THREE-CP
Three Hadron Reactions to Estimate the Effects of CP

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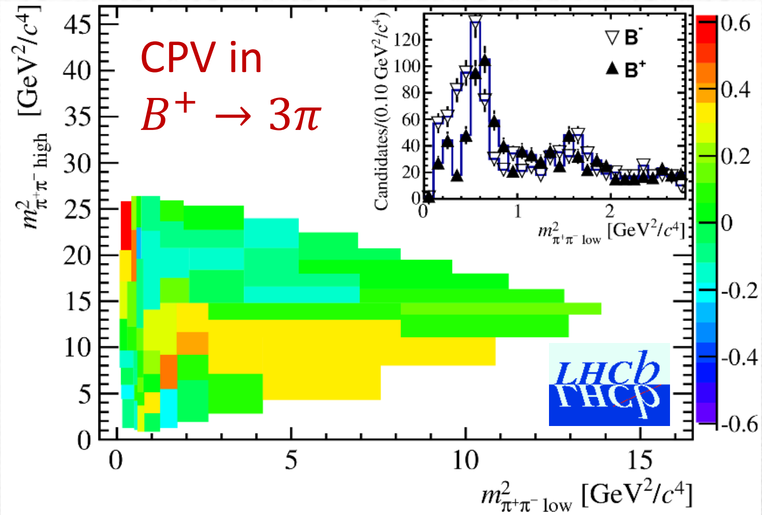
Fellini general meeting, March 5th, 2021

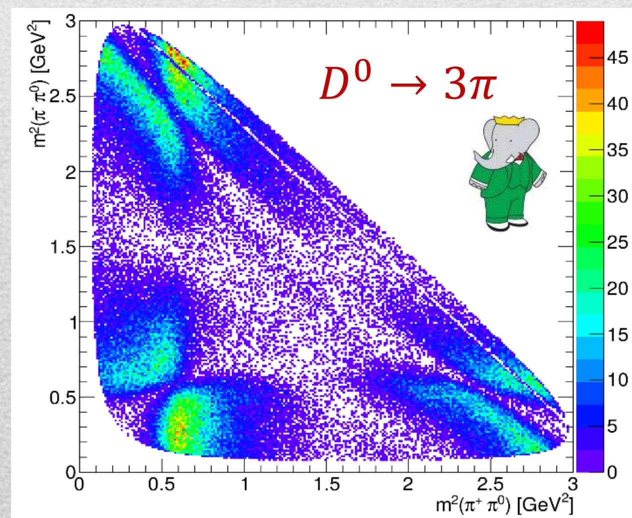
Istituto Nazionale di Fisica Nucleare

Fellowship for Innovation at INFN

H2020 MSCA COFUND G.A. 754496

Heavy meson decays



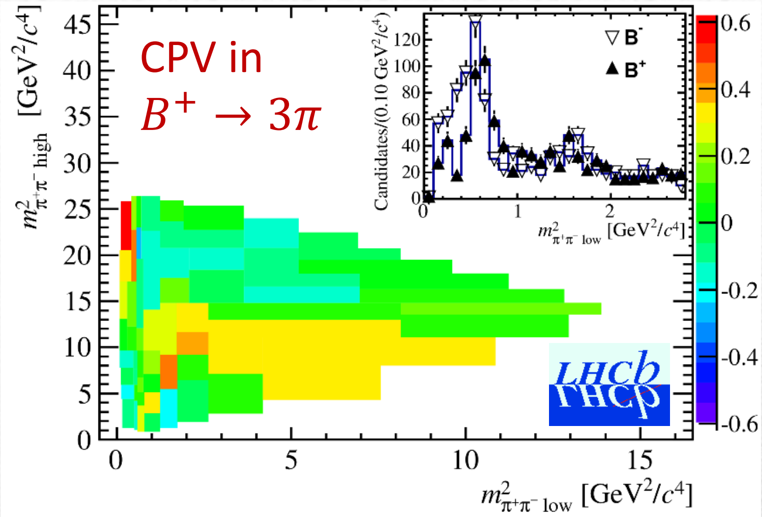


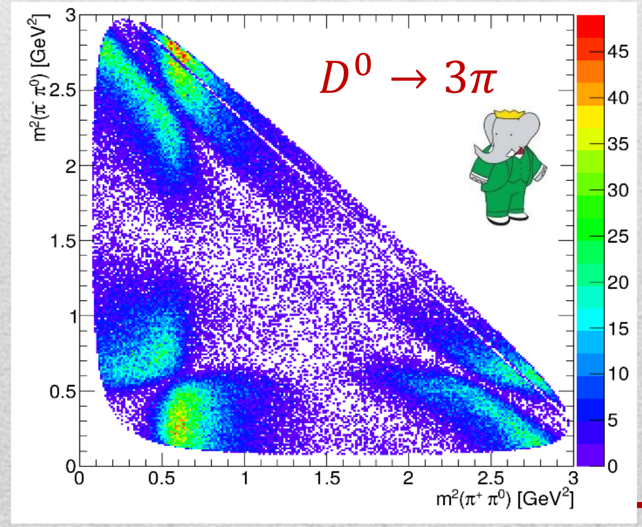
Recently, lots of attention has been devoted to the multibody decays of heavy mesons

CP-violating phases can be extracted by amplitude analyses to the Dalitz plots which represent decays into 3+ particles These could provide more contraints to the CKM matrix elements

The interference between different resonances makes these phases measurable

Nonperturbative QCD at work





«However, the weak decays of beauty & charm hadrons are produced mostly with 3- & 4-body Final States.

Those transitions are shaped by the impact of non-perturbative QCD with $\sim O(1)$ GeV. [...] It is affected by thresholds, resonances – in particular for broad ones – etc.»

I. Bigi

More precise measurements will require a more refined description of the hadronic matrix elements.

