
New ideas for a precision measurement of the W boson mass at the LHC

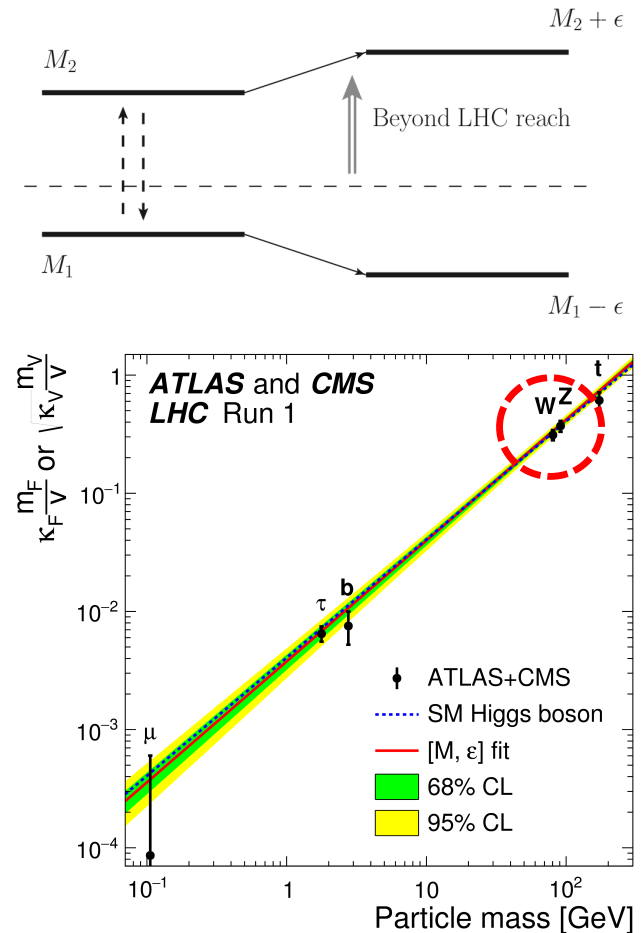
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European Research Council
Established by the European Commission

Why M_W matters

- W mass is an **Electroweak Precision Observable**
 - Stress-test of the SM \rightarrow indirect search of NP
- Why is M_W remarkable?
 - **Phenomenology**
 - High sensitivity and robustness of SM prediction.
 - **Opportunity**
 - Theory more precise than experiment.
 - **Case**
 - Slight tension with SM prediction.



What we can learn from M_W

- SM at tree-level $\rightarrow M_W$ is a function of 3 parameters: $G_\mu, M_Z, \alpha_{\text{EM}}(M_Z)$

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha_{\text{EM}}(M_Z)}{\sqrt{2} G_F (1 - \Delta r)}$$

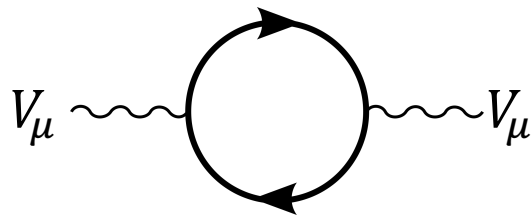
- M_h and m_T enter via **radiative corrections**: $\Delta r \approx 3.6\%$ [$\rightarrow +500$ MeV on M_W]

- This relation entails *custodial symmetry* (and the breaking of it):

$$\Delta r = \underbrace{-\frac{3G_F m_t^2}{8\sqrt{2}\pi^2} \frac{\cos^2 \theta_W}{\sin^2 \theta_W} + \frac{11G_F M_W^2}{24\sqrt{2}\pi^2} \log \frac{M_h^2}{M_W^2}}_{\text{SM}} + \underbrace{\left\{ \begin{array}{l} \bullet \text{ Higgs multiples with } T > \frac{1}{2} \\ \bullet \text{ Non-degenerate doublets} \\ \bullet \text{ Degenerate chiral fermions} \\ \bullet \text{ U(1)'} \\ \bullet \dots \end{array} \right\}}_{\text{BSM ?}}$$

M_W as a probe of NP

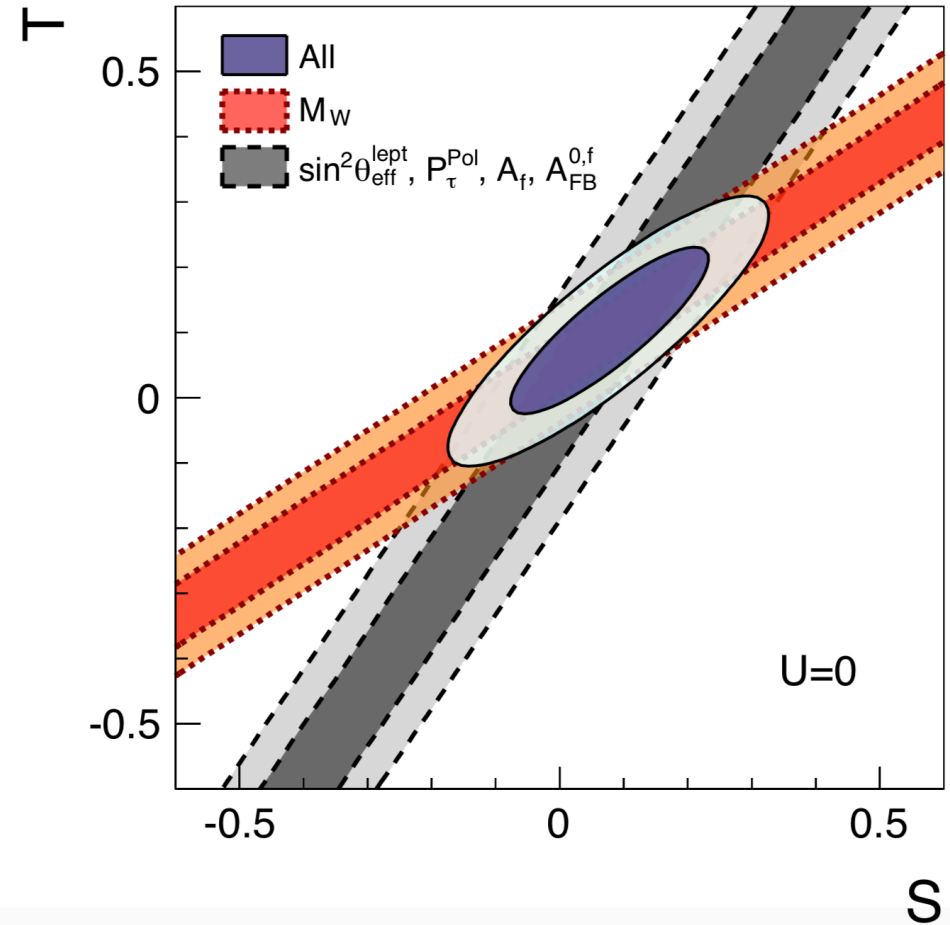
- Pivotal role in the determination of **oblique parameters S, T, U**



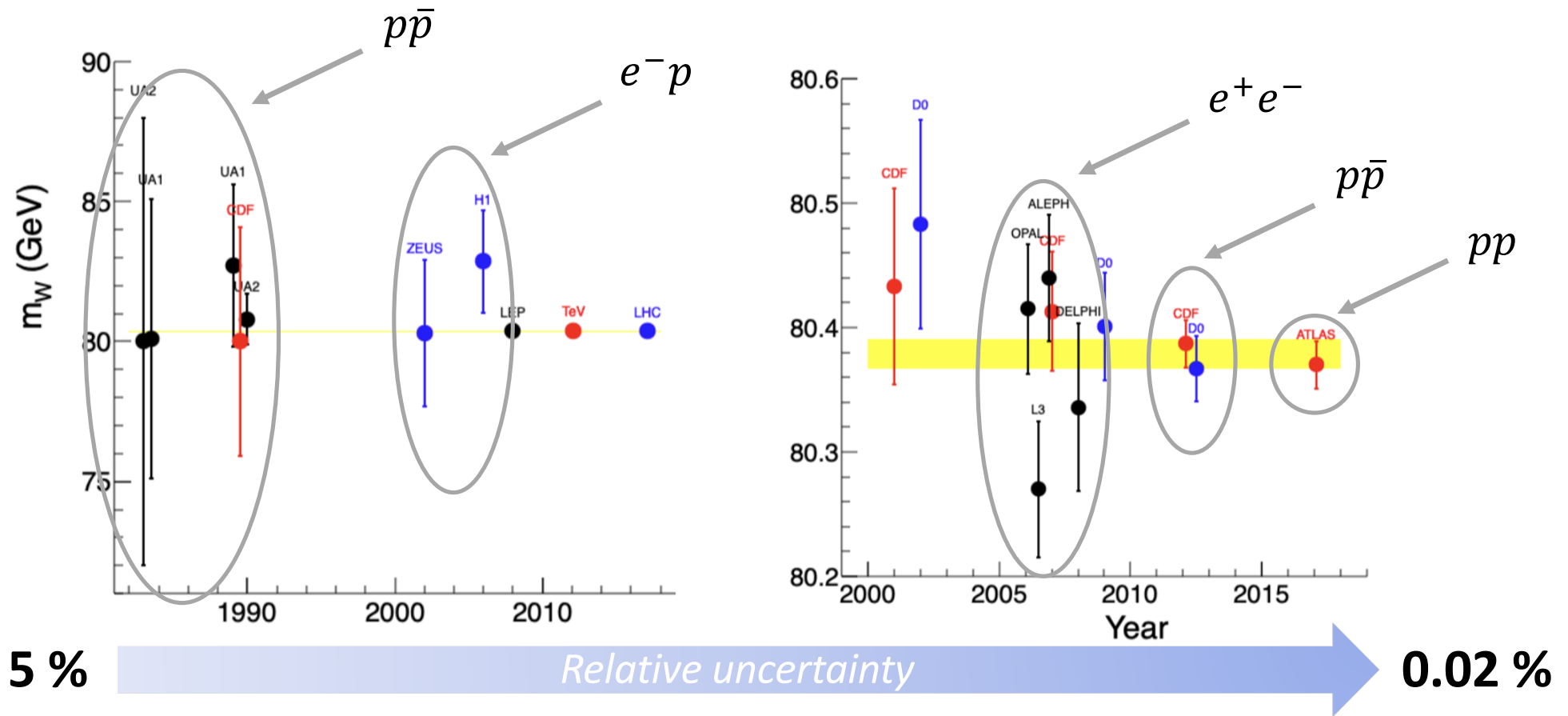
$$M_Z^2 = M_{Z0}^2 \frac{1 - \hat{\alpha}(M_Z)T}{1 - G_F M_{Z0}^2 S / 2\sqrt{2}\pi},$$

$$M_W^2 = M_{W0}^2 \frac{1}{1 - G_F M_{W0}^2 (S + U) / 2\sqrt{2}\pi},$$

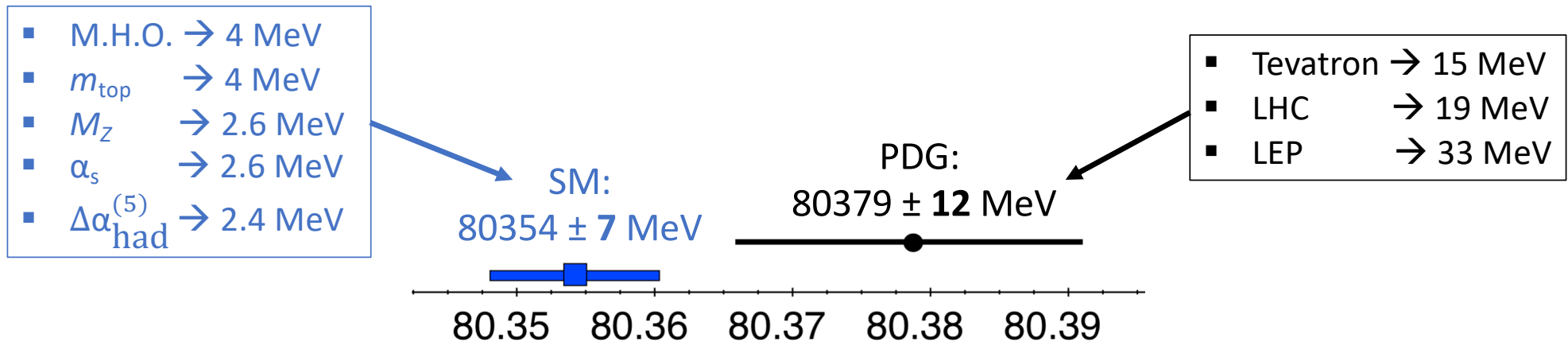
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M_W in the history of colliders



What we know today and what is a useful target

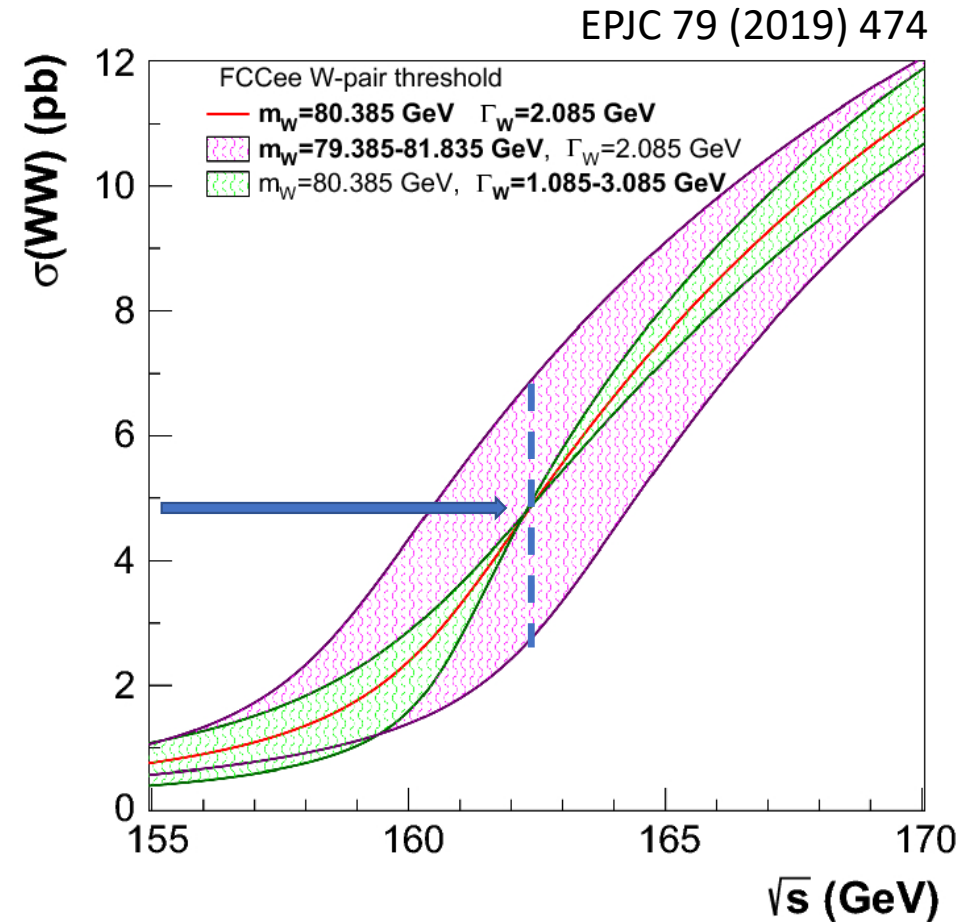


- Consistent at $\sim 2\sigma$. *Fluctuation? Missing systematic? Emerging anomaly...?*

➔ Target for a new measurement: ≤ 10 MeV

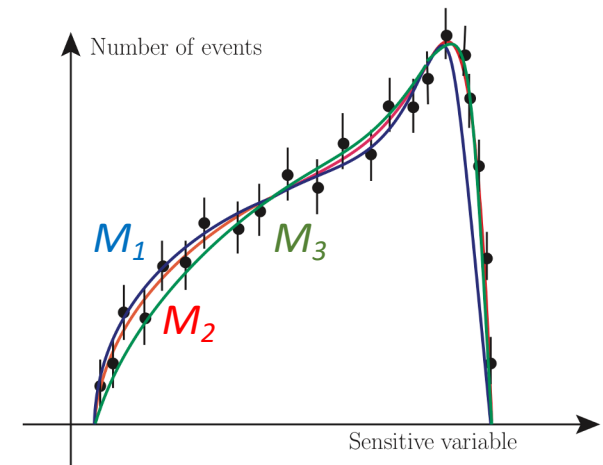
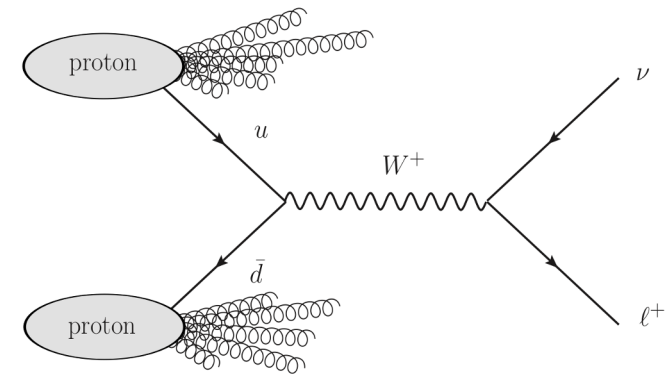
Which future for M_W ?

