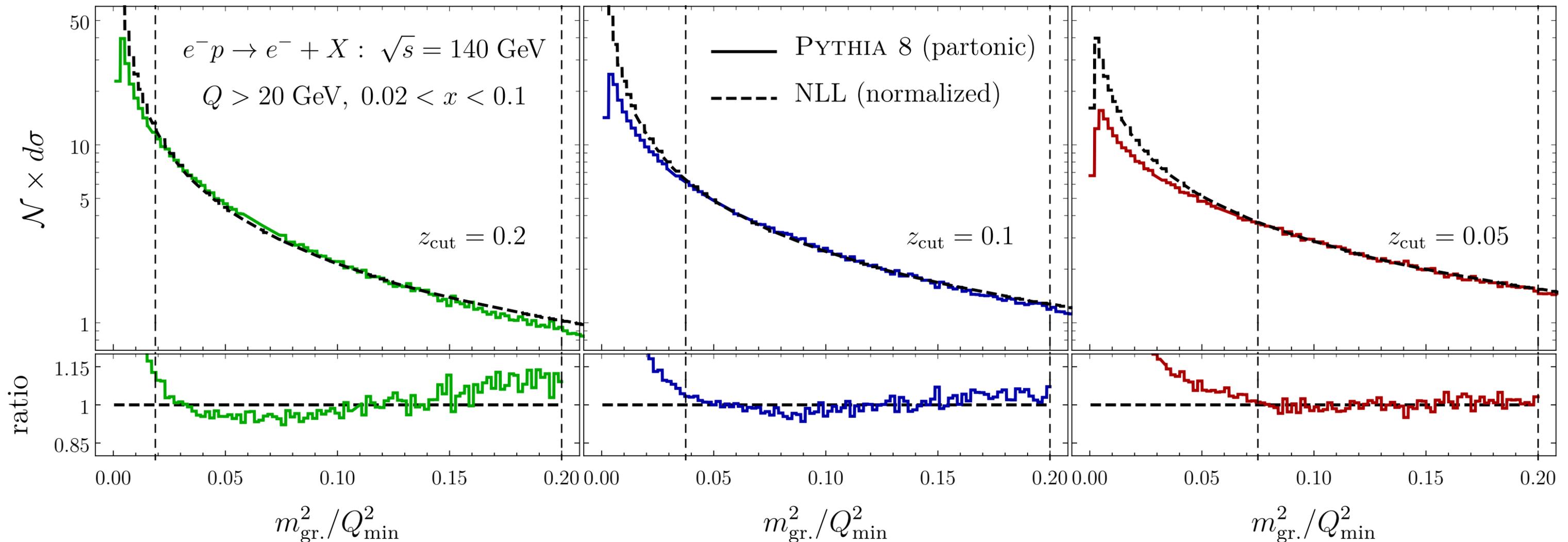
$$e_{\min} = \frac{\Lambda_{\text{NP}}^2}{z_{\text{cut}}}$$

Power corrections to factorization:

$$e_{\max} = 0.2$$

The effective region shrinks quickly with decreasing  $z_{\text{cut}}$ .

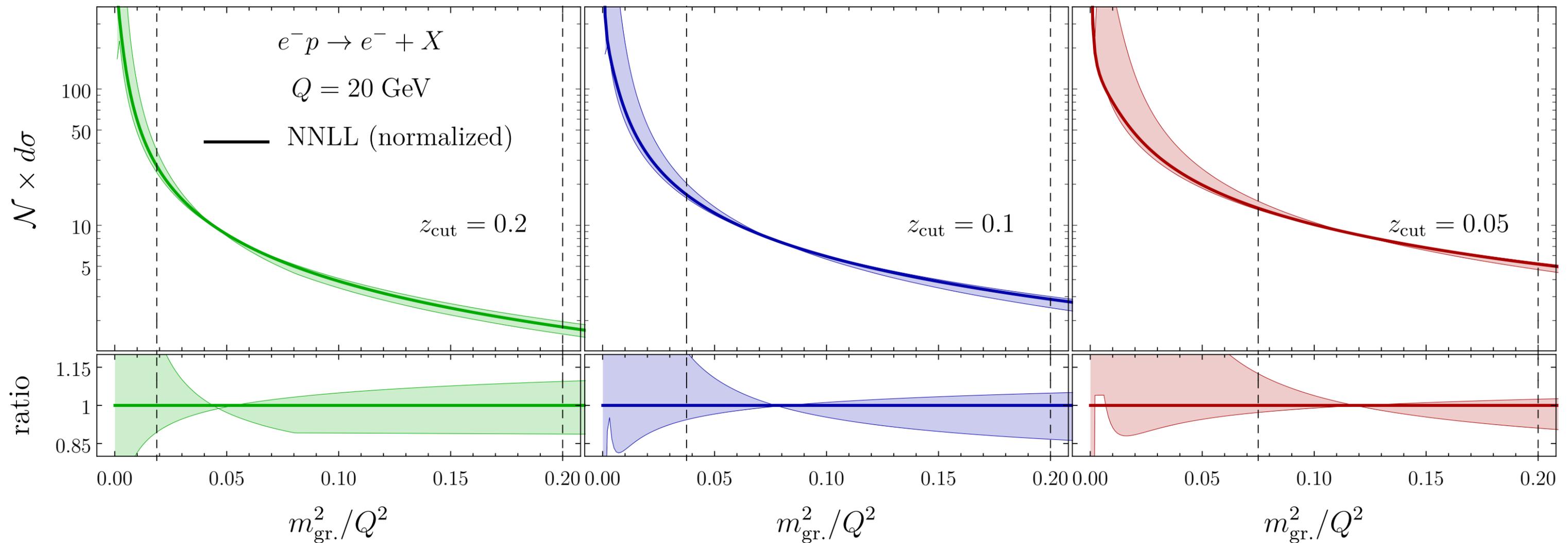
This will only get worse at EIC expected energies where  $Q \sim 10$  GeV