Example: QCD factorization

For example, many calculations to date have been performed assuming QCD factorization

$$\mathcal{A}(D^+ \to K^-\pi^+\pi^+) = \frac{G_F}{\sqrt{2}}\cos^2\theta_C(a_1\mathcal{A}_1 + a_2\mathcal{A}_2) + (\pi_1^+ \leftrightarrow \pi_2^+)$$

$$= \frac{G_F}{\sqrt{2}}\cos^2\theta_C[a_1\langle K^-\pi_1^+|\bar{s}\gamma^\mu(1-\gamma_5)c|D^+\rangle\langle\pi_2^+|\bar{u}\gamma_\mu(1-\gamma_5)d|0\rangle$$

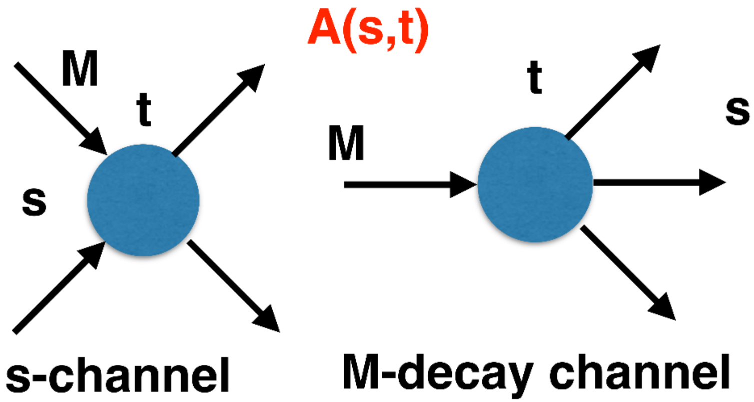
$$+a_2\langle K^-\pi_1^+|\bar{s}\gamma^\mu(1-\gamma_5)d|0\rangle\langle\pi_2^+|\bar{u}\gamma_\mu(1-\gamma_5)c|D^+\rangle] + (\pi_1^+ \leftrightarrow \pi_2^+)$$

Boito and Escribano, PRD80, 054007

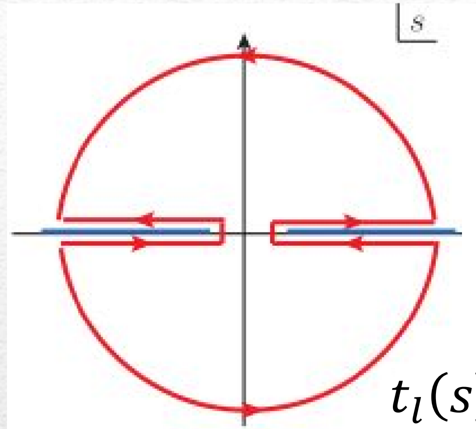
$K\pi_1$ and π_2 do not talk with each other,
isobar approximation

The errors are expected to be $O(\Lambda_{QCD}/m_{c,b})$,
but not uniform on the Dalitz plot