- **Ultimate precision** from next-generation of lepton colliders (>2040)
 - FCC-ee + 2y at threshold \rightarrow 0.5 MeV
 - Beyond the reach of hadron colliders
- LHC has analyzed just a **tiny fraction** of its data for M_W
 - Strong mandate to probe its limits.



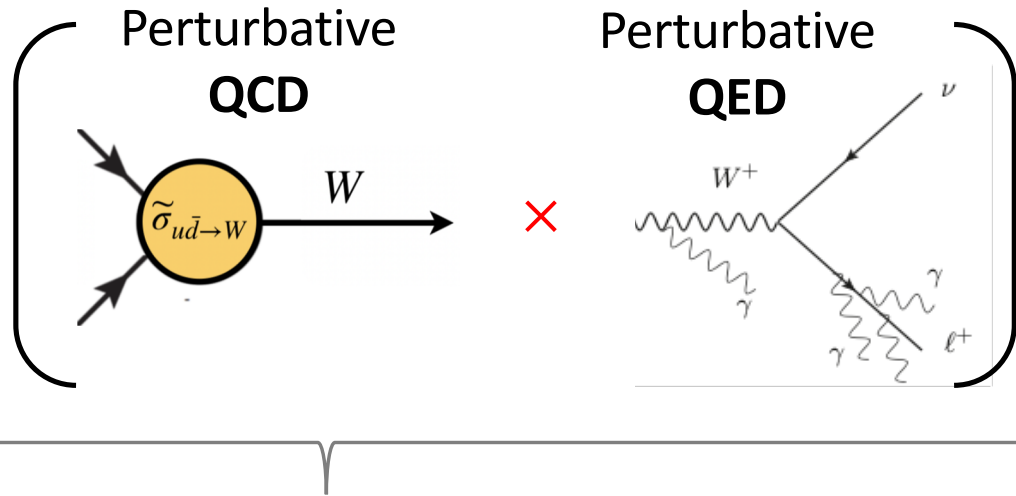
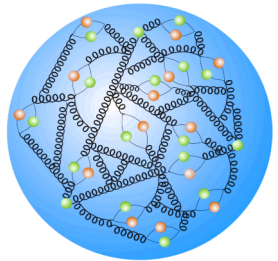
M_W at hadron colliders

- Direct production: $pp \rightarrow W^\pm \rightarrow l^\pm \nu$
 - Continuous spectrum of W momenta
 - Neutrino p_4 unreconstructed
- No “invariant mass” estimator
 - Use of kinematic variables sensitive to M_W (NOT Lorentz-invariant)
- Comparison of experimental distributions to **model-dependent** templates
 - Fit for the “best” M_W



Models of W production

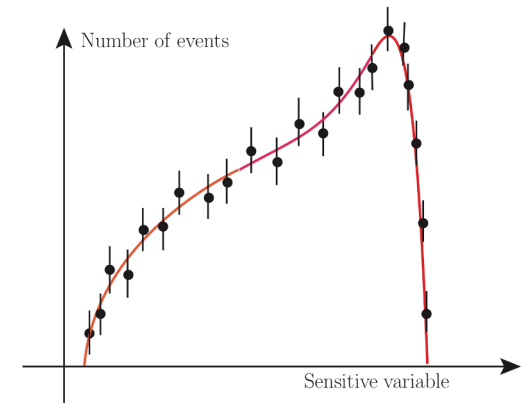
Proton PDFs



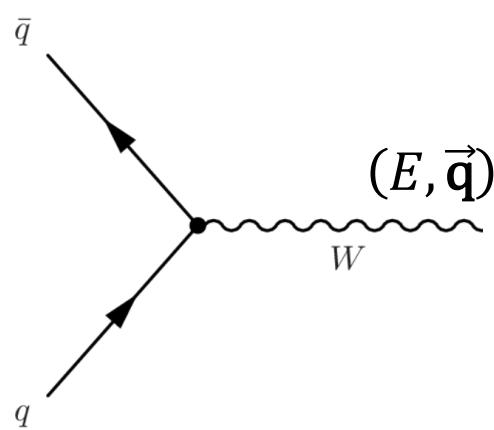
$$\frac{d^6\sigma}{d^3\mathbf{p}_l d^3\mathbf{p}_\nu}$$



p.d.f. of sensitive variable



Interlude: notation



Virtuality
($\sim M_W$)

Rapidity
(\sim velocity along beam)

$$(E, \vec{q}) = \left(\sqrt{q_T^2 + Q^2} \cosh y, \mathbf{q}_T, \sqrt{q_T^2 + Q^2} \sinh y \right)$$

Transverse momentum
(to the beam)

W boson dynamics in the lab

▪ Longitudinal dynamics:

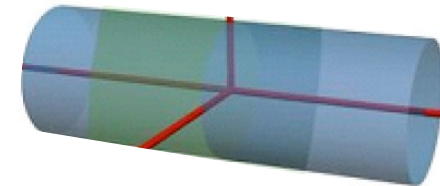
$$\frac{d\sigma}{dy} \sim \int dx_1 dx_2 \delta\left(x_1 x_2 - \frac{Q^2}{s}\right) \delta\left(y - \ln \frac{x_1}{x_2}\right) [u(x_1)\bar{d}(x_2) + \dots]$$

- “||” momenta → PDFs

▪ Transverse dynamics:

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2} \left[1 + o\left(\frac{q_T}{Q}\right) + o\left(\frac{q_T^2}{Q^2}\right) + \dots \right]$$

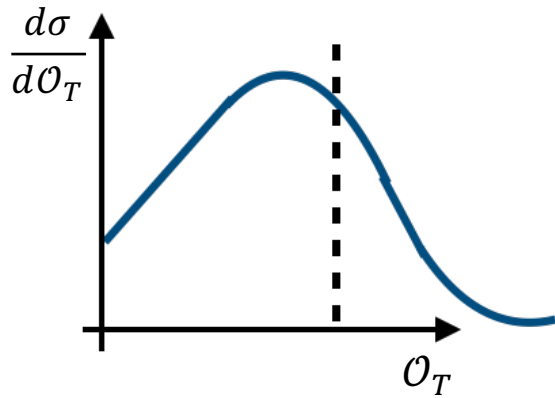
- “⊥” momenta → W decay



→ Transverse variables best suited for studying M_W

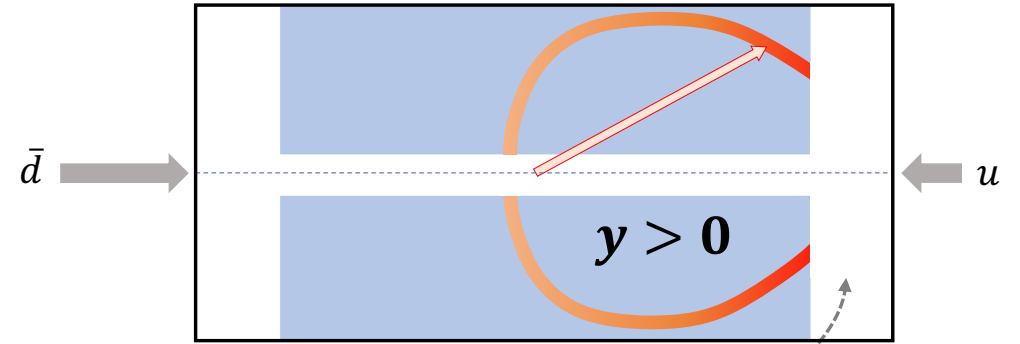
The PDF uncertainty

- Why longitudinal dynamics matters?
 - Mostly an **acceptance** artefact



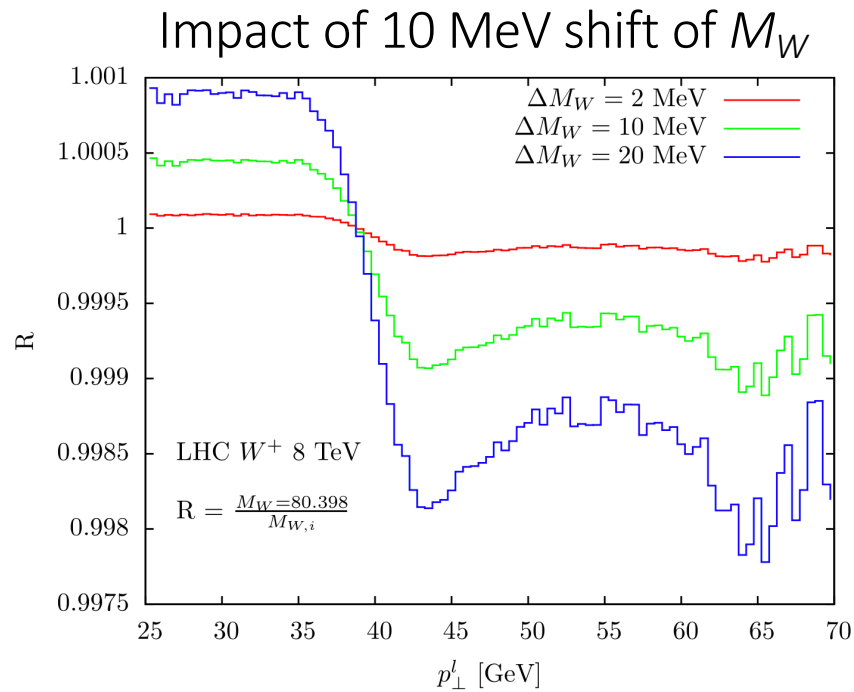
$$\frac{d\sigma}{d\theta_T} = \left[\frac{d\sigma}{d\theta_T} \right]_{\text{ideal}} \times F(\theta_T)$$

PDF dependence from here!

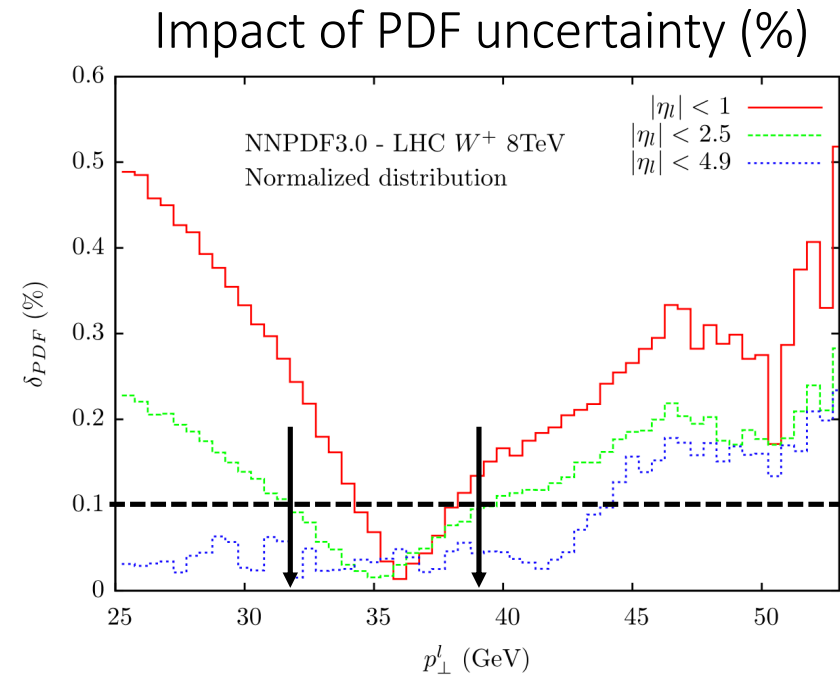


➔ How large $\frac{\Delta F}{F}$ for a **10 MeV precision** on M_W ?

Impact of PDF uncertainty on p_T^l spectrum



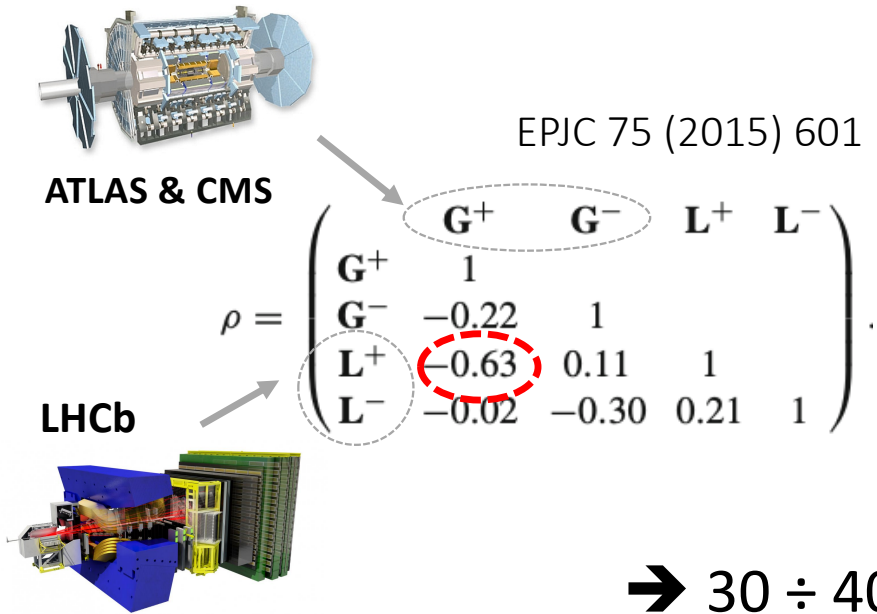
Green curve: 10 MeV \rightarrow $< 0.1\%$



