Single-particle spectral function of the Λ-hyperon in finite nuclei

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Purpose

✓ Calculation of finite nuclei Λ spectral function

✧ From its self-energy derived within a perturbative many-body approach

✧ Using traditional meson-exchange (Juelich/ Nijmegen) & chiral (at LO & NLO) YN interactions

Results for the chiral potentials in collaboration with Johann Haidenbauer

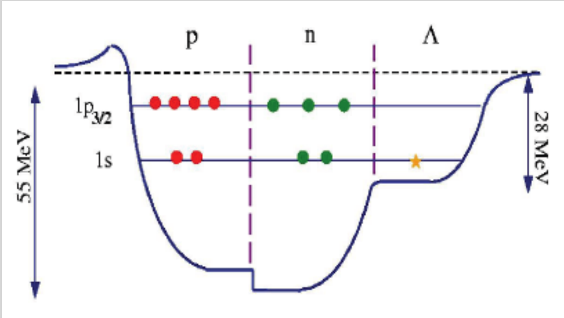
For details see:

I.V., NPA 958, 48 (2017)

J. Haidenbauer & I.V., EPJA 56, 55 (2020)

Mean field picture and Correlations

✧ Most of the theoretical descriptions of single Λ-hypernuclei rely on the **validity of the mean field picture**



✧ **Correlations induced by the YN interaction** can, however, substantially change this picture and, therefore, **should not be ignored**

✧ The knowledge of the **single-particle spectral function of the Λ in finite nuclei** is fundamental to determine:

- ✓ To which extent the mean field description of hypernuclei is valid
- ✓ To describe properly the cross section of different production mechanisms of hypernuclei

$$d\sigma_A \propto \int d\vec{p}_N\, dE_N\, d\sigma S_N\left(\vec{p}_N, E_N\right) S_\Lambda(\vec{p}_\Lambda, E_\Lambda)$$

✧ Information on the **Λ spectral function** can be obtained from a combined analysis of data provided by e.g., (e,e'K$^+$) reactions or other experiments with theoretical calculations

Scheme of the Calculation

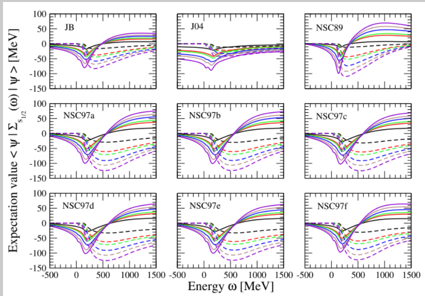
$$G_{NM} = V + V\left(\frac{Q}{E}\right)_{NM} G_{NM}$$

Nuclear matter G-matrix

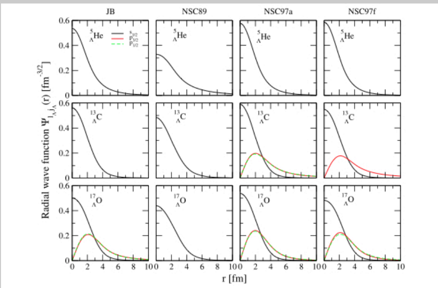
$$G_{FN} = G_{NM} + G_{NM}\left[\left(\frac{Q}{E}\right)_{FN} - \left(\frac{Q}{E}\right)_{NM}\right] G_{FN}$$

Finite nuclei G-matrix

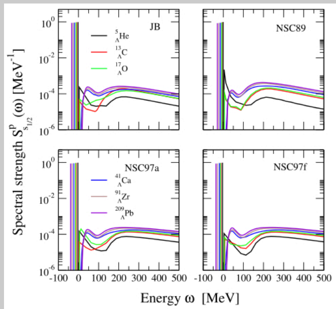
Λ irreducible self-energy in finite nuclei

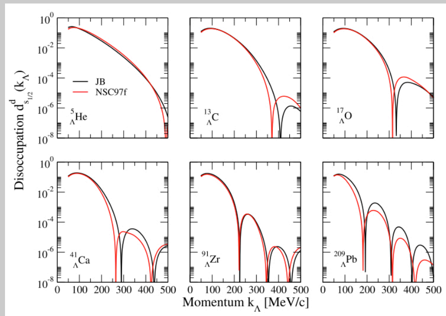


Binding energies, wave functions of s.p. bound states



Finite nuclei Λ spectral function & disoccupation





Finite nuclei hyperon-nucleon G-matrix

- Finite nuclei G-matrix

$$G_{FN} = V + V\left(\frac{Q}{E}\right)_{FN} G_{FN}$$

- Nuclear matter G-matrix

$$G_{NM} = V + V\left(\frac{Q}{E}\right)_{NM} G_{NM}$$

Eliminating V:

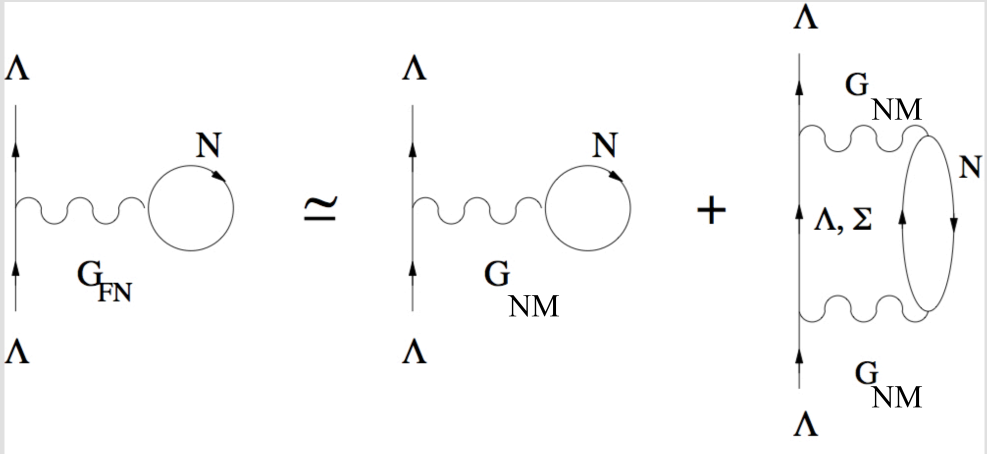
$$G_{FN} = G_{NM} + G_{NM}\left[\left(\frac{Q}{E}\right)_{FN} - \left(\frac{Q}{E}\right)_{NM}\right]G_{FN}$$

Truncating the expansion up second order:

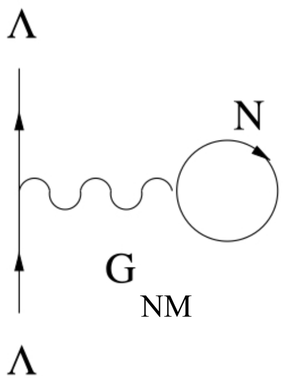
$$G_{FN} \approx G_{NM} + G_{NM}\left[\left(\frac{Q}{E}\right)_{FN} - \left(\frac{Q}{E}\right)_{NM}\right]G_{NM}$$

Finite nucleus Λ self-energy in the BHF approximation

Using G_{FN} as an effective YN interaction, the finite nucleus Λ self-energy is given as sum of a 1st order term & a 2p1h correction



✧ 1st order term

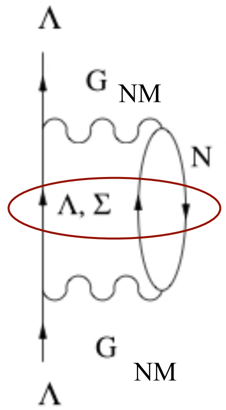


$$\mathcal{V}_1(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda) = \frac{1}{2j_\Lambda + 1} \sum_{\mathcal{J}} \sum_{n_h l_h j_h t_{z_h}} (2\mathcal{J}+1) \times \langle (k'_\Lambda l_\Lambda j_\Lambda)(n_h l_h j_h t_{z_h})\mathcal{J} | G | (k_\Lambda l_\Lambda j_\Lambda)(n_h l_h j_h t_{z_h})\mathcal{J} \rangle$$

This contribution is real & energy-independent

N.B. most of the effort is on the basis transformation $\left|(k_\Lambda l_\Lambda j_\Lambda)(n_h l_h j_h t_{z_h})J\right\rangle \rightarrow \left|KLqLSJTM_T\right\rangle$

✧ 2p1h correction



$$\left[\left(\frac{Q}{E}\right)_{FN} - \left(\frac{Q}{E}\right)_{NM}\right]$$

This contribution is the sum of two terms:

- The first, due to the piece $G_{NM}(Q/E)_{FN}G_{NM}$, gives rise to an imaginary energy-dependent part in the Λ self-energy

$$\mathcal{W}_{2p1h}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda, \omega)$$
$$= -\frac{\pi}{2j_\Lambda+1} \sum_{n_h l_h j_h t_{z_h}} \sum_{\mathcal{L}LSJ\mathcal{J}} \sum_{Y'=\Lambda\Sigma} \int dq q^2 \int dK K^2 (2\mathcal{J}+1)$$
$$\times \langle (k'_\Lambda l_\Lambda j_\Lambda)(n_h l_h j_h t_{z_h})\mathcal{J}|G|K\mathcal{L}qLSJ\mathcal{J}TM_T\rangle$$
$$\times \langle K\mathcal{L}qLSJ\mathcal{J}TM_T|G|(k_\Lambda l_\Lambda j_\Lambda)(n_h l_h j_h t_{z_h})\mathcal{J}\rangle$$
$$\times \delta\left(\omega + \varepsilon_h - \frac{\hbar^2K^2}{2(m_N+m_{Y'})} - \frac{\hbar^2q^2(m_N+m_{Y'})}{2m_N m_{Y'}} - m_{Y'} + m_\Lambda\right)$$

From which can be obtained the contribution to the real part of the self-energy through a dispersion relation

$$\mathcal{V}^{(1)}_{2p1h}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda, \omega) = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} d\omega' \frac{\mathcal{W}_{2p1h}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda, \omega')}{\omega' - \omega}$$