S-Matrix principles



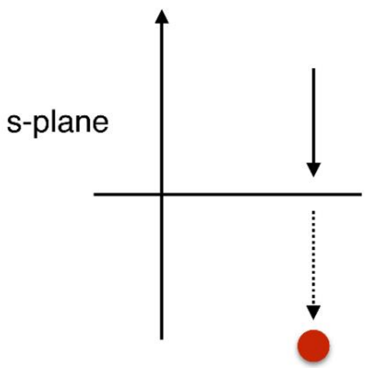
Crossing

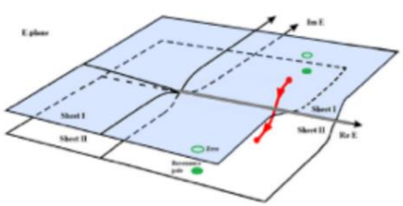


$$t_l(s) = \lim_{\epsilon\to 0} t_l(s+i\epsilon)$$

Analyticity

$$\mathrm{Im}\, t_l = \rho_l |t_l|^2$$





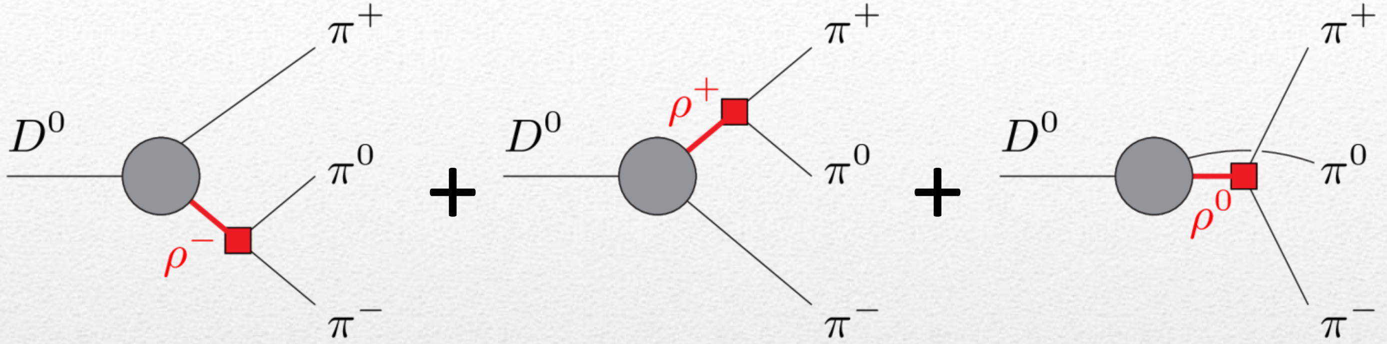
Unitarity

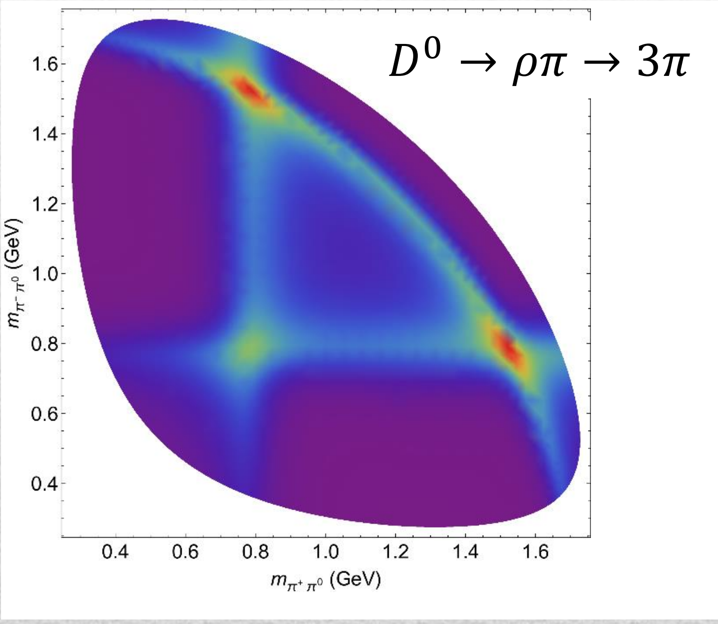
+ Lorentz, discrete & global symmetries

Even though the solution of QCD is unknown, there exists general principles of the S-Matrix which the amplitudes must satisfy

These allow us to write dispersion relations which connect different channels in different kinematical regimes

The isobar model





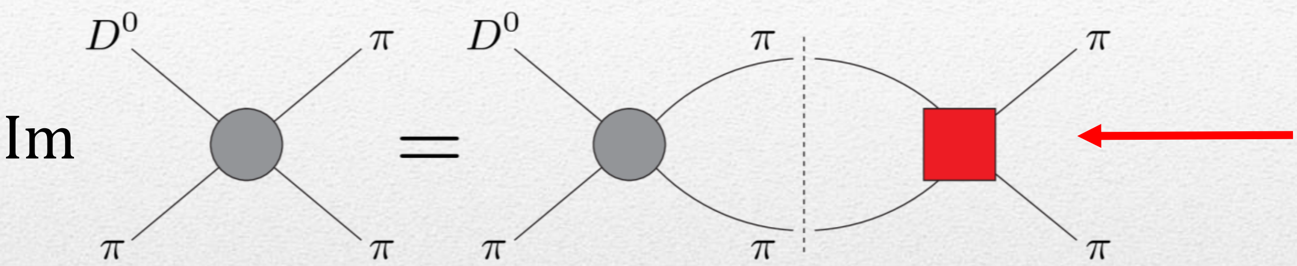
$$f(s,t,u) = g_{(s)} e^{i\phi_{(s)}} A^{(s)}(s) + g_{(t)} e^{i\phi_{(t)}} A^{(t)}(t) + g_{(u)} e^{i\phi_{(u)}} A^{(u)}(u)$$

The isobars A are usually Breit-Wigners.
The sum breaks unitarity

This specifically affects interference pattern between resonances, and so the extraction of CPV phases from mass-dependent fits

Khuri-Treiman equations

Solution : assume the same decomposition, but imposing unitarity to it



Input: $\pi\pi$ phase shift
well known till 1.4 GeV
GKPY, PRD83, 074004
can be extracted from
$e^+e^- \to \pi^+\pi^-$ (BaBar)
and $J/\psi \to \gamma\, \pi^0\pi^0$ (BESIII)

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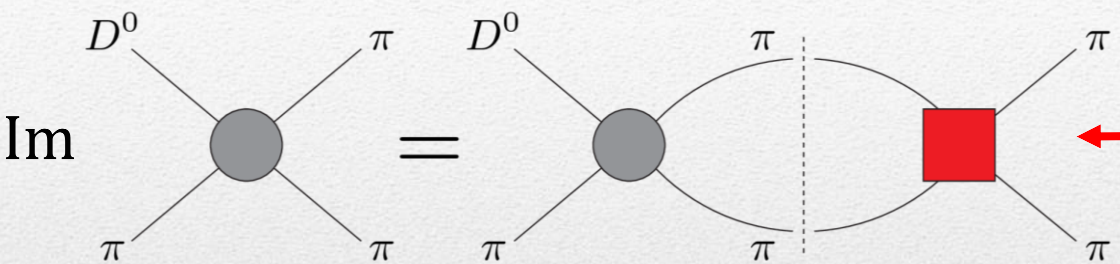

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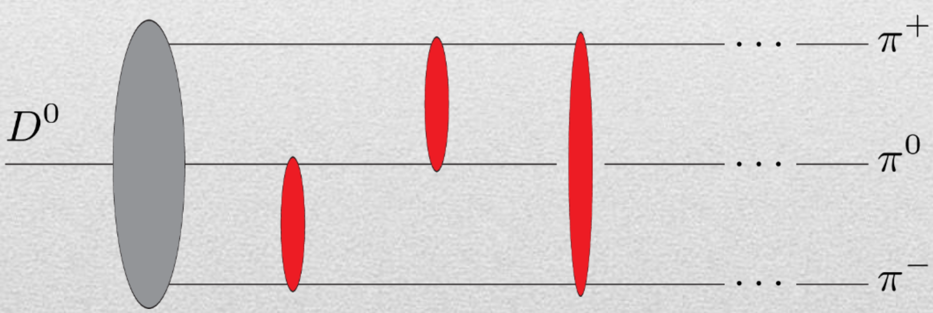


Khuri-Treiman formalism was introduced to describe $K \to 3\pi$

Khuri and Treiman, PR119, 1115

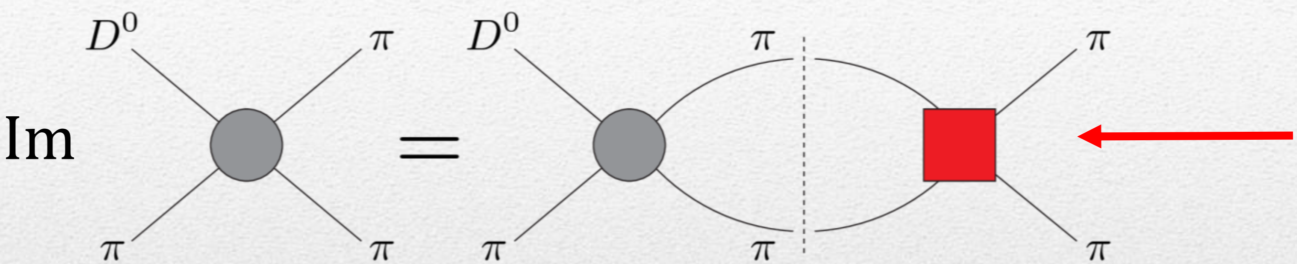
Used recently for several reactions,
but never for CPV ones

