# ANALYTIC TOOLS FOR PRECISION IN PARTICLE PHYSICS

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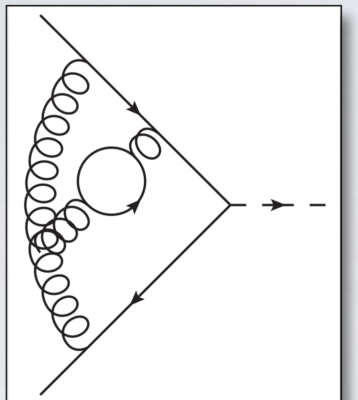
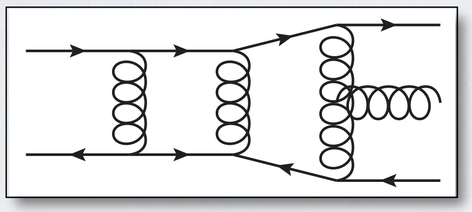
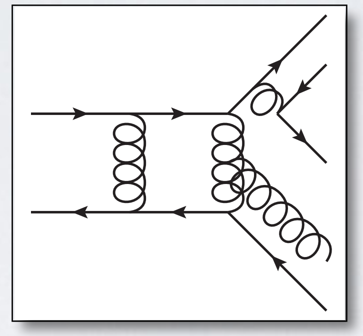
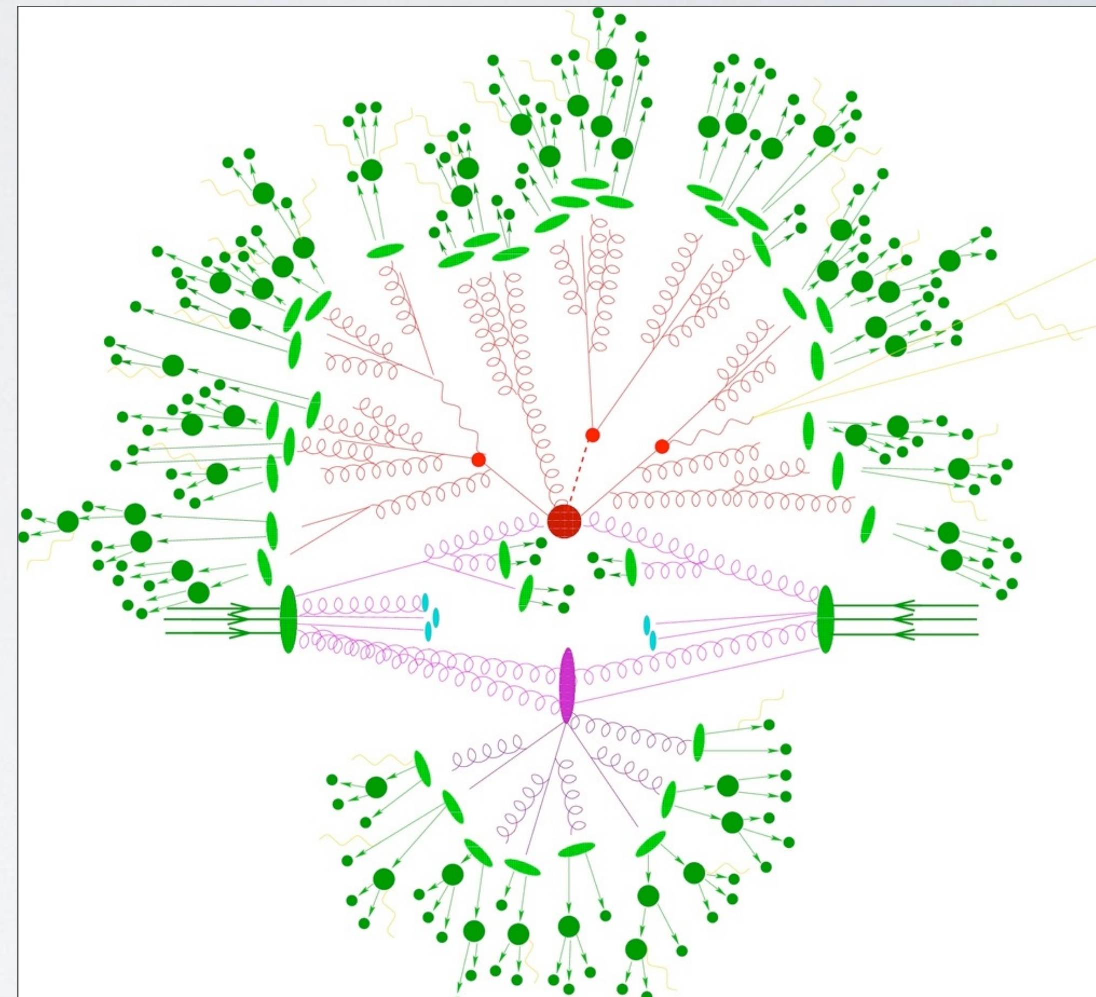
# PRECISION FOR COLLIDER PHYSICS

- Hard scattering processes are calculated in perturbation theory.









- Going beyond NNLO and N3LO turns out to be incredibly difficult, yet necessary to match the precision of current and forthcoming experiments!
- Loop and phase space integrals:
  - Analytic vs numerical evaluation
  - Space of functions
  - Infrared divergences
  - Large logarithms

}

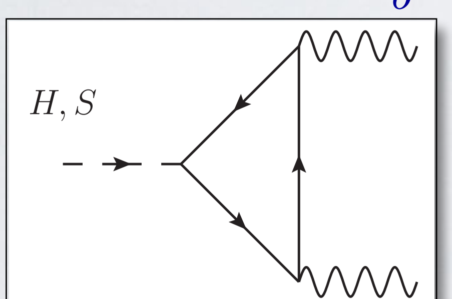
***This talk***

# ANALYTIC TOOLS: RESUMMATION (NLP)

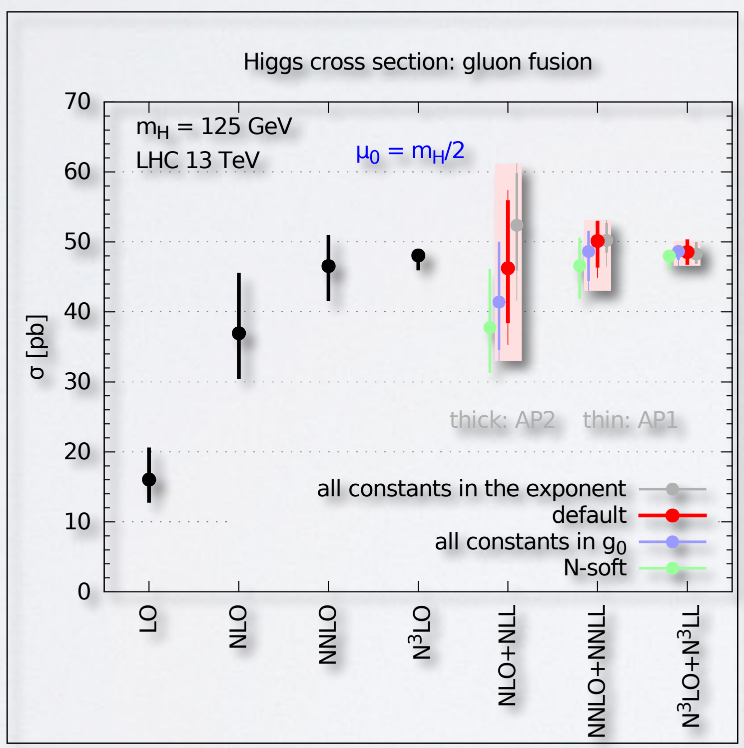
- Large logarithms spoil the convergence of the perturbative series, and need to be resummed:

$$d\sigma = 1 + \alpha_s(L^2 + L + 1) + \alpha_s(L^4 + L^3 + L^2 + L + 1) + \ldots$$

