Echoes in accelerators

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Echoes are a fascinating phenomenon:

• Acoustic echo, measure sound speed	[?, <1000 B.C.]
• Spin echo, Nuclear Magnetic Resonance	[Hahn, 1950]
• Echo in plasma	[Baker, Ahern, Wong, 1968]
• Echo in accelerators	[Stupakov, 1992]

Particle motion in an accelerator has 4 dimensions:

- Transverse:
  - horizontal = x
  - vertical = y
- Longitudinal: z
- Spin: S

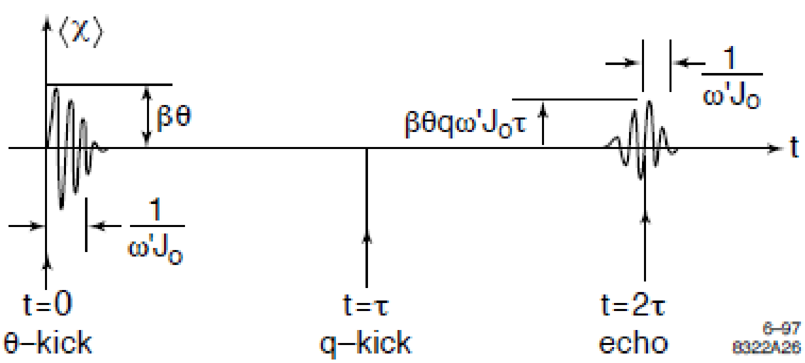
There is an echo effect in each of these 4 dimensions, each with its own special characteristics.
=> Echoes are everywhere in an accelerator!

A rich subject in accelerator physics and technology, and is rapidly evolving.

1. Introduction

Consider a thought experiment in a proton storage ring:

- A beam is stored for a long time in a steady state. Its centroid signal reads $<x>=0$.
- The beam is kicked at $t = 0$ by a dipole kicker. The beam's centroid executes a free betatron oscillation.
- Due to a spread in the betatron frequencies, the centroid signal decoheres in a short time. Decoherence time ~ (frequency spread)$^{-1}$ ~ 1ms.
- Long after the kick, say 1 s later, the centroid signal is dead zero. At this point, $t = \tau$, we give the beam a second, quadrupole kick. Such a kick does not affect the beam centroid, and thus $<x>$ stays zero.
- Now we wait exactly 1 s after the quadrupole kick. The beam centroid will see a sudden and pronounced blip. The blip lasts ~1 ms, and quiet afterwards.



This sudden blip is an echo. It occurs at exactly $t = 2\tau$. It is a result of the correlation and interplay between the two kicks and a long memory of the intricate beam dynamics in the phase space.

Above is transverse echo. There is also longitudinal echo: an rf phase shift and an rf amplitude jump play the roles of the dipole and quadrupole kicks, respectively (bunched beam).

In addition to the transverse and longitudinal orbital motions, there is also a spin echo.

Recently, echo has also been suggested as a clever way to generate microbunching in beam distribution for generating X-ray FELs.

[Stupakov 2008]

All echo effects involve intricate microstructures in phase space. They come about because memory in phase space lasts forever. The long memory is due to Liouville theorem, i.e. you cannot destroy phase space!

2. Transverse decoherence ($0 < t < \tau$)

Consider the setup for transverse echo mentioned earlier.

First concentrate the decoherence period $(0 < t < \tau)$ after the dipole kick but before the quadrupole kick.

- Transform phase space $(x, p) \rightarrow (\phi, J)$
- Initial distribution in (x,p) is centered Gaussian $\psi_0(J)=exp[-J/J_0]$
- dipole kick at $t = 0$ => displaced Gaussian
- after dipole kick, distribution filaments because $\omega_\beta(J)= \omega_0 + \omega' J$. RMS spread = $\omega' J_0$.
- Now calculate centroid signal $<x>(t)$. It decoheres from $t = 0$ to $t = \tau$,

$$\langle x\rangle(t) \approx \frac{\beta\theta}{1+\Theta^2}\sin(\omega_0 t + 2\tan^{-1}\Theta)$$

$$\Theta \equiv \omega' J_0 t$$

$\omega_0 t$ is fast time variable, Θ is slow time variable.