Niecknig and Kubis, JHEP 10, 142
Colangelo, *et al.*, PRL118, 022001
Albaladejo, ... AP et al., EPJC

Equivalent to implementing the all-order rescattering in all the 3 channels at once

Khuri-Treiman equations

Solution : assume the same decomposition, but imposing unitarity to it



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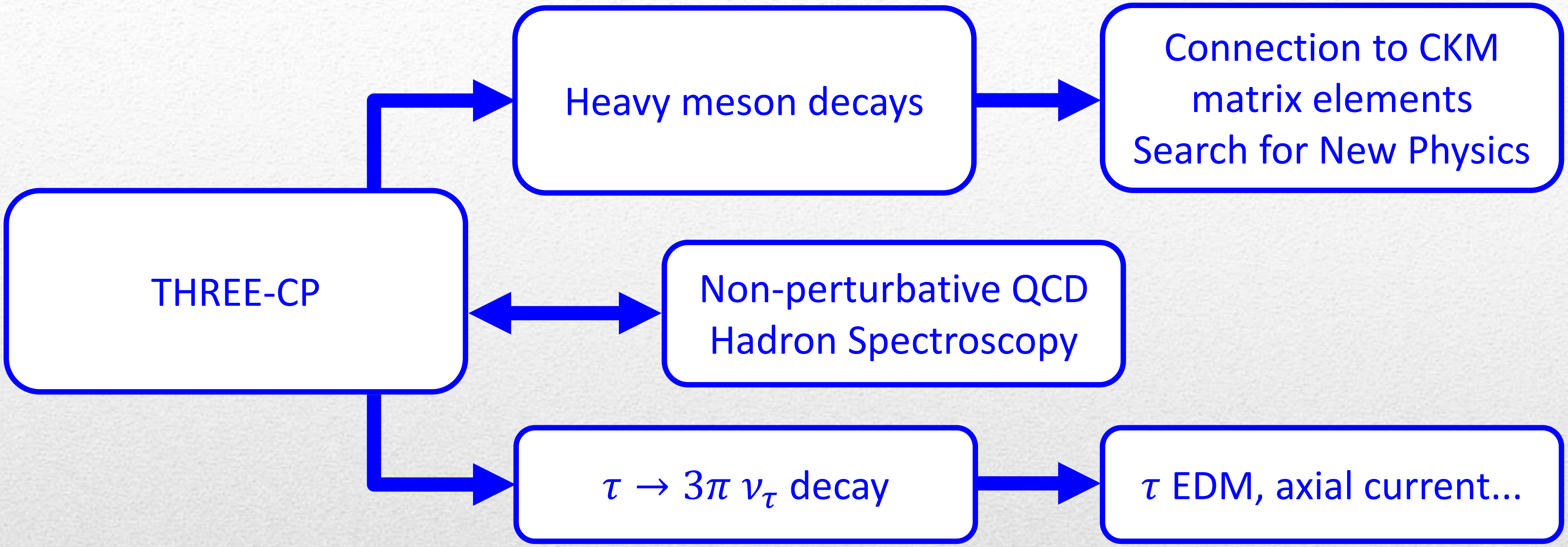
$$A_\ell^{(s)}(s) = \Omega_\ell(s)\left(\mathrm{Pol}^{(n-1)}(s) + \frac{s^n}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\sin\delta_\ell(s')}{|\Omega_\ell(s')|\, s'^n (s' - s - i\epsilon)}\right.$$
$$\left.\times \int d\cos\theta_s \sum_{\ell'}^{L_{\max}} P_\ell(\cos\theta_s) P_{\ell'}\left(\cos\theta_t(s',\cos\theta_s)\right) A_{\ell'}^{(t)}\left(t(s',\cos\theta_s)\right) + (t \to u)\right)$$

$$\Omega_\ell(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta_\ell(s')}{s'(s'-s)}\right)$$