- The second, due to the piece $G_{NM}(Q/E)_{NM}G_{NM}$, gives also a real & energy-independent contribution to the Λ self-energy and avoids double counting of Y'N states

$$
\begin{aligned}
&\mathcal{V}^{(2)}_{2p1h}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda) \\
&\quad = \frac{1}{2j_\Lambda+1} \sum_{n_h l_h j_h t_{z_h}} \sum_{\mathcal{L}LSJ\mathcal{J}} \sum_{Y'=\Lambda\Sigma} \int dq q^2 \int dK K^2 (2\mathcal{J}+1) \\
&\qquad \times \langle (k'_\Lambda l_\Lambda j_\Lambda)(n_h l_h j_h t_{z_h})\mathcal{J} | G | K\mathcal{L}qLSJ\mathcal{J}TM_T\rangle \\
&\qquad \times \langle K\mathcal{L}qLSJ\mathcal{J}TM_T | G | (k_\Lambda l_\Lambda j_\Lambda)(n_h l_h j_h t_{z_h})\mathcal{J}\rangle \\
&\qquad \times Q_{Y'N}\left(\Omega - \frac{\hbar^2K^2}{2(m_N+m_{Y'})} - \frac{\hbar^2 q^2(m_N+m_{Y'})}{2m_N m_{Y'}} - m_{Y'} + m_\Lambda\right)^{-1}
\end{aligned}
$$

Summarizing, in the BHF approximation the finite nucleus Λ self-energy is given by:

$$\Sigma_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) = \mathcal{V}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) + i\mathcal{W}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega)$$

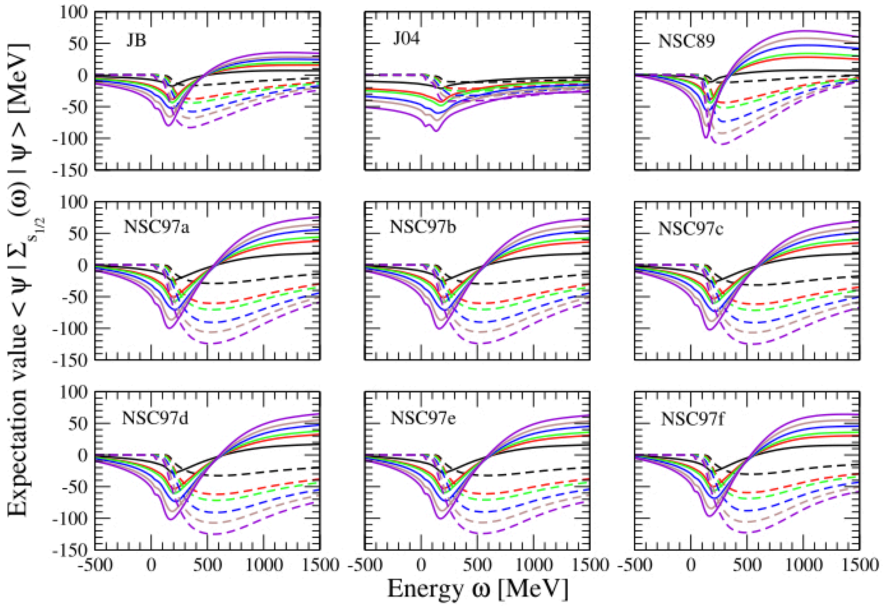
with

$$\mathcal{V}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) = \mathcal{V}_1(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda) + \mathcal{V}^{(1)}_{2p1h}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda, \omega) - \mathcal{V}^{(2)}_{2p1h}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda)$$

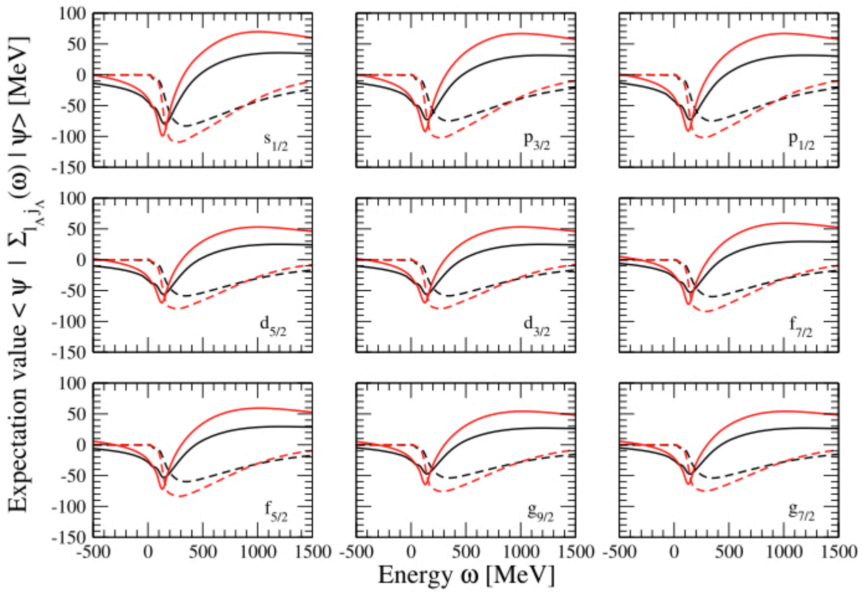
$$\mathcal{W}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) = \mathcal{W}_{2p1h}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda, \omega)$$

Λ self-energy in finite nuclei

s-wave state: He (black), C (red), O (green), Ca (blue), Zr (brown) & Pb (violet)

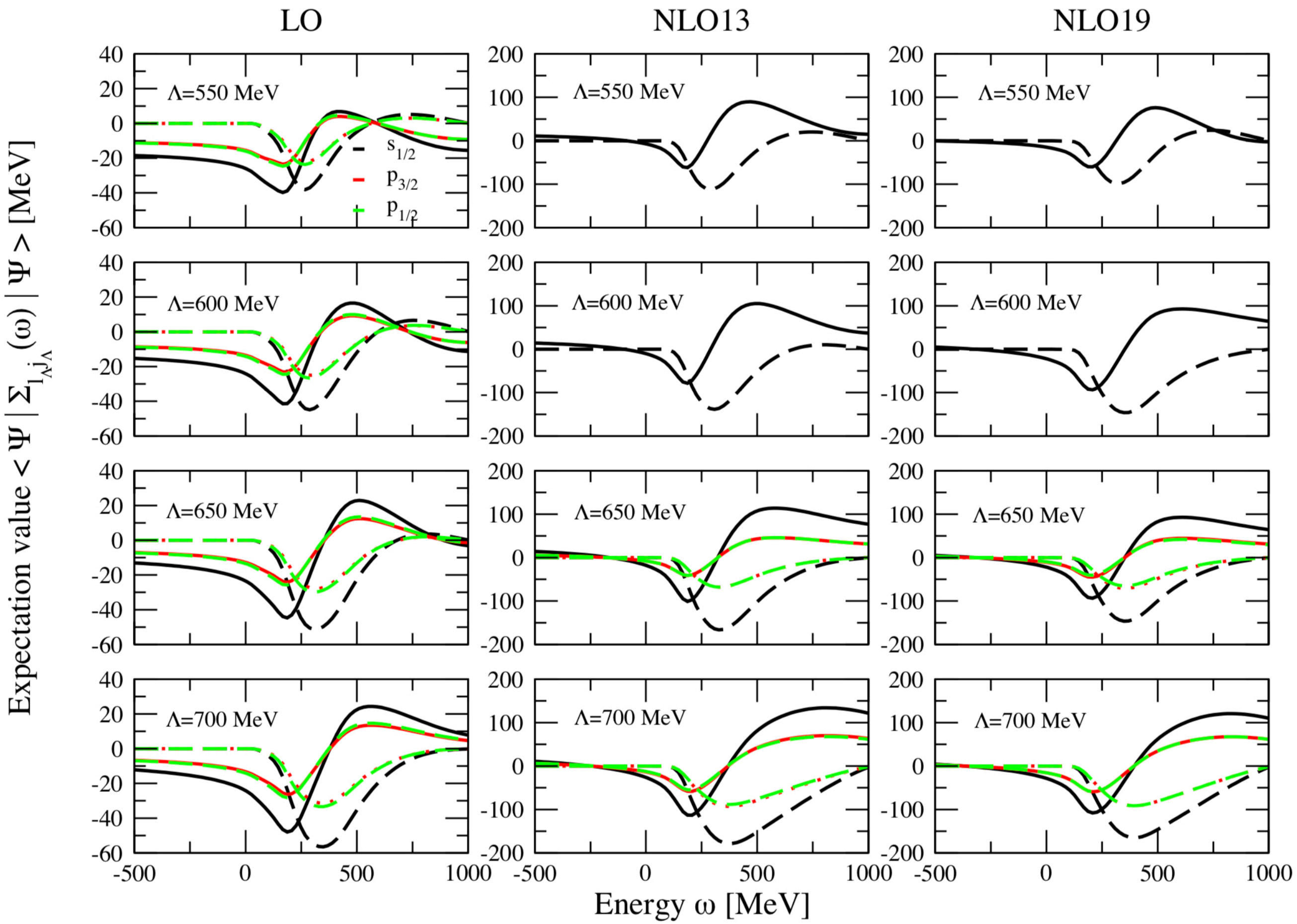


s-, p-, d-, f- and g- wave states for Pb
JB (black) & NSC89 (red)



- |Im <Ψ |Σ|Ψ>| larger in Nijmegen models ⟶ strong ω dependence of Re <Ψ |Σ|Ψ>
- Im <Ψ |Σ|Ψ> ≠ 0 only for ω > 0 & always negative
- Im <Ψ |Σ|Ψ> behaves almost quadratically for energies close to ω = 0
- Re <Ψ |Σ|Ψ> attractive for ω < 0 up to a given value of ω turning repulsive at high ω
- Up to 500-600 MeV Re <Ψ |Σ|Ψ> more attractive for heavier hypernuclei. At higher ω more repulsive than that of lighter ones

Λ self-energy in $^{17}{}_{\Lambda}O$ with chiral YN potentials



- ✧ In general similar qualitative behavior but:
  - ➢ Im <Ψ |Σ|Ψ> is not always negative for ω > 0 with the lower cut-off
  - ➢ Re <Ψ |Σ|Ψ> slightly repulsive for very negatives ω < 0 in the case of NLO

