A triangle singularity showing effects of the Λ(1405) above the K^- N threshold in the $K^- d \rightarrow p\,\Sigma^-$ reaction

E. Oset, L.R. Dai, N. Ikeno, R. Molina, A. Feijoo, IFIC, U. Valencia- CSIC

Triangle singularities

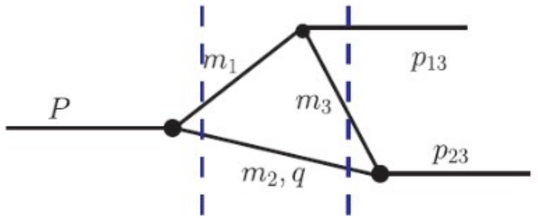
TS in the pp $\rightarrow \pi^+ d$ reaction

A new interpretation of the “d*(2380) dibaryon” peak

TS in the $K^- d \rightarrow p\,\Sigma^-$ reaction

Triangle singularities: Introduced by Landau

L. D. Landau, Nucl. Phys. **13**, 181 (1959).



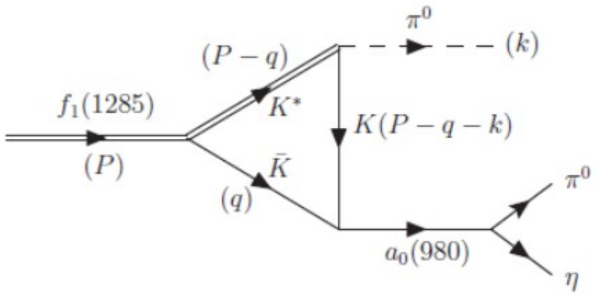
Some reaction mechanism involve a Feynman diagram with a loop of three particles. Sometimes this mechanism develops a singularity (becomes infinity) .

No experimental confirmation was found at that time

Nowadays we find many experimental cases

The search goes on….

THE $\pi a_0(980)$ DECAY MODE OF THE $f_1(1285)$



$$t_T = i\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\,\vec{\epsilon}_{f_1}\cdot\vec{\epsilon}_{K^*}\;\vec{\epsilon}_{K^*}\cdot(2\vec{k}+\vec{q})\,\frac{1}{q^2-m_K^2+i\epsilon}\,\frac{1}{(P-q)^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}}\,\frac{1}{(P-q-k)^2-m_K^2+i\epsilon}$$

$$\begin{aligned}\widetilde{t}_T &= \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\left(2+\frac{\vec{k}\cdot\vec{q}}{\vec{k}^2}\right)\frac{1}{8\,\omega(q)\omega'(q)\omega^*(q)}\,\frac{1}{k^0-\omega'(q)-\omega^*(q)+i\epsilon}\,\frac{1}{P^0-\omega^*(q)-\omega(q)+i\epsilon}\\ &\quad\times\frac{2P^0\omega(q)+2k^0\omega'(q)-2(\omega(q)+\omega'(q))(\omega(q)+\omega'(q)+\omega^*(q))}{(P^0-\omega(q)-\omega'(q)-k^0+i\epsilon)(P^0+\omega(q)+\omega'(q)-k^0-i\epsilon)},\end{aligned}$$

$$\omega(q) = \sqrt{\vec{q}^{\,2}+m_K^2},\;\; \omega'(q) = \sqrt{(\vec{q}+\vec{k})^2+m_K^2},\;\; \omega^*(q) = \sqrt{\vec{q}^{\,2}+m_{K^*}^2}$$

Poles in the integration

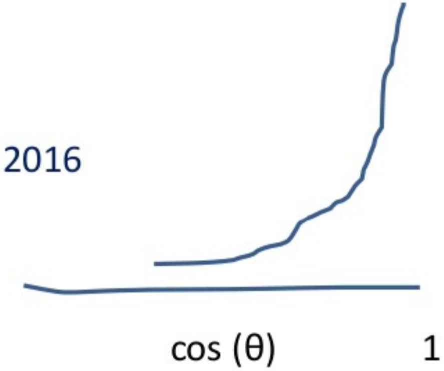
$$P^0-\omega^*(q)-\omega(q)+i\epsilon = 0, \qquad q_{\mathrm{on}+} = q_{\mathrm{on}} + i\,\epsilon \quad \text{with} \quad q_{\mathrm{on}} = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)}$$

$$P^0-\omega(q)-\omega'(q)-k^0+i\epsilon = 0$$

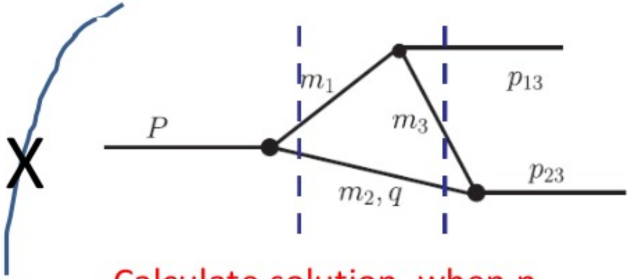
$$P^0 - \omega(q) - \omega'(q) - k^0 + i\epsilon \;=\; 0 \qquad \omega'(q) \;=\; \sqrt{(\vec{q}+\vec{k})^2 + m_K^2}$$

If we fix cos (θ)= ± 1 and we make this expression zero, then in the integral of cos(θ) one cannot cancel the divergence with the principal value, and the divergence remains. Θ is the angle between $\vec{k}$ and $\vec{q}$.