# N<sup>2</sup>LL predictions

scale variations:  $\times 2 / \times 0.5$  ( $\mu_{\mathcal{J}}, \mu_{\mathcal{C}}, \mu = Q_{\min}$ )

$\Lambda_{\text{NP}} = 1.4 \text{ GeV}$   
uncertainty = 10-20 %

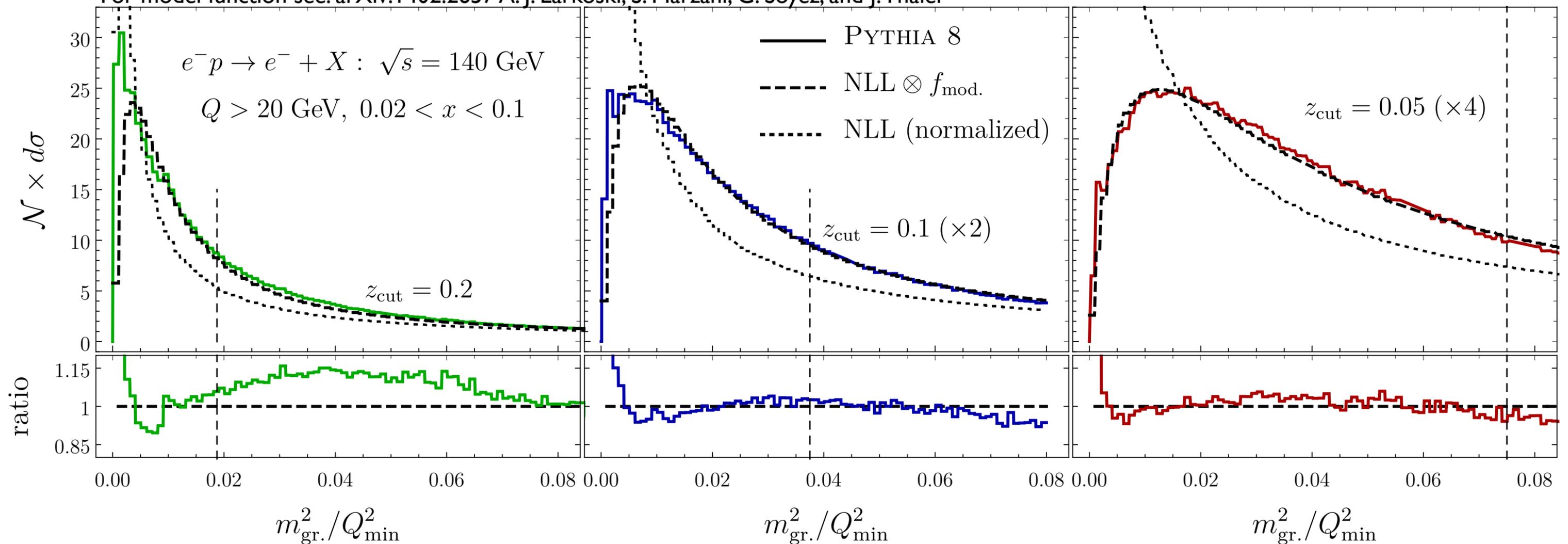


# Sensitivity to hadronization effects

$$\frac{d\sigma_{\text{had.}}}{dx dQ^2 dm_{\text{gr.}}^2} = \int d\epsilon \frac{d\sigma}{dx dQ^2 dm_{\text{gr.}}^2} \left( m_{\text{gr.}}^2 - \frac{\epsilon^2}{z_{\text{cut}}} \right) f_{\text{mod.}}(\epsilon)$$

$$f_{\text{mod.}}(\epsilon) = N_{\text{mod.}} \frac{4\epsilon}{\Omega^2} \exp\left(\frac{2\epsilon}{\Omega}\right)$$

For model function see: arXiv:1402.2657 A. J. Larkoski, S. Marzani, G. Soyez, and J. Thaler



# Outlook

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Hadronization corrections for jet mass with Centauro and anti-kT

Event (groomed) substructure studies

FastJet framework: effects of soft radiation (optimization, Grooming, WTA, e.t.c)



The necessary files can be found here:

<https://github.com/miguelignacio/CentauroJetAlgorithm>

Other...?

underlying principle:

utilize the asymmetric nature of the process

## In this talk:

Asymmetric jet clustering in deep-inelastic scattering

[arXiv: 2006.10751](#)

In collaboration with:

M. Arratia, D. Neill, F. Ringer,  
N. Sato

Grooming algorithms for DIS: example 1-jettiness

[arXiv 2101.02708](#)

## See also:

(Backward) energy-energy correlation (B-EEC) in DIS

[arXiv: 2102.05669](#)

In collaboration with:

H.T. Li and I. Vitev