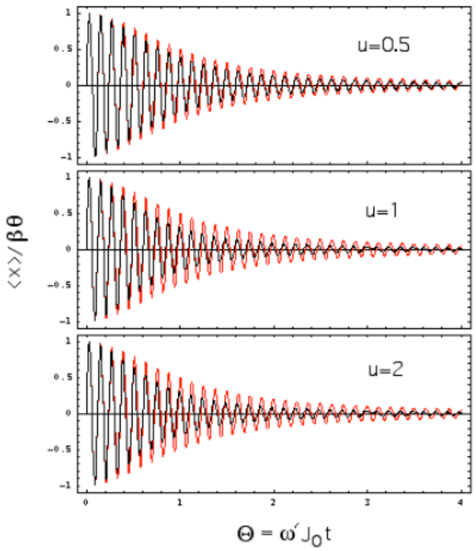
This signal has a slowly varying oscillation amplitude

$$\langle x\rangle^{\mathrm{ampl}}(t) \quad \approx \quad \frac{\beta\theta}{1+\Theta^2}$$

This amplitude starts with the value βθ at t=0, but slowly decreases with time. Decoherence time:

$$\tau_{\mathrm{decoh}} \approx \frac{1}{\omega' J_0}$$

Typically frequency spread $\omega' J_0 \sim 10^{-3}$, meaning τ_{decoh} ~10^3 turns, or ~ 1ms, which is very short.



3. Transverse echo (t > τ)

We now let the beam decohere for a time $\tau >> \tau_{decoh}$, e.g. $\tau \sim 1$ s. The beam centroid signal completely vanishes due to decoherence.

At time $t = \tau$, we give the beam a quadrupole kick, $q = \beta/f$ with f = quadrupole's focal length.

After the quadrupole kick $(t > \tau)$, the beam distribution can be calculated:

$$\langle x \rangle (t) \approx 2\beta\theta q\tau \int_0^{\infty} J^2 dJ \omega'(J) \psi_0'(J) \times \int_0^{2\pi} d\phi \cos\phi \sin[2\phi - 2\omega(J)t + 2\omega(J)\tau] \cos[\phi - \omega(J)t]$$

This integration over ϕ is the key to the echo phenomenon. Magically, the two kicks, the frequency spread, and the particle evolution in phase space conspire to produce a recoherence at time $t = 2\tau$!

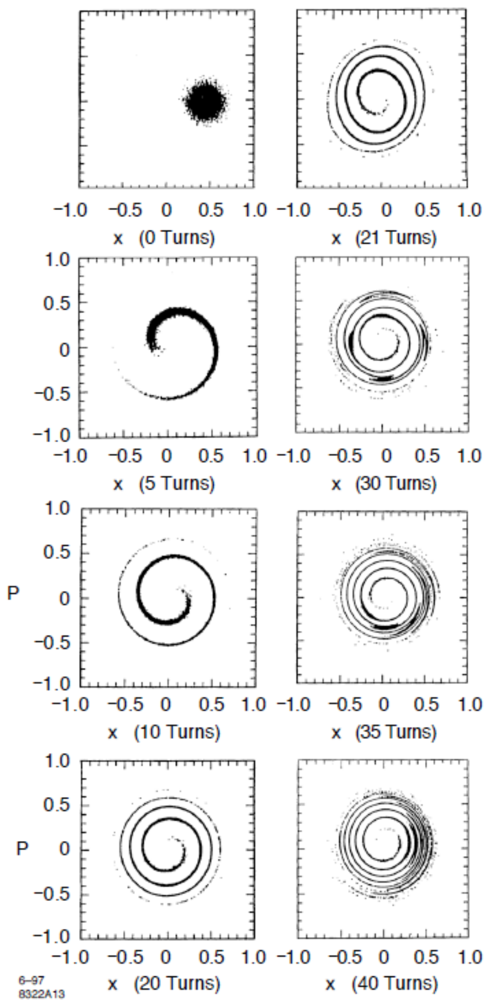
$$\langle x \rangle (t) \approx 2\beta\theta q\tau \int_0^{\infty} J^2 dJ \omega'(J) \psi_0'(J) \left(-\frac{\pi}{2} \sin[\omega(J)(t-2\tau)] \right)$$

The recoherence, and the fact that it occurs at $t = 2\tau$, do not depend on the exact form of $\psi_0(J)$ or $\omega_\beta(J)$!