Bayar, Aceti, Guo, E. O, PRD 2016



For cos(θ)=-1



Calculate solution when p_{23} is at rest (p_2*) and make a boost

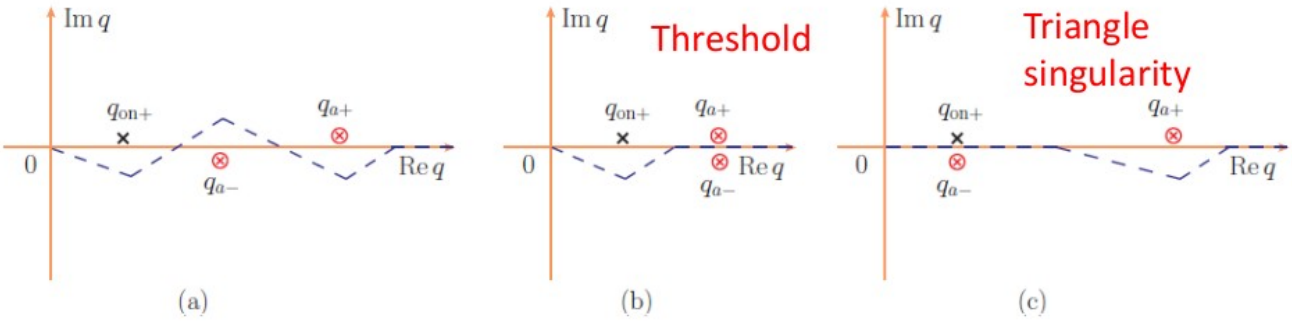
$$q_{a+} = \gamma\,(v\,E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{a-} = \gamma\,(v\,E_2^* - p_2^*) - i\,\epsilon$$

$$v = \frac{k}{E_{23}}\,, \qquad \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E_{23}}{m_{23}}\,,$$

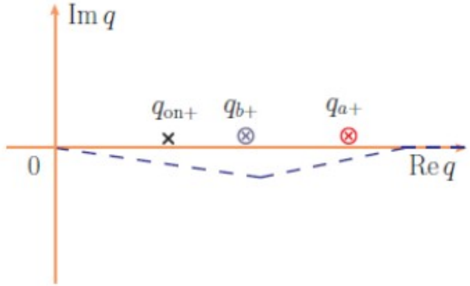
$$E_2^* = \frac{1}{2m_{23}}\left(m_{23}^2 + m_2^2 - m_3^2\right), \qquad p_2^* = \frac{1}{2m_{23}}\sqrt{\lambda(m_{23}^2, m_2^2, m_3^2)}$$

For cos(θ)=1 $$q_{b+} = \gamma\,(-v\,E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{b-} = -\gamma\,(v\,E_2^* + p_2^*) - i\,\epsilon$$

For cos(θ)=-1



For cos(θ)=1



Triangle singularity

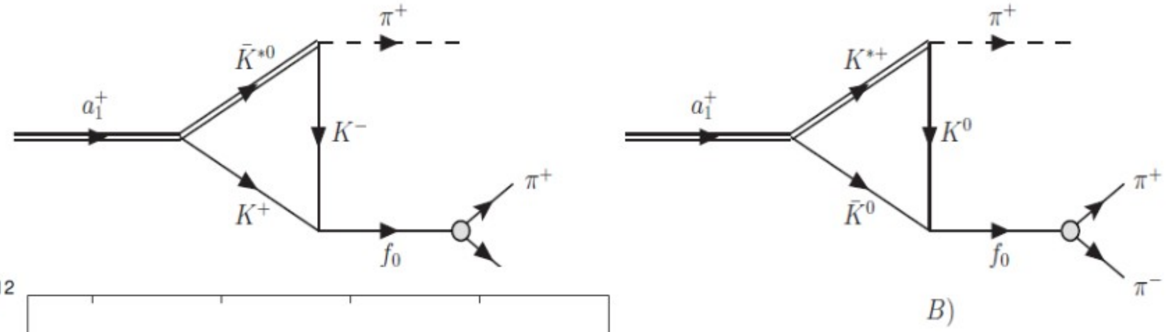
$$\lim_{\epsilon \to 0} (q_{\mathrm{on}+} - q_{a-}) = 0$$

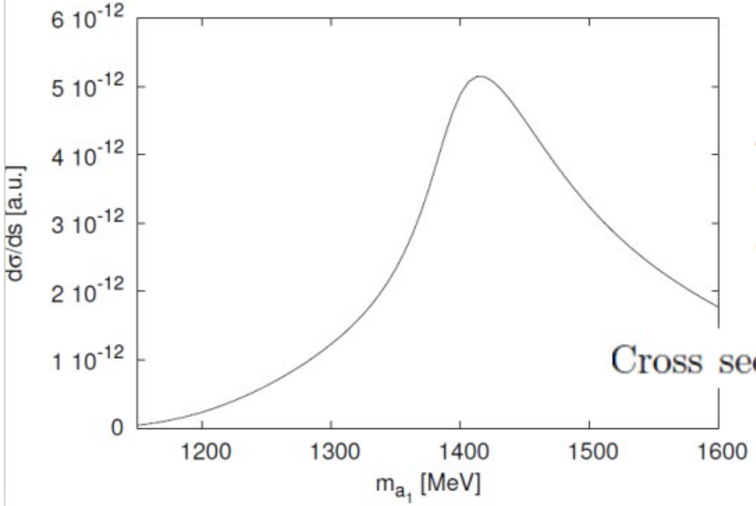
Very simple expression to see where the TS appears , and to explain the Coleman-Norton theorem, Nuovo Cim. 1965, (TS appears when the decays in the loop can occur at the classical level).

These results are a replica of what was found for the "$a_1(1420)$"

In M. Mikhasenko, B. Ketzer, A. Sarantsev, PRD 2015
F. Aceti, L.R. Dai, E. O., PRD 2016
suggested in X.H. Liu, M. Oka, Q. Zhao, PLB 2016

The "a1(1420)" peak is a manifestation of the decay of the $a_1(1260)$ into $\pi\ f_0(980)$