Λ single-particle bound states

Λ s.p. bound states can be obtained using the real part of the Λ self-energy as an effective hyperon-nucleus potential in the Schroedinger equation

$$\sum_{i=1}^{N_{max}} \left[\frac{\hbar^2 k_i^2}{2m_\Lambda} + \mathcal{V}_{l_\Lambda j_\Lambda}(k_n, k_i, \omega = \varepsilon_{l_\Lambda j_\Lambda}) \right] \Psi_{i l_\Lambda j_\Lambda m_{j_\Lambda}} = \varepsilon_{l_\Lambda j_\Lambda} \Psi_{n l_\Lambda j_\Lambda m_{j_\Lambda}}$$

solved by diagonalizing the Hamiltonian in a complete & orthonormal set of regular basis functions within a spherical box of radius R_{box}

$$\Phi_{n l_\Lambda j_\Lambda m_{j_\Lambda}}(\vec{r}) = \langle \vec{r} | k_n l_\Lambda j_\Lambda m_{j_\Lambda} \rangle = N_{n l_\Lambda} \, j_{l_\Lambda}(k_n r) \psi_{l_\Lambda j_\Lambda m_{j_\Lambda}}(\theta, \phi)$$

- $N_{nl\Lambda}$ $\longrightarrow$ normalization constant
- N_{max} $\longrightarrow$ maximum number of basis states in the box
- $j_{j\Lambda}(k_n r)$ $\longrightarrow$ Bessel functions for discrete momenta ($j_{j\Lambda}(k_n R_{box})=0$)
- $\psi_{l\Lambda j\Lambda m j\Lambda}(\theta,\phi)$ $\longrightarrow$ spherical harmonics the including spin d.o.f.
- $\Psi_{nl\Lambda j\Lambda m j\Lambda} = \langle k_n l_\Lambda j_\Lambda m_{j\Lambda} | \Psi \rangle$ $\longrightarrow$ projection of the state $|\Psi\rangle$ on the basis $|k_n l_\Lambda j_\Lambda m_{j\Lambda}\rangle$

N.B. a self-consistent procedure is required for each eigenvalue

Λ single-particle bound states: Energy (meson-exchange YN potentials)

Nuclei	$l_\Lambda j_\Lambda$	JB	J04	NSC89	NSC97a	NSC97b	NSC97c	NSC97d	NSC97e	NSC97f	Exp.
$^{5}_{\Lambda}$He											($^{5}_{\Lambda}$He)
	$s_{1/2}$	−2.28	−5.89	−0.58	−3.16	−3.38	−3.94	−4.24	−4.20	−3.59	−3.12(2)
$^{13}_{\Lambda}$C											($^{13}_{\Lambda}$C)
	$s_{1/2}$	−9.48	−18.94	−5.69	−11.46	−11.79	−12.76	−13.08	−12.82	−11.37	−11.69(12)
	$p_{3/2}$		−3.66		−0.24	−0.32	−0.63	−0.68	−0.54	−0.01	−0.8(3) (p)
	$p_{1/2}$		−4.07		−0.12	−0.14	−0.37	−0.35	−0.19		
$^{17}_{\Lambda}$O											($^{16}_{\Lambda}$O)
	$s_{1/2}$	−11.83	−23.40	−7.39	−14.31	−14.65	−15.70	−15.99	−15.68	−14.02	−13.0(2)
	$p_{3/2}$	−0.87	−8.16		−2.57	−2.72	−3.24	−3.33	−3.10	−2.17	−2.5(2) (p)
	$p_{1/2}$	−1.06	−8.03		−2.16	−2.22	−2.61	−2.57	−2.30	−1.41	
$^{41}_{\Lambda}$Ca											($^{40}_{\Lambda}$Ca)
	$s_{1/2}$	−19.60	−36.16	−15.04	−23.09	−23.42	−24.60	−24.74	−24.20	−21.96	−18.7(1.1)
	$p_{3/2}$	−9.64	−23.81	−6.92	−12.37	−12.57	−13.40	−13.35	−12.84	−11.09	−11.0(5) (p)
	$p_{1/2}$	−9.92	−23.78	−6.29	−12.10	−12.23	−12.95	−12.78	−12.22	−10.45	
	$d_{5/2}$	−0.70	−11.72		−2.80	−2.93	−3.47	−3.38	−3.00	−1.83	−1.0(5) (d)
	$d_{3/2}$	−1.01	−11.65		−2.43	−2.46	−2.85	−2.61	−2.18	−1.04	
$^{91}_{\Lambda}$Zr											($^{89}_{\Lambda}$Y)
	$s_{1/2}$	−25.80	−46.30	−22.77	−31.38	−31.73	−33.05	−33.06	−32.33	−29.56	−23.6(5)
	$p_{3/2}$	−18.19	−37.73	−17.08	−23.92	−24.20	−25.28	−25.22	−24.58	−22.25	−17.7(6) (p)
	$p_{1/2}$	−18.30	−38.01	−16.68	−23.82	−24.06	−25.07	−24.92	−24.23	−21.88	
	$d_{5/2}$	−11.16	−28.35	−9.05	−14.41	−14.58	−15.36	−15.09	−14.42	−12.41	−10.9(6) (d)
	$d_{3/2}$	−11.17	−28.44	−8.49	−14.30	−14.40	−15.12	−14.77	−14.06	−11.99	
	$f_{7/2}$	−3.05	−18.45	−1.56	−5.46	−5.52	−6.03	−5.59	−4.93	−3.27	−3.7(6) (f)
	$f_{5/2}$	−2.99	−18.76	−1.00	−5.28	−5.26	−5.69	−5.20	−4.52	−2.86	
$^{209}_{\Lambda}$Pb											($^{208}_{\Lambda}$Pb)
	$s_{1/2}$	−31.36	−59.95	−29.52	−38.85	−39.23	−40.63	−40.44	−39.50	−39.30	−26.9(8)0
	$p_{3/2}$	−27.13	−55.21	−26.01	−33.49	−33.91	−35.13	−34.80	−33.86	−31.03	−22.5(6) (p)
	$p_{1/2}$	−27.18	−55.40	−25.72	−33.38	−33.78	−34.94	−34.54	−33.56	−30.72	
	$d_{5/2}$	−21.70	−45.08	−17.85	−23.23	−23.54	−24.38	−23.79	−22.858	−20.60	−17.4(7) (d)
	$d_{3/2}$	−21.77	−45.07	−17.65	−23.17	−23.45	−24.27	−23.68	−22.75	−20.51	
	$f_{7/2}$	−13.00	−37.15	−9.67	−15.38	−15.43	−16.04	−15.05	−13.81	−10.98	−12.3(6) (f)
	$f_{5/2}$	−13.13	−37.16	−9.31	−15.35	−15.33	−15.90	−14.87	−13.61	−10.76	
	$g_{9/2}$	−8.14	−29.91	−5.27	−10.07	−10.14	−10.68	−9.80	−8.71	−6.28	−7.2(6) (g)
	$g_{7/2}$	−8.26	−30.16	−4.80	−10.01	−10.00	−10.46	−9.49	−8.37	−5.91	

- ✧ Qualitatively good agreement with experiment, except for J04 (unrealistic overbinding)
- ✧ Zr & Pb overbound also for NSC97a-f models. These models predict $U_\Lambda(0) \sim$ -40 MeV compared with -30 MeV extrapolated from data
- ✧ Splitting of p-, d-, f- and g-waves of ~ few tenths of MeV due to the small spin-orbit strength of YN interaction

Λ single-particle bound states: Energy (chiral YN potentials)