=>

In spite of the intricacy in phase space, which implies sensitivity to minute details, echo is a robust phenomenon.

This really should be a movie…..



[G. Stupakov, S. Kauffmann, 1992]

A Gaussian beam receives a dipole kick at 0-th turn and a quadrupole kick at 20-th turn. From 0-th turn to 20-th turn the beam decoheres. By about the 30-th turn, several ``kinks" develop in the beam's phase space distribution. These kinks come from an interplay of the two kicks. Each of the kicks occurs at a specific amplitude, which are most conspiring in the sense that, with the amplitude-dependent frequency, they all come to coherence simultaneously at the 40-th turn as they migrate in the phase space relative to one another, thus yielding an echo.

<u>Example</u> If $\omega_\beta(J) = \omega_0 + \omega' J$, and $\psi_0(J)$ = Gaussian, then

$$\langle x\rangle^{\text{echo ampl}}(t) = \beta\theta q \frac{\omega' J_0 \tau}{(1+\xi^2)^{3/2}}$$

$$\xi = \omega' J_0 (t - 2\tau)$$

The maximum echo amplitude occurs at $t = 2\tau$, and the maximum value is

$$\langle x\rangle^{\text{echo ampl max}} = \beta\theta q \omega' J_0 \tau$$

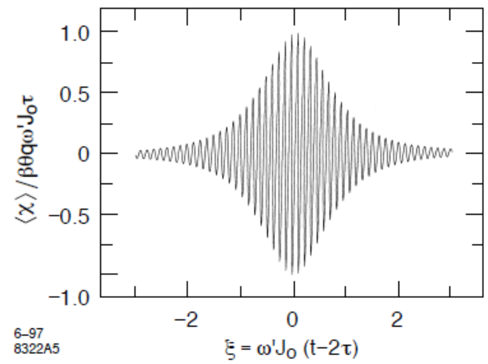


Figure 4: A close-up observation of the echo signal.

Echo duration is $\sim \dfrac{1}{\omega' J_0} \sim \tau_{\text{decoh}}$.

If we compare the maximum echo amplitude to the initial kick amplitude $\beta\theta$, we see that the echo amplitude is weaker by a factor of $q\omega' J_0 \tau$ which has been assumed << 1.

Simulation shows the as q is increased, the echo maximum echo amplitude is ~40% of the initial kick amplitude (case 3).

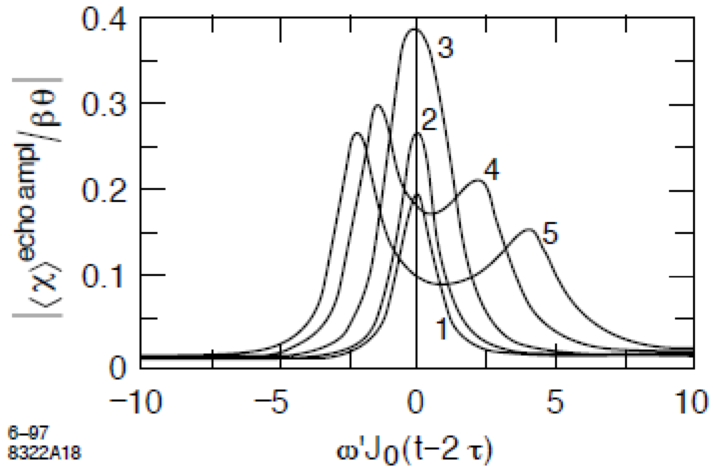


Figure 6: Echo amplitude signal obtained in a simulation. Curves 1 to 5 correspond to $q = 0.02, 0.03, 0.08, 0.2, 0.3$ respectively.

4. Longitudinal echo (unbunched beam)

The first kick at time $t = 0$ is to impose an rf energy kick

$$\Delta\delta(z) = \frac{eV_1}{E_0}\sin\left(h_1\frac{z}{R} + \phi_1\right)$$

where h_1 is the harmonic number of the rf system.

After the first kick, the beam begins to bunch up into h_1 bunches, but due to the energy spread of the beam, this bunching signal decoheres quickly.