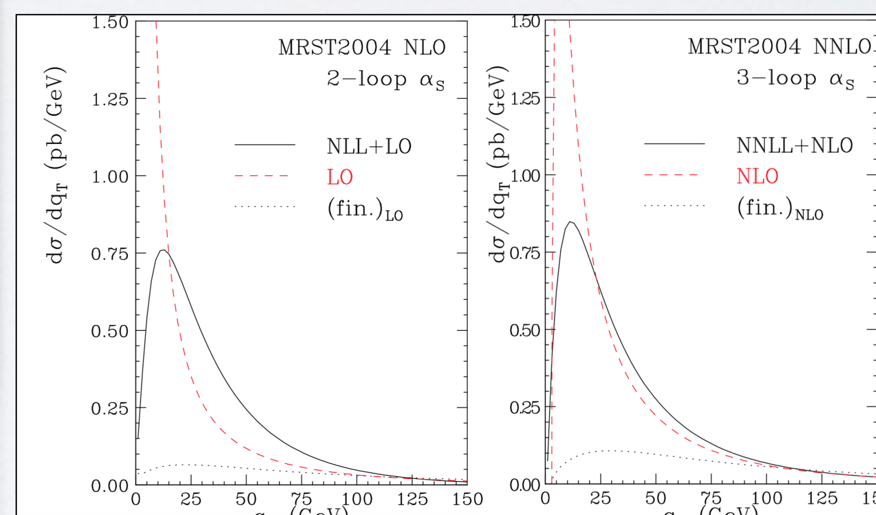
$$\sim \log^2 \frac{m_H^2}{m_b^2}$$



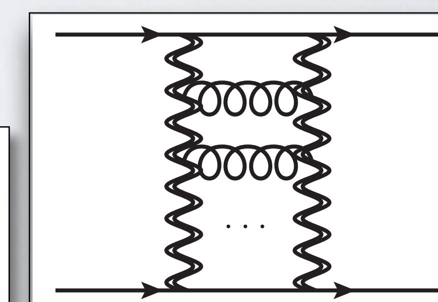

$$\sim \log^2(1-z)$$




$$\sim \log \frac{m_H^2}{p_T^2}$$




$$\sim \log \frac{s}{-t}$$



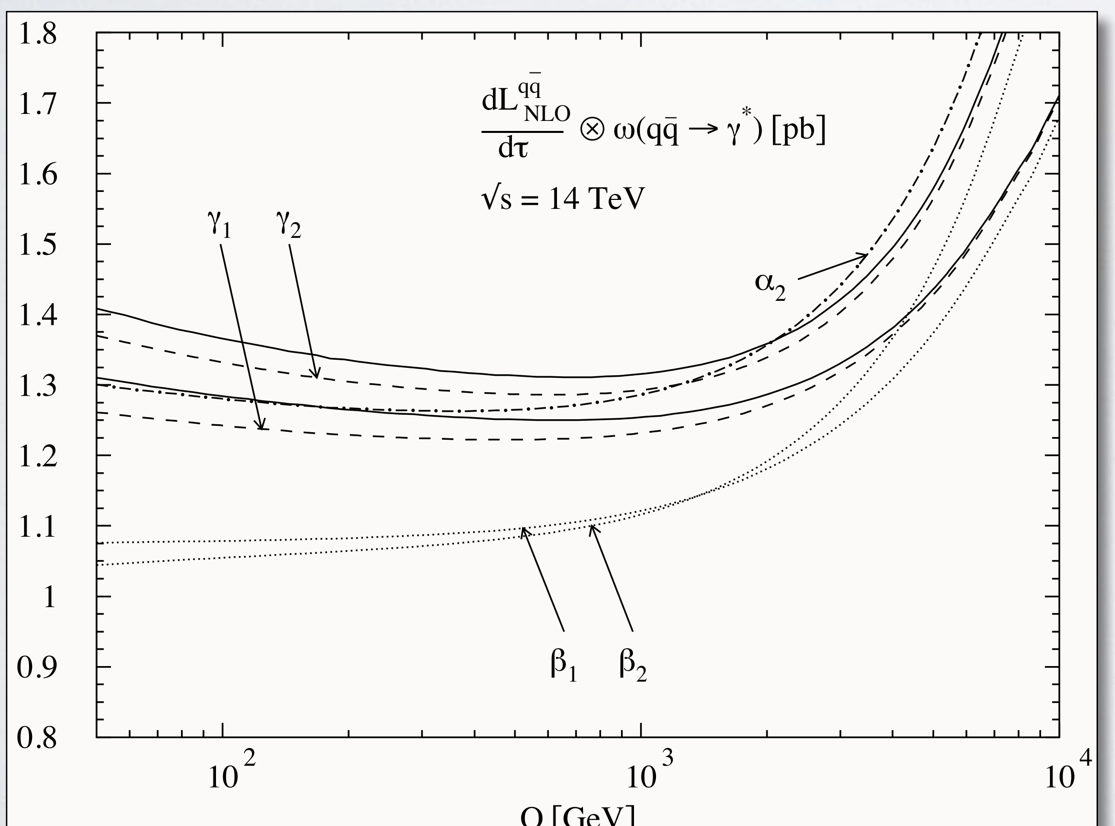
- Resummation is well established for several kinematic limits, at leading power.

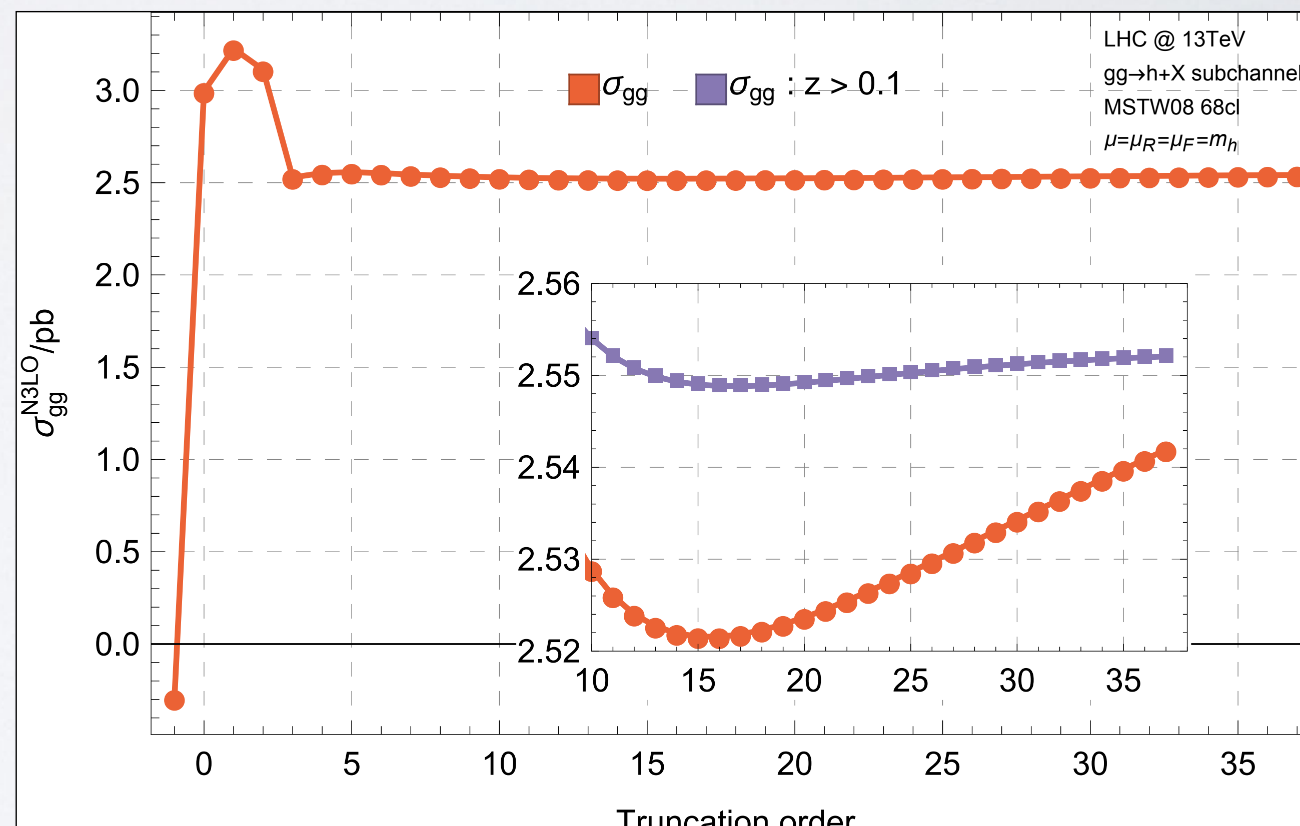
# ANALYTIC TOOLS: RESUMMATION (NLP)