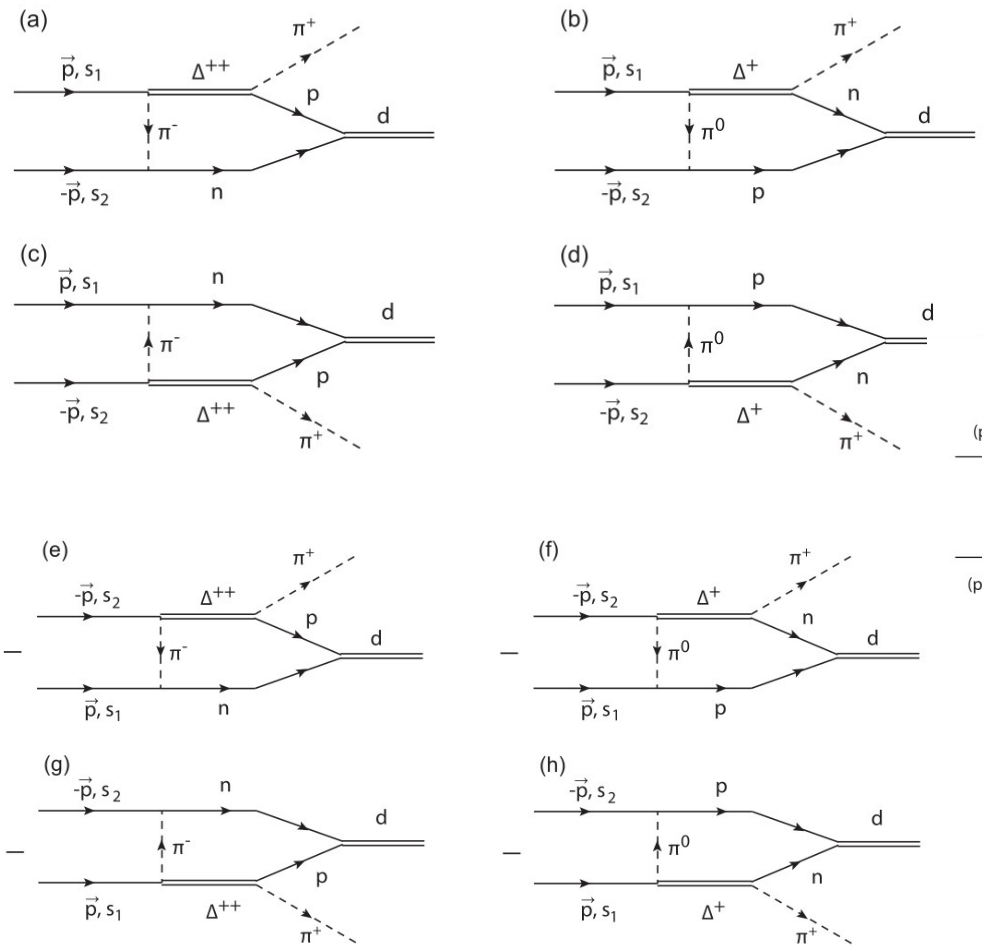
A Triangle Singularity as the Origin of the $a_1(1420)$

COMPASS Collaboration • M.G. Alexeev (Turin U. and INFN, Turin) et al. (Jun 9, 2020)
e-Print: 2006.05342 [hep-ph]

Cross section for the decay $a_1^+(1260) \to \pi^+ f_0(980)$

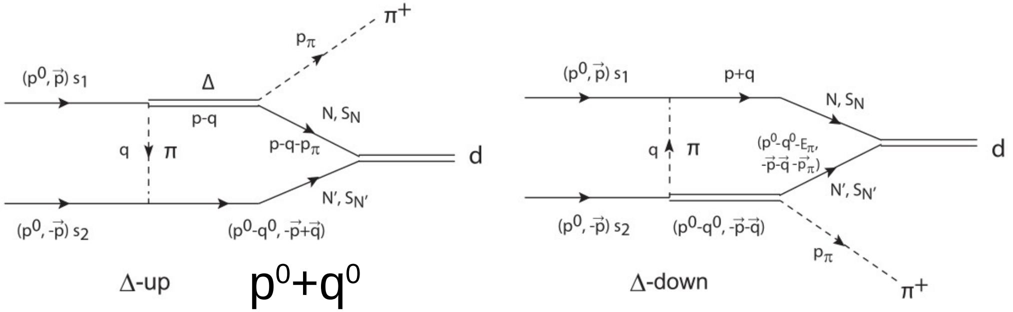
The pp $\rightarrow \pi^+$ d reaction

N. Ikeno, R. Molina, E. Oset



$$-i\delta H_{\pi NN} = \frac{f}{m_\pi}\vec{\sigma}\cdot\vec{q}\,\tau^{\lambda}; \qquad f = 1.00,$$

$$-i\delta H_{\pi N\Delta} = \frac{f^*}{m_\pi}\vec{S}^{\dagger}\cdot\vec{q}\,T^{\dagger\lambda}; \qquad f^* = 2.13,$$



p^0+q^0

The triangle singularity condition is found at $\sqrt{s}$=2179 MeV

The time reversed reaction was much studied in the past, using a Quantum Mechanics formalism, not Field Theory, and the triangle singularity was not identified.

It explains why the cross section is much bigger than expected for fusion reactions.

Pionic Disintegration of the Deuteron

D.O. Riska (Michigan State U.), M. Brack (SUNY, Stony Brook), W. Weise (SUNY, Stony Brook) (1976)

Published in: *Phys.Lett.B* 61 (1976) 41-44

P Wave Meson Production in p p --> d pi+

Anthony M. Green (Helsinki U.), J.A. Niskanen (Helsinki U.) (1976)