Long after the signal has decohered, at $t = \tau$, a second kick is applied,

$$\Delta\delta(z) = \frac{eV_2}{E_0}\sin\left(h_2\frac{z}{R} + \phi_2\right)$$

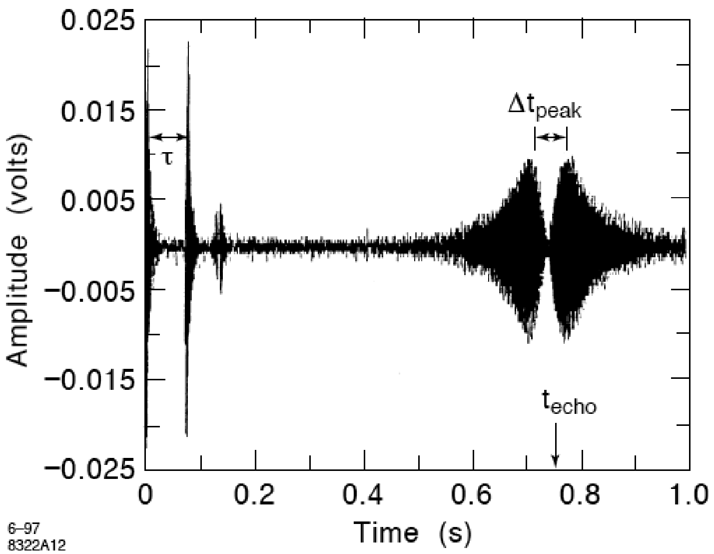
with harmonic number h_2.

At a much later time, $t = t^{echo}$, one observes a sudden echo with harmonic number h_2-h_1.

$$I^{\text{echo ampl}}(t) = -\frac{1}{2}\text{sgn}(h_2) I_0 \frac{eV_1}{E_0}\frac{eV_2}{E_0}\frac{h_1 \eta c \tau}{R\sigma_\delta}\ \xi e^{-\xi^2/2}$$

$$t^{\text{echo}} = \frac{|h_2|\tau}{|h_2| - |h_1|}$$

$$\xi = \frac{\eta c \sigma_\delta (|h_2| - |h_1|)}{R}(t - t^{\text{echo}})$$



A beautiful experiment at CERN AA
[L. Spenzouris, J.-F. Ostigy, P. Colestock, 1996]

First kick t = 0, $h_1 = 9$
Second kick t = τ, h_2=10
=>
echo at t = $h_2\tau$ / (h_2- h_1) = 10τ

The shape of the echo also comes out right!

5. Longitudinal echo with diffusion

The echo signal is very susceptible to diffusion effects. Any small diffusion can mess up the intricate phase space that echo relies on.

This is why echoes occur only in proton storage rings. Electrons don't echo because of synchrotron radiation.

With a diffusion coefficient D_0,

$$I^{\text{echo ampl}} = \frac{1}{2} I_0 \frac{eV_1}{E_0} \frac{eV_2}{E_0} \frac{h_1 \eta c \tau}{R \sigma_\delta} \, \xi e^{-\xi^2/2} \exp\left[-\frac{1}{3} \frac{\eta^2 c^2}{R^2} D_0 \frac{h_1^2 h_2}{h_2 - h_1} \tau^3 \right]$$

$$\xi = \frac{\eta c \sigma_\delta (|h_2| - |h_1|)}{R} (t - t^{\text{echo}})$$

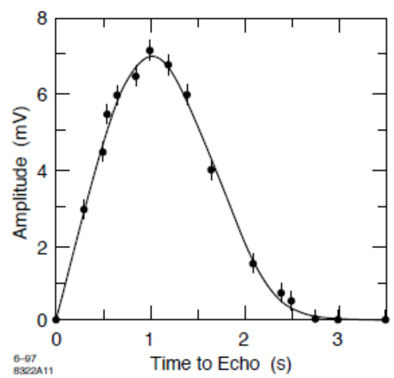


Figure 11: Measured maximum echo amplitude as a function of t^{echo} in the AA ring. Diffusion is playing an important role.

Fitting the AA data above yields $D_0 = 1.3 \times 10^{-10}$/s. (Would be a straight line in case of no diffusion.)