$$\frac{d\sigma}{d\xi} \sim \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n\left[c_n\delta(\xi)+\sum_{m=0}^{2n-1}\left(c_{nm}\left[\frac{\ln^m(\xi)}{\xi}\right]_+ + d_{nm}\ln^m(\xi)\right)+\dots\right].$$

**LP terms** **NLP terms**

- The ratio of largely different scales provides an expansion parameter. Resummation is known only for the first term in the series (at leading power, LP).
- I have been working toward the development of resummation for large logarithm contributing at next-to-leading power, which is relevant for precision physics.

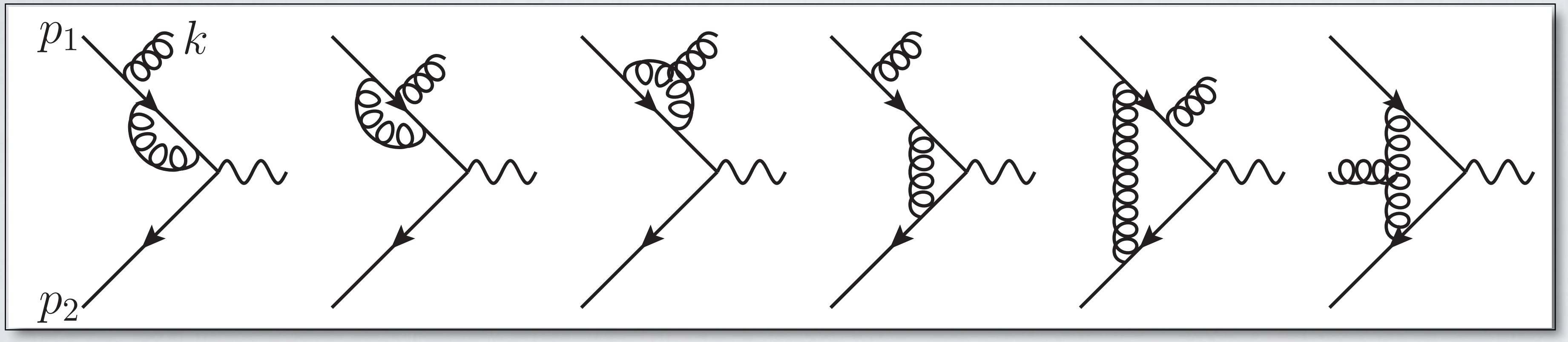



*Kramer, Laenen, Spira, 1998*

*Anastasiou, Duhr, Dulat, Herzog, Mistlberger, 2015*

# ANALYTIC TOOLS: RESUMMATION (NLP)




$$\hat{s} = (p_1 + p_2)^2, \quad t = (p_1 - k)^2, \quad u = (p_2 - k)^2.$$

- NLP:

$$|\mathcal{M}|^2 \propto C_F^2 \Bigg\{ \overset{\text{NLP}}{\frac{\hat{s}(t+u)}{tu}} \overset{\text{hard}}{\left(\frac{\mu^2}{-\hat{s}}\right)^{\epsilon}} \left( -\frac{2}{\epsilon^2} - \frac{1}{\epsilon} + \ldots \right) + \Bigg[ \overset{\text{NLP}}{\frac{s}{t}} \overset{\text{coll.}}{\left(\frac{\mu^2}{-t}\right)^{\epsilon}} + \overset{\text{NLP}}{\frac{\hat{s}}{u}} \overset{\text{anti-coll.}}{\left(\frac{\mu^2}{-u}\right)^{\epsilon}} \Bigg] \left( -\frac{2}{\epsilon} + \ldots \right) \Bigg\}$$

$$+ C_A C_F \overset{\text{NLP}}{\frac{\hat{s}(t+u)}{tu}} \overset{\text{soft}}{\left(\frac{\hat{s}\,\mu^2}{t\,u}\right)^{\epsilon}} \left( -\frac{1}{\epsilon^2} + \ldots \right) + \Bigg[ \overset{\text{NLP}}{\frac{\hat{s}}{t}} \overset{\text{coll.}}{\left(\frac{\mu^2}{-t}\right)^{\epsilon}} + \overset{\text{NLP}}{\frac{\hat{s}}{u}} \overset{\text{anti-coll.}}{\left(\frac{\mu^2}{-u}\right)^{\epsilon}} \Bigg] \left( -\frac{5}{2} + \ldots \right) \Bigg\} + \ldots$$

**Factorisation?**

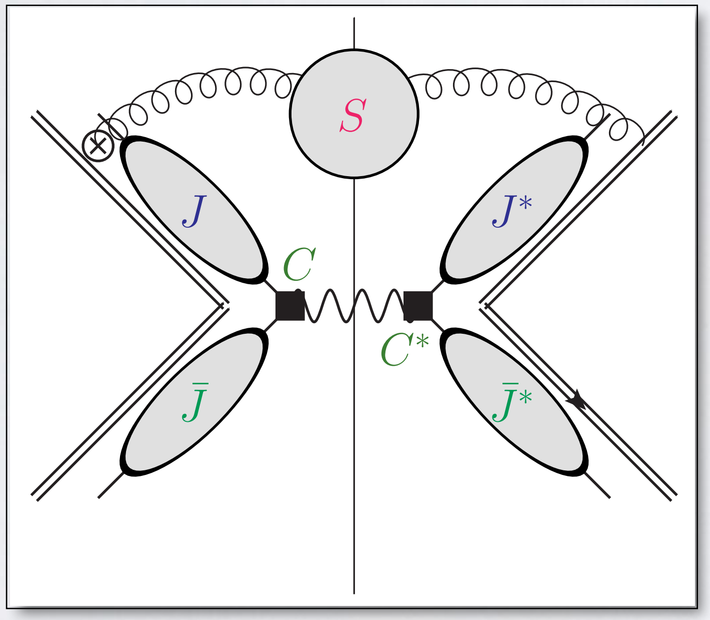
***Bonocore, Laenen, Magnea, LV, White, 2014,2015,2016***

$$S\left[\frac{\hat{s}\,\mu^2}{t\,u}, \epsilon\right] \times J\left[\frac{\mu^2}{-t}, \epsilon\right] \times \bar{J}\left[\frac{\mu^2}{-u}, \epsilon\right] \times H\left[\frac{\mu^2}{-\hat{s}}, \epsilon\right]$$

- Need an effective approach to take into account hard, collinear and soft modes.