Λ (MeV)	LO 550	LO 600	LO 650	LO 700	NLO13 500	NLO13 550	NLO13 600	NLO13 650	NLO13 700	NLO19 500	NLO19 550	NLO19 600	NLO19 650	NLO19 700	Exp.
$^{5}_{\Lambda}$He															$^{5}_{\Lambda}$He
$s_{1/2}$	−4.04	−3.32	−3.06	−3.26	−0.73	−0.15	−0.63	−2.36	−4.90	−2.16	−1.36	−1.77	−3.42	−5.63	−3.12(2)
$^{13}_{\Lambda}$C															$^{13}_{\Lambda}$C
$s_{1/2}$	−12.33	−11.01	−10.54	−10.93	−4.44	−2.24	−3.72	−8.91	−13.40	−8.91	−6.42	−7.22	−10.81	−14.98	−11.69(12)
$p_{3/2}$	–	–	–	–	–	–	–	–	−1.22	–	–	–	−0.12	−1.76	−0.8(3) (p)
$p_{1/2}$	−1.11	−0.58	−0.45	−0.72	–	–	–	–	−0.97	–	–	–	–	−1.40	
$^{17}_{\Lambda}$O															$^{16}_{\Lambda}$O
$s_{1/2}$	−16.12	−14.64	−14.13	−14.65	−6.07	−3.46	−5.35	−10.51	−16.37	−11.46	−8.61	−9.55	−13.60	−18.18	−13.0(2)
$p_{3/2}$	−3.16	−2.29	−2.02	−2.30	–	–	–	−1.22	−4.04	−1.26	−0.14	−0.53	−2.40	−4.89	−2.5(2) (p)
$p_{1/2}$	−3.47	−2.64	−2.41	−2.76	–	–	–	−0.66	−3.31	−0.51	–	–	−1.69	−4.10	
$^{41}_{\Lambda}$Ca															$^{40}_{\Lambda}$Ca
$s_{1/2}$	−24.83	−23.17	−22.66	−23.26	−12.37	−8.78	−11.24	−17.56	−24.36	−19.51	−15.86	−16.80	−21.30	−26.47	−18.7(1.1)
$p_{3/2}$	−14.50	−13.05	−12.54	−12.95	−4.95	−2.54	−3.98	−8.82	−13.43	−9.91	−6.93	−7.48	−11.04	−15.06	−11.0(5) (p)
$p_{1/2}$	−14.70	−13.28	−12.81	−13.25	−4.37	−2.08	−3.50	−7.73	−12.87	−9.13	−6.23	−6.82	−10.42	−14.47	
$d_{5/2}$	−4.61	−3.45	−3.01	−3.23	–	–	–	−0.40	−3.59	−1.47	–	–	−1.99	−4.67	−1.0(5) (d)
$d_{3/2}$	−6.91	−5.64	−5.18	−5.51	–	–	–	−0.50	−4.02	−0.56	–	–	−1.20	−3.84	
$^{91}_{\Lambda}$Zr															$^{89}_{\Lambda}$Y
$s_{1/2}$	−31.27	−29.22	−28.48	−29.11	−19.36	−14.66	−17.83	−25.10	−32.50	−27.72	−22.57	−23.19	−28.94	−34.61	−23.6(5)
$p_{3/2}$	−24.31	−22.43	−21.72	−22.22	−14.24	−10.59	−13.27	−19.27	−25.45	−20.59	−16.24	−16.94	−22.05	−26.96	−17.7(6) (p)
$p_{1/2}$	−24.80	−22.96	−22.28	−22.80	−13.95	−10.39	−13.05	−19.07	−25.31	−20.45	−15.96	−16.67	−21.86	−26.82	
$d_{5/2}$	−16.60	−14.79	−14.09	−14.30	−6.21	−3.33	−5.24	−10.30	−15.27	−11.92	−8.10	−8.44	−12.68	−16.78	−10.9(6) (d)
$d_{3/2}$	−17.57	−15.80	−15.06	−15.40	−5.80	−2.98	−4.88	−9.70	−14.97	−11.65	−7.61	−7.98	−12.27	−16.40	
$f_{7/2}$	−8.69	−7.04	−6.25	−6.36	–	–	–	−1.68	−5.63	−4.04	−0.98	−0.89	−3.97	−7.04	−3.7(6) (f)
$f_{5/2}$	−10.17	−8.58	−7.85	−8.04	–	–	–	−1.28	−5.23	−3.59	−0.33	−0.28	−3.39	−6.54	
$^{209}_{\Lambda}$Pb															$^{208}_{\Lambda}$Pb
$s_{1/2}$	−40.97	−38.18	−36.91	−37.32	−25.75	−21.41	−25.09	−32.28	−39.51	−36.28	−29.50	−29.60	−35.84	−41.58	−26.9(8)
$p_{3/2}$	−37.62	−34.85	−33.42	−33.50	−21.88	−15.77	−18.33	−25.13	−31.83	−33.72	−26.73	−25.27	−30.26	−34.71	−22.5(6) (p)
$p_{1/2}$	−37.81	−35.05	−33.62	−33.69	−21.55	−15.53	−18.14	−25.00	−31.74	−33.58	−26.57	−25.13	−30.17	−34.64	
$d_{5/2}$	−29.57	−26.94	−25.45	−25.29	−14.47	−8.79	−9.96	−14.78	−19.98	−25.49	−19.28	−16.84	−20.08	−23.15	−17.4(7) (d)
$d_{3/2}$	−30.03	−27.44	−26.00	−25.87	−14.35	−8.71	−9.83	−14.62	−19.83	−25.29	−18.98	−16.57	−19.85	−22.97	
$f_{7/2}$	−22.85	−20.33	−18.92	−18.79	−4.46	–	–	−5.91	−12.57	−16.23	−10.15	−7.91	−11.90	−15.80	−12.3(6) (f)
$f_{5/2}$	−23.45	−20.95	−19.57	−19.48	−4.42	–	–	−5.60	−12.24	−15.96	−9.70	−7.47	−11.47	−15.38	
$g_{9/2}$	−20.33	−17.75	−16.23	−15.96	−1.87	–	–	−3.23	−9.21	−13.72	−7.55	−5.18	−8.92	−12.32	−7.2(6) (g)
$g_{7/2}$	−21.47	−18.96	−17.50	−17.29	−1.38	–	–	−2.91	−8.94	−13.38	−7.03	−4.69	−8.53	−12.00	

✧ LO

Tendency for overbinding

✧ NLO13

Predict less bound states. Underbinds most of the considered hypernuclei

✧ NLO19

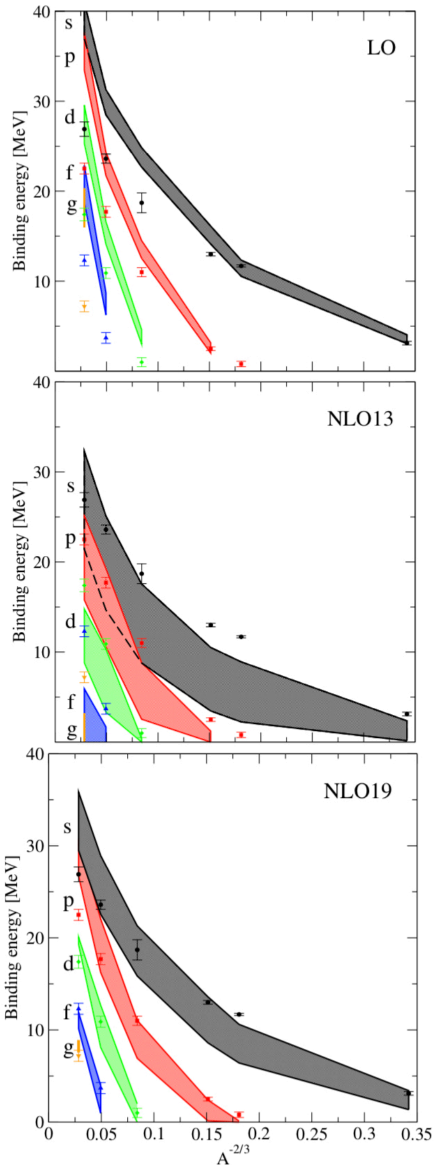
Qualitatively good agreement considering the uncertainty of the regulator dependence

✧ Small splitting of p-, d-, f- and g-waves as in the case of meson-exchange potentials

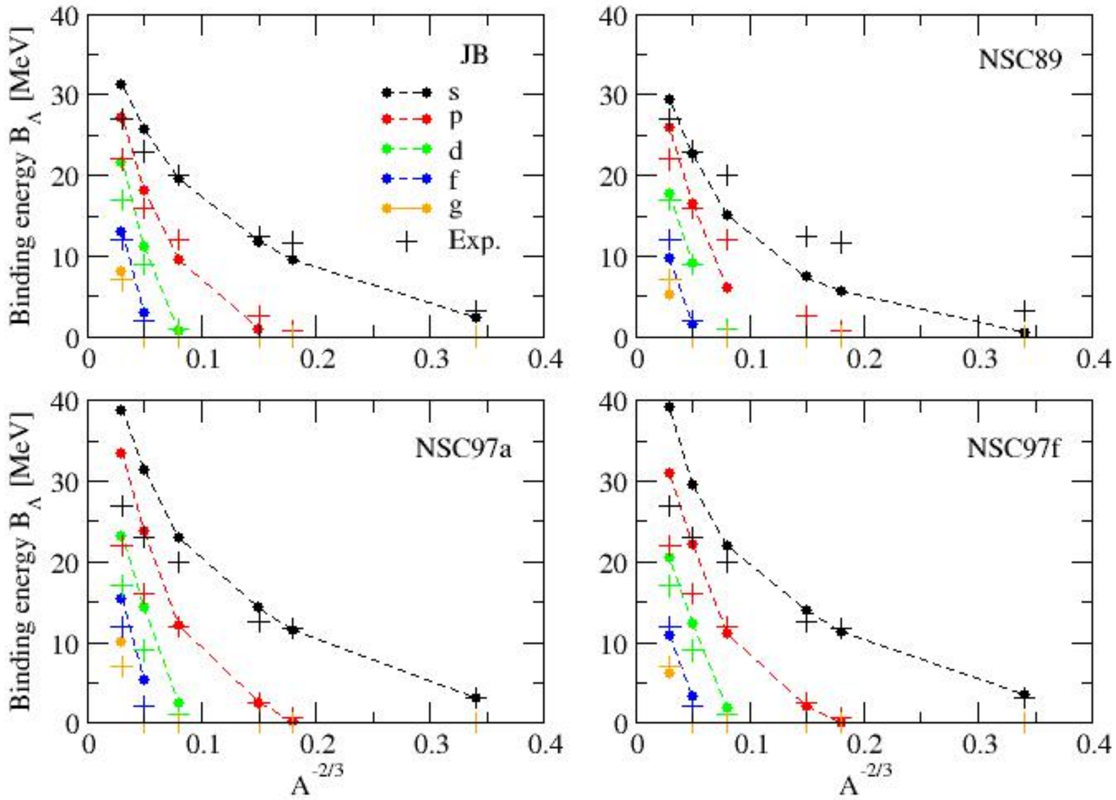
J. Haidenbauer & I.V., EPJA 56, 55 (2020)