Published in: *Nucl.Phys.A* 271 (1976) 503-524

Pionic Disintegration of the Deuteron

M. Brack (SUNY, Stony Brook), D.O. Riska (Michigan State U.), W. Weise (Regensburg U.) (1977)

Published in: *Nucl.Phys.A* 287 (1977) 425-450

A covariant theory of the pionic disintegration of the deuteron

D. Schiff, J. Tran Thanh Van (1968)

Published in: *Nucl.Phys.B* 5 (1968) 529-559

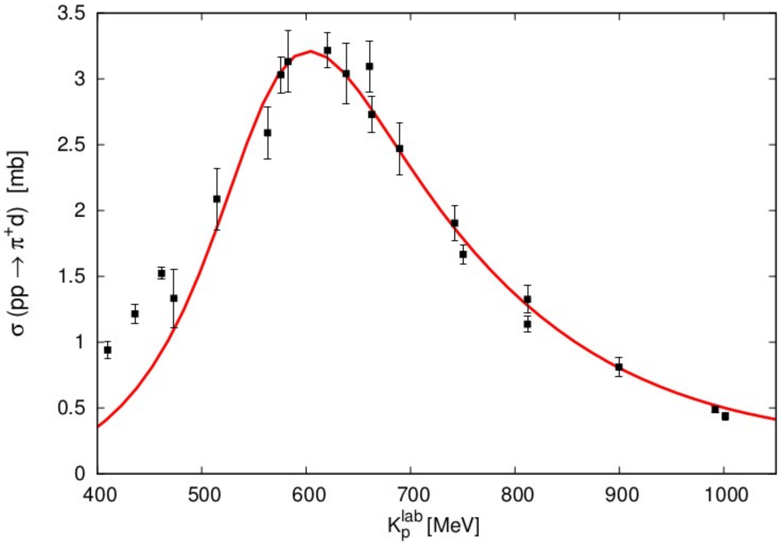
$$
\begin{aligned}
-it^{\pi} &= -g_d \frac{4\sqrt{2}}{3}\left(\frac{f^*}{m_\pi}\right)^2\left(\frac{f}{m_\pi}\right)\int \frac{d^3q}{(2\pi)^3}\left(\frac{\Lambda^2-m_\pi^2}{\Lambda^2+\vec{q}^{\,2}}\right)^2 \\
&\cdot \left\{\vec{S}_1\cdot\vec{p}_\pi\ \vec{S}_1\cdot\vec{q}\ \vec{\sigma}_2\cdot\vec{q}\ F(\vec{p},\vec{q},\vec{p}_\pi)\ -\vec{\sigma}_1\cdot\vec{q}\ \vec{S}_2\cdot\vec{p}_\pi\ \vec{S}_2^{\dagger}\cdot\vec{q}\ F(-\vec{p},\vec{q},\vec{p}_\pi)\ \right\}
\end{aligned}
$$

$$
\begin{aligned}
F(\vec{p},\vec{q},\vec{p}_\pi) &= \frac{M_N}{E_N(-\vec{p}+\vec{q})}\ \frac{M_N}{E_N(\vec{p}-\vec{q}-\vec{p}_\pi)}\ \frac{M_\Delta}{E_\Delta(\vec{p}-\vec{q})}\ \frac{1}{2\omega(q)}\ \frac{1}{2p^0-E_\pi-E_N(-\vec{p}+\vec{q})-E_N(\vec{p}-\vec{q}-\vec{p}_\pi)+i\epsilon} \\
&\cdot \left\{\frac{1}{p^0-\omega(q)-E_\Delta(\vec{p}-\vec{q})+i\frac{\Gamma_\Delta}{2}}\ \frac{1}{p^0-\omega(q)-p_\pi^0-E_N(\vec{p}-\vec{q}-\vec{p}_\pi)+i\epsilon}\right. \\
&+\frac{1}{p^0-\omega(q)-E_\Delta(\vec{p}-\vec{q})+i\frac{\Gamma_\Delta}{2}}\ \frac{1}{2p^0-E_\Delta(\vec{p}-\vec{q})-E_N(-\vec{p}+\vec{q})+i\frac{\Gamma_\Delta}{2}} \\
&+\left.\frac{1}{p^0-\omega(q)-E_N(-\vec{p}+\vec{q})+i\epsilon}\ \frac{1}{2p^0-E_\Delta(\vec{p}-\vec{q})-E_N(-\vec{p}+\vec{q})+i\frac{\Gamma_\Delta}{2}}\right\}\cdot\theta(q_{\max}-|\vec{p}-\vec{q}-\frac{\vec{p}_\pi}{2}|).
\end{aligned}
$$

g_d is the coupling of NN to the deuteron together with the θ(.) function.

q_{max} is chosen such as to reproduce the np triplet scattering length.

ρ exchange is also taken into account and reduces the π exchange contribution.