# RECENT DEVELOPMENTS

- Effective field theories (Soft-collinear effective field theory, SCET, in case of collider physics) provide a systematic tool for describing the factorisation of soft and collinear radiation.
- The hard scattering kernel is described in terms of effective operators; Momentum modes in the theory are integrated out, giving rise to short-distance coefficients.
- ➔ obtained a factorisation theorem for Drell-Yan at general subleading power.




$$\hat{\sigma}_{q\bar{q}}^{\text{NLP}} = \sum_{\text{terms}} |C|^2 \, J \otimes \bar{J} \otimes J^* \otimes \bar{J}^* \otimes S.$$

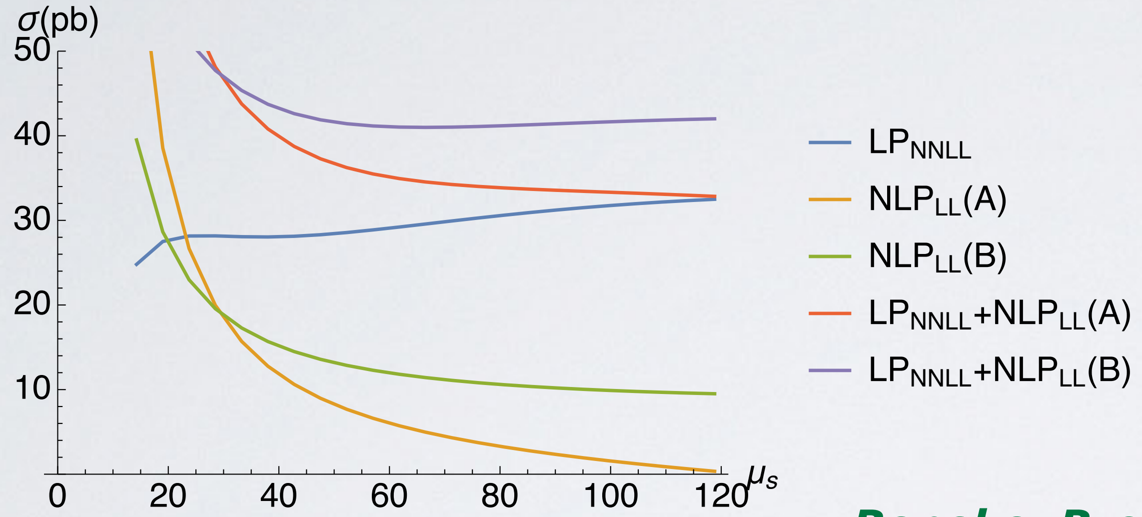
***Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018; Beneke, Broggio, Jaskiewicz, LV, 2019***

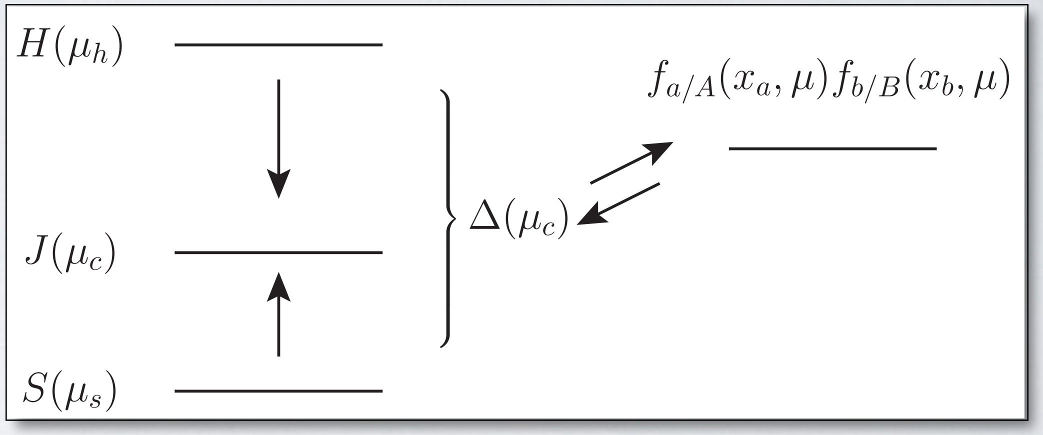
- Complementary work on a diagrammatic approach - somewhat simpler and less technical, although less systematic.

***Laenen, Sinninghe Damsté, LV, Waalewijn, Zoppi, 2020***

# RECENT DEVELOPMENTS

- Resummation is obtained within an EFT by determining the renormalisation group equation of a short distance coefficient.



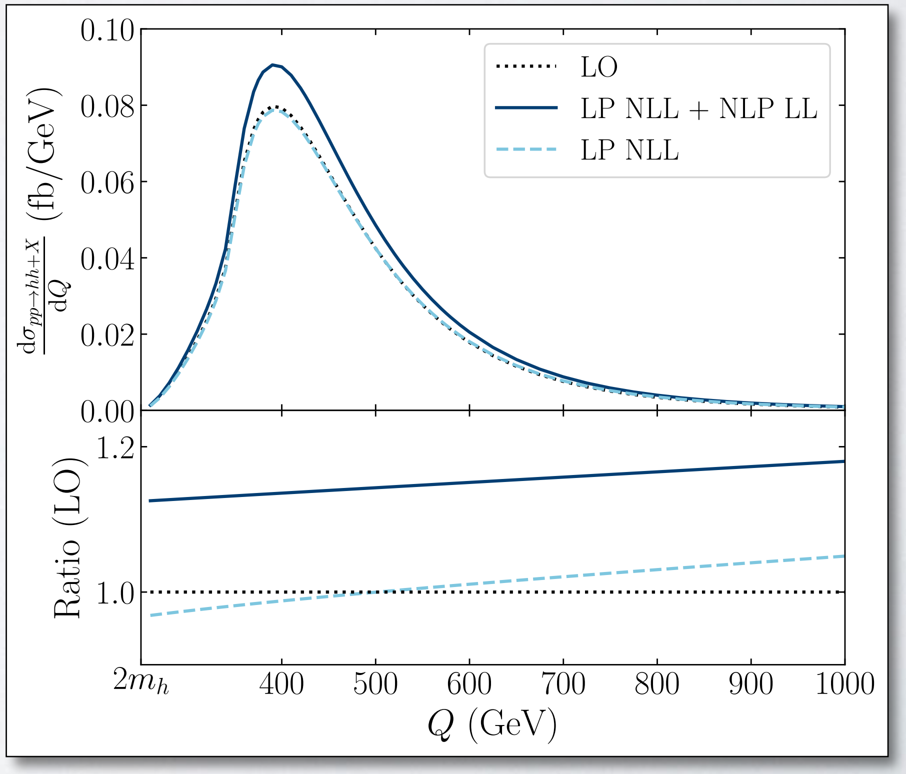




***Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018;***
***Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019;***

- Equivalent results are obtained within a “direct QCD” approach. Comparison between D-QCD and SCET.

- ➔ resummed large threshold logarithms at leading logarithmic accuracy at NLP for electroweak annihilation processes. Effects are relevant for precision physics; confirmed several past analyses based on ansatz.