Λ Binding Energy

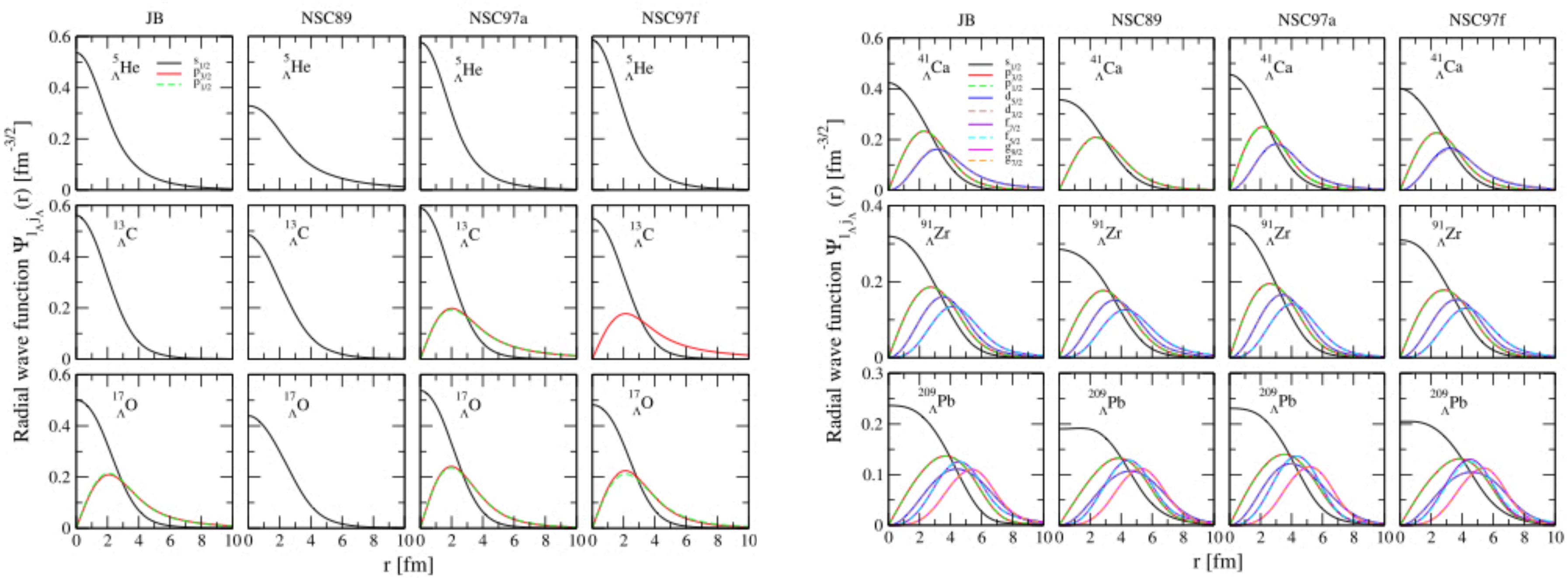
YN chiral potentials



YN meson-exchange potentials



Λ single-particle bound states: Radial Wave Function

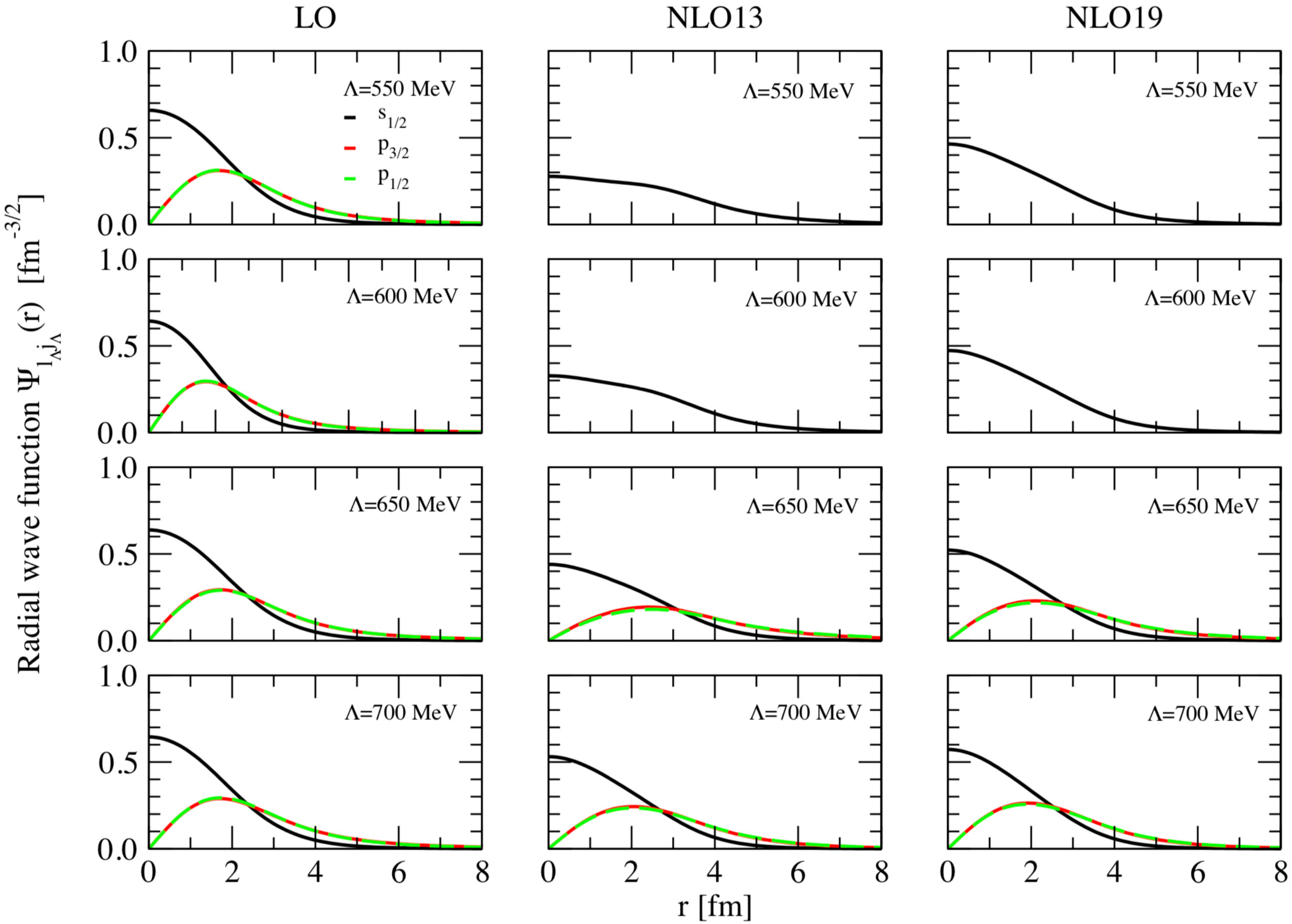


- ✧ $\Psi_{s1/2}$ state more and more spread when going from light to heavy hypernuclei ⟶ probability of finding the Λ at the center of the hypernuclei ($|\Psi_{s1/2}(r=0)|^2$) decreases.

- ✧ Only He falls out this pattern because the energy of the $s_{1/2}$ state is too low, therefore, resulting in a very extended wave function

- ✧ The small spin-orbit splitting of the p-, d-, f- and g-wave states cannot be resolved in the corresponding wave functions

Λ single-particle bound states: Radial Wave Function
$^{17}_{\Lambda}O$ with chiral YN potentials



✧ s- and p-wave states more extended for the NLO pontentials.

General Remarks on the s.p. Spectral Function

Single-particle Green's function (Lehmann representation):

$$g_{\alpha\beta}(\omega) = \int_{E_0^{N+1}-E_0^N}^{\infty} d\omega' \frac{S^p_{\alpha\beta}(\omega')}{\omega - \omega' + i\eta} + \int_{-\infty}^{E_0^N - E_0^{N-1}} d\omega' \frac{S^h_{\alpha\beta}(\omega')}{\omega - \omega' - i\eta}$$

Describes propagation of a particle or a hole added to a N-particle system

being

$$S^p_{\alpha\beta}(\omega) = \sum_m \langle\Psi_0^N|\hat{c}_\alpha|\Psi_m^{N+1}\rangle\langle\Psi_m^{N+1}|\hat{c}^\dagger_\beta|\Psi_0^N\rangle\delta(\omega - (E_m^{N+1} - E_0^N)), \;\; \omega > E_0^{N+1} - E_0^N$$

$$S^h_{\alpha\beta}(\omega) = \mp\sum_n \langle\Psi_0^N|\hat{c}^\dagger_\beta|\Psi_n^{N-1}\rangle\langle\Psi_n^{N-1}|\hat{c}_\alpha|\Psi_0^N\rangle\delta(\omega - (E_0^N - E_n^{N-1})), \;\; \omega < E_0^N - E_0^{N-1}$$

Particle & hole part of the s.p. spectral function

Diagonal parts $S^p_{\alpha\alpha}$ & $S^h_{\alpha\alpha}$ = probability density of adding or removing a particle to the ground state of the N-particle system & finding the resulting N+1 (N-1) one with energy $\omega-(E^{N+1}_0-E^N_0)$ or $(E^N_0-E^{N-1}_0)-\omega$

The case of the single-particle Λ spectral function

In the case of a Λ hyperon that is added to a pure nucleonic system (e.g., infinite nuclear matter or an ordinary nuclei), it is clear, that since there are no other Λ's in the N-particle pure nucleonic system, the Λ can only be added to it and, therefore, **the hole part of its spectral function is zero**

The Lehmann representation of the single-Λ propagator is simply:

$$g^{\Lambda}_{\alpha\beta}(\omega) = \int_{E_0^{N+\Lambda}-E_0^N}^{\infty} d\omega' \frac{S^{\Lambda p}_{\alpha\beta}(\omega')}{\omega - \omega' + i\eta}$$

Λ Spectral Strength

In any production mechanism of single-Λ hypernuclei a Λ can be formed in a bound or in a scattering state $\longrightarrow$ the Λ particle spectral function is sum of a discrete & a continuum contribution

✧ Discrete contribution

$$S^{p(d)}_{l_\Lambda j_\Lambda}(k_n, \omega) = Z_{l_\Lambda j_\Lambda} |\langle k_n l_\Lambda j_\Lambda m_{j_\Lambda} | \Psi \rangle|^2 \delta(\omega - \varepsilon_{l_\Lambda j_\Lambda})$$

is a delta function located at the energy of the s.p. bound state with strength given by the Z-factor

$$Z_{l_\Lambda j_\Lambda} = \left(1 - \frac{\partial \langle \Psi | \Sigma_{l_\Lambda j_\Lambda}(\omega) | \Psi \rangle}{\partial \omega} \Big|_{\omega = \varepsilon_{l_\Lambda j_\Lambda}} \right)^{-1}$$

The **discrete contribution to the total Λ spectral strength** is obained by summing over all discrete momenta k_n

$$S^{p(d)}_{l_\Lambda j_\Lambda}(\omega) = Z_{l_\Lambda j_\Lambda} \delta(\omega - \varepsilon_{l_\Lambda j_\Lambda})$$