System of coupled integral equations
Input: $\pi\pi$ phase shift
Output: isobars

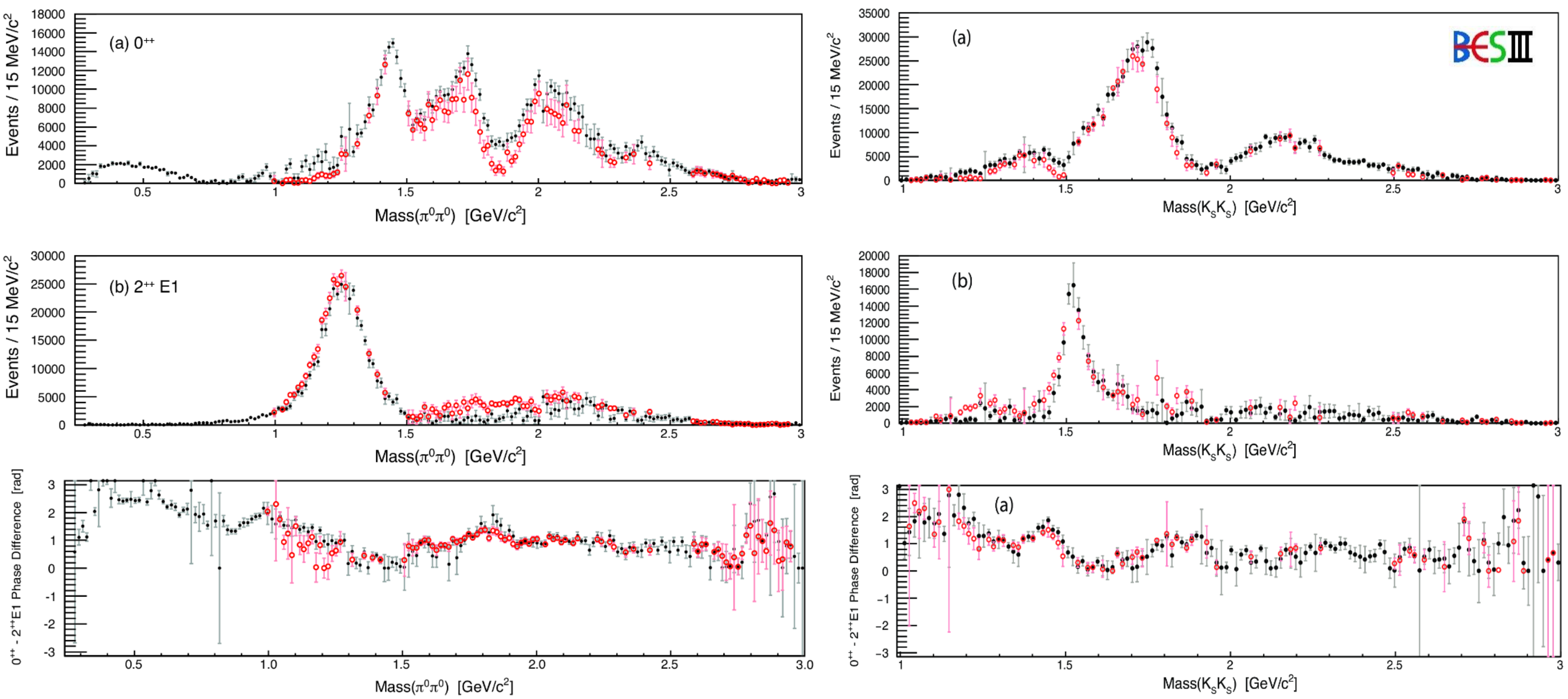
The project



	Year 1	Year 2	Year 3
Modelling of $\pi\pi$			
$D^0 \to \pi^+ \pi^- \pi^0$			
$B^+ \to \pi^+ \pi^- \pi^+$			
$B^+ \to \pi^+ K^+ K^-$			
$\tau \to 3\pi\, \nu_\tau$			

$J/\psi \to \gamma\, \pi^0\pi^0$ and $\to \gamma\, K_S^0 K_S^0$: in progress

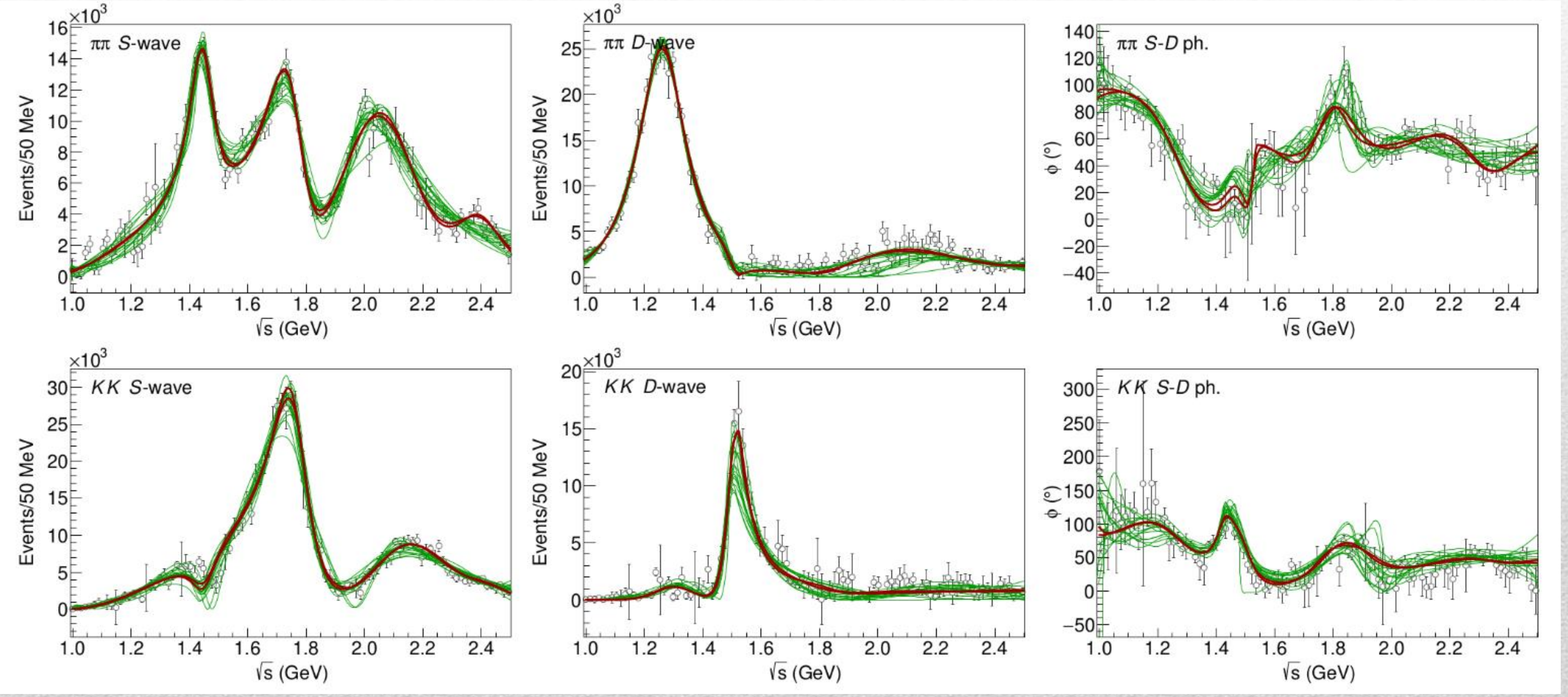
This process gives us $\pi\pi$ scattering shift in S and D-wave up to 3 GeV
This is a gluon-rich process, golden channel for identifying of the scalar glueball