***Van Beekveld, Eric Laenen, Sinninghe Damsté, LV, 2021.***



# RECENT DEVELOPMENTS

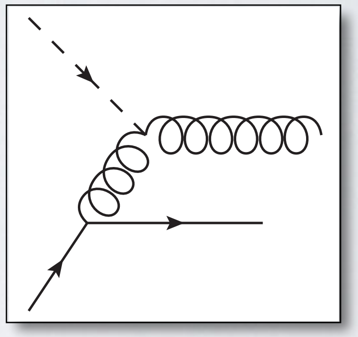
- Resummation at NLP beyond leading logarithmic accuracy cannot be achieved with current methods. Requires the development of new tools.
- RGEs cannot (yet) be formulated in 4 dimensions, due to endpoint divergences.

$$\int d\omega \underbrace{\left(n_+p\,\omega\right)^{-\epsilon}}_{\text{collinear piece}} \underbrace{\frac{1}{\omega^{1+\epsilon}} \frac{1}{(\Omega-\omega)^{\epsilon}}}_{\text{soft piece}} .$$

***Beneke, Broggio, Jaskiewicz, LV, 2019***

- ➔ explored resummation in dimensional regularization for the off-diagonal channel in deep inelastic scattering; provides new ideas for systematic resummation at NLP, beyond leading logarithmic accuracy.

$$\tilde{C}^{NLP,LL}_{\phi,q}\Big|_{\epsilon\to 0} = \frac{1}{2N\ln N}\frac{C_F}{C_F-C_A}\Bigg(\mathcal{B}_0(a)\exp\left[C_A\frac{\alpha_s}{\pi}\left(\frac{1}{2}\ln^2 N+\ln N\ln\frac{\mu^2}{Q^2}\right)\right] - \exp\left[\frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}\ln^2 N+\ln N\ln\frac{\mu^2}{Q^2}\right)\right]\Bigg),$$



$$\mathcal{B}_0(a) = F_{\rm fin}(0,a), \qquad F_{\rm fin}(w,a) = \sum_{k\geq 0} w^k \sum_{n\geq k}\frac{B_n}{n!(n-k)!}a^{n-k}, \qquad a = \frac{\alpha_s}{\pi}(C_F-C_A)\ln^2 N.$$

***Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019***

- Solving this problem would constitute a breakthrough in SCET, with applications in several fields beyond collider physics.

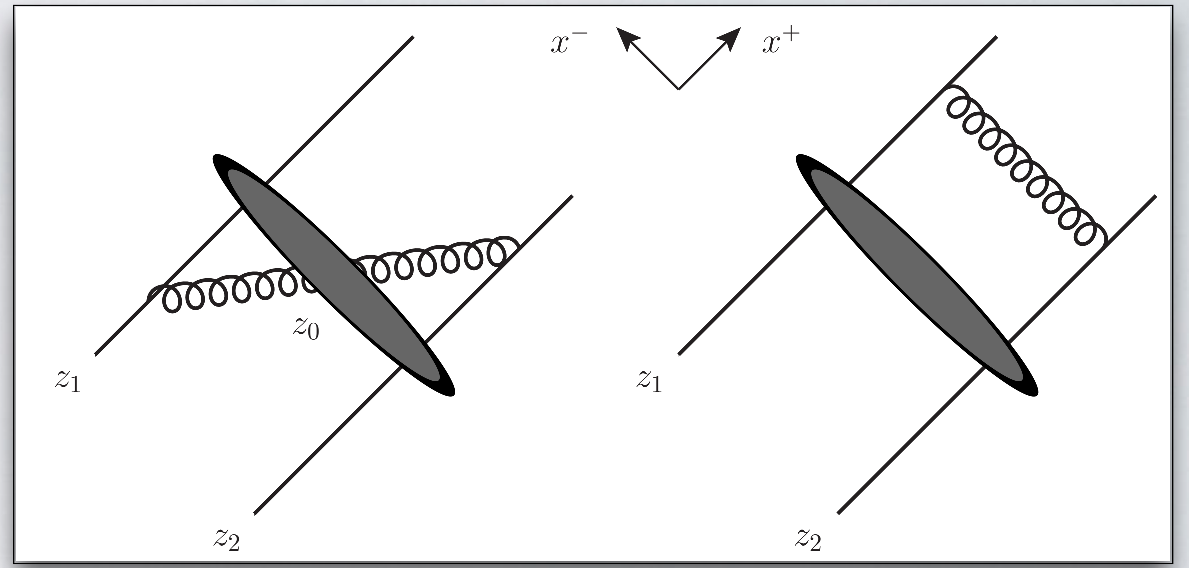
# ANALYTIC TOOLS: HIGH-ENERGY LIMIT

***MRK in N=4 SYM:***
***Dixon, Pennington, Duhr, 2012;***
***Del Duca, Dixon, Pennington, Duhr, 2013;***
***Del Duca, Druc, Drummond, Duhr, Dulat, Marzucca, Papathanasiou, Verbeek 2016;***
***…***

- Very interesting theoretical problem:
  - retain rich dynamic in the 2d transverse plane,
  - toy model for the full amplitude,
  - non-trivial function spaces,
  - predict amplitudes in overlapping limits: → soft limit, infrared divergences.
- Relevant for phenomenology at the LHC and future colliders:
  - perturbative phenomenology of forward scattering, e.g.
    - → Deep inelastic scattering/saturation (small $x$ = Regge, large $Q^2$ = perturbative),
    - → Mueller-Navelet: $pp \to X+2$jets, forward and backward.

# ANALYTIC TOOLS: SHOCKWAVE FORMALISM

- ➔ developed tools to evaluate scattering amplitudes in the high-energy limit by means of the shockwave formalism:
- High-energy limit = forward scattering: to leading power, the fast projectile and target described in terms of Wilson lines:




$$U(z_\perp) = \mathcal{P}\exp\left[ig_s \int_{-\infty}^{+\infty} A_+^a(x^+, x^-{=}0, z_\perp)dx^+ T^a\right].$$