Λ Spectral Strength

✧ Continuum contribution

$$S^{p(c)}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) = \lim_{\eta\to 0^+} \frac{i}{2\pi}\left(g^\Lambda_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega + i\eta) - g^\Lambda_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega - i\eta)\right) \xrightarrow[\text{Lehmann rep.}]{\text{using}} -\frac{1}{\pi}\mathrm{Im}\, g^\Lambda_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega)$$

where the single-Λ propagator can be derived from the following form of the Dyson equation

$$g^\Lambda_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) = \frac{\delta(k_\Lambda - k'_\Lambda)}{k^2_\Lambda}\underbrace{g^{(0)}_\Lambda(k_\Lambda, \omega)}_{\text{Free s.p. propagator}} + g^{(0)}_\Lambda(k_\Lambda, \omega)\underbrace{\Sigma^{red}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega)}_{\text{Reducible } \Lambda \text{ self-energy}} g^{(0)}_\Lambda(k'_\Lambda, \omega)$$

$$\Sigma^{red}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) = \Sigma_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) + \int dq_\Lambda q^2_\Lambda \Sigma_{l_\Lambda j_\Lambda}(k_\Lambda, q_\Lambda, \omega) g^{(0)}_\Lambda(q_\Lambda, \omega)\Sigma^{red}_{l_\Lambda j_\Lambda}(q_\Lambda, k'_\Lambda, \omega)$$

Λ Spectral Strength

Due to the delta function in the Dyson equation is numerically more convenient to obtain the continuum contribution of the Λ spectral function in coordinate space

$$S^{p(c)}_{l_\Lambda j_\Lambda}(r_\Lambda, r'_\Lambda, \omega) = \frac{2}{\pi}\int_0^\infty dk_\Lambda k_\Lambda^2 \int_0^\infty dk'_\Lambda k'^2_\Lambda j_{l_\Lambda}(k_\Lambda r_\Lambda) S^{p(c)}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) j_{l_\Lambda}(k'_\Lambda r'_\Lambda)$$

The **continuum contribution to the total Λ spectral strength** is obtained from the following double folding of the spectral function

$$S^{p(c)}_{l_\Lambda j_\Lambda}(\omega) = \int_0^\infty dr_\Lambda r_\Lambda^2 \int_0^\infty dr'_\Lambda r'^2_\Lambda \Psi_{l_\Lambda j_\Lambda}(r_\Lambda) S^{p(c)}_{l_\Lambda j_\Lambda}(r_\Lambda, r'_\Lambda, \omega) \Psi_{l_\Lambda j_\Lambda}(r'_\Lambda)$$

Λ Spectral Strength

where

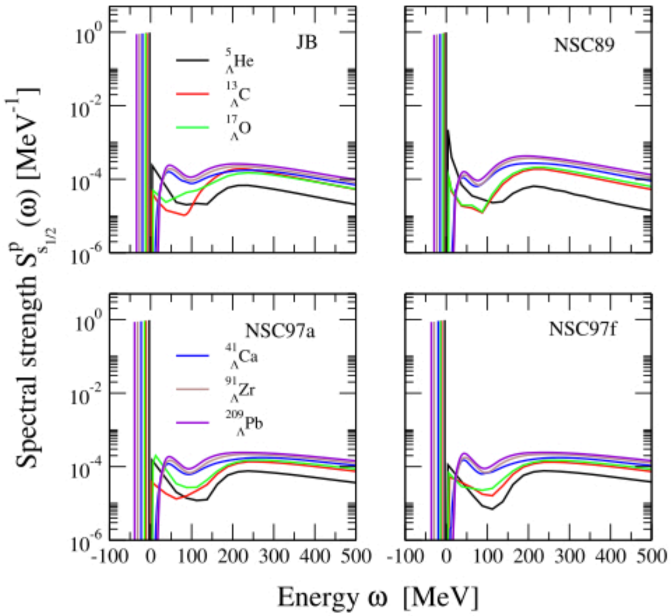
$$
\begin{aligned}
S^{p(c)}_{l_\Lambda j_\Lambda}(r_\Lambda, r'_\Lambda, \omega) = {} & \frac{2}{\pi}\frac{m_\Lambda k_0}{\hbar^2} j_{l_\Lambda}(k_0 r_\Lambda) j_{l_\Lambda}(k_0 r'_\Lambda) \\
& + 2\left(\frac{m_\Lambda k_0}{\hbar^2}\right)^2 j_{l_\Lambda}(k_0 r_\Lambda) j_{l_\Lambda}(k_0 r'_\Lambda) \mathrm{Im}\, \Sigma^{red}_{l_\Lambda j_\Lambda}(k_0, k_0, \omega) \\
& + \frac{2}{\pi}\frac{m_\Lambda k_0}{\hbar^2} j_{l_\Lambda}(k_0 r_\Lambda) \mathcal{P}\int_0^\infty dk'_\Lambda k'^2_\Lambda \frac{j_{l_\Lambda}(k'_\Lambda r'_\Lambda) \mathrm{Re}\, \Sigma^{red}_{l_\Lambda j_\Lambda}(k_0, k'_\Lambda, \omega)}{\omega - \frac{\hbar^2 k'^2_\Lambda}{2m_\Lambda}} \\
& + \frac{2}{\pi}\frac{m_\Lambda k_0}{\hbar^2} j_{l_\Lambda}(k_0 r'_\Lambda) \mathcal{P}\int_0^\infty dk_\Lambda k^2_\Lambda \frac{j_{l_\Lambda}(k_\Lambda r_\Lambda) \mathrm{Re}\, \Sigma^{red}_{l_\Lambda j_\Lambda}(k_\Lambda, k_0, \omega)}{\omega - \frac{\hbar^2 k^2_\Lambda}{2m_\Lambda}} \\
& - \frac{2}{\pi^2}\mathcal{P}\int_0^\infty dk_\Lambda k^2_\Lambda \frac{j_{l_\Lambda}(k_\Lambda r_\Lambda)}{\omega - \frac{\hbar^2 k^2_\Lambda}{2m_\Lambda}} \mathcal{P}\int_0^\infty dk'_\Lambda k'^2_\Lambda \frac{j_{l_\Lambda}(k'_\Lambda r'_\Lambda) \mathrm{Im}\, \Sigma^{red}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega)}{\omega - \frac{\hbar^2 k'^2_\Lambda}{2m_\Lambda}}
\end{aligned}
$$

Total Λ spectral strength

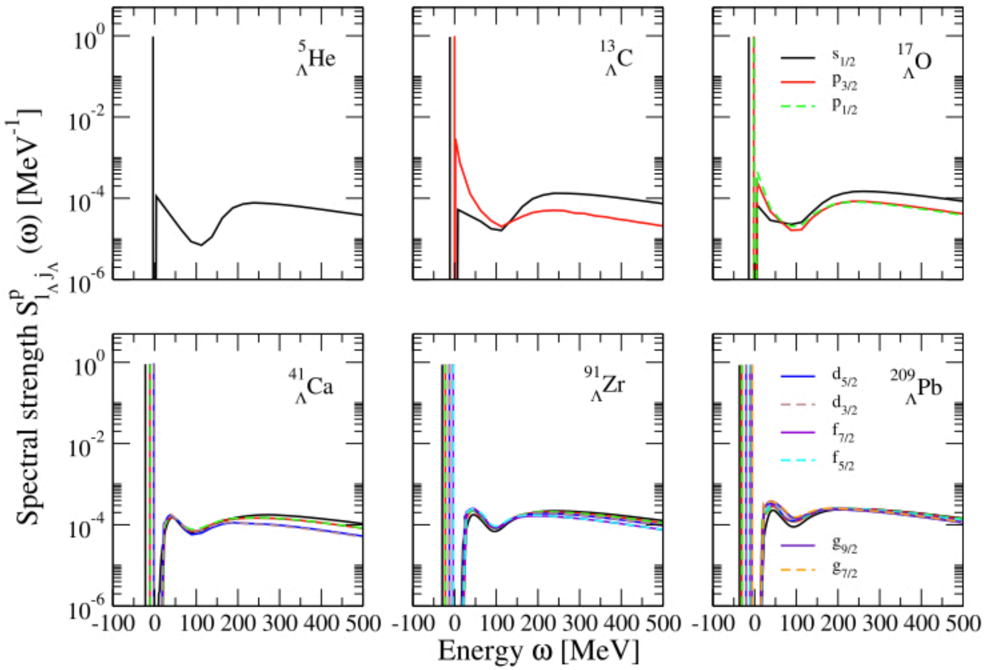
$$
S^{p}_{l_\Lambda j_\Lambda}(\omega) = S^{p(d)}_{l_\Lambda j_\Lambda}(\omega) + S^{p(c)}_{l_\Lambda j_\Lambda}(\omega)
$$

Λ Spectral Strength: Results

s-wave state: He (black), C (red), O (green), Ca (blue), Zr (brown) & Pb (violet)



s-, p-, d-, f- and g- wave states (NSC97f)