$J/\psi \to \gamma\, \pi^0\pi^0$ and $\to \gamma\, K_S^0 K_S^0$: in progress

Analysis at advanced stage, draft in preparation, should be submitted in 1 month

Rodas, AP, in preparation



Heavy mesons

$D^0 \to 3\pi$

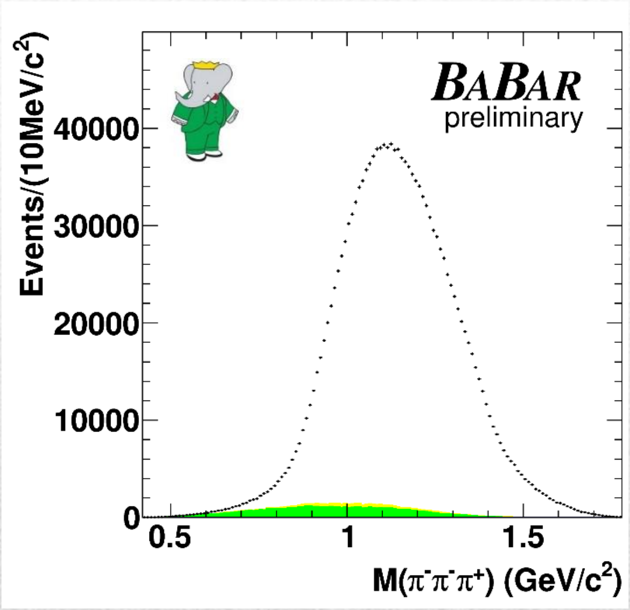
- (Mostly) CP conserving
- Dominated by $I = 0$
- All ingredients ready

$B^+ \to 3\pi, \pi K \bar{K}$

- Requires CPV in KT : to do
- Larger phase space, requires asymptotic $\pi\pi$, Regge + KT : in progress

Timely to be LHCb THCp theory affiliate!

Heavy leptons: $\tau \to 3\pi\, \nu_\tau$



The τ sector is ideal for the search of New Physics

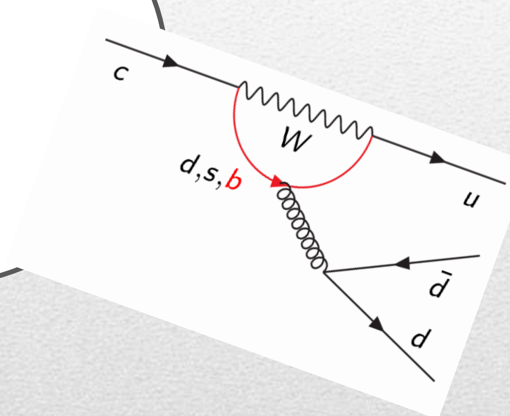
The $\tau \to 3\pi\, \nu_\tau$ system is particularly interesting:

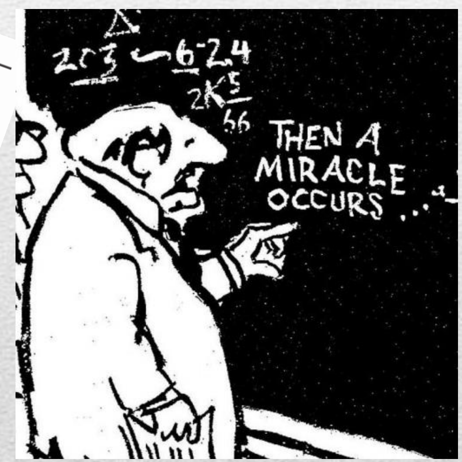
- One can extract the 3-body axial current, and use it to describe the neutrino-nucleon interaction
- The decay enters the determination from the spectral function to extract $\alpha_s(m_\tau)$
- This channel is promising to get a more refined measurement of the τ Electric Dipole Moment
- The 3π data are dominated by the $a_1(1260)$ which is still not well established in the PDG

- Extension of KT to running 3π masses studied already,
  Mikhasenko, ... AP et al., JHEP 08 (2019) 080
- In touch with N. Neri (UniMi) to implement this in EDM measurement at LHCb with the SELDOM ERC project