6. Echo for X-ray free electron laser

Echo can be used to generate microbunching for free electron laser!

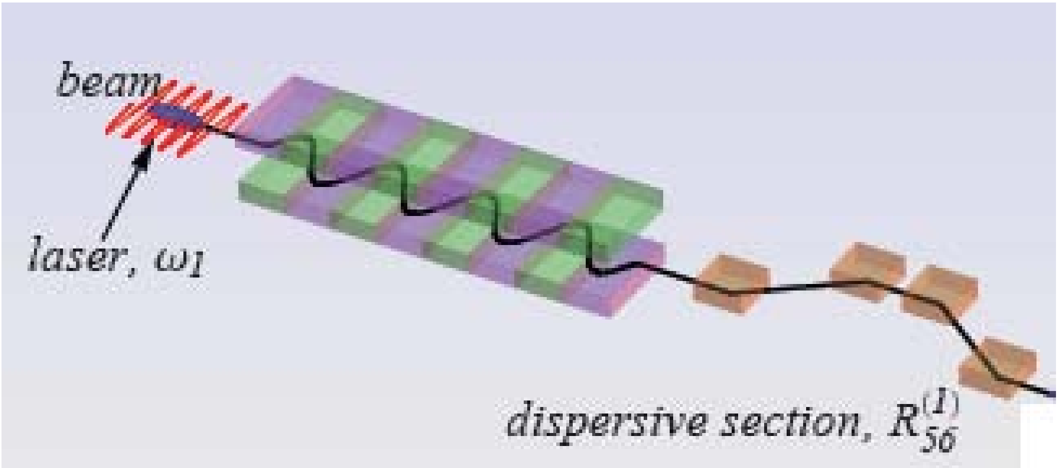
[Stupakov, 2008]

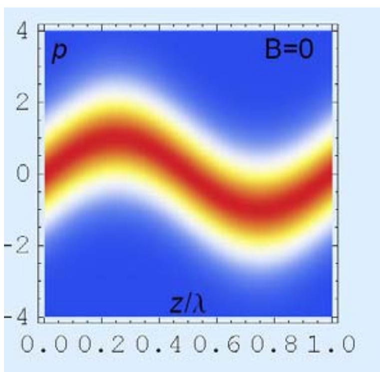
- FEL echo is analogous to the longitudinal echo except that it is now applied to a linac instead of a storage ring. ➔ It now applies to electrons (no synchrotron radiation in linacs).

- Our longitudinal echo analysis calculates only the linear effects to first order in V_1 and V_2. As a result, echo occurs only at low harmonic h_2-h_1. For X-ray FEL echo, we wish to generate echo frequency at high harmonics $n_1h_2+n_2h_1$. This requires terms nonlinear in V_1 and V_2. ➔ FEL echo is therefore intrinsically a nonlinear device.

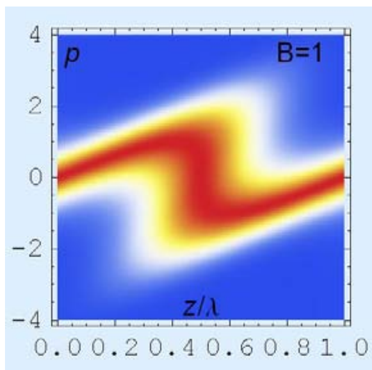
- Being a nonlinear effect, it opens up another possibility of a "steady state microbunching" in an electron storage ring, which has the advantage of a very high repetition rate. [Ratner, Chao, 2009]

Without echo scheme, we have the usual HGHG:

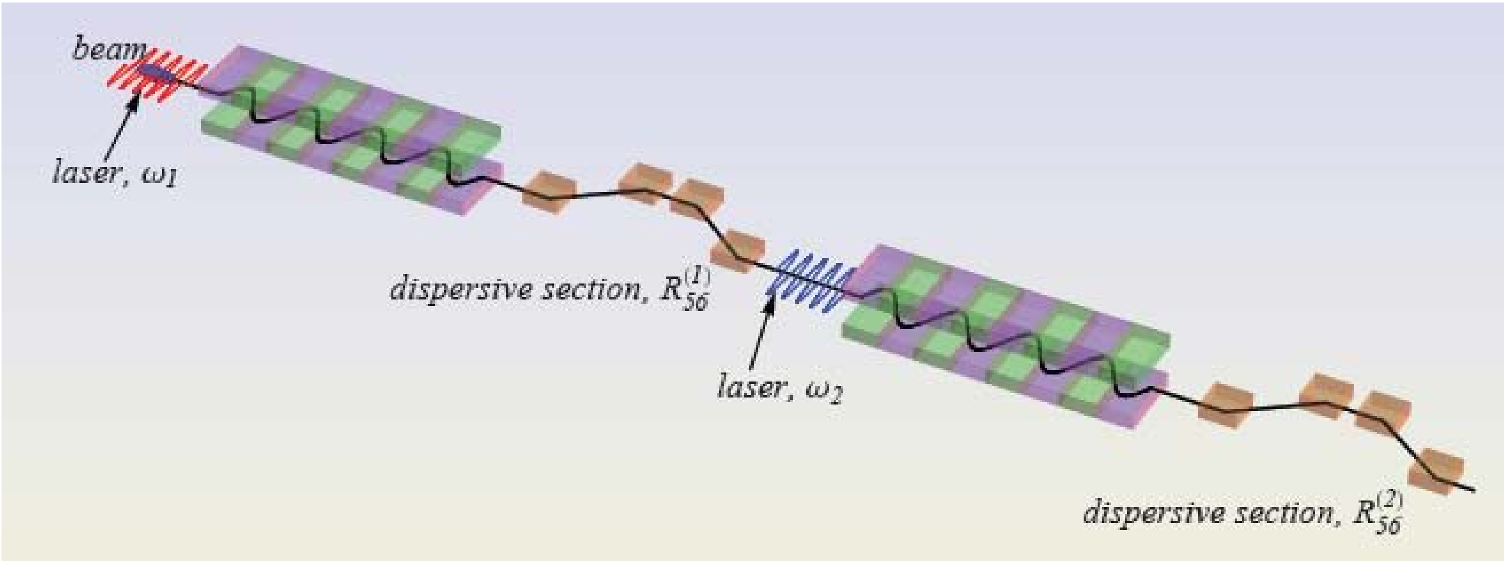


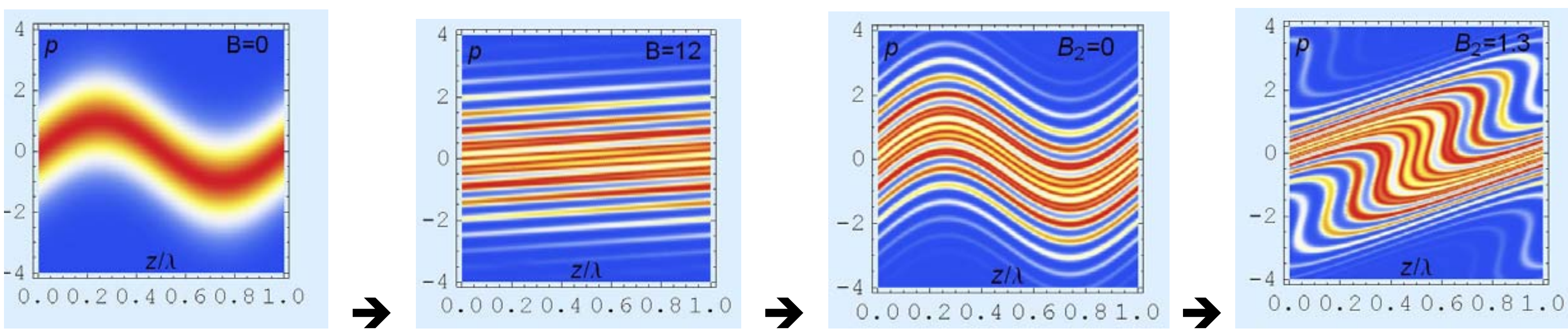


➔



With echo scheme, we have:





Experiments are being performed at SLAC and SSRF. Results expected 2010!

7. **Spin motion near a depolarization resonance**

Consider a depolarization resonance

spin tune $G\gamma = \kappa$
(= integer, or integer ± ν_y, etc)

where $G = (g\text{-}2)/2$ is anomalous magnetic moment, γ is Lorentz energy factor.