**\rightarrow At the limit for standard detectors
 ($|\eta| < 2.5$)**

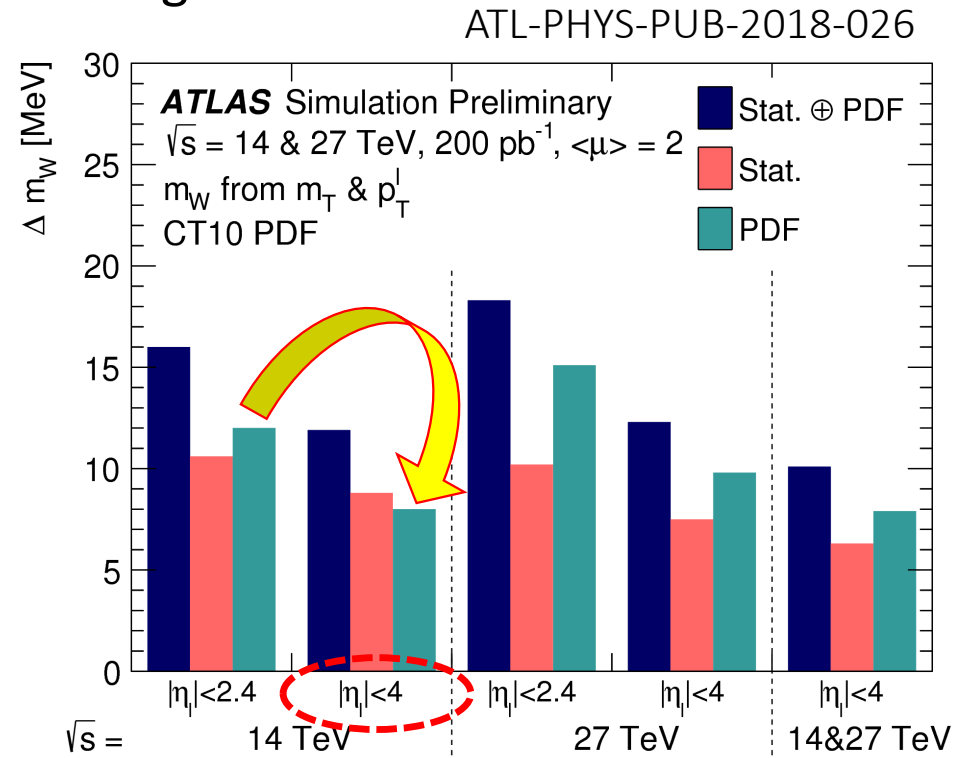
PDF mitigation: experimental perspective

Joining experiments:



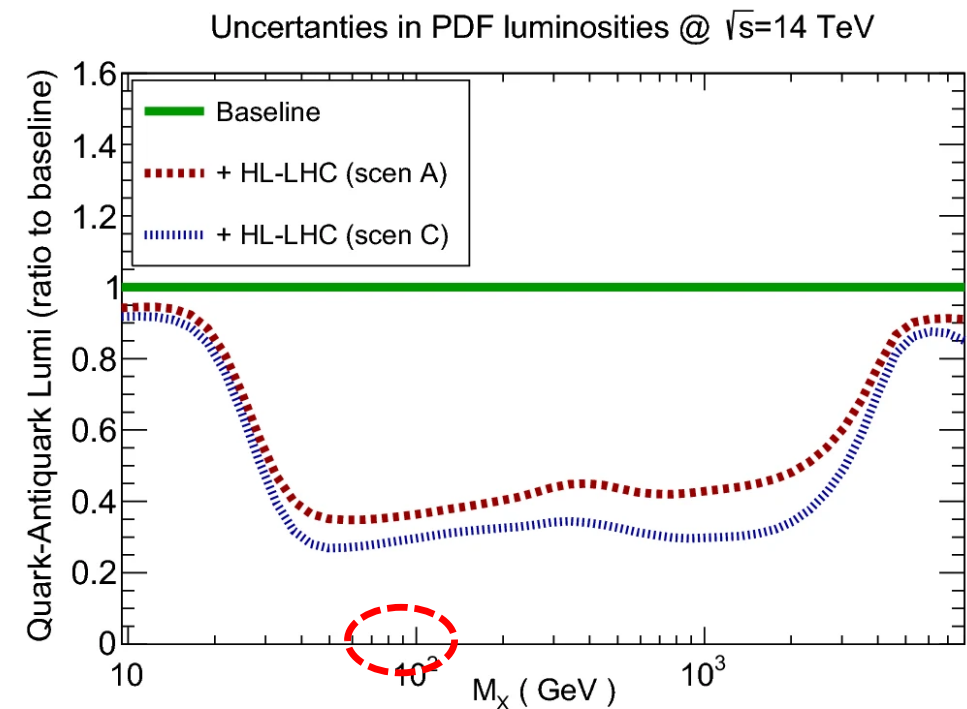
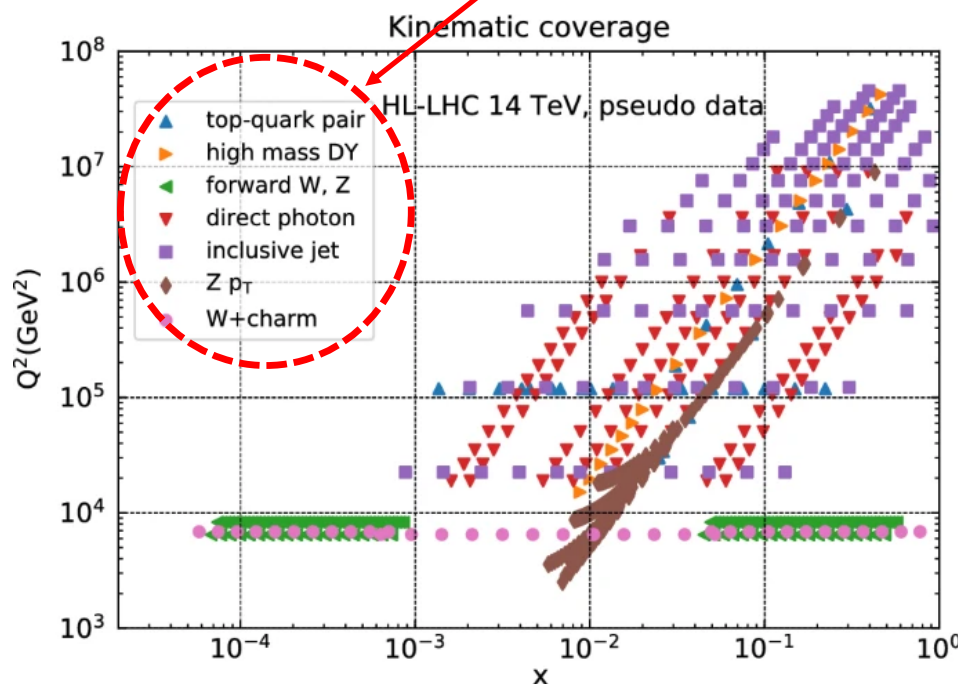
→ 30 ÷ 40%
reduction on δ_{PDF}

Doing *in situ*:

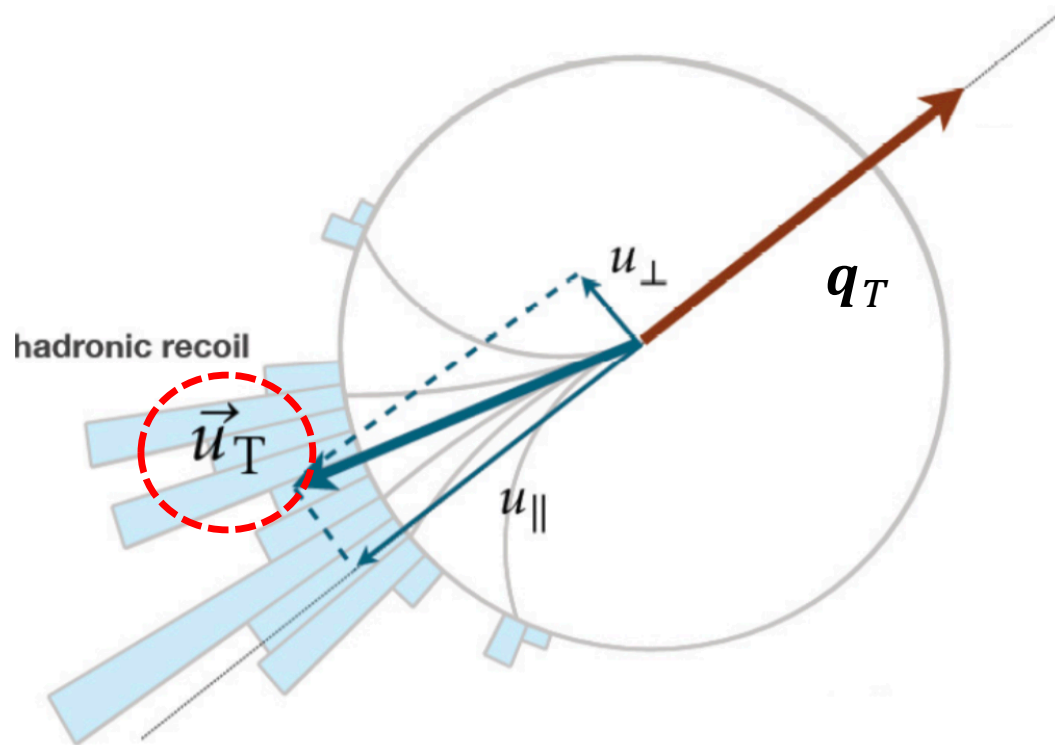


PDF mitigation: experimental perspective

- PDFs will also evolve with **luminosity**
 - Projecting **these** measurements at HL-LHC $\rightarrow \approx 50\%$ reduction on δ_{PDF}



Transverse motion



- Hadronic recoil \mathbf{u}_T is a proxy of \mathbf{q}_T
 $m_T \equiv m(\mathbf{p}_T^l, \mathbf{p}_T^v) = m(\mathbf{p}_T^l, \mathbf{u}_T - \mathbf{p}_T^l)$
- \mathbf{u}_T resolution degraded by high pile-up (and \sqrt{s}) @LHC:

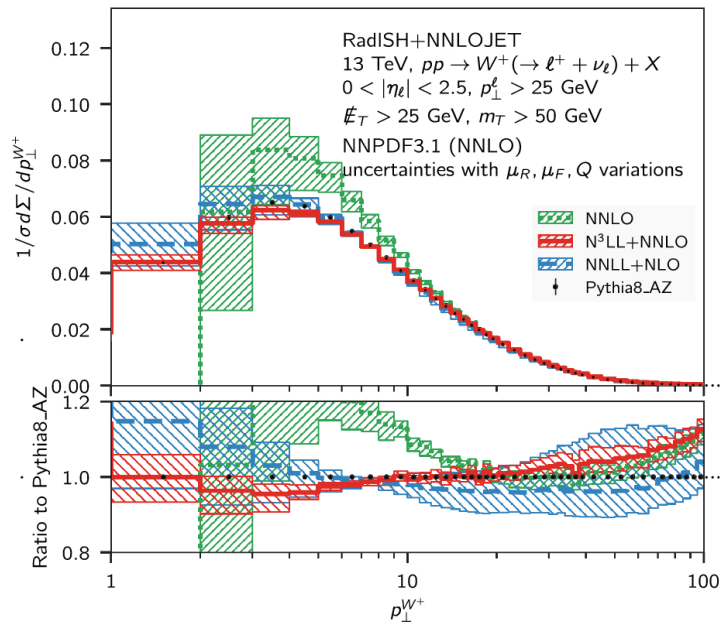
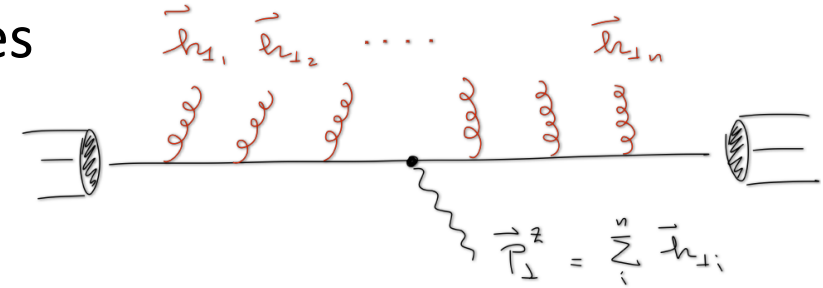
Optimal weights for combining $m_T: p_T^l$ fits at Tevatron and LHC:

$m_T: p_T^l$	
CDF @ Tevatron	ATLAS @ LHC7
0.53 : 0.47	0.14 : 0.86

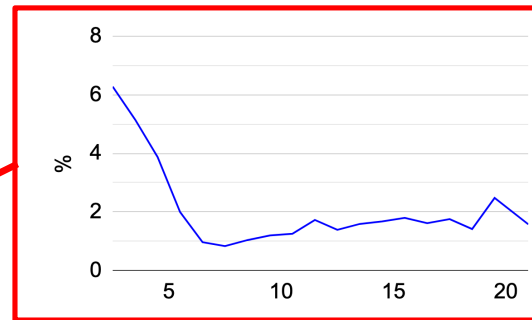
➔ Modeling of q_T is critical at LHC!

Theoretical prediction

- All-order resummation of log-divergent series
 - plus, matching to fixed-order at finite q_T
 - State-of-the-art: N³LL + NNLO



Best theoretical precision (%)



➔ Which precision needed for **10 MeV** on M_W ?

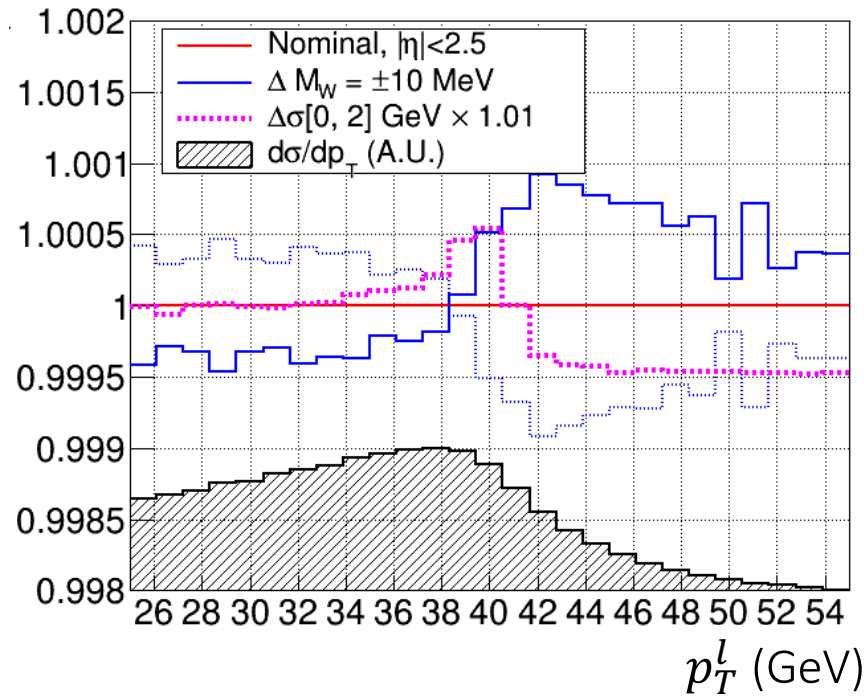
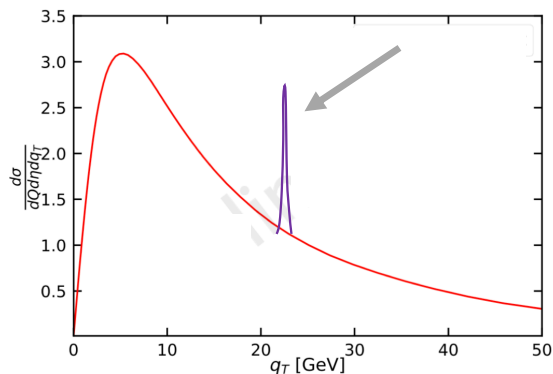
A qualitative assessment

BLUE

10 MeV shift of M_W

MAGENTA

1% perturbation of $\frac{\Delta\sigma}{\Delta q_T}$



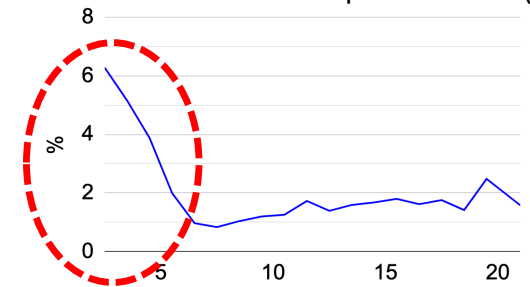
Rule-of-thumb:

2% for $q_T \lesssim 5$ GeV

→ $\mathcal{O}(10)$ MeV on M_W

REMINDER

Best theoretical precision (%)



Enhancements to q_T model: theory

- W^\pm/Z ratio

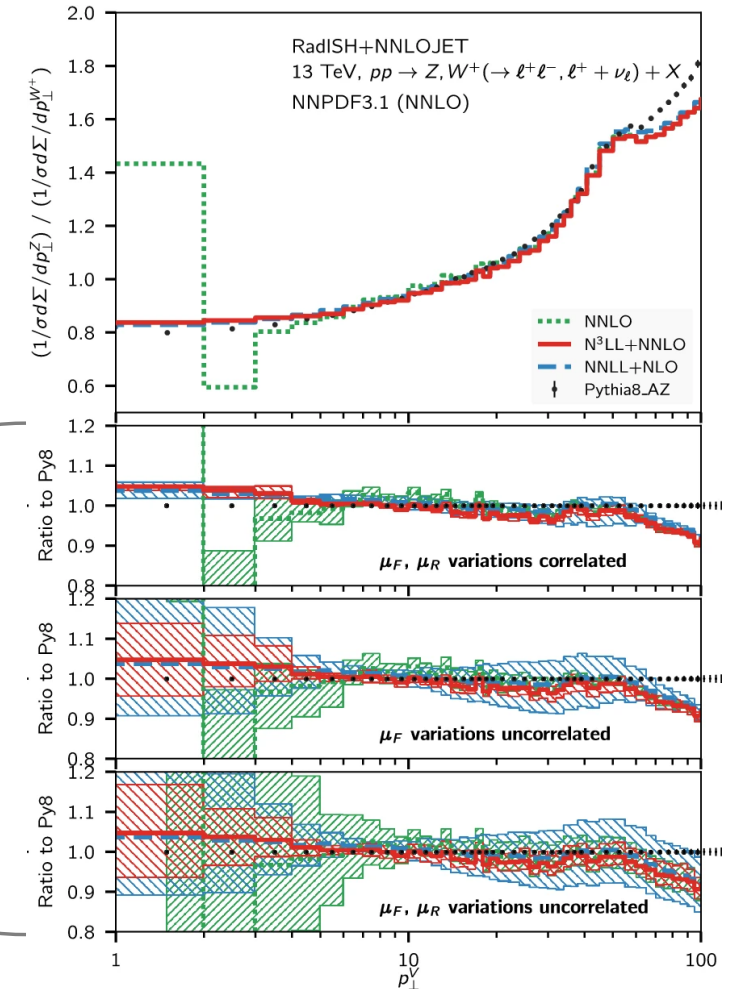
- Common uncertainties **cancel in the ratio**
- q_T of Z boson measured to < 1% precision

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- Correlation scheme matters

- One choice vs another \rightarrow \sim tens of MeV on M_W
- Reliable and agreed prescription still **missing**.

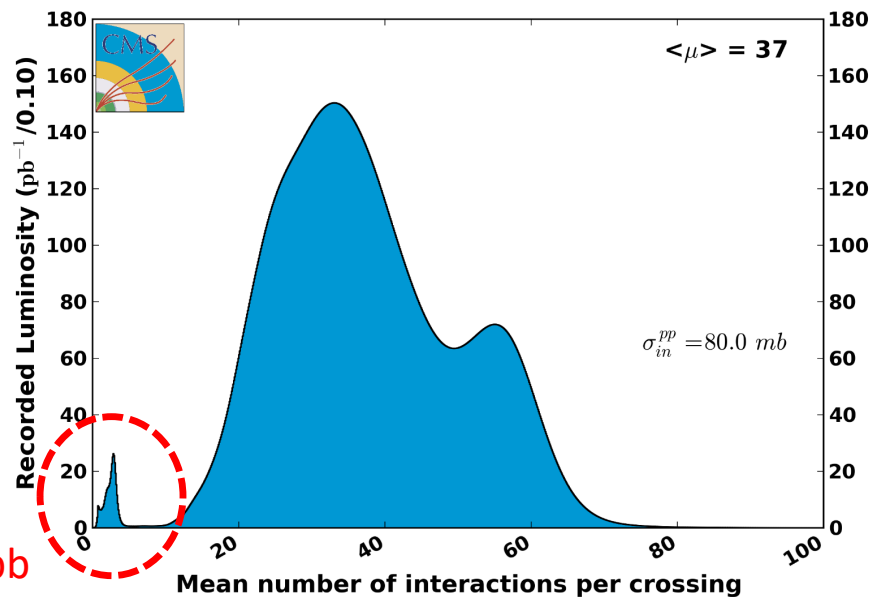
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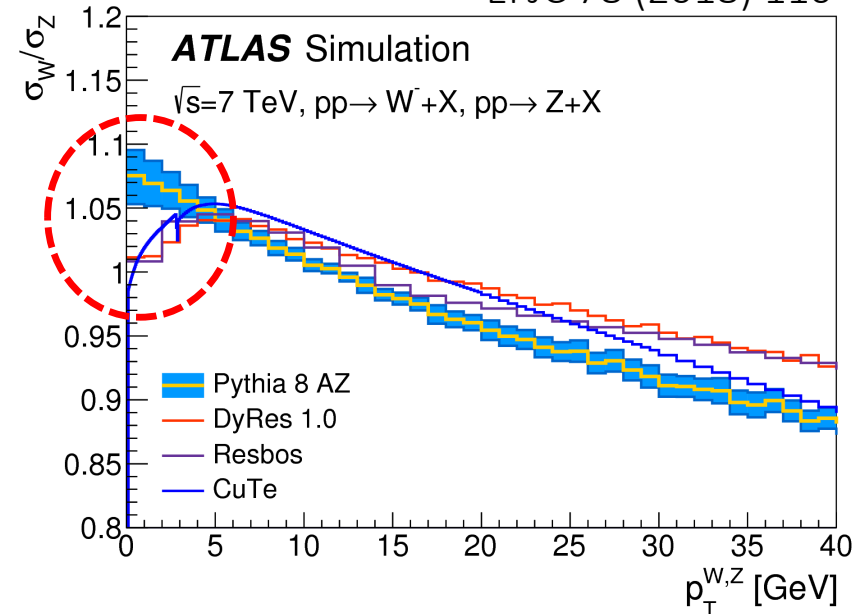
Enhancements to q_T model: experiment

- Precision measurement of $W q_T$
 - Special LHC runs can improve $\sigma(u_T)$
- Focus on critical region $q_T \lesssim 5$ GeV
 - **Challenging** detector resolution even at low PU

CMS Average Pileup, pp, 2017, $\sqrt{s} = 13$ TeV



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Wrapping up on model uncertainties

- ATLAS measurement @ 7 TeV → predominance of **model uncertainties**
 - PDF, QCD contribute by $\simeq 8, 9$ MeV.

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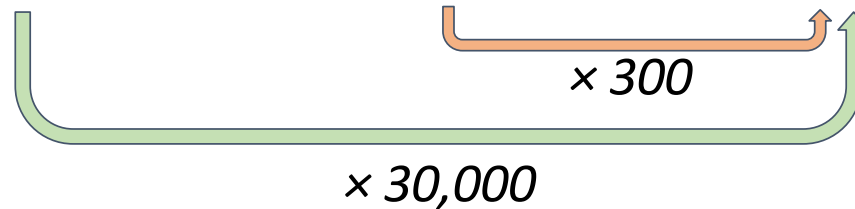
Combined categories	Value [MeV]	Stat. Unc.	Muon Unc.	Elec. Unc.	Recoil Unc.	Bckg. Unc.	QCD Unc.	EW Unc.	PDF Unc.	Total Unc.	χ^2/dof of Comb.
$m_T-p_T^\ell, W^\pm, e-\mu$	80369.5	6.8	6.6	6.4	2.9	4.5	8.3	5.5	9.2	18.5	29/27

→ Some **breakthrough** needed to reach the **10 MeV target!**

The breakthrough: data!

- LHC → high-luminosity discovery machine
 - unprecedented **statistical power** for producing EWK bosons

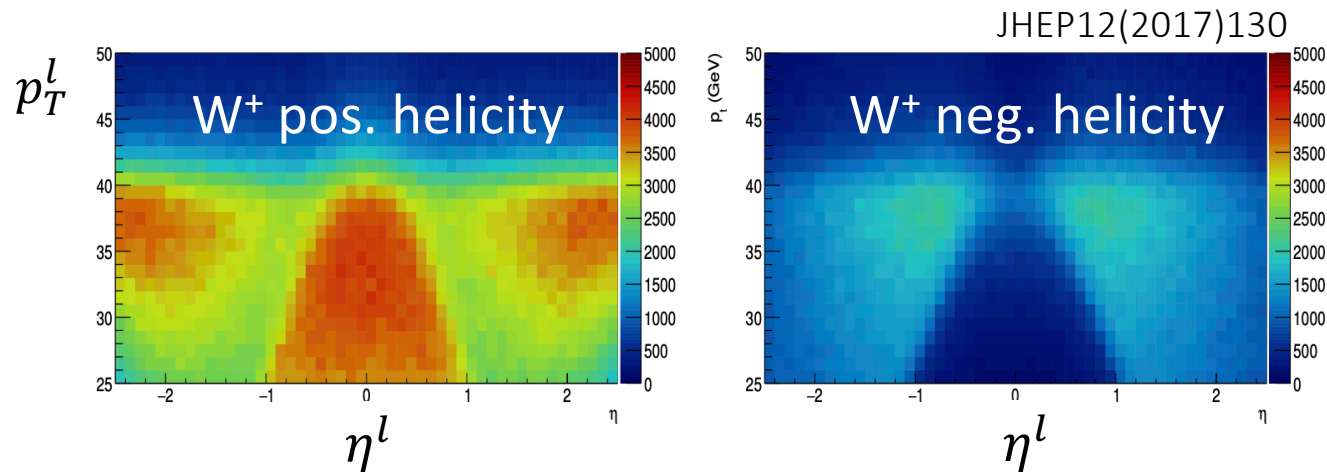
	LEP	Tevatron	LHC
# of W bosons	80,000	3,000,000	1,000,000,000



How to make the best of it?

Ideas for an ancillary measurement

- Measurement of the **rapidity spectrum** for the two **helicity states** of W^\pm



See also:
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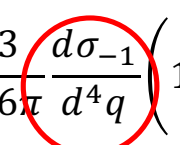
- Observation: 2D spectrum (p_T^l, η^l) provides simultaneous information on rapidity and helicity of the W^\pm .
- y e h depend on quark flavor and momentum \rightarrow constraint *in situ* of PDFs
 - Corroborated by recent CMS work [PRD 102 (2020) 092012]

Towards an agnostic model

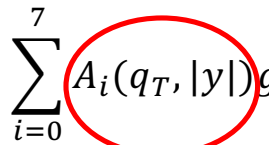
- Precision measurement of differential spectrum → model constraint
 - Can one push this **paradigm** to its limit?
- First principles: generic cross-section for $pp \rightarrow V \rightarrow l\nu$:

$$[1] \quad \frac{d\sigma}{d^4q d\cos\vartheta d\varphi} = \frac{3}{16\pi} \sum_{i=-1}^7 \frac{d\sigma_i}{d^4q} g_i(\theta, \varphi)$$

$$= \frac{3}{16\pi} \frac{d\sigma_{-1}}{d^4q} \left(1 + \cos^2\theta + \sum_{i=0}^7 A_i(q_T, |y|) g_i(\theta, \varphi) \right)$$

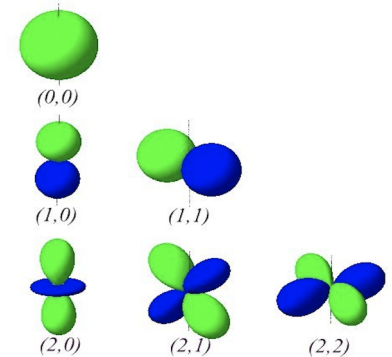


Unpolarized
cross-section



Angular
coefficients

$$\begin{aligned}
 g_{-1}(\theta, \varphi) &= 1 + \cos^2\theta, \\
 g_0(\theta, \varphi) &= 1 - \cos^2\theta, \\
 g_1(\theta, \varphi) &= \sin(2\theta) \cos\varphi, \\
 g_2(\theta, \varphi) &= \frac{1}{2} \sin^2\theta \cos(2\varphi), \\
 g_3(\theta, \varphi) &= \sin\theta \cos\varphi, \\
 g_4(\theta, \varphi) &= \cos\theta, \\
 g_5(\theta, \varphi) &= \sin^2\theta \sin(2\varphi), \\
 g_6(\theta, \varphi) &= \sin(2\theta) \sin\varphi, \\
 g_7(\theta, \varphi) &= \sin\theta \sin\varphi.
 \end{aligned}$$



— Towards an agnostic model

- Eq. [1] \rightarrow joint p.d.f. (p_T^l, η^l) as a linear combination of a **finite** and **complete** set of templates:

$$\frac{\Delta^2 \sigma}{\Delta p_T^l \Delta \eta^l} = \sum_{\Delta q_T, \Delta |y|} \frac{\Delta^2 \sigma_{-1}}{\Delta q_T \Delta |y|} \left(T_{-1}(p_T, \eta | \mathbf{M}_W) + \sum_{i=0 \dots 4} A_{i, \Delta q_T, \Delta |y|} \times T_i(p_T, \eta | \mathbf{M}_W) \right)$$

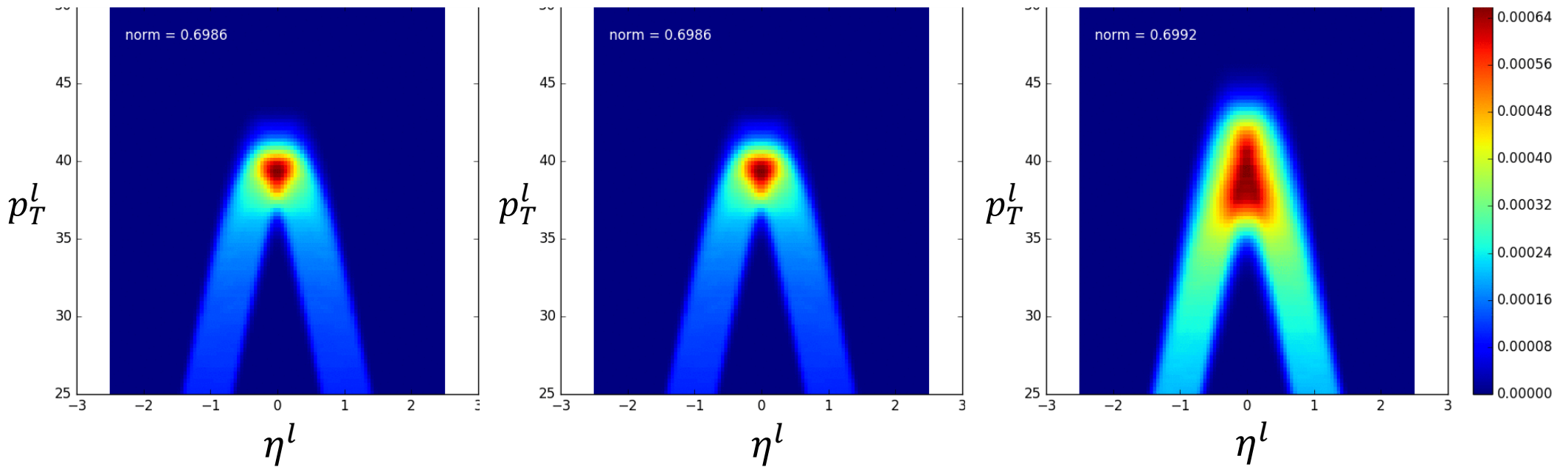
- **Normalizations** \rightarrow W production & decay dynamics
- **Template shape** $\rightarrow \mathbf{M}_W$