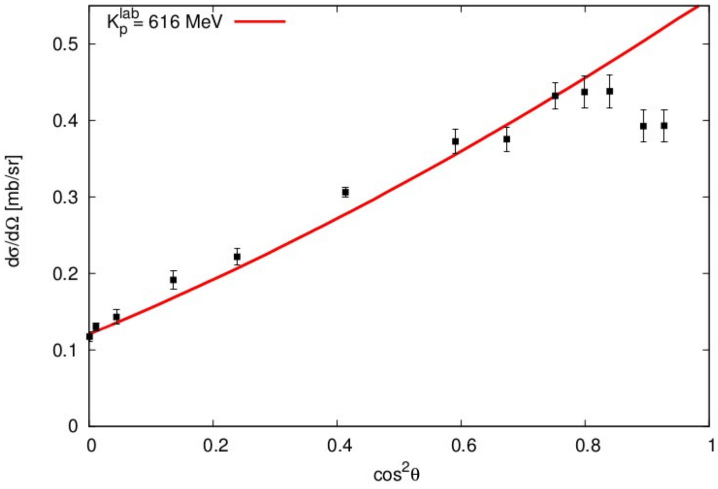
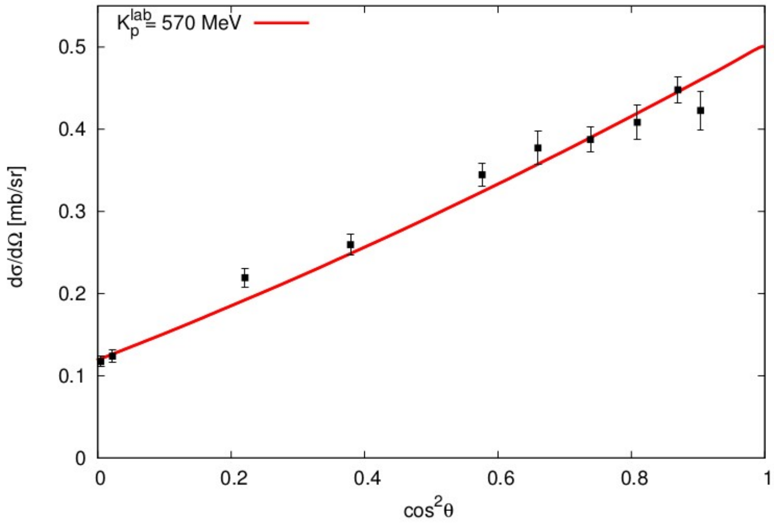
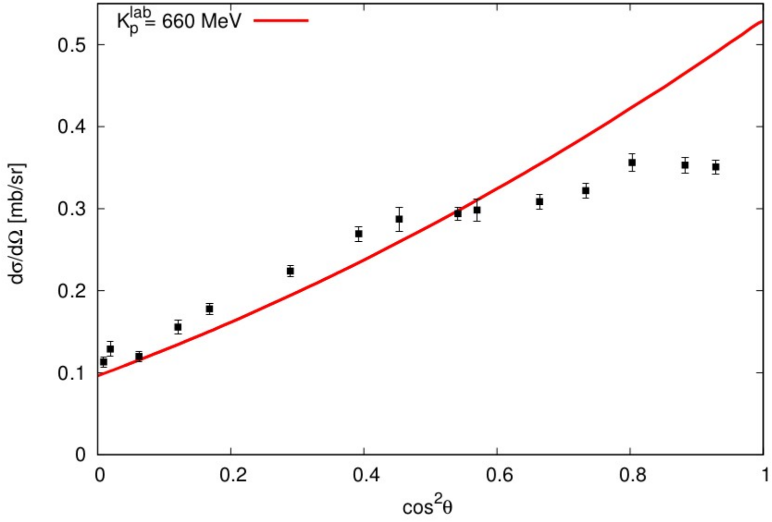
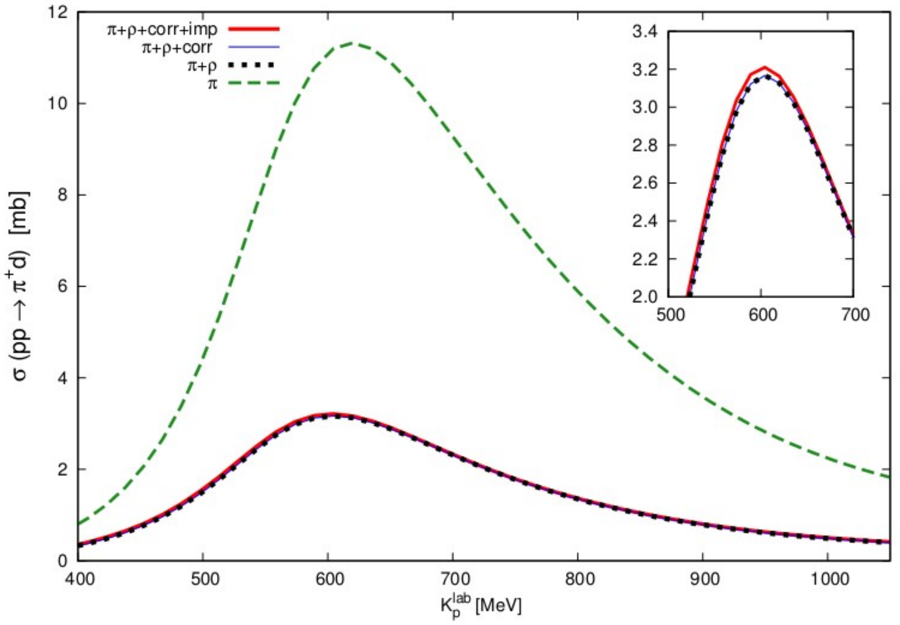


FIG. 6: $d\sigma/d\Omega = \frac{1}{2\pi}\frac{d\sigma}{d\cos\theta_\pi}$ as a function of $\cos^2\theta_\pi$ for $K_p^{\rm lab} = 616$ MeV.

Data from Richard-Serre et al. , NP B 20, 413 (1970)





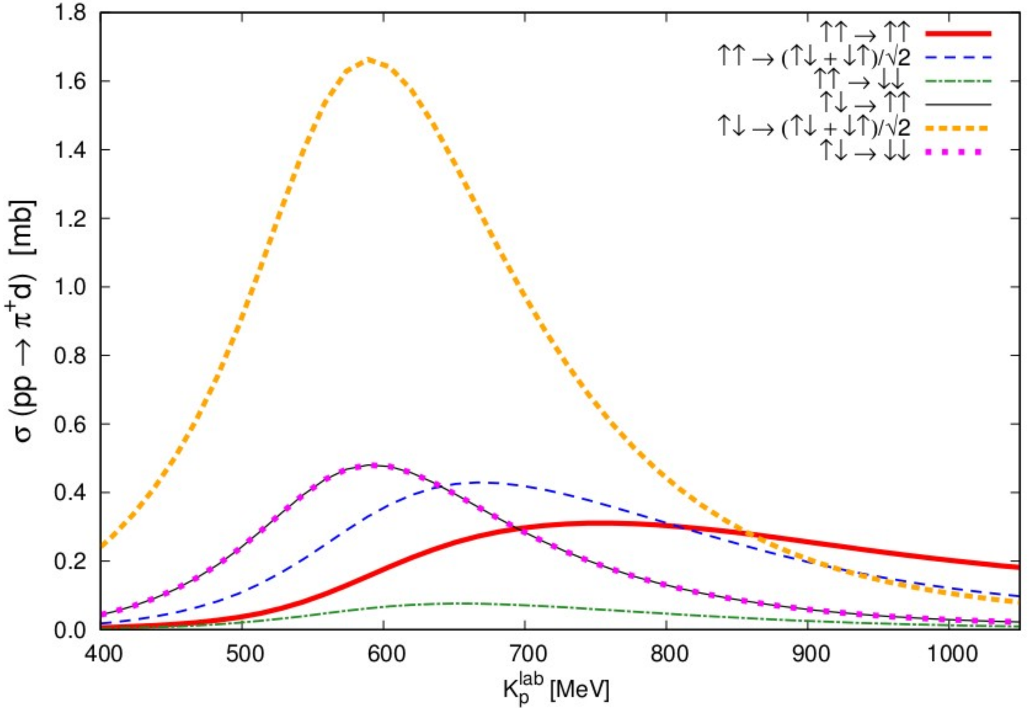


Note the dominance of $\uparrow\downarrow \rightarrow \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$