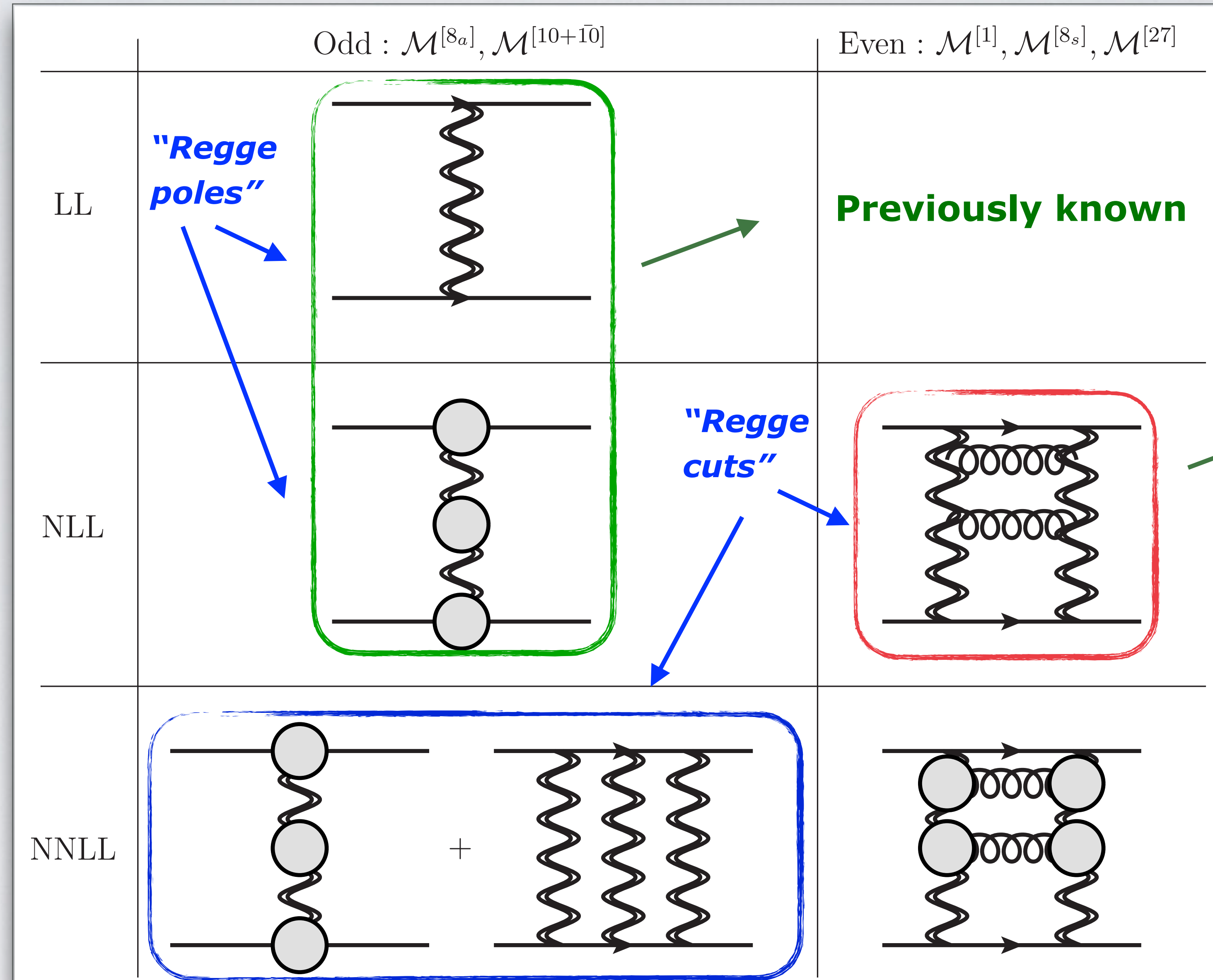
- The Wilson lines obeys the (non linear!) Balitsky-JIMWLK evolution equation:

$$-\frac{d}{d\eta}\Big[U(z_1)\ldots U(z_n)\Big] = \sum_{i,j=1}^{n} H_{ij}\cdot\Big[U(z_1)\ldots U(z_n)\Big],$$

Scattering amplitudes are calculated as matrix elements of Wilson lines evolved to equal rapidity:

$$\mathcal{M} \sim \langle U|e^{-HL}|U\rangle.$$

# HIGH-ENERGY LIMIT: RECENT DEVELOPMENTS



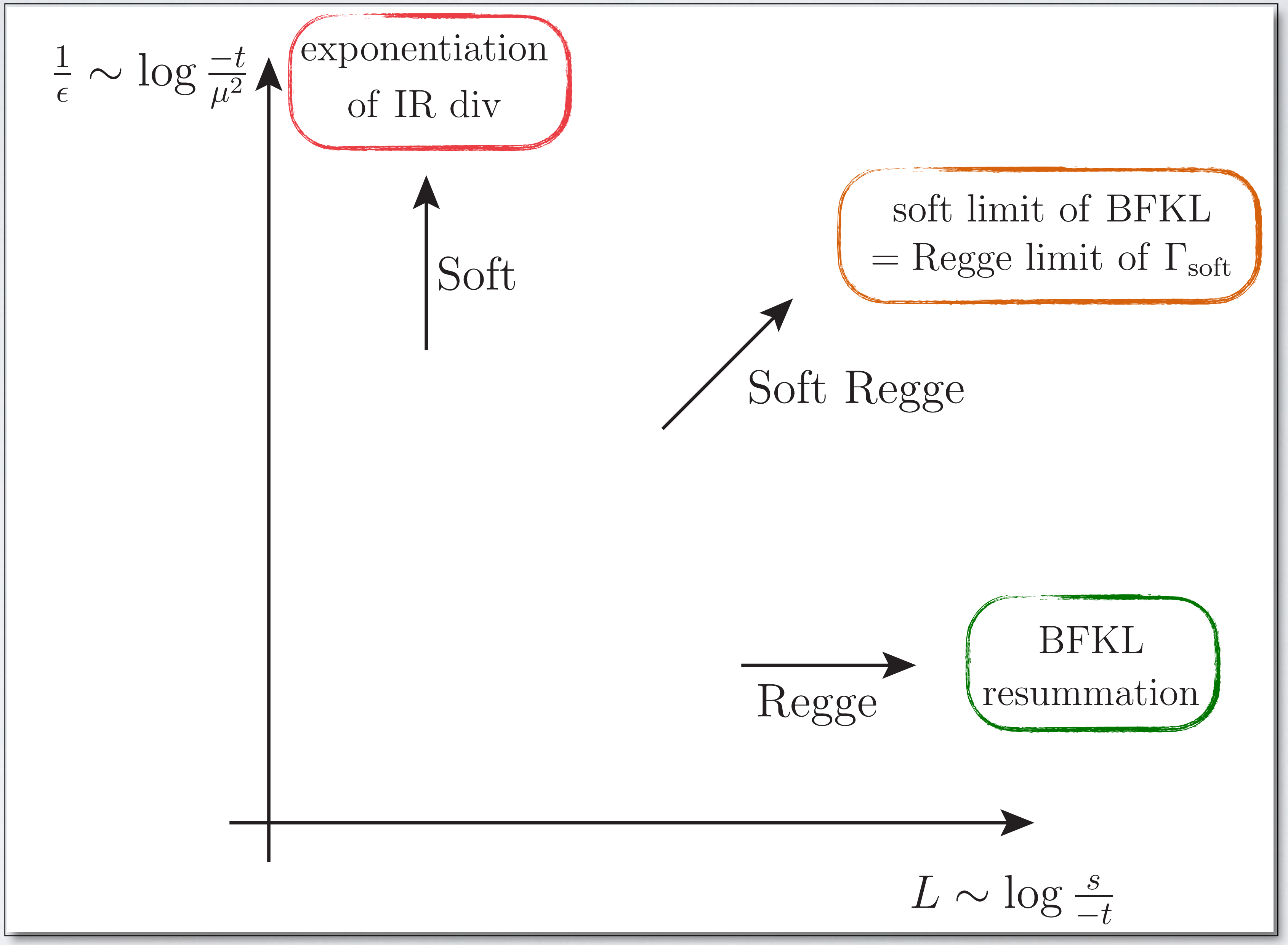

Determined to all orders: Caron-Huot, Gardi, Reichel, LV, 2017,2020

*(See also Caron-Huot, 2013; Del Duca, Falcioni, Magnea, LV, 2014)*

**Determined to three and four loops: Caron-Huot, Gardi, LV, 2017; Gardi, Falcioni, Milloy, LV, 2020**

# REGGE VS INFRARED FACTORISATION

- A tale of two factorisations:




- Application: test (and predict) the analytic structure of infrared divergences.