Towards an agnostic model

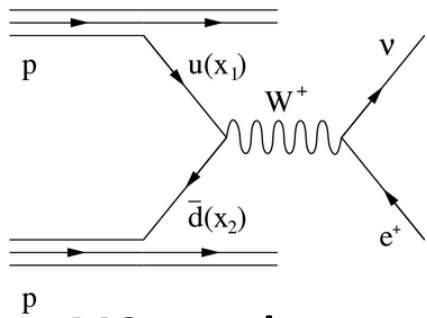
$\uparrow q_T$

$\longleftrightarrow y$

$\uparrow \varphi^*$
 $\rightarrow \cos \theta^*$

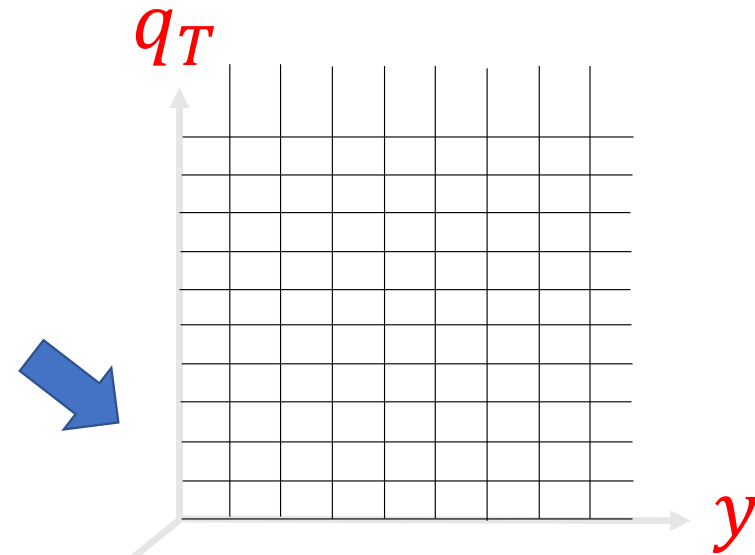


In practice



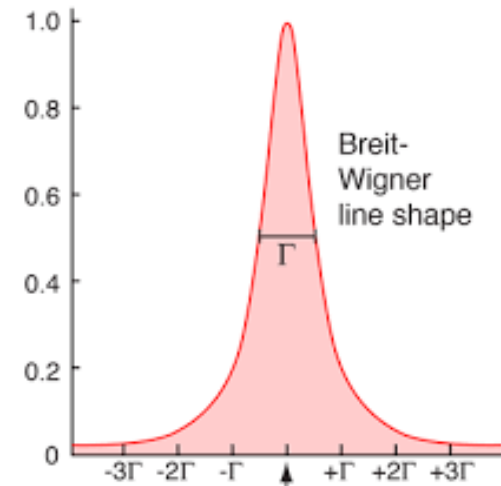
MC sample

- $pp \rightarrow W^\pm \rightarrow l^\pm \nu$
- QED
- Detector simulation



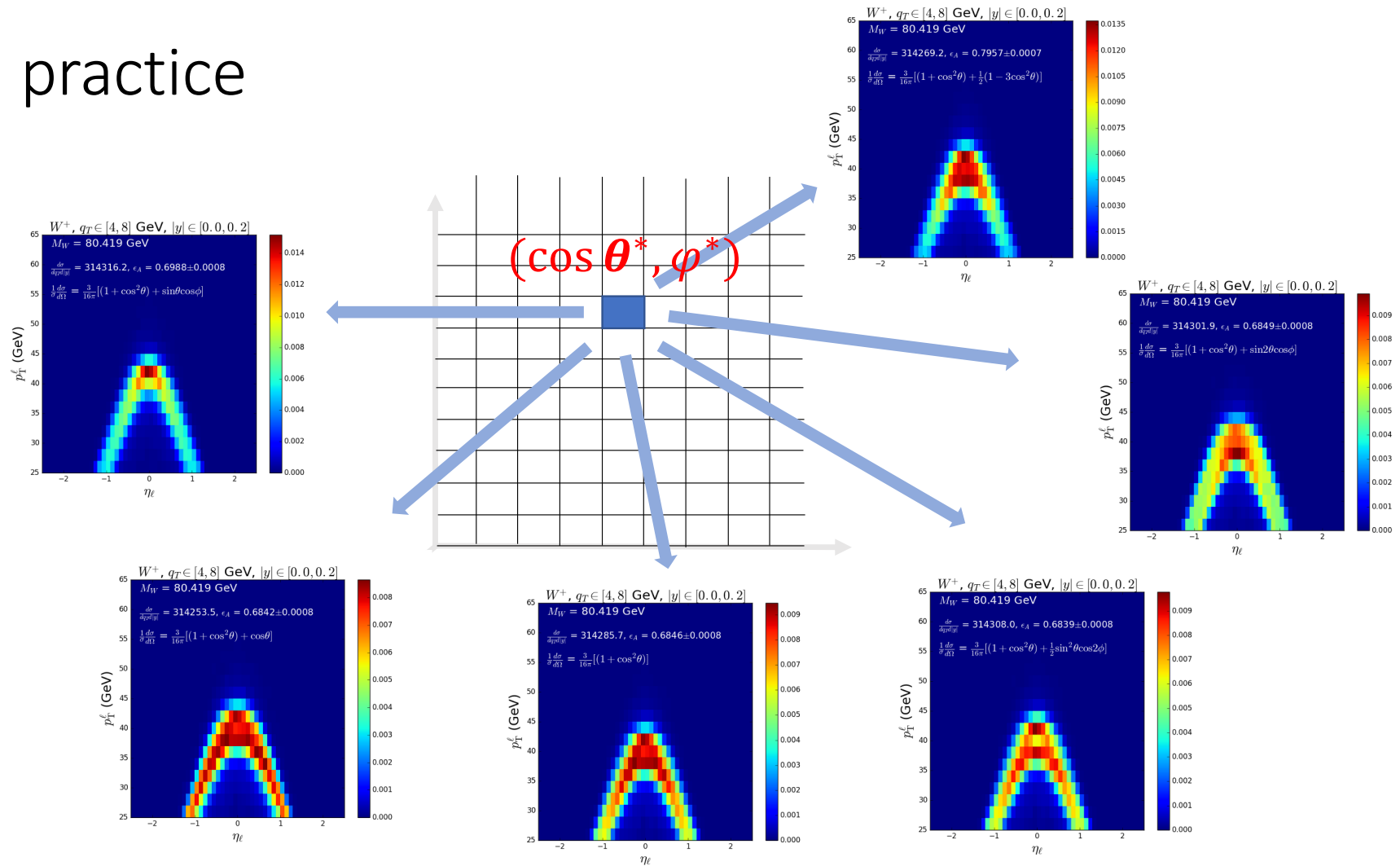
Q

Use MC to populate this dimension



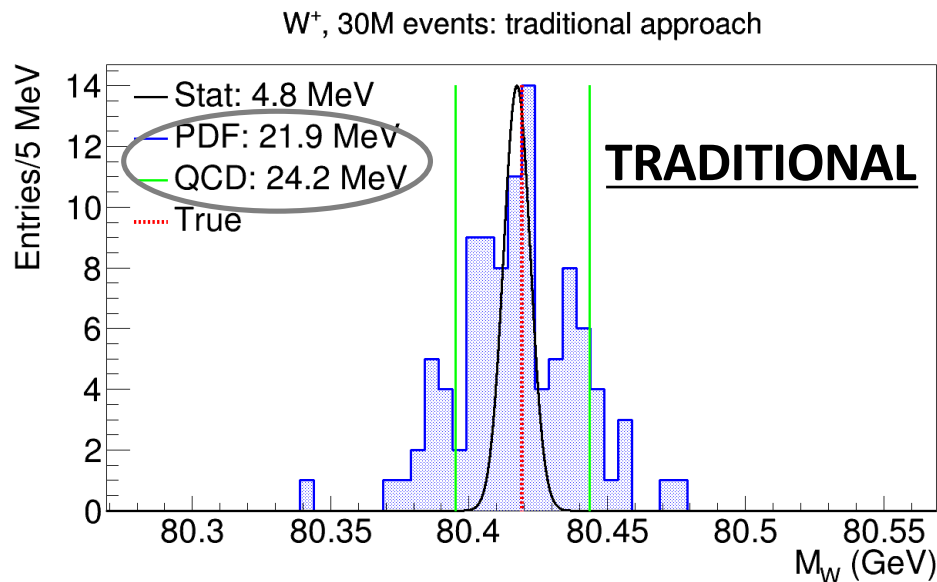
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In practice

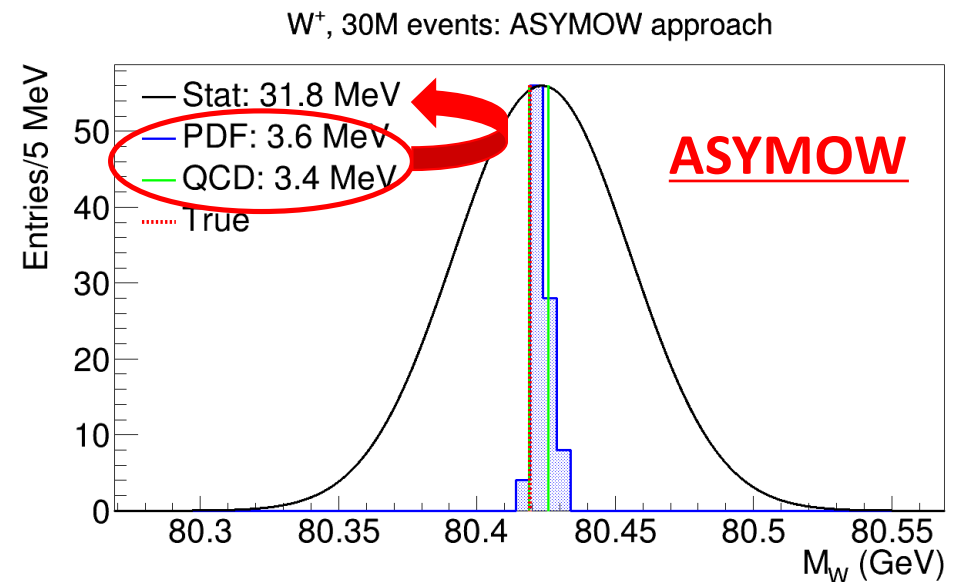


Proof of concept

- Which **statistical precision** on M_W ?
 - Studied on MC simulation (~ 1300 templates, ~ 300 free parameters)



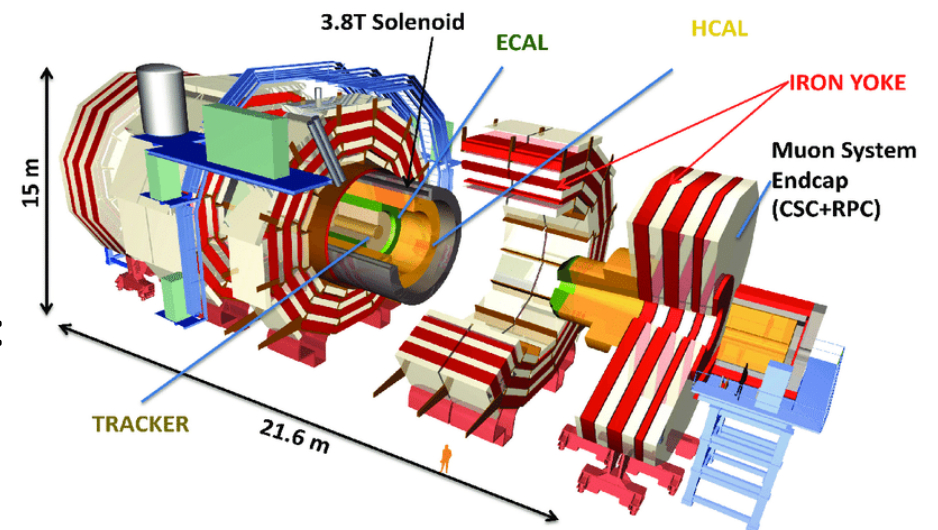
Model uncertainties →
systematic bias > **10 MeV**



Model uncertainties →
statistical uncertainty

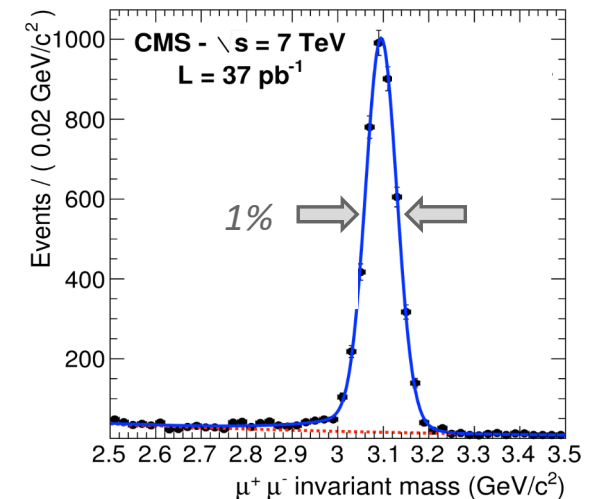
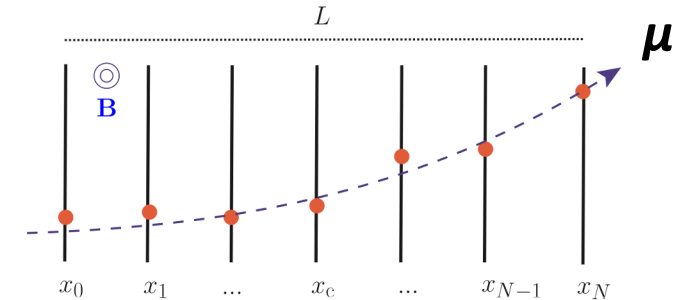
Experimental challenges

- $\Delta M_W(\text{stat.}) = 4 \text{ MeV} \rightarrow L_{\text{int}} \simeq 300/\text{fb.}$
 - Possible with Run2 + Run3
 - $\mathcal{O}(100)$ more events than any M_W measurement to date
- Challenge: keep **experimental systematics** under control
 - Background subtraction
 - Detector calibration
 - Residual theory uncertainty
- Experiment \rightarrow **CMS**
 - Superior performance Tracker + μ -system:
 \rightarrow Priority to **measurement with muons**



Example: momentum scale calibration

- **Target:** $\Delta p/p < 10^{-4}$
 - ~ 300 nm biases on mean track curvature;
 - unique use-case for such a precision!
- Intense B-field + silicon tracker
 - J/Ψ as standard candle \rightarrow closure on the Z
 - $\sim 2 \times 10^{-4}$ achieved at 7 TeV [CMS-PAS-SMP-14-007]
- An historical & ongoing effort driven by CMS group @ SNS