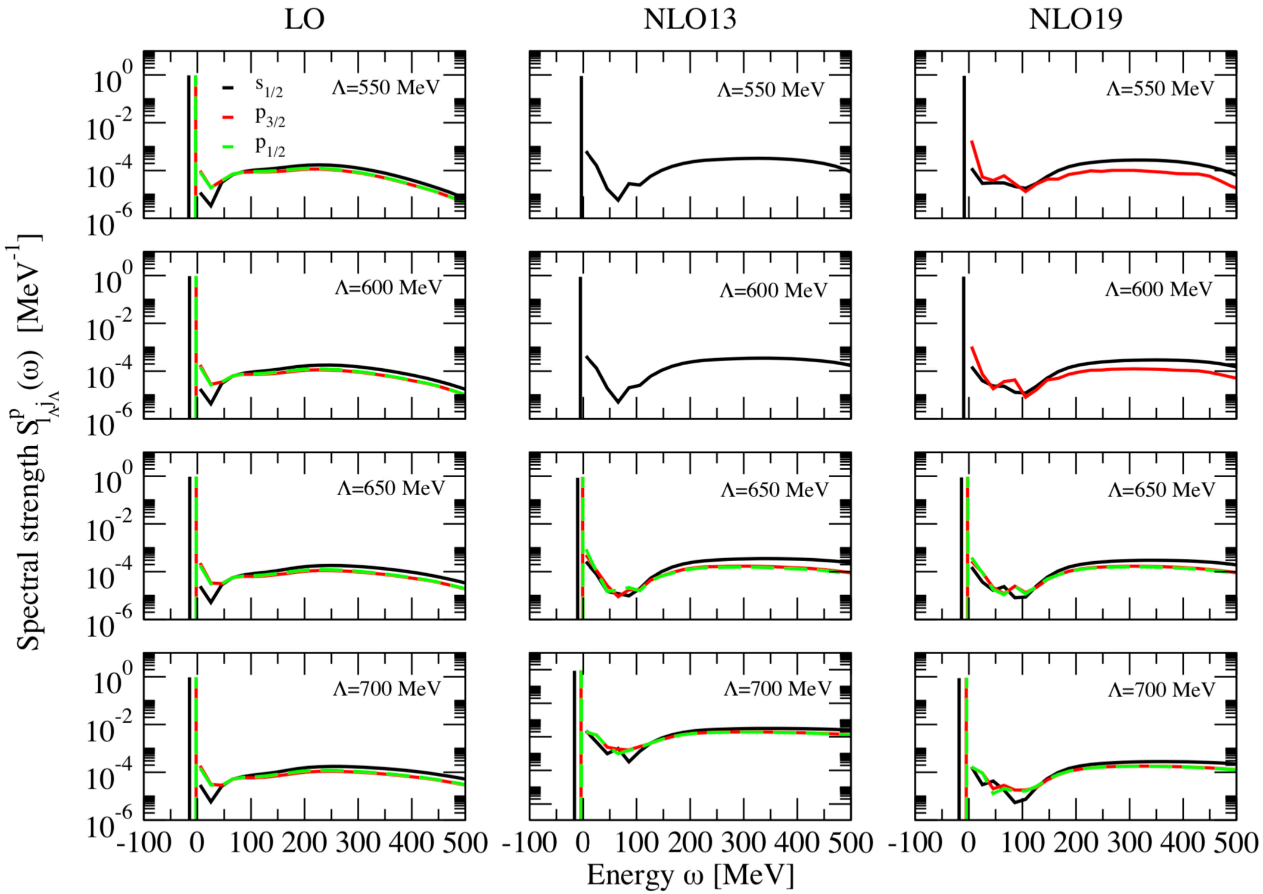


- ✧ Discrete contribution: delta function located at the energy of the s.p. bound state with strength given by the Z-factor. Decreases when moving from light to heavy nuclei $\longrightarrow$ ΛN correlations more important when the size of the nuclear core increases

- ✧ Continuum contribution: strength spread over all positive energies. Structure for $\omega < 100$ MeV reflects the behavior of self-energy. Monotonically reduction for $\omega > 200$

Λ Spectral Strength: Results

$^{17}_{\Lambda}$O with chiral YN potentials



✧ Results qualitatively similar to those obtained with meson-exchange potentials

ΛN correlations: Z-factor

$$Z_{l_\Lambda j_\Lambda} = \left(1 - \frac{\partial\langle\Psi|\Sigma_{l_\Lambda j_\Lambda}(\omega)|\Psi\rangle}{\partial\omega}\Big|_{\omega=\varepsilon_{l_\Lambda j_\Lambda}}\right)^{-1}$$

Z measures the importance of correlations. The smaller the value of Z the more important are the correlations of the system

- Z is relatively large for all hypernuclei $\longrightarrow$ Λ keeps its identity inside the nucleus & is less correlated than nucleons
- Z decreases from light to heavy hypernuclei $\longrightarrow$ ΛN correlations increase when the size of the nuclear core increases
- Z increases when increasing the partial wave $\longrightarrow$ ΛN correlations become less important for higher partial waves

Nuclei	$l_\Lambda j_\Lambda$	JB	NSC89	NSC97a	NSC97f
${}^{5}_{\Lambda}$He	$s_{1/2}$	0.976	0.983	0.965	0.964
${}^{13}_{\Lambda}$C	$s_{1/2}$	0.950	0.940	0.933	0.933
	$p_{3/2}$	–	–	0.975	0.979
	$p_{1/2}$	–	–	0.976	
${}^{17}_{\Lambda}$O	$s_{1/2}$	0.942	0.930	0.923	0.924
	$p_{3/2}$	0.973	–	0.956	0.959
	$p_{1/2}$	0.971	–	0.957	0.961
${}^{41}_{\Lambda}$Ca	$s_{1/2}$	0.920	0.896	0.898	0.898
	$p_{3/2}$	0.930	0.915	0.911	0.914
	$p_{1/2}$	0.929	0.914	0.910	0.912
	$d_{5/2}$	0.952	–	0.932	0.938
	$d_{3/2}$	0.949	–	0.931	0.939
${}^{91}_{\Lambda}$Zr	$s_{1/2}$	0.904	0.870	0.879	0.876
	$p_{3/2}$	0.906	0.875	0.884	0.883
	$p_{1/2}$	0.907	0.876	0.885	0.883
	$d_{5/2}$	0.910	0.886	0.891	0.893
	$d_{3/2}$	0.911	0.886	0.891	0.891
	$f_{7/2}$	0.919	0.903	0.903	0.906
	$f_{5/2}$	0.920	0.905	0.902	0.907
${}^{209}_{\Lambda}$Pb	$s_{1/2}$	0.884	0.846	0.857	0.856
	$p_{3/2}$	0.885	0.847	0.858	0.857
	$p_{1/2}$	0.885	0.847	0.858	0.857
	$d_{5/2}$	0.896	0.858	0.870	0.869
	$d_{3/2}$	0.896	0.857	0.869	0.867
	$f_{7/2}$	0.891	0.852	0.863	0.857
	$f_{5/2}$	0.891	0.851	0.863	0.855
	$g_{9/2}$	0.892	0.855	0.869	0.862
	$g_{7/2}$	0.892	0.854	0.868	0.860

ΛN correlations: Z-factor of $^{17}{}_{\Lambda}O$ with chiral YN potentials

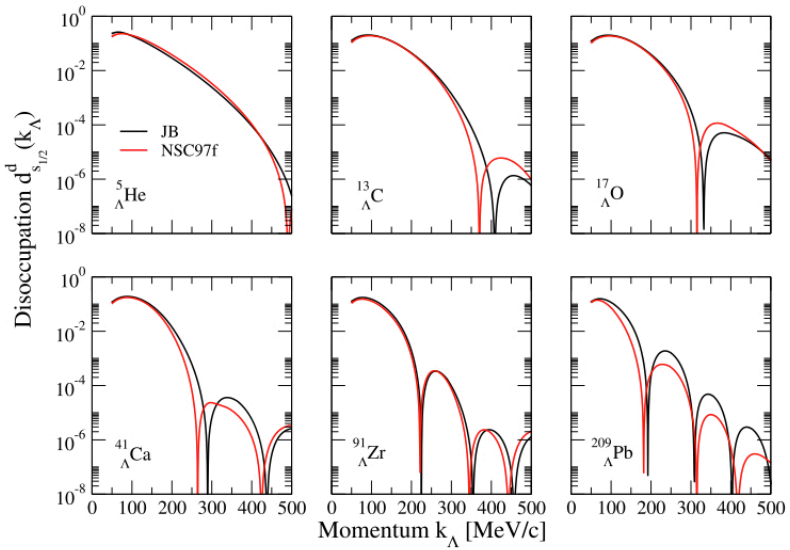
	$l_\Lambda j_\Lambda$	550	600	650	700
LO	$s_{1/2}$	0.957	0.951	0.945	0.941
	$p_{3/2}$	0.970	0.968	0.965	0.962
	$p_{1/2}$	0.967	0.965	0.963	0.959
NLO13	$s_{1/2}$	0.903	0.880	0.861	0.854
	$p_{3/2}$	—	—	0.933	0.912
	$p_{1/2}$	—	—	0.939	0.914
NLO19	$s_{1/2}$	0.927	0.905	0.886	0.873
	$p_{3/2}$	0.973	0.959	0.936	0.919
	$p_{1/2}$	—	—	0.940	0.920

- ✧ Results qualitatively similar to those obtained with meson-exchange potentials
- ✧ Correlations more important for the NLO13 potential

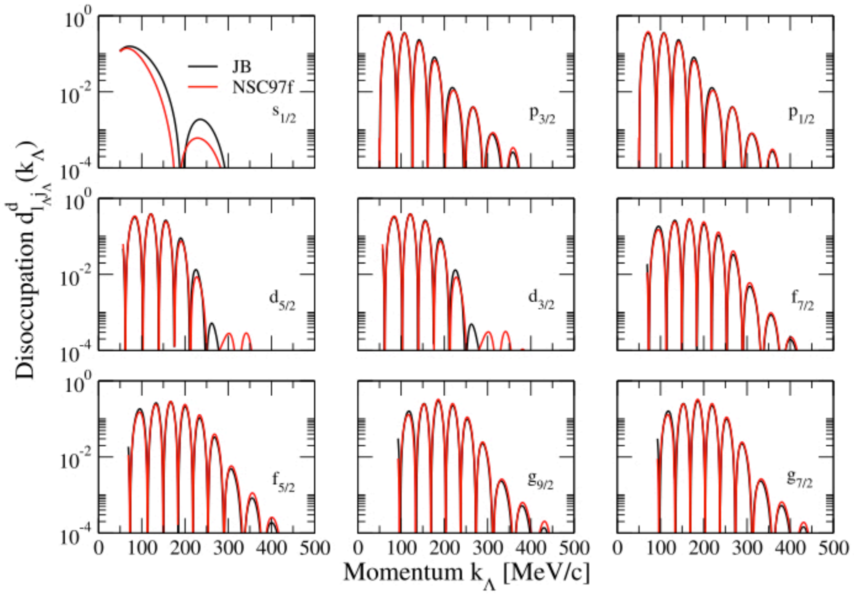
Disoccupation
(discrete contribution)

$$d^d_{l_\Lambda j_\Lambda}(k_\Lambda) = \int_{\mu_\Lambda}^{\infty} d\omega\, S^{p(d)}_{l_\Lambda j_\Lambda}(k_\Lambda, \omega) = Z_{l_\Lambda j_\Lambda} |\langle k_\Lambda l_\Lambda j_\Lambda m_{j_\Lambda} | \Psi \rangle|^2$$

s-wave state: He, C, O, Ca, Zr & Pb
JB (blak) & NSC89 (red)