But we see that the initial state is

$\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$ S=0 for pp

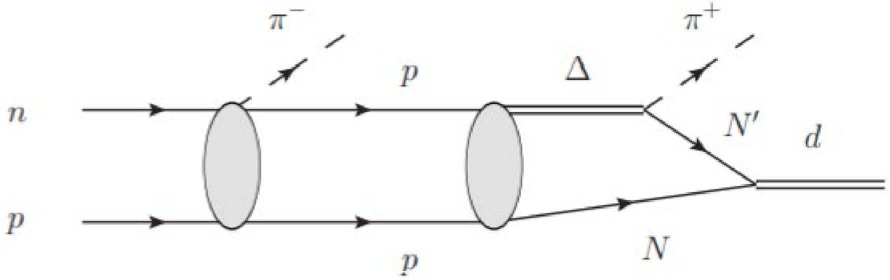
And also find L=2 dominance in pp,
in agreement with experiment
Albrow et al. PLB 34 (1971) 337



Sequential single pion production explanining the dibaryon "$d^*(2380)$" peak

R. Molina,[1,*] Natsumi Ikeno,[1,2,†] and Eulogio Oset[1,‡]

Arxiv 2102.05575



Idea taken from

Analysis of the reaction n p ---> d pi+ pi- below 3.5 gev/c

#11

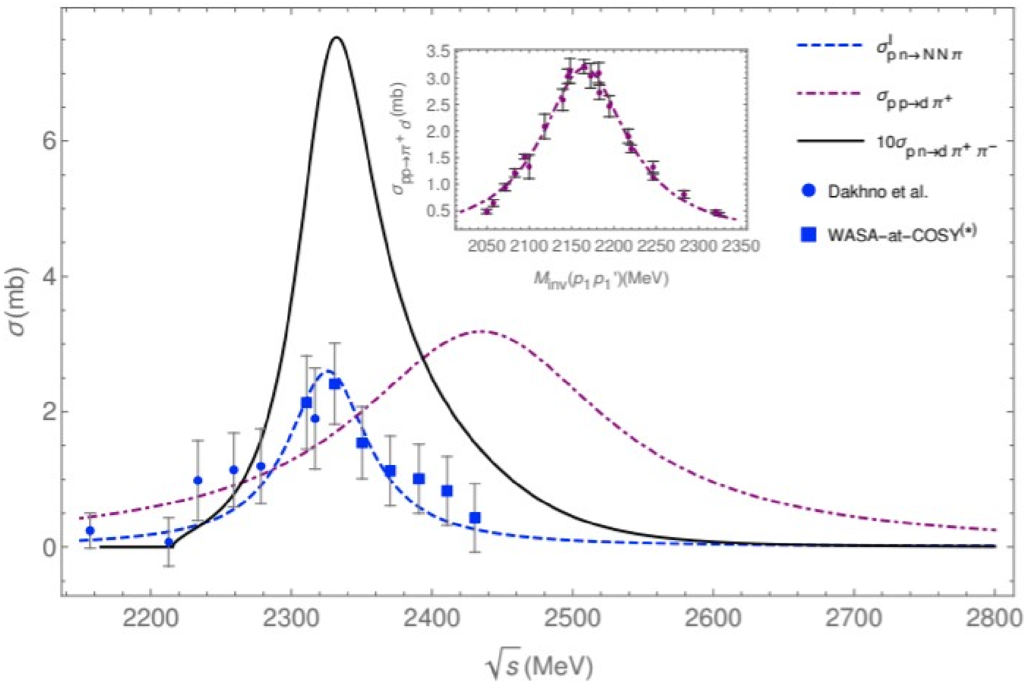
I. Bar-Nir (Heidelberg U.), E. Burkhardt (Heidelberg U.), H. Filthuth (Heidelberg U.), H. Oberlack (Heidelberg U.), A. Putzer (Heidelberg U.) et al. (1973)

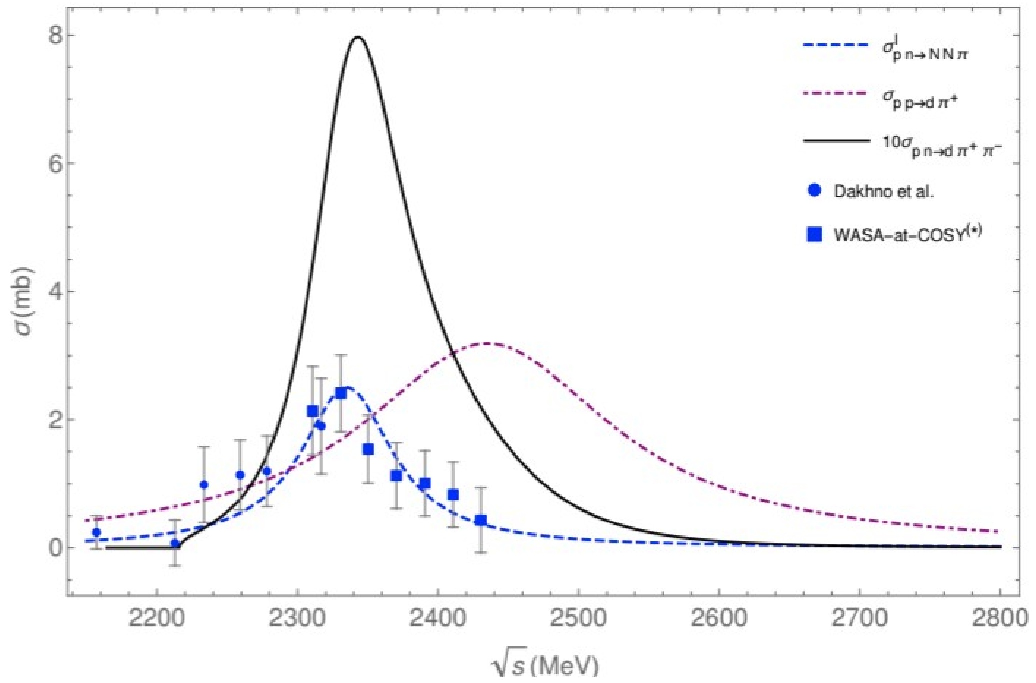
Published in: *Nucl.Phys.B* 54 (1973) 17-28

