Nuclear Equation of State and Astrophysical Applications

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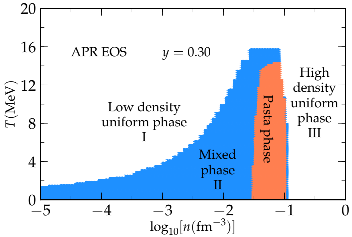
	Core-collapse supernovae	Proto-neutron stars	Mergers of compact binary stars
Baryon Density(n_0)	$10^{-8} - 10$	$10^{-8} - 10$	$10^{-8} - 10$
Temperature(MeV)	$0 - 30$	$0 - 50$	$0 - 100$
Entropy(k_B)	$0.5 - 10$	$0 - 10$	$0 - 100$
Proton Fraction	$0.35 - 0.45$	$0.01 - 0.3$	$0.01 - 0.6$



- Phase diagram corresponding to the APR EOS at a lepton fraction of 0.3. From Eur. Phys. J. A (2019) 55 :10

Supranuclear EOS - Phenomenological Approaches

- Skyrme: $\hat{V}_{NN} = \sum_{i<j} \hat{V}_{ij} + \sum_{i<j<k} \hat{V}_{ijk}$, zero-range.
  Evaluated in the Hartree-Fock approximation $\Rightarrow \mathcal{H} = \frac{\hbar^2}{2m^*}\tau + V(n)$.

- Akmal-Pandharipande-Ravenhall (APR): $V_{NN} = v_{18,ij} + V_{IX,ijk} + \delta v(\mathbf{P}_{ij})$.
  Extended to infinite matter using Variational Chain Summation then fitted to a Skyrme-like functional.

- Relativistic meson exchange in the mean-field approxiation (= negligible meson-field fluctuations, uniform and static system).

- Momentum-dependent interactions of the Yukawa type, borrowed from heavy-ion physics.

- High-precision interactions fitted to NN scattering data
  - meson-exchange models
    e.g. Nijmegen, Paris, Juelich-Bonn
  - sums of local operators
    e.g. Urbana, Argonne
- Interactions from chiral EFT
- RG-evolved potentials

Extension of the above to bulk matter by a variety of techniques: SCGF, BHF, variational, etc.

Subnuclear EOS

- Lattimer-Swesty (LS): $F_{total} = F_N + F_\alpha + F_{bulk} + F_e + F_\gamma$
  - Single representative species of heavy nucleus [single-nucleus approximation (SNA)]; described by the compressible liquid-drop model: $F_N = F_{bulk,in} + F_s + F_C + F_{tr}$.
  - α-particles represent light nuclei; treated as non-interacting Boltzmann gas having hard sphere interactions with nuclei [excluded volume (EV)]: $F_\alpha = (1-u)n_\alpha f_\alpha$
  - Nucleons have hard-sphere interactions with α's and nuclei: $F_{bulk} = (1-u)(1-v_\alpha n_\alpha)n_{out} f_{bulk}$
  - Pasta : Use smooth generalized functions which modify F_C and F_S appropriately.
- Virial expansion:
  - Nondegenerate limit expansion of the grand potential in small fugacity, $z = \exp[(\mu - m)/T] \ll 1$.
  - Coefficients depend on experimental scattering phase shifts $\Rightarrow$ Model-independent predictions for the equation of state.
- Nuclear Statistical Equilibrium (NSE):
  - Statistical ensemble of nucleons and nuclei in thermodynamic equilibrium
  - Maxwell-Boltzmann statistics $n_a = \gamma_a \exp(\frac{\mu_a}{T})\lambda^{-3}$; $\lambda = \left(\frac{h^2}{2\pi m_a T}\right)^{1/2}$; $\mu_a = N_a\mu_n + Z_a\mu_p$
  - Abundances determined by Saha equation with nuclear binding energies as input $X_a = \gamma_a \frac{m_a}{\sum_a m_a n_a}\lambda^{-3}\exp\left(\frac{\mu_a + B_a}{T}\right)$; $B_a = Z_a m_p + N_a m_n - m_a$.