THREE-CP

D^0





π^+

π^-

π^0

BACKUP

My story

PostDoc Virginia (USA) 2015-2018

Jefferson Lab

PostDoc Trento 2018-2020

ECT*

PhD Roma 2012-2015

SAPIENZA UNIVERSITÀ DI ROMA

RTD Roma 2020-2023

INFN

Fellini – Fellowship for Innovation at INFN

Theorist with strong connections with experiments

JLab: clas, GlueX

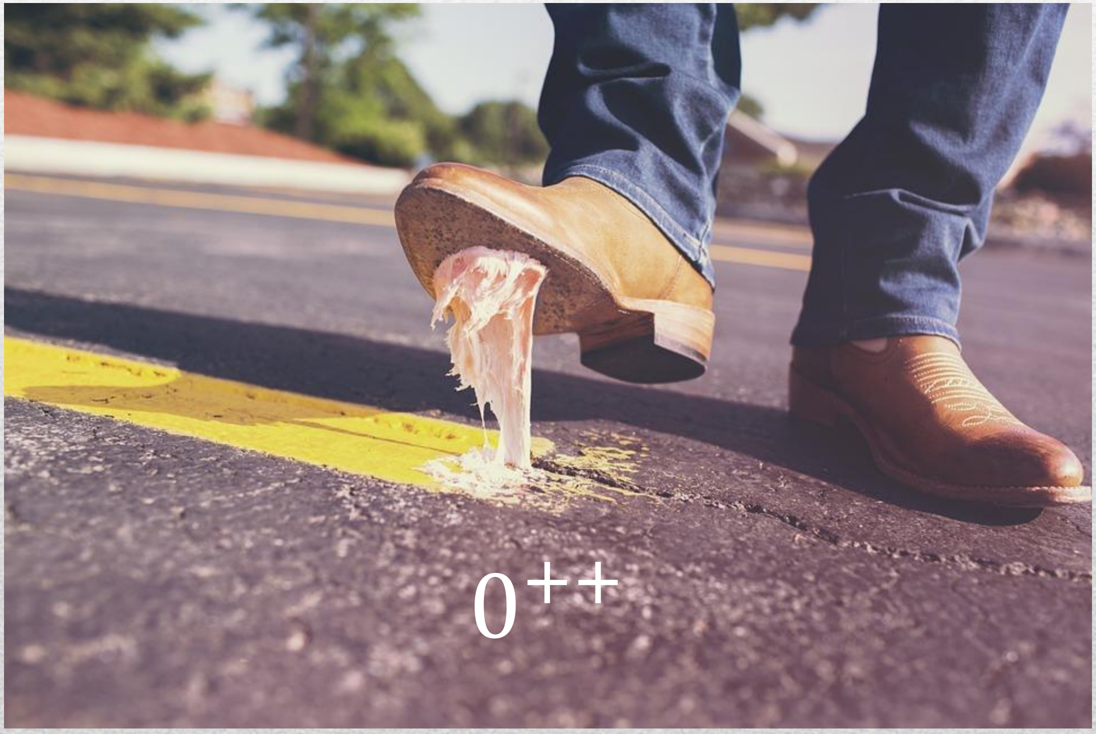
CERN: LHCb, COMPASS

BABAR

EIC^2

The scalar glueball

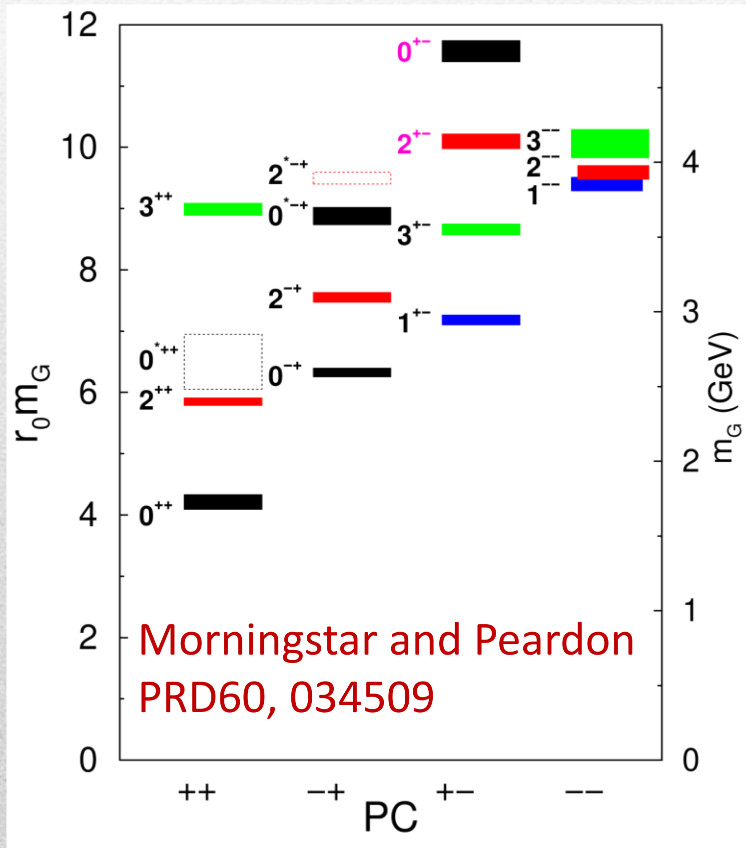
A. Rodas, AP *et al.* (JPAC) in progress



Glueballs

The clearest sign of confinement in pure Yang-Mills

The worst state to search in real life



Morningstar and Peardon PRD60, 034509

J^{PC}	Mass MeV			
	Unquenched	Quenched		
	This work	M&P	Ky	Meyer
0^{-+}		2590(40)(130)	2560(35)(120)	2250(60)(100)
2^{-+}	3460(320)	3100(30)(150)	3040(40)(150)	2780(50)(130)
0^{-+}	4490(590)	3640(60)(180)		3370(150)(150)
2^{-+}				3480(140)(160)
5^{-+}				3942(160)(180)
0^{--} (exotic)	5166(1000)			
1^{--}		3850(50)(190)	3830(40)(190)	3240(330)(150)
2^{--}	4590(740)	3930(40)(190)	4010(45)(200)	3660(130)(170)
2^{--}				3.740(200)(170)
3^{--}		4130(90)(200)	4200(45)(200)	4330(260)(200)
1^{+-}	3270(340)	2940(30)(140)	2980(30)(140)	2670(65)(120)
3^{+-}	3850(350)	3550(40)(170)	3600(40)(170)	3270(90)(150)
3^{+-}				3630(140)(160)
2^{+-} (exotic)		4140(50)(200)	4230(50)(200)	
0^{+-} (exotic)	5450(830)	4740(70)(230)	4780(60)(230)	
5^{+-}				4110(170)(190)
0^{++}	1795(60)	1730(50)(80)	1710(50)(80)	1475(30)(65)
2^{++}	2620(50)	2400(25)(120)	2390(30)(120)	2150(30)(100)
0^{++}	3760(240)	2670(180)(130)		2755(30)(120)
3^{++}		3690(40)(180)	3670(50)(180)	3385(90)(150)
0^{++}				3370(100)(150)
0^{++}				3990(210)(180)
2^{++}				2880(100)(130)
4^{++}				3640(90)(160)
6^{++}				4360(260)(200)