# REGGE VS INFRARED FACTORISATION

- Infrared divergences are calculated in terms of the so-called soft anomalous dimension:

$$\mathcal{M}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right) = \mathbf{Z}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right)\mathcal{H}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right),$$

$$\mathbf{Z}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right) = \mathcal{P}\exp\left\{-\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\,\mathbf{\Gamma}_n\left(\{p_i\},\lambda,\alpha_s(\lambda^2)\right)\right\}.$$

| Re | $L^0$ | $L^1$ | $L^2$ | $L^3$ | $L^4$ | $L^5$ | $L^6$ |
|---|---|---|---|---|---|---|---|
| $\alpha_s^1$ | $\frac{1}{4}\widehat{\gamma}_K^{(1)}\ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i+\sum_{i=1}^4\gamma_i^{(1)}$ | $\frac{1}{2}\widehat{\gamma}_K^{(1)}\mathbf{T}_t^2$ | | | | | |
| $\alpha_s^2$ | $\frac{1}{4}\widehat{\gamma}_K^{(2)}\ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i+\sum_{i=1}^4\gamma_i^{(2)}$ | $\frac{1}{2}\widehat{\gamma}_K^{(2)}\mathbf{T}_t^2$ | 0 | | | | |
| $\alpha_s^3$ | $\frac{1}{4}\widehat{\gamma}_K^{(3)}\ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i+\sum_{i=1}^4\gamma_i^{(3)}+\Delta^{(+,3,0)}$ | $\frac{1}{2}\widehat{\gamma}_K^{(3)}\mathbf{T}_t^2$ | 0 | 0 | | | |
| $\alpha_s^4$ | | | $\Delta^{(+,4,2)}$ | 0 | 0 | | |
| $\alpha_s^5$ | | | | | 0 | 0 | |
| $\alpha_s^6$ | | | | | | 0 | 0 |

| Im | $L^0$ | $L^1$ | $L^2$ | $L^3$ | $L^4$ | $L^5$ | $L^6$ |
|---|---|---|---|---|---|---|---|
| $\alpha_s^1$ | $\frac{1}{2}\widehat{\gamma}_K^{(1)}i\pi\mathbf{T}_{s-u}^2$ | 0 | | | | | |
| $\alpha_s^2$ | $\frac{1}{2}\widehat{\gamma}_K^{(2)}i\pi\mathbf{T}_{s-u}^2$ | 0 | 0 | | | | |
| $\alpha_s^3$ | $\frac{1}{2}\widehat{\gamma}_K^{(3)}i\pi\mathbf{T}_{s-u}^2+\Delta^{(-,3,0)}$ | $\Delta^{(-,3,1)}$ | 0 | 0 | | | |
| $\alpha_s^4$ | | | | $\Delta^{(-,4,3)}$ | 0 | | |
| $\alpha_s^5$ | | | | | $\Delta^{(-,5,4)}$ | 0 | |
| $\alpha_s^6$ | | | | | | $\Delta^{(-,6,5)}$ | 0 |

***Caron-Huot, Gardi, LV, 2017***

***Caron-Huot, Gardi, Reichel, LV, 2017;***

**Gardi, Falcioni, Milloy, LV, 2020 and in preparation**

# ANALYTIC TOOLS: HIGH-ENERGY LIMIT

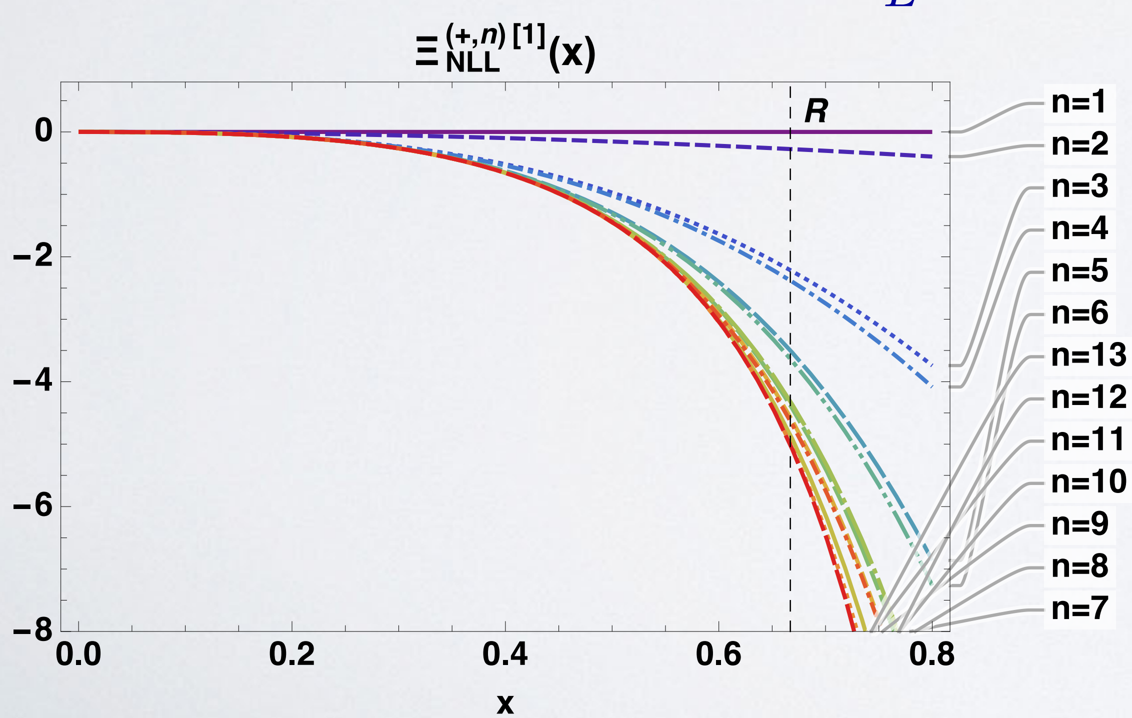
- Knowledge of the amplitude to high-orders allows to perform several interesting studies:
- Number theory properties of an amplitude;

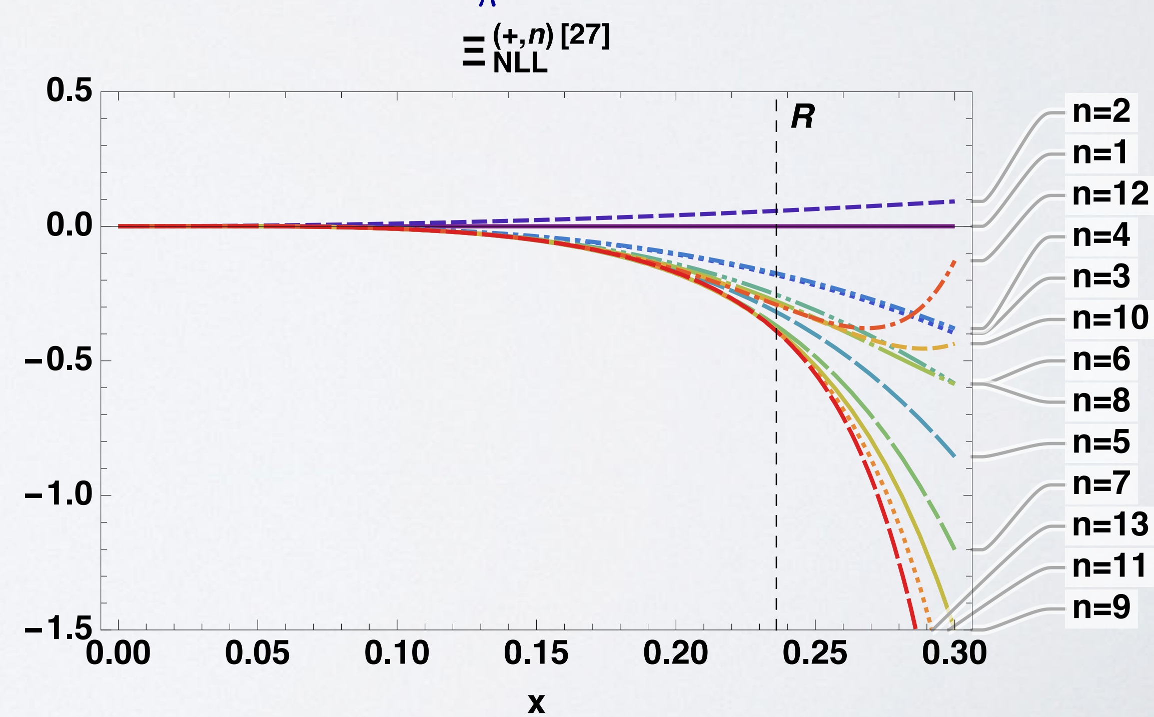
$$\hat{\mathcal{M}}^{(+,11)}_{\rm NLL} \supset \Big( -\frac{149}{688128000}(2C_A-\mathbf{T}_t^2)^8(C_A-\mathbf{T}_t^2)^2 + \frac{26209}{6193152000}(2C_A-\mathbf{T}_t^2)^7(C_A-\mathbf{T}_t^2)^3 - \frac{14813}{442368000}(2C_A-\mathbf{T}_t^2)^6(C_A-\mathbf{T}_t^2)^4 + \frac{210383}{1548288000}(2C_A-\mathbf{T}_t^2)^5(C_A-\mathbf{T}_t^2)^5 - \frac{7549}{25804800}(2C_A-\mathbf{T}_t^2)^4(C_A-\mathbf{T}_t^2)^6 + \frac{39257}{129024000}(2C_A-\mathbf{T}_t^2)^3(C_A-\mathbf{T}_t^2)^7 - \frac{11}{102400}(2C_A-\mathbf{T}_t^2)^2(C_A-\mathbf{T}_t^2)^8 \Big) \times g_{5,3,3}.$$