Quark Phase

- MIT Bag, $\mathcal{L} = \sum_i [\overline{\psi}_i (i\partial\!\!\!/ - m_i - B)\psi_i + \mathcal{L}_{\rm int}]\Theta$
- vMIT, $\mathcal{L}_{\rm int} = -G_v \sum_i \; \overline{\psi}\gamma_\mu V^\mu \psi + (m_V^2/2) V_\mu V^\mu$
- vBag, $\mathcal{L}_{\rm int} = G_v \sum_i \; (\overline{\psi}\gamma^\mu \psi)^2$
- vNJL, $\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - \hat{m}_0)\psi + G_s \sum_{k=0}^{8} [(\overline{\psi}\lambda_k \psi)^2 + (\overline{\psi} i\gamma_5 \lambda_k \psi)^2]$
  $-K[\det_f(\overline{\psi}(1+\gamma_5)\psi) + \det_f(\overline{\psi}(1-\gamma_5)\psi)] + G_v \sum_i \; (\overline{\psi}\gamma^\mu \psi)^2$
- Quarkyonic: Above a transition density, the low-momentum degrees of freedom inside the Fermi sea behave as noninteracting quarks, whereas at higher momenta they are subjected to confining forces resulting in baryons.
- CSS, $\epsilon(p) = \begin{cases} \epsilon_{\rm NM}(p) & p < p_{\rm trans} \\ \epsilon_{\rm NM}(p_{\rm trans}) + \Delta\epsilon + c_{\rm QM}^{-2}(p - p_{\rm trans}) & p > p_{\rm trans} \end{cases}$
- Phase transition
  - 1st order: Maxwell ($\sigma_s \to \infty$) vs. Gibbs ($\sigma_s = 0$)
  - higher order/crossover

- Matter in β-equilibrium supported against gravitational collapse by neutron degeneracy.
- Structure determined by simultaneous solution of:
  - Interior mass, $m(r) = 4\pi \int_0^r \epsilon(r') r'^2 dr'$
  - Hydrostatic equilibrium, $\frac{dp}{dr} = -\frac{Gm(r)\epsilon(r)}{r^2}\left[1 + \frac{p(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 p(r)}{m(r)}\right]\left[1 - \frac{2Gm(r)}{r}\right]^{-1}$
  - EOS, $p = p(\epsilon)$
- Constraints
  - Largest observed mass, (binaries) $M = 2.01 M_\odot$
  - Largest observed frequency, (pulsars) $\Omega = 114$ rad/s
  - Inferred radius range, (photospheric emission, thermal spectra) $9\text{ km} \leq R \leq 15\text{ km}$

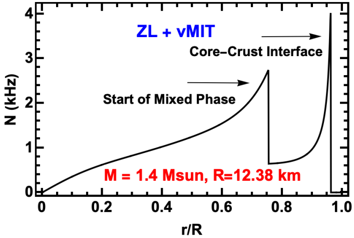
Applications - Core-Collapse Supernovae (CCSN)

- A massive star ($M > 8M_\odot$) forms an iron core; it cannot produce energy via fusion and contraction ensues.
- As the density increases, electron capture becomes favorable and neutrinos are produced: $p^+ + e^- \to n + \nu_e$
- When the Fermi energy of neutrinos gets large enough, they become trapped and β-equilibrium is achieved.
- At this point the entropy per nucleon is $S \simeq 1$. Dripped neutrons require $S \simeq 8$ thus nuclei persist until the core contracts to $n \sim n_o$. Then nucleonic matter emerges.
- Nucleons are compressed to $n \sim 3n_o$ where the repulsive core of the strong interaction dominates their attraction and inhibits further contraction.
- The core rebounds and creates a shock wave that disrupts the star.
- EOS relevance
  - EOS controls electron capture rates and therefore the neutrino signal.
  - GW amplitude related to PNS compactness and the high-density properties of the EOS.
  - Homologous core, time to collapse
  - Reaction networks

Applications - Binary Neutron Star Mergers (BNSM)

- Relativistic binaries not in equilibrium : Gravitational wave (GW) emission leads to orbital decay.
- Early stage: only gravitational interactions, GW signal contains information for the masses of the components.
- Coalescence stage: Tidal disruption of the lower-mass star, mass transfer onto the more massive one. Mass transfer rate depends on $\mathcal{C} = M_{NS}/R_{NS}$ and reflected in GW signal. Ejected matter is very neutron-rich and can lead to heavy element formation via the r-process.
- Late stage: Black hole or hypermassive neutron star formation.
- EOS relevance
  - Tidal disruption of NS during coalesence of BH-NS binary depends on the stiffness of the EOS. GW frequency sensitive to orbital frequency at disruption.
  - r-process production rates and abundances depend on the composition of the ejecta and thus the EOS.
  - Tidal deformability, $\Lambda = \frac{2}{3} k_2 \left(\frac{R\,c^2}{G\,m} \right)^5$.
  - g-mode frequencies: $N^2 = g^2 \left(\frac{1}{c_e^2} - \frac{1}{c_s^2} \right) \mathrm{e}^{\nu - \lambda}$

    $g = -\nabla[p/(\epsilon + p)]$



Plans

- Pions: thermally-excited and collective modes.
- Quarks: Identify binary-neutron-star-merger observables that can establish the presence of deconfined quarks in neutron star interiors.
- Beyond mean-field: Needed for the study of phase transitions, quantum fluctuations at lower densities, and the transport properties of hot and dense matter.
- Phase-equivalent potentials: to address the limitations of the virial expansion and the excluded volume approximation.
- Relax single-nucleus approximation: so that processes requiring a full nuclear ensemble can be accommodated.
- Microscopic treatment of nuclear properties: to replace liquid drop model and explore the role of deformed nuclei in thermal effects.
- Astrophysical applications.