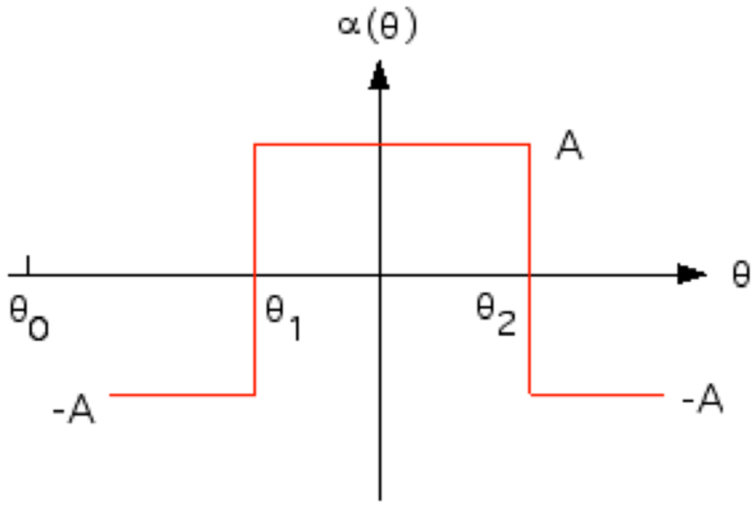
Let the resonance strength = ε (a complex dimensionless quantity, Fourier component of perturbing magnetic fields)

Let the spin tune of the particle be near the resonance according to

$$G\gamma = \kappa + \alpha(\theta)$$

$\alpha(\theta)$ is deviation from resonance, and is function of time θ = (number of turns) x 2π

The simplest crossing of a resonance is a sudden spin tune jump. Successively jumping across a depolarization resonance twice produces interesting spin dynamics effects.

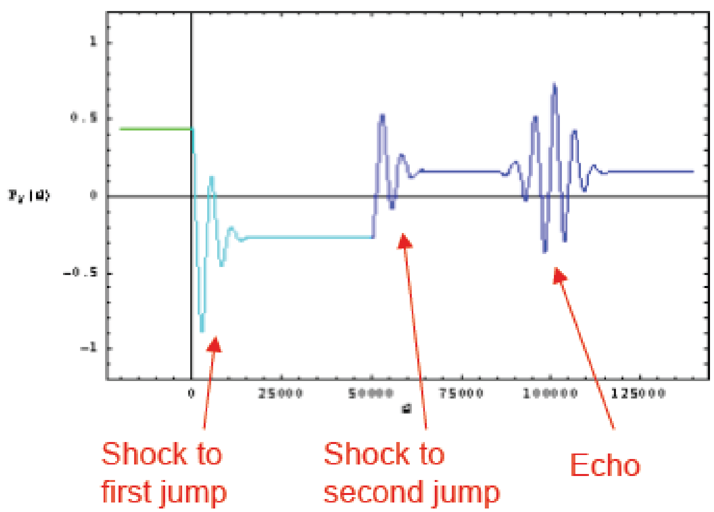


By adjusting these parameters, the beam polarization after the second jump exhibits a wealth of effects of constructive interference, destructive interferences and spin echo [Chao, Courant, 2007].

8. Proposed spin echo experiment proposals

No spin echo or spin interference experiments have been done so far. Proposal for COSY:

Polarized protons initially 100% polarized, brought adiabatically to launching position $-A$ below the resonance ε_0, jump across to $+A$, wait for a long time τ, jump back to $-A$, then measure final polarization. Adjustable parameters: A, τ, $|\varepsilon_0|$.



COSY synchrotron (Germany)
2.1 GeV/*c* polarized protons
$\sigma_\delta = 10^{-4}$ (with electron cooling)
$\kappa = 4.4$
$f_c = 1.5$ MHz
$N_{\text{jump}} < 100$

- To dramatize the echo, choose a large τ.
- Echo signal maximized by $A = |\varepsilon_0|/2$ => 57%.

Echo experiment proposal for RHIC

[M. Bai et al, 2008]

Computer simulation of the RHIC experiment:

Left: $\tau = 2\pi$ x 40,000.

Right: $\tau = 2\pi$ x 300,000, the amplitude of the echo signal is still present but reduced.