An on shell approximation was done, suggesting that this should be improved in the future.
We prove that it is quite good, but go beyond.

With some good approximations we find:

$$\sigma_{np\to\pi^+\pi^-p} = \frac{M_{\rm inv}(p_1p_1')}{6\pi}\,\frac{\sigma^I_{np\to NN\pi}\,\sigma_{pp\to\pi^+d}}{M_{\rm inv}(\pi\pi)}\,\frac{\tilde{p}_1^2}{p_\pi p_\pi'}\,p_d\tilde{p}_\pi$$


$\sigma^{I}_{pn\to NN\pi}$
$\sigma_{pp\to d\pi^+}$
$10\sigma_{pn\to d\pi^+\pi^-}$
Dakhno et al.
WASA-at-COSY(*)
σ(mb)
$\sqrt{s}$(MeV)
$\sigma_{pp\to\pi^+d}$(mb)
$M_{inv}(p_1 p_1')$(MeV)


$\sigma^{I}_{pn\to NN\pi}$
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$10\sigma_{pn\to d\pi^+\pi^-}$
Dakhno et al.
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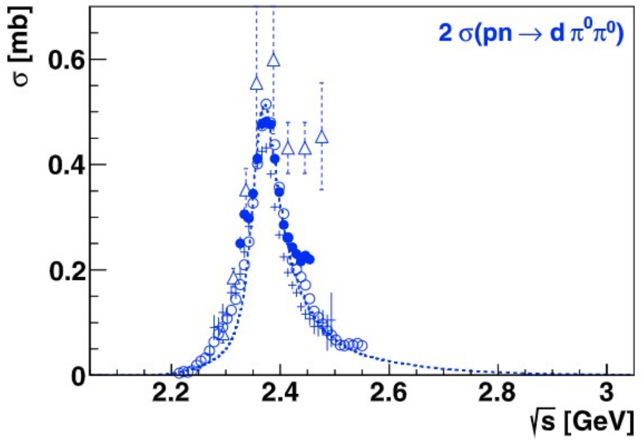
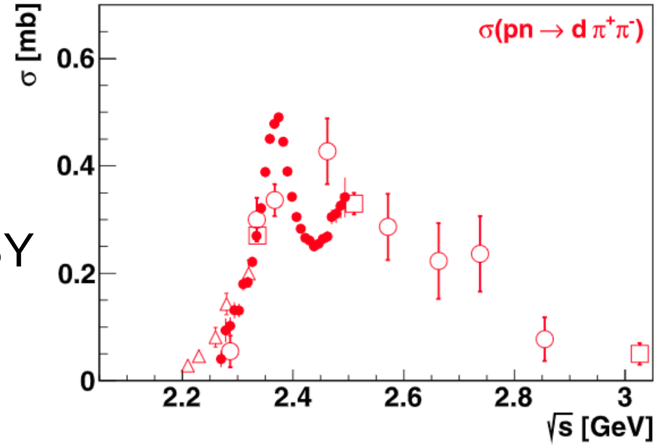
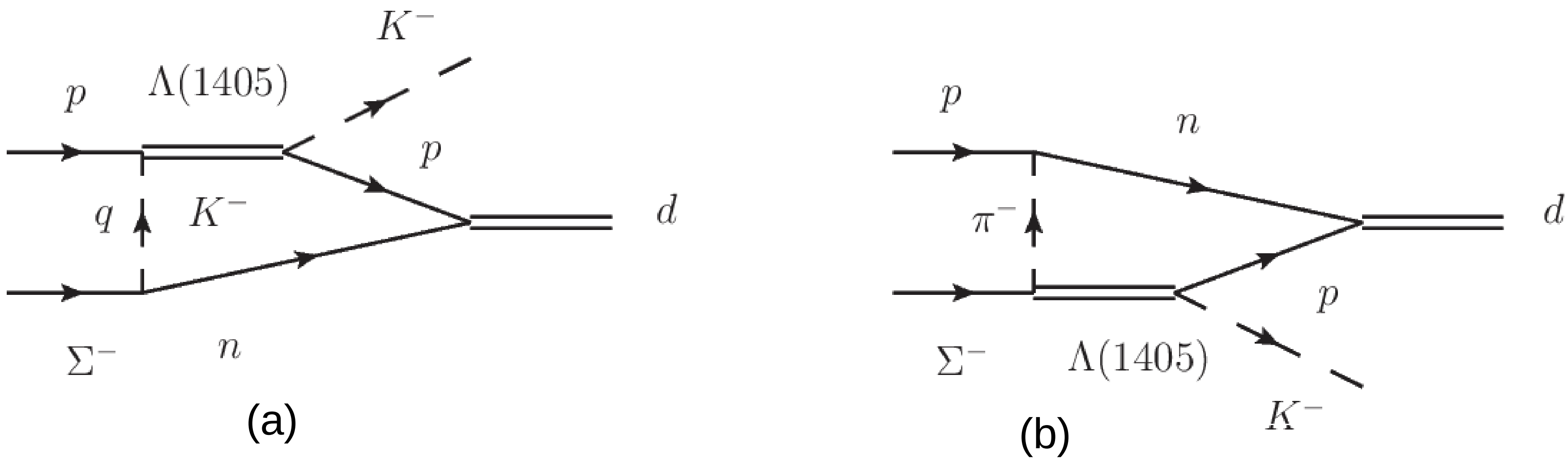
σ(mb)
$\sqrt{s}$(MeV)