Expected performances

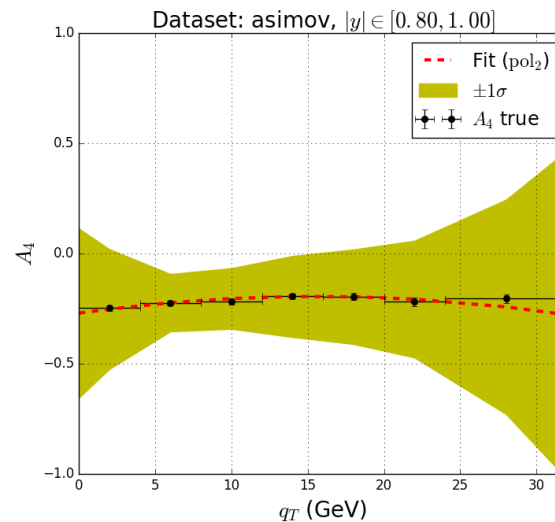
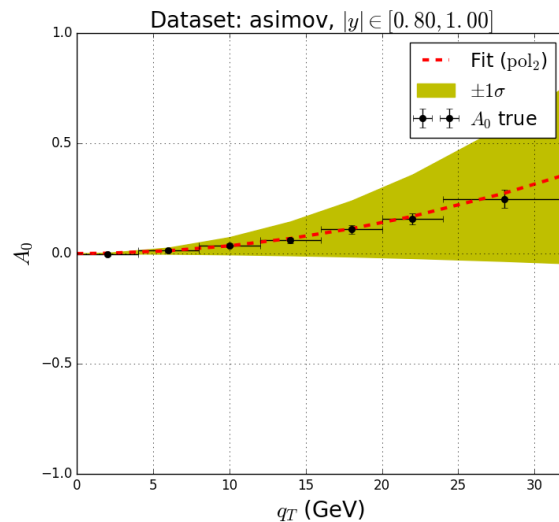
- Three scenarios of increasing experimental success:
 - $\Delta M_W \simeq 10 \text{ MeV}$ appears to be a robust deliverable
 - ➔ Possible by overcoming the model-dependence bottleneck

		Stat.	Exp.	Bkg.	QCD	EW	PDF	Tot.
Reference (ATLAS @7 TeV)		7	6	5	8	6	9	19
ASYMOW	Conservative	4	8	5	3	3	3	11
	Intermediate	4	4 / 8	5 / 3	3	3	3	9 / 10
	Aggressive	4	4	3	3	3	3	8

~2 better than
**single-
experiment to
date**

Not just M_W

- Multi-differential model is a “nuisance”... but also a by-product
 - Tighter **PDF** constraints
 - Benchmark for **precision calculations**
 - e.g., a recoil-free measurement of q_T



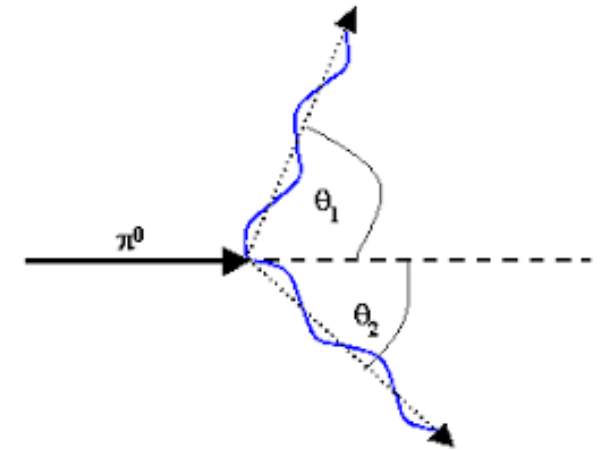
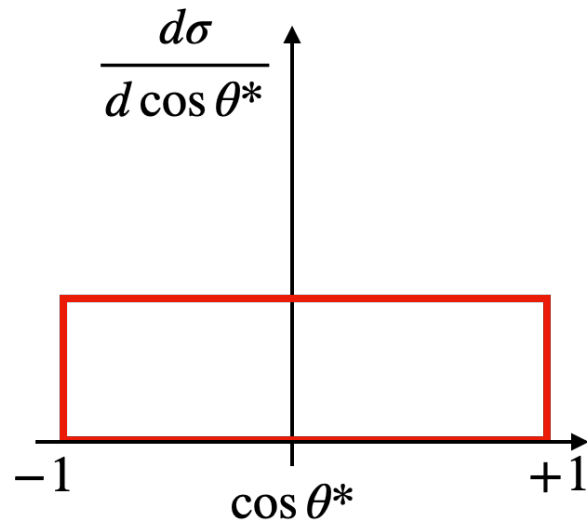
Examples of **angular coefficients** extracted together with M_W

— One last idea...

- ASYMOW → profile the full underlying production mechanism while measuring M_W
 - Can it be done with **fewer nuisances**?

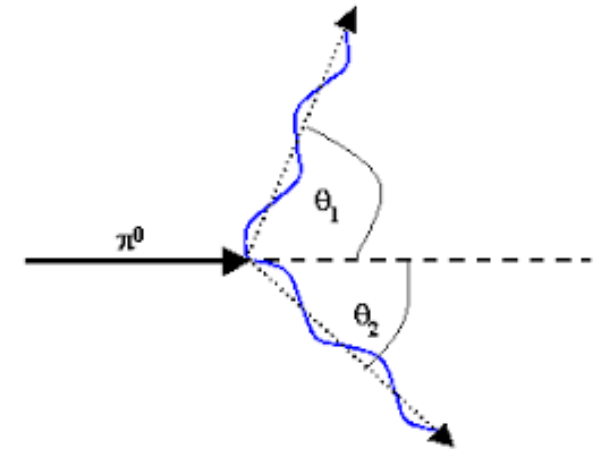
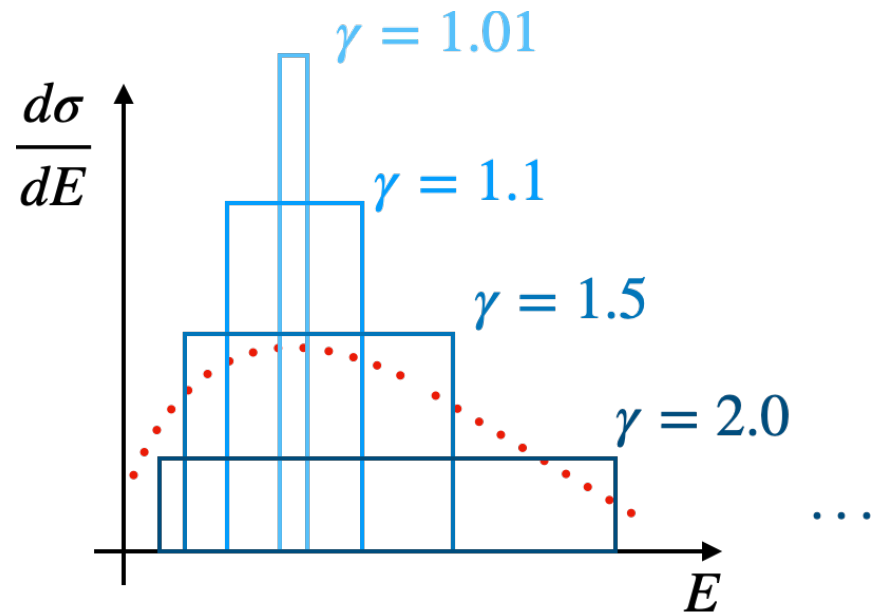
A subtle point of invariance

- Remember the famous π^0 decay...



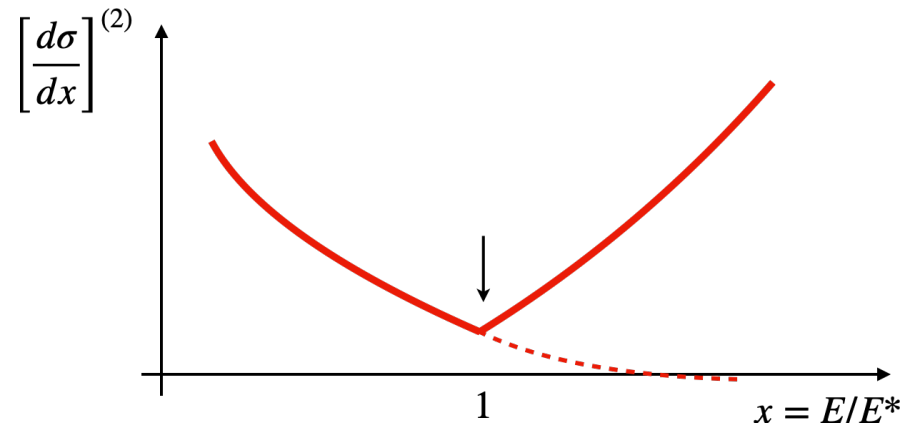
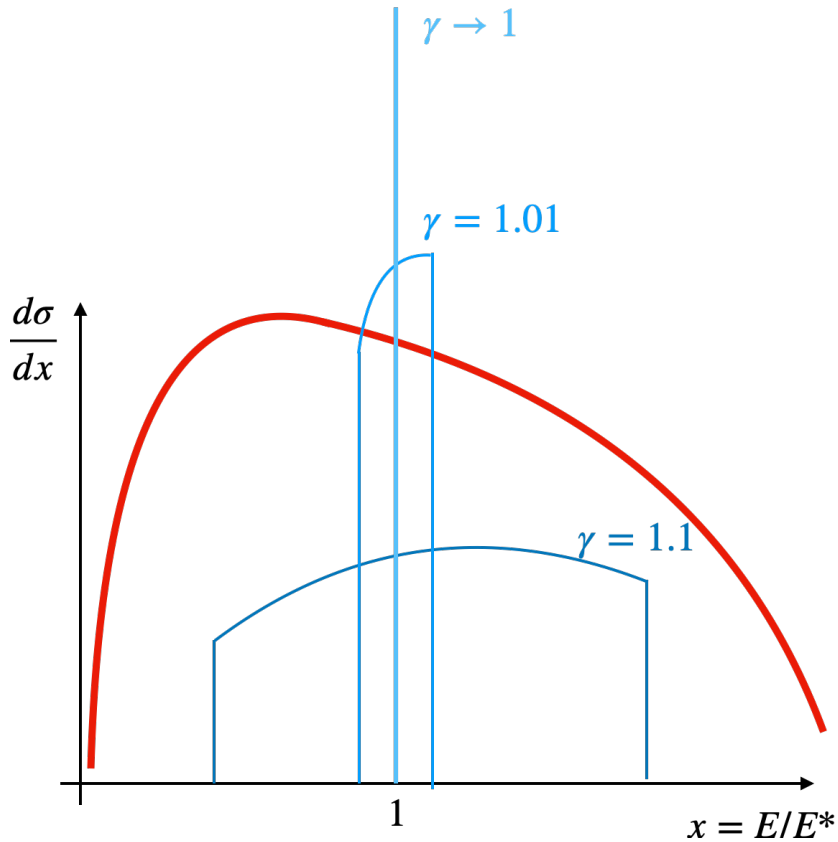
A subtle point of invariance

- Remember the famous π^0 decay...



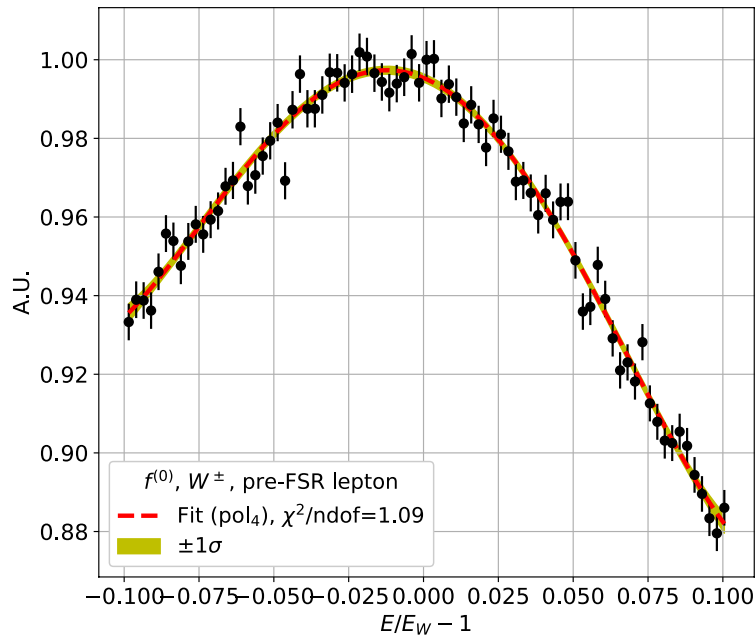
A subtle point of invariance

- What about a spin-1 particle?
 - “footprint” (kink) in the second-order derivative $f^{(2)}$ of the **energy spectrum**
 - ➔ Lorentz-invariant estimator of M_W

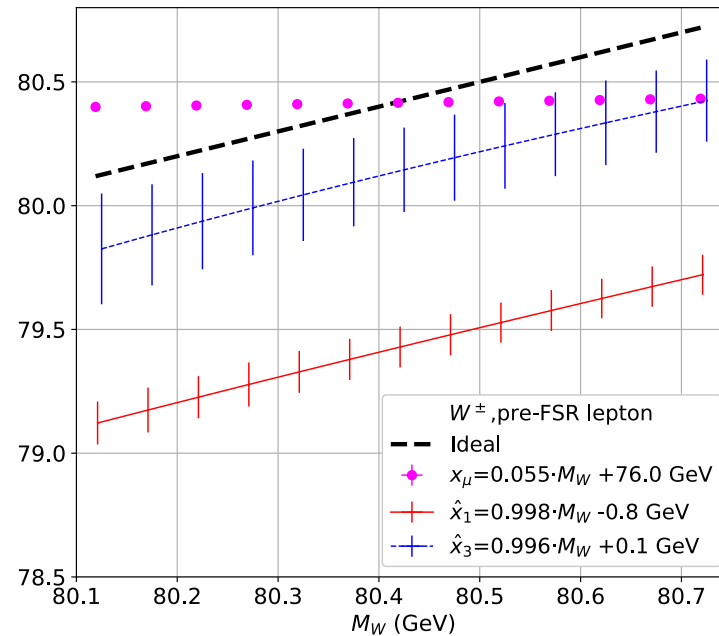


An energy-only M_W measurement?

E -spectrum fit to 4th order polynomial.
Take root of 3rd-order derivative (\hat{x}_3)



MC calibration needed to account for finite-width and QED radiation



← **~15 MeV (stat.)**
feasible with full LHC data.

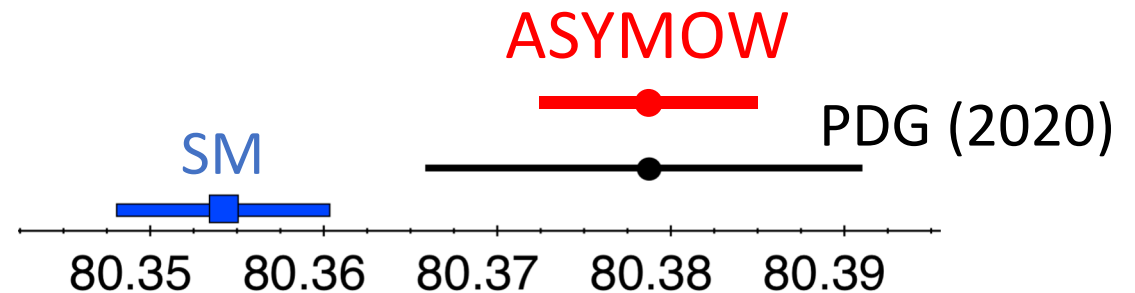
Further studies needed to assess feasibility.