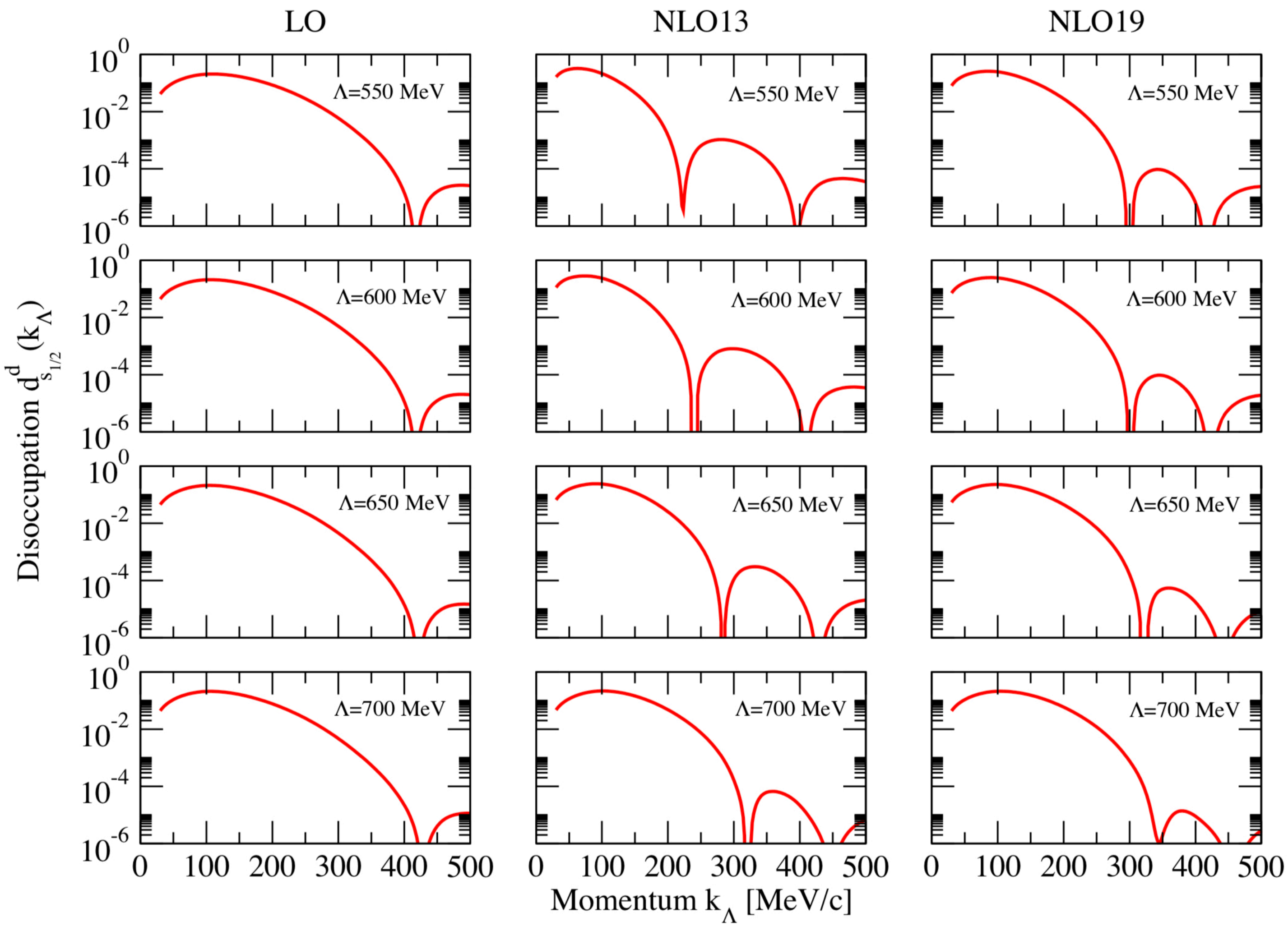


s-, p-, d-, f- and g- wave states for Pb
JB (black) & NSC89 (red)



- ✧ $d^d_{l_\Lambda j_\Lambda}(k_\Lambda)$ gives the probability of adding a Λ of momentum k_Λ in the s.p. state $l_\Lambda j_\Lambda$ of the hypernucleus
- ✧ Intuitively one expects that if k_Λ is large the Λ can easily escape from the nucleus & the probability of binding it must be small. Both plots show in fact that $d^d_{l_\Lambda j_\Lambda}(k_\Lambda)$ decreases when increasing k_Λ and is almost negligible for very large values $\longrightarrow$ In hypernuclear production reactions the Λ is mostly formed in a quasi-free state

Disoccuparion of $^{17}{}_{\Lambda}O$ with chiral YN potentials



Total Disoccupation Number

The total spectral strength of the Λ hyperon fulfills the sum rule

$$\int_{\mu_\Lambda}^{\infty} d\omega S^{p}_{l_\Lambda j_\Lambda}(\omega) = \underbrace{\int_{\mu_\Lambda}^{\infty} d\omega S^{p(d)}_{l_\Lambda j_\Lambda}(\omega)}_{\text{discrete}} + \underbrace{\int_{\mu_\Lambda}^{\infty} d\omega S^{p(c)}_{l_\Lambda j_\Lambda}(\omega)}_{\text{continuum}} = 1$$

The total disoccupation number is 1 $\longrightarrow$ is always possible to add a Λ either in a bound or a scattering state of a given ordinary nucleus

Nuclei		$s_{1/2}$	$p_{3/2}$	$p_{1/2}$	$d_{5/2}$	$d_{3/2}$	$f_{7/2}$	$f_{5/2}$	$g_{9/2}$	$g_{7/2}$
${}^{5}_{\Lambda}$He	Discrete	0.964	–	–	–	–	–	–	–	–
	Continuum	0.023	–	–	–	–	–	–	–	–
	Total	0.987	–	–	–	–	–	–	–	–
${}^{13}_{\Lambda}$C	Discrete	0.933	0.979	–	–	–	–	–	–	–
	Continuum	0.040	0.017	–	–	–	–	–	–	–
	Total	0.973	0.996	–	–	–	–	–	–	–
${}^{17}_{\Lambda}$O	Discrete	0.924	0.959	0.961	–	–	–	–	–	–
	Continuum	0.053	0.037	0.036	–	–	–	–	–	–
	Total	0.977	0.996	0.997	–	–	–	–	–	–
${}^{41}_{\Lambda}$Ca	Discrete	0.898	0.914	0.912	0.938	0.939	–	–	–	–
	Continuum	0.071	0.063	0.064	0.048	0.047	–	–	–	–
	Total	0.969	0.977	0.976	0.986	0.986	–	–	–	–
${}^{91}_{\Lambda}$Zr	Discrete	0.876	0.883	0.883	0.893	0.891	0.906	0.907	–	–
	Continuum	0.120	0.113	0.113	0.103	0.105	0.089	0.090	–	–
	Total	0.996	0.996	0.996	0.996	0.996	0.995	0.997	–	–
${}^{209}_{\Lambda}$Pb	Discrete	0.856	0.857	0.857	0.869	0.867	0.857	0.855	0.862	0.860
	Continuum	0.138	0.142	0.142	0.129	0.130	0.140	0.141	0.137	0.139
	Total	0.994	0.999	0.999	0.998	0.997	0.997	0.996	0.999	0.999

Total Disoccupation Number of $^{17}{}_{\Lambda}O$ with chiral YN potentials

		LO			NLO13			NLO19		
		$s_{1/2}$	$p_{3/2}$	$p_{1/2}$	$s_{1/2}$	$p_{3/2}$	$p_{1/2}$	$s_{1/2}$	$p_{3/2}$	$p_{1/2}$
	Discrete	0.957	0.970	0.967	0.903	–	–	0.927	0.973	–
550	Continuum	0.042	0.029	0.032	0.096	–	–	0.067	0.025	–
	Total	0.999	0.999	0.999	0.999	–	–	0.994	0.998	–
	Discrete	0.951	0.968	0.965	0.880	–	–	0.905	0.959	–
600	Continuum	0.048	0.031	0.034	0.110	–	–	0.093	0.039	–
	Total	0.999	0.999	0.999	0.990	–	–	0.998	0.998	–
	Discrete	0.945	0.965	0.963	0.861	0.933	0.939	0.886	0.936	0.940
650	Continuum	0.052	0.034	0.036	0.136	0.066	0.060	0.112	0.062	0.058
	Total	0.997	0.999	0.999	0.997	0.999	0.999	0.998	0.998	0.998
	Discrete	0.941	0.962	0.959	0.854	0.912	0.914	0.873	0.919	0.920
700	Continuum	0.053	0.036	0.039	0.144	0.086	0.085	0.126	0.079	0.078
	Total	0.994	0.998	0.998	0.998	0.998	0.995	0.999	0.998	0.998

The final message of this talk

- ✧ Purpose:
  - ✓ Calculation of finite nuclei Λ spectral function from its self-energy derived within a perturbative many-body approach with realistic (meson-exchange & chiral) YN interactions
- ✧ Results & Conclusions
  - ✓ Binding energies in qualitatively good agreement with experiment
  - ✓ Z-factor relatively large ⟶ Λ less correlated than nucleons
  - ✓ Discrete cont. to disoc. numb decreases with k_Λ ⟶ Λ is mostly formed in a quasi-free state in production reactions
  - ✓ Scattering reactions such as (e,e',K^+) at JLAB & MAMI-C can provide valuable information on the disoccupation of Λ s.p. bound states

✧ You for your time & attention