- Convergence of the perturbative expansion.

***Caron-Huot, Gardi, Reichel, LV, 2020***

$$\hat{\mathcal{M}}^{(+)}_{\rm NLL} = \frac{i\pi}{L}\,\Xi^{(+)}_{\rm NLL}\,\mathbf{T}^2_{s-u}\mathcal{M}^{(\rm tree)}, \qquad x = \frac{L\alpha_s}{\pi}.$$



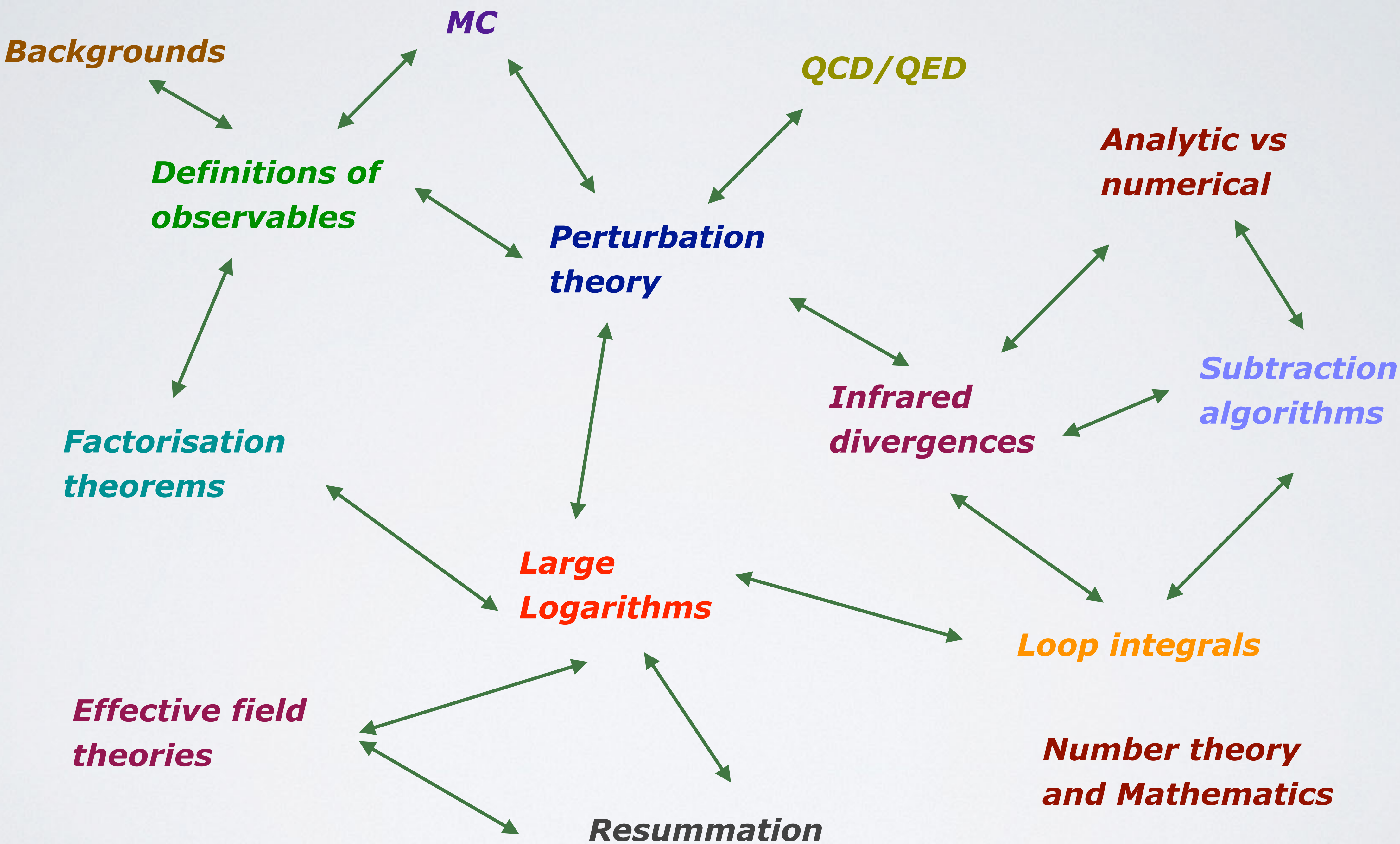
# CONCLUSION

- Development of analytic tools for precision in particle physics:
  - diagrammatic and effective field theory methods for resummation of large logarithms at next-to-leading power;
  - analytic structure of scattering amplitudes: calculation of scattering amplitudes in the high-energy limit and determination of the structure of infrared divergences.
- Recent developments:
  - Factorisation theorem for threshold logarithms at next-to-leading power in SCET;
  - First example of resummation in d dimension.
  - Calculation of two-parton scattering amplitudes in the high energy limit:
    - Imaginary part of the amplitude, to all orders in perturbation theory, at NLL;
    - Real part of the amplitude, at NNLL through four loops.

# QCD & TOP

- *Michele Faucci Giannelli*, (Roma Tor Vergata) "Top physics and fast calorimeter simulation in ATLAS";
- *Nello Bruscino*, (Roma1) "Probing Higgs false vacuum at LHC: measurement of the top Yukawa coupling in tHq production channel";
- *Leonardo Vernazza*, (Torino) "Analytic tools for precision in particle physics";
- *Leandro Javier Cieri*, (Firenze) "N3LOPHYSICS";
- *Yiannis Makris*, (Pavia) "SCET-Q Progress report";
- *Manoj Kumar Mandal*, (Padova) "Scattering Amplitudes: Role of Intersection theory".

# QCD & TOP



MC
Backgrounds
QCD/QED
Definitions of observables
Analytic vs numerical
Perturbation theory
Subtraction algorithms
Infrared divergences
Factorisation theorems
Large Logarithms
Loop integrals
Effective field theories
Number theory and Mathematics
Resummation