Conclusions

- Three ways towards a precision measurement:

- enhancements to theory models,
- enhancements to the experiments,
- a *Third Way*:
 - Novel
 - Challenging
 - Competitive

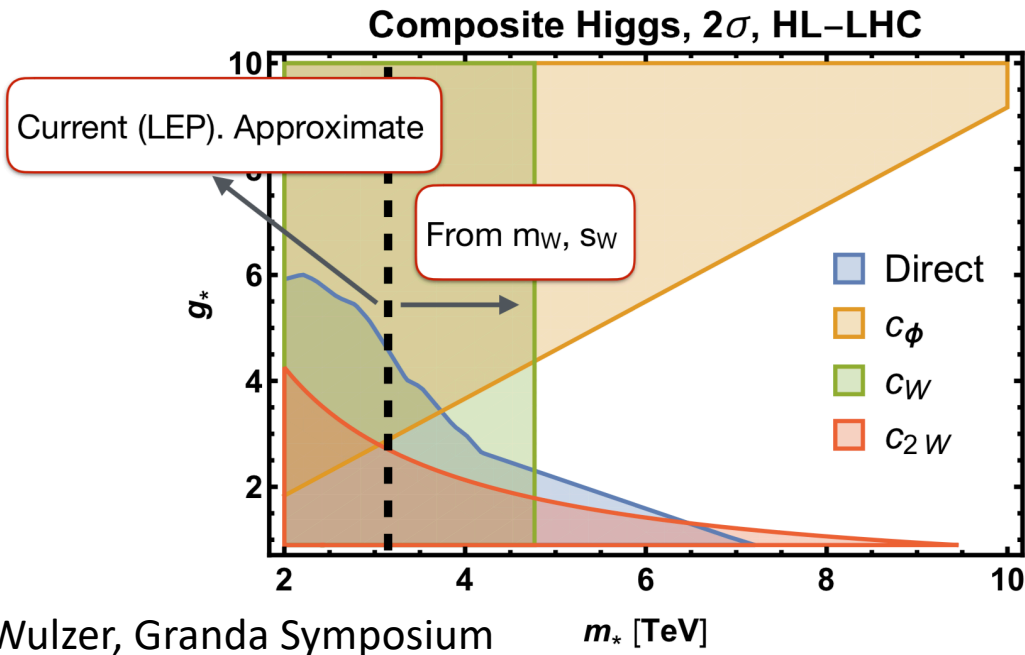
A project for the next years!



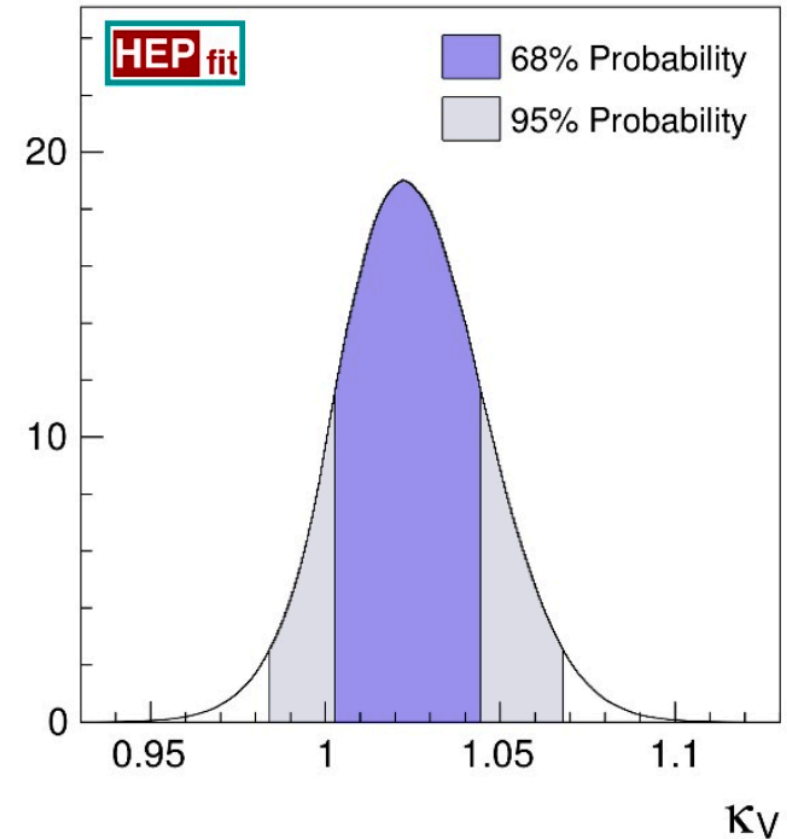
Grazie per la vostra attenzione!

Cosa possiamo imparare da mW

- Interpretazione in modelli specifici
 - complementarità con misure dirette (e.g. sez. d'urto dell'Higgs)



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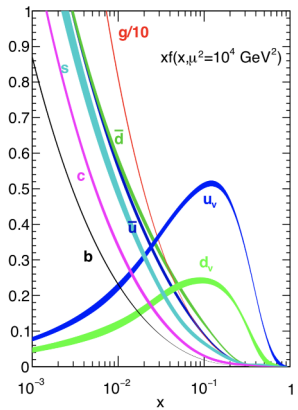


—● Predizione teorica

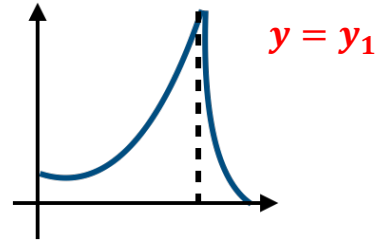
$$80.3535 \pm 0.0027_{m_t} \pm 0.0030_{\delta_{\text{theo}} m_t} \\ \pm 0.0026_{M_Z} \pm 0.0026_{\alpha_S} \\ \pm 0.0024_{\Delta\alpha_{\text{had}}} \pm 0.0001_{M_H} \pm 0.0040_{\delta_{\text{theo}} M_W}$$

The PDF uncertainty

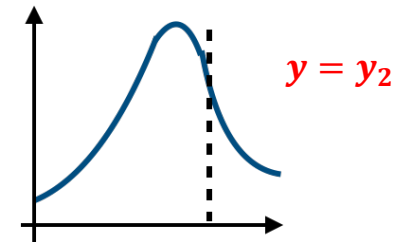
Marginalize over all y values and q (\bar{q}) pairs



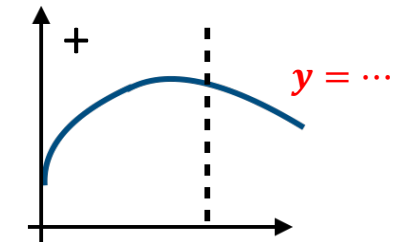
$$u\left(\frac{Q}{\sqrt{s}}e^{+y_1}\right)\bar{d}\left(\frac{Q}{\sqrt{s}}e^{-y_1}\right) \times$$



$$u\left(\frac{Q}{\sqrt{s}}e^{+y_2}\right)\bar{d}\left(\frac{Q}{\sqrt{s}}e^{-y_2}\right) \times$$

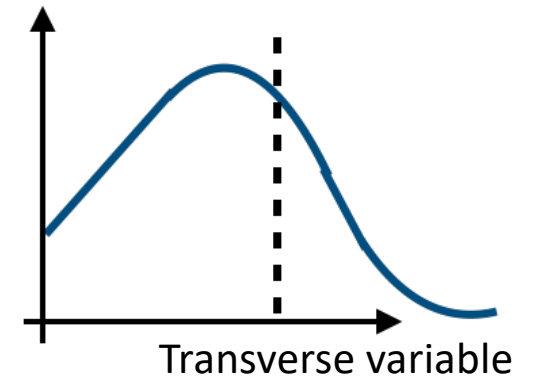


$$\bar{d}\left(\frac{Q}{\sqrt{s}}e^{+y_2}\right)u\left(\frac{Q}{\sqrt{s}}e^{-y_2}\right) \times$$



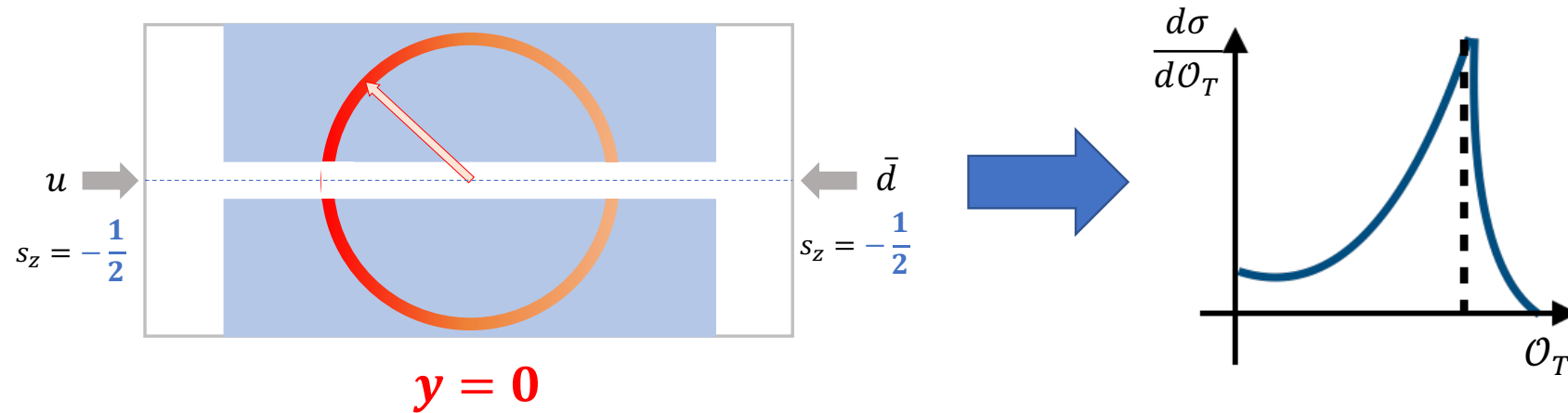
...

...



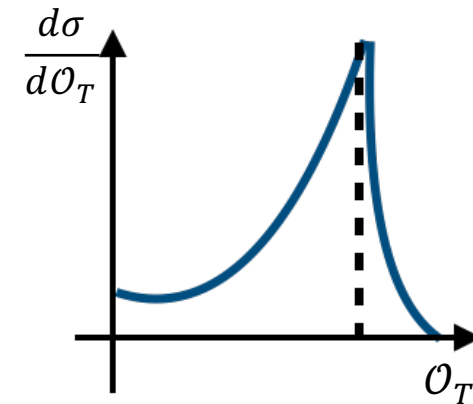
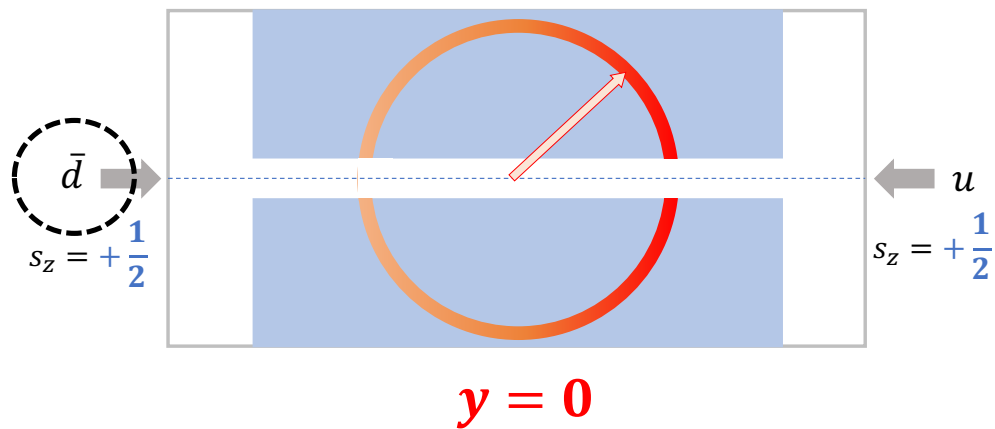
The PDF uncertainty

- Why then longitudinal dynamics also matters?
 - Mostly an **acceptance** artefact



The PDF uncertainty

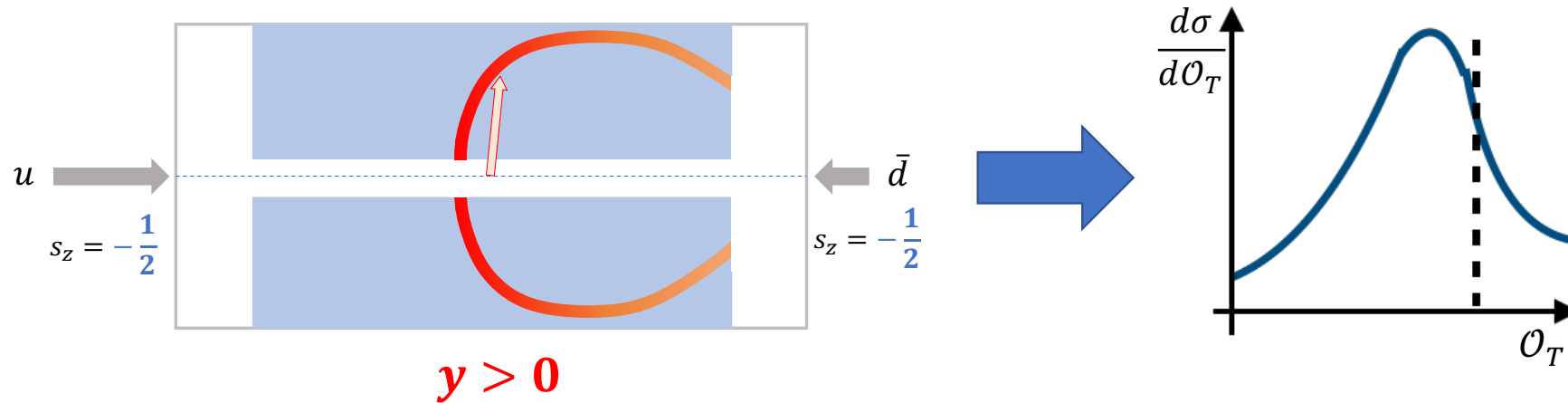
- Why then longitudinal dynamics also matters?
 - Mostly an **acceptance** artefact



→ Invariant under $q \leftrightarrow \bar{q}$

The PDF uncertainty

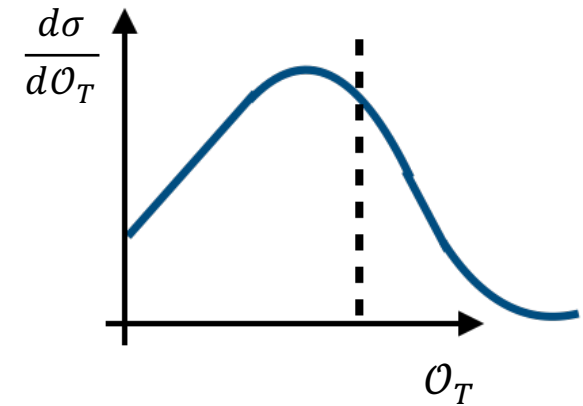
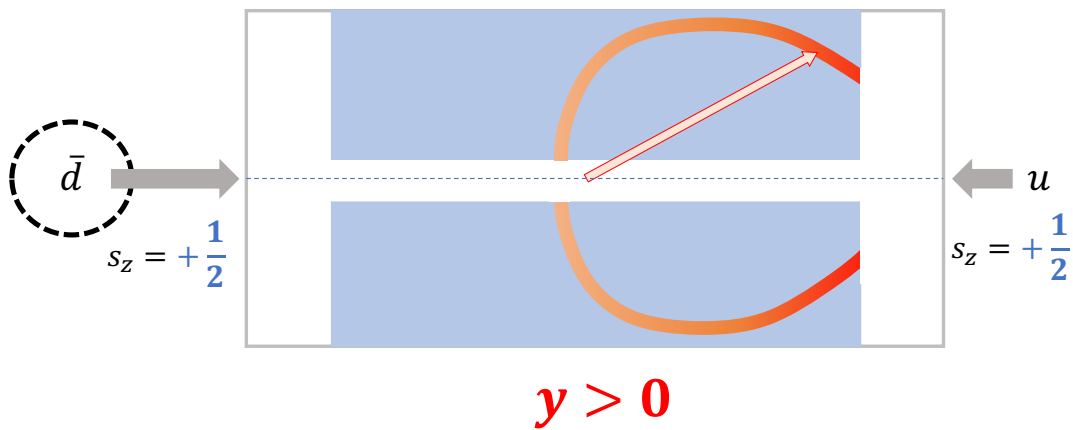
- Why then longitudinal dynamics also matters?
 - Mostly an **acceptance** artefact



→ Sculpted by the detector

The PDF uncertainty

- Why then longitudinal dynamics also matters?
 - Mostly an **acceptance** artefact