Gregory *et al.* JHEP1210, 170

Same model as before

Two channels, $i, k = \pi\pi, KK$

Two waves, $J = S, D$

52 parameters

$$D^J_{ki}(s) = \left[K^J(s)^{-1}\right]_{ki} - \frac{s}{\pi}\int_{s_k}^{\infty} ds' \frac{\rho N^J_{ki}(s')}{s'(s'-s-i\epsilon)}$$

$$K^J_{ki}(s) = \sum_R \frac{g_k^{(R)} g_i^{(R)}}{m_R^2 - s} + c^J_{ki} + d^J_{ki}\, s$$

3 K-matrix pole for the S-wave
3 K-matrix poles for the D-wave

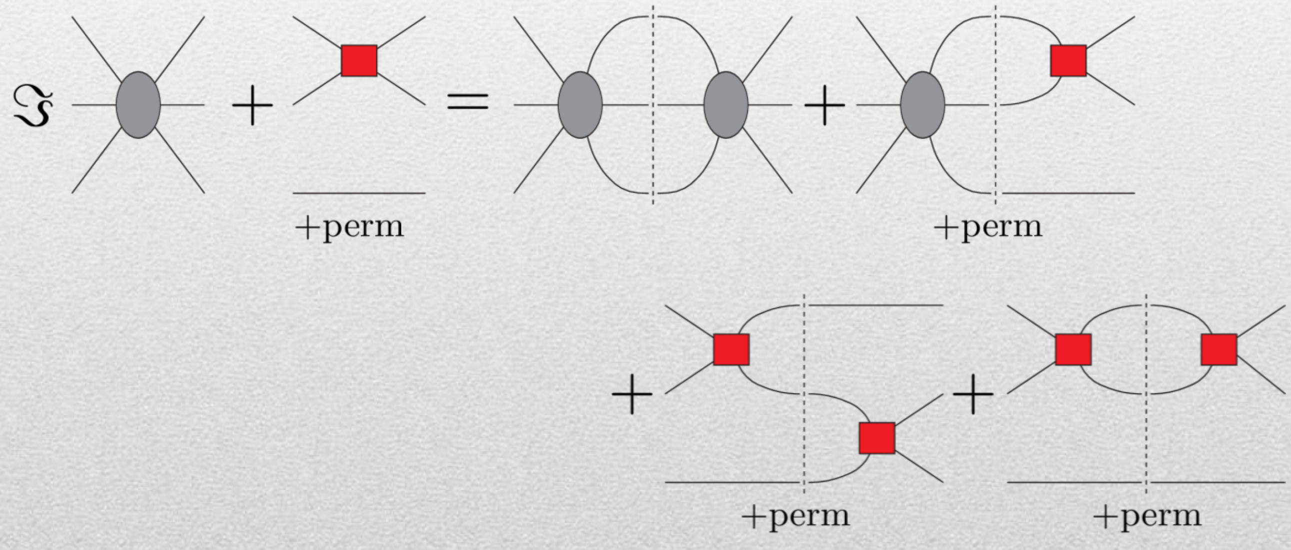
$$\rho N^J_{ki}(s') = \delta_{ki} \frac{\lambda^{J+1/2}\left(s', m^2_{\eta^{(\prime)}}, m^2_\pi\right)}{(s'+s_R)^{2J+1+\alpha}}$$

$$n^J_k(s) = \sum_{n=0}^{3} a^{J,k}_n T_n\left(\frac{s}{s+s_0}\right)$$

3-body unitarity

Mai, AP, *et al.* EPJA53, 177
Jackura, AP, et al., in progress
Alarcon, Passemar, AP, Weiss, in progress

The KT equations have no control over the total 3π invariant mass dependence.
For heavy mesons, this is a fix number, but for the $\tau \to 3\pi$ one has to consider the full 3-body scattering



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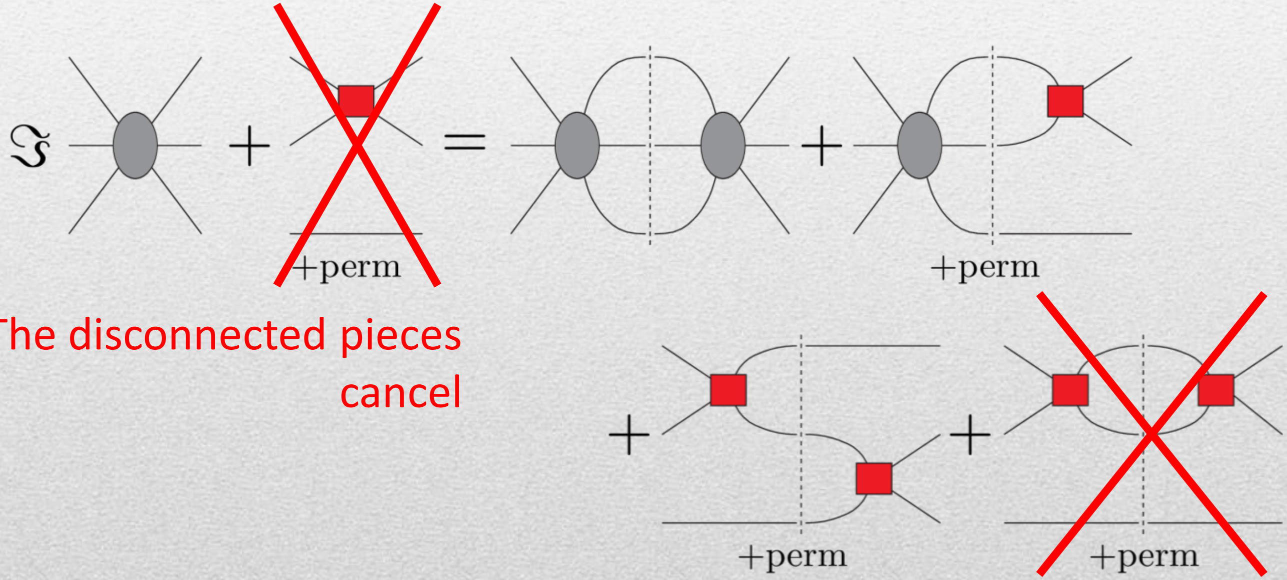

The disconnected pieces cancel