$2\,\sigma(pn \to d\,\pi^0\pi^0)$
σ [mb]
$\sqrt{s}$ [GeV]

WASA at COSY

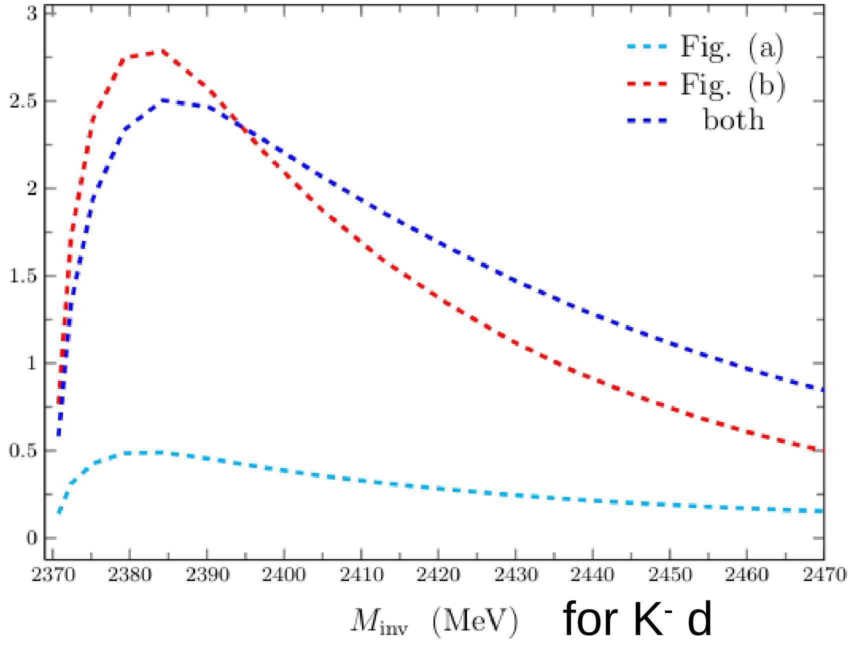

$\sigma(pn \to d\,\pi^+\pi^-)$
σ [mb]
$\sqrt{s}$ [GeV]

Triangle singularity in the $K^- d \to p \Sigma^-$ reaction L. R. Dai, R. Molina, A. Feijoo and E. O.



The $p \Sigma^- \to K^- d$ reaction cross section can be obtained from the other via the detailed balance theorem

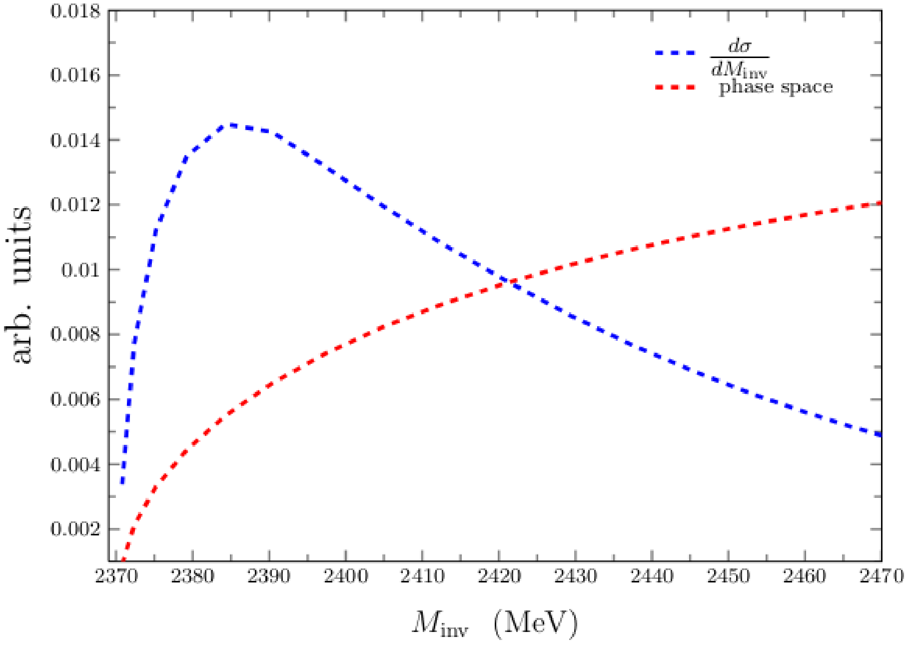
This diagram generates a triangle singularity close to the K^- N threshold, but above it, at an energy around 2380 MeV, p_{k-} = 70 MeV/c, within DAFNE acceptance (140 MeV/c)



for K^- d

Cross section for p Σ^- → K^- d

Preliminary results



The existence of the Λ(1405) (at 1420 MeV) below the K^- p threshold repercutes in a peak above threshold. Thus, looking above threshold with this reaction one is learning about the K^- p amplitude below threshold. A MUCH SEARCHED FOR INFORMATION.

Conclusions

Triangle singularities are catching up the attention of the community, explaining some features formely associated to resonances.

The $a_1(1420)$ case is one of them

We showed that the $pp \to \pi^+ d$ reaction offers a clear case, not identified before as a TS

The sequential one pion production in $np(I=0) \to \pi^- pp$ followed by $pp \to \pi^+ d$ offers a natural explanation of the peak so far associated to the dibaryon $d^*(2380)$.

The $K^- d \to p \Sigma^-$ reaction shows again a TS with a peak above the Kbar N threshold, as a consequence of the presence of the $\Lambda(1405)$ below threshold. By looking at the structure of the cross section above threshold one is learning about the Kbar N amplitudes below threshold, where the different theoretical models differ from each other. THIS WILL BE A NEW SOURCE OF INFORMATION ON THE Kbar N AMPLITUDES BELOW THRESHOLD