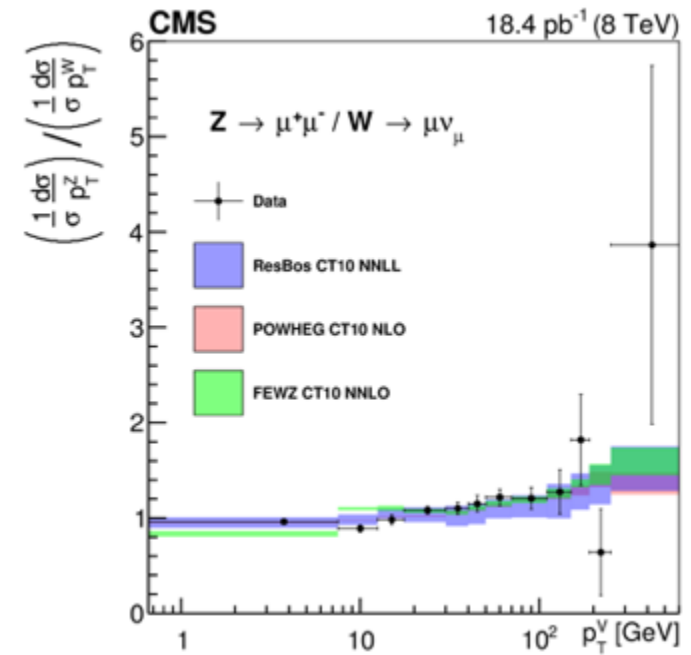
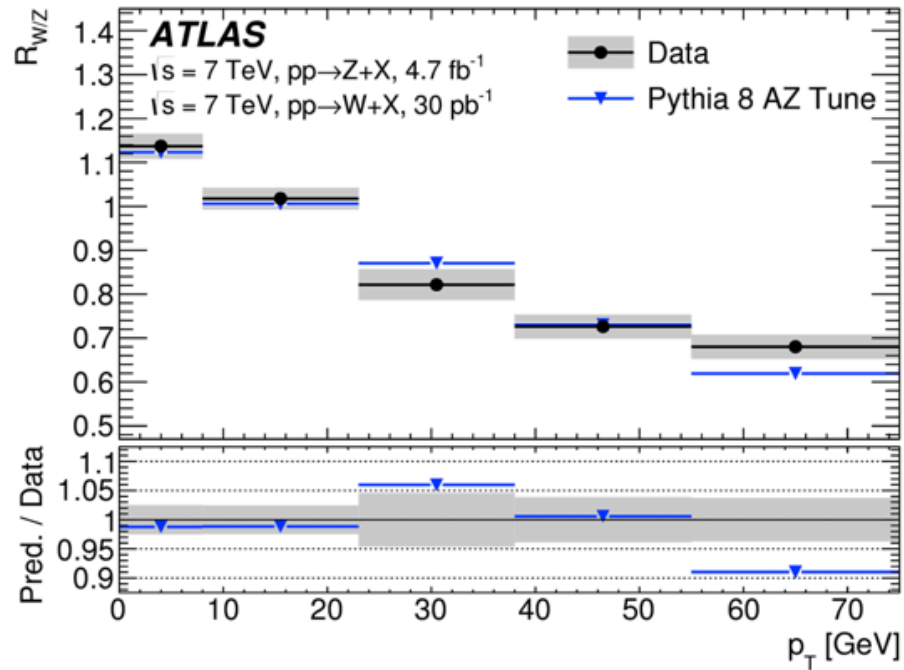


→ Different under $q \leftrightarrow \bar{q}$

Measurements of q_T spectra

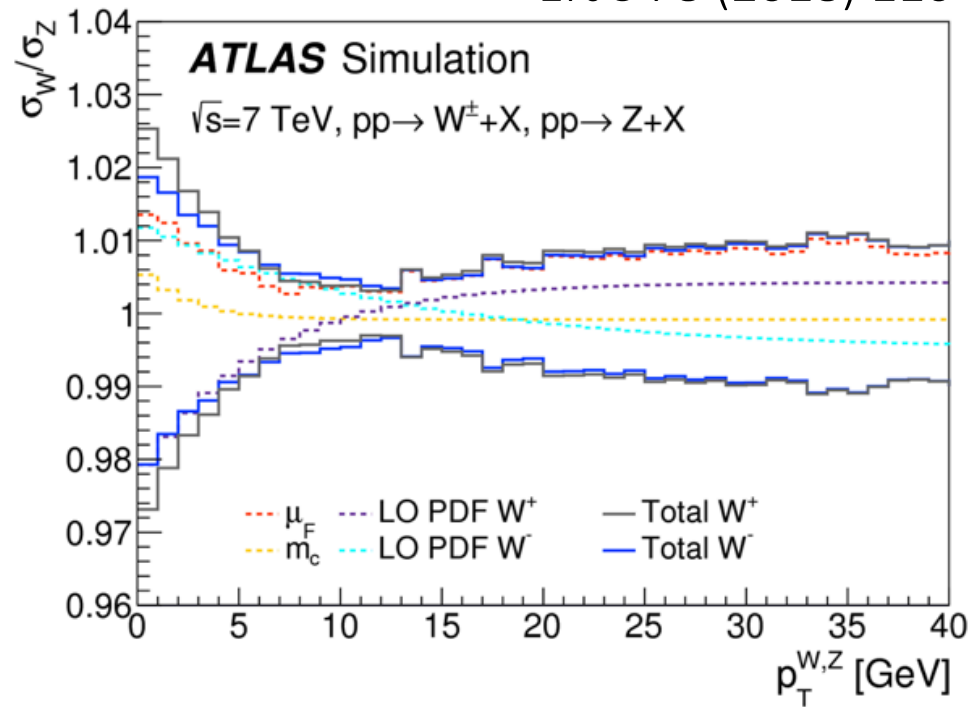
PRD 85 (2012) 012005

JHEP 02 (2017) 096

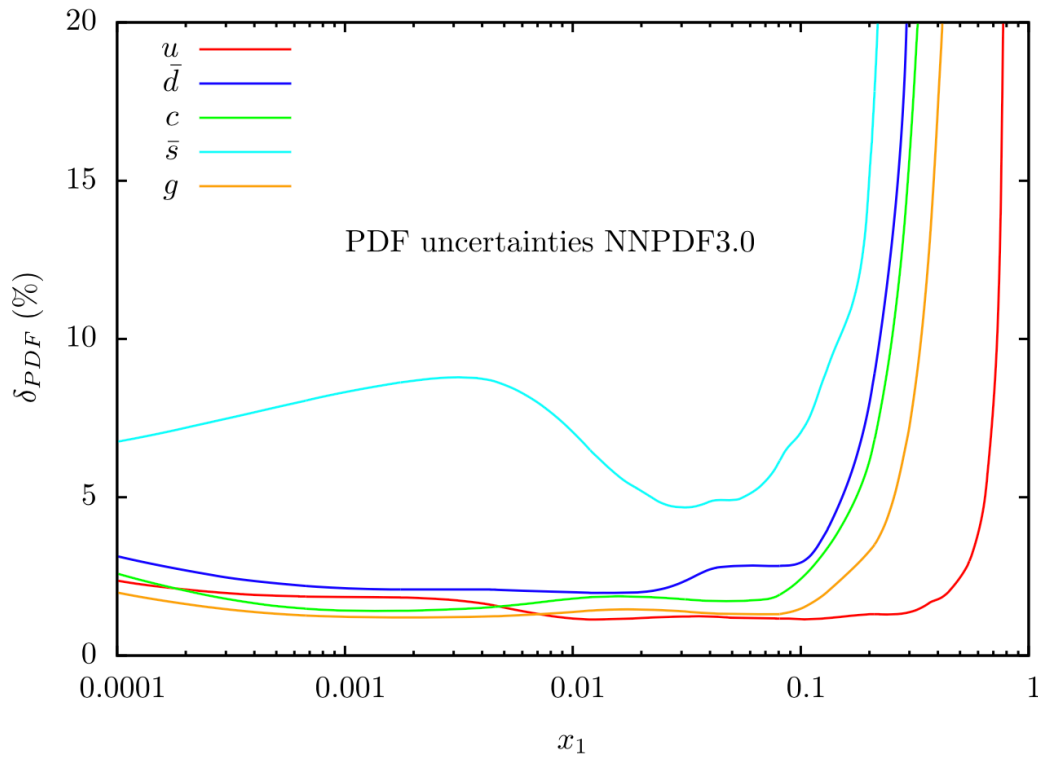


Measurements of q_T spectra

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PDF uncertainty vs acceptance



PRD 91 (2015) 113005

Normalized distributions			
Cut on p_{\perp}^W	Cut on $ \eta_l $	CT10	NNPDF3.0
Inclusive	$ \eta_l < 2.5$	$80.400 + 0.032 - 0.027$	80.398 ± 0.014
$p_{\perp}^W < 20$ GeV	$ \eta_l < 2.5$	$80.396 + 0.027 - 0.020$	80.394 ± 0.012
$p_{\perp}^W < 15$ GeV	$ \eta_l < 2.5$	$80.396 + 0.017 - 0.018$	80.395 ± 0.009
$p_{\perp}^W < 10$ GeV	$ \eta_l < 2.5$	$80.392 + 0.015 - 0.012$	80.394 ± 0.007
$p_{\perp}^W < 15$ GeV	$ \eta_l < 1.0$	$80.400 + 0.032 - 0.021$	80.406 ± 0.017
$p_{\perp}^W < 15$ GeV	$ \eta_l < 2.5$	$80.396 + 0.017 - 0.018$	80.395 ± 0.009
$p_{\perp}^W < 15$ GeV	$ \eta_l < 4.9$	$80.400 + 0.009 - 0.004$	80.401 ± 0.003
$p_{\perp}^W < 15$ GeV	$1.0 < \eta_l < 2.5$	$80.392 + 0.025 - 0.018$	80.388 ± 0.012

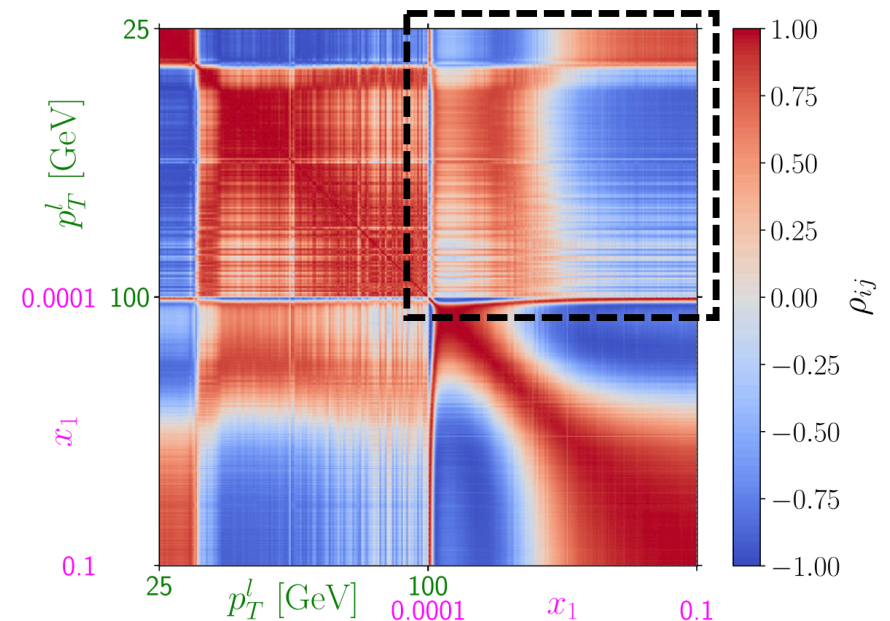
A new look at PDF uncertainty

- p_T^l **spectrum alone** enough to constrain all the PDF uncertainty at high lumi
 - Just use **correlation** pattern with p_T^l

$$\chi_{k,\min}^2 = \sum_{(r,s) \in \text{bins}} (\mathcal{T}_{0,k} - \mathcal{D}^{\text{exp}})_r (C^{-1})_{rs} (\mathcal{T}_{0,k} - \mathcal{D}^{\text{exp}})_s$$

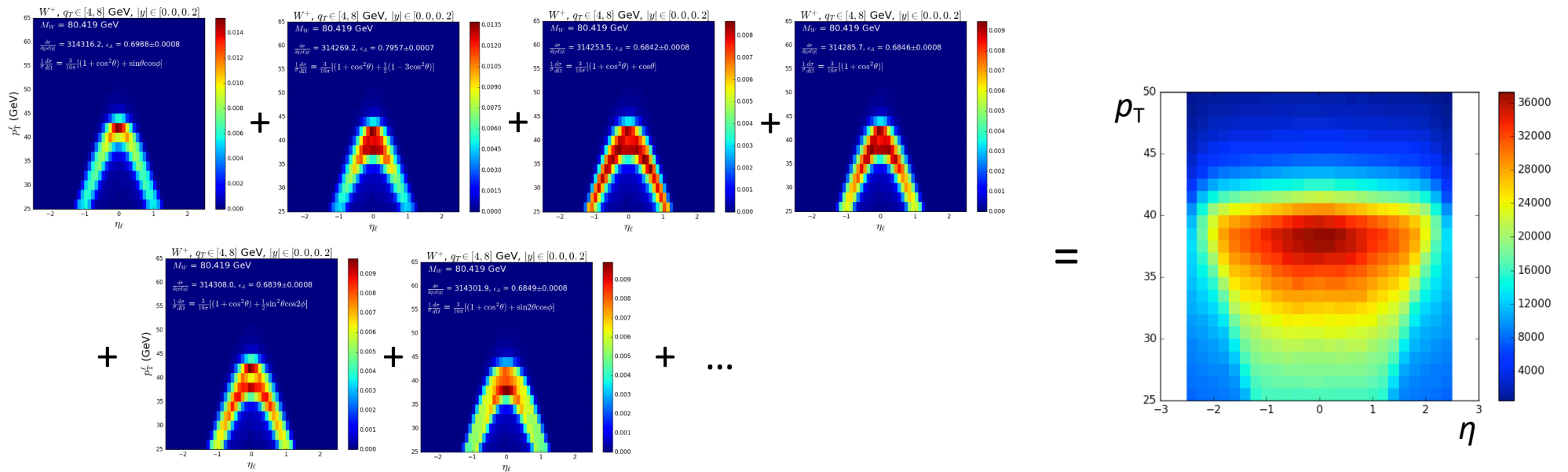
→ $\delta_{\text{PDF}} \rightarrow 1 \text{ MeV}$ with 300/fb!

- Looser constraints when detector included
 - Also handling of q_T not straightforward in this setup



Towards an agnostic model

- From Eq. [1]: joint p.d.f. (p_T, η) as a linear combination of a finite and complete set of templates:



Master formula for spin-1 energy spectrum

$$f(x) = \int \frac{dy}{y} h(y) \int_{\frac{1}{2}\left(\frac{x}{y} + \frac{y}{x}\right)}^{+\infty} d\gamma g(\gamma) \\ \times \frac{3}{8} \left[\frac{1 + \frac{1}{2}A_0(\gamma)}{(\gamma^2 - 1)^{\frac{1}{2}}} + \frac{A_4(\gamma)}{(\gamma^2 - 1)} \left(\frac{x}{y} - \gamma\right) + \frac{1 - \frac{3}{2}A_0(\gamma)}{(\gamma^2 - 1)^{\frac{3}{2}}} \left(\frac{x}{y} - \gamma\right)^2 \right]$$

$$f(1 + \epsilon) \approx A + B\epsilon + C|\epsilon| + D\epsilon^2 + E|\epsilon|\epsilon + F\epsilon^3 + G|\epsilon^3| + H\epsilon \ln |\epsilon| + \mathcal{O}(\epsilon^3)$$

Testing on MC

