A new method to measure scattering length with threshold cusp

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Fundamental Physics at the Strangeness Frontier at DAΦNE

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Scattering length

- Low energy elastic scattering
  - dominated by S-wave
  - S-wave phase shift: δ_0 ~ ak for k ~ 0
  - a characterizes low energy scattering
    → scattering length
- a>0: “attractive”, a<0: “repulsive”
  - However, attractive interaction gives a < 0 if there is a bound state.
- Become complex if there is lower channel
  - E.g., $\bar{K}N$ vs $\Lambda\pi, \Sigma\pi$ channel

Experimental methods (1)

- Low energy scattering
  - pp, pn, nA, K^-p, …
  - $\sigma=4\pi a^2$ at the low energy limit
  - Both beam & target must be stable – rather limited
- Exotic atom – shift from Coulomb potential
  - (Improved) Desar formula:
    shift & width $\propto$ complex scattering length
  - E.g., Kaonic atom experiments at J-PARC & DAΦNE
  - Low yield & large width especially for K^-d atom

K^{bar}N scattering length

Deser-Type relation

U.-G. Meißner, U.Raha, A.Rusetsky, Eur. phys. J. C35 (2004) 349
next-to-leading order, including isospin breaking

$$\varepsilon_{1s} - \frac{i}{2}\Gamma_{1s} = -2\alpha^3\mu_c^2 \boxed{a_{K^-p}} \left(1 - 2\alpha\mu_c(\ln\alpha - 1)a_{K^-p}\right)$$

Shift Width Scattering length

Kaonic hydrogen (K$^-$p)

$$a_{K^-p} = \frac{1}{2}\left[a_0 + a_1\right]$$

$$a_{K^-n} = a_1$$

no atom

Kaonic deuterium (K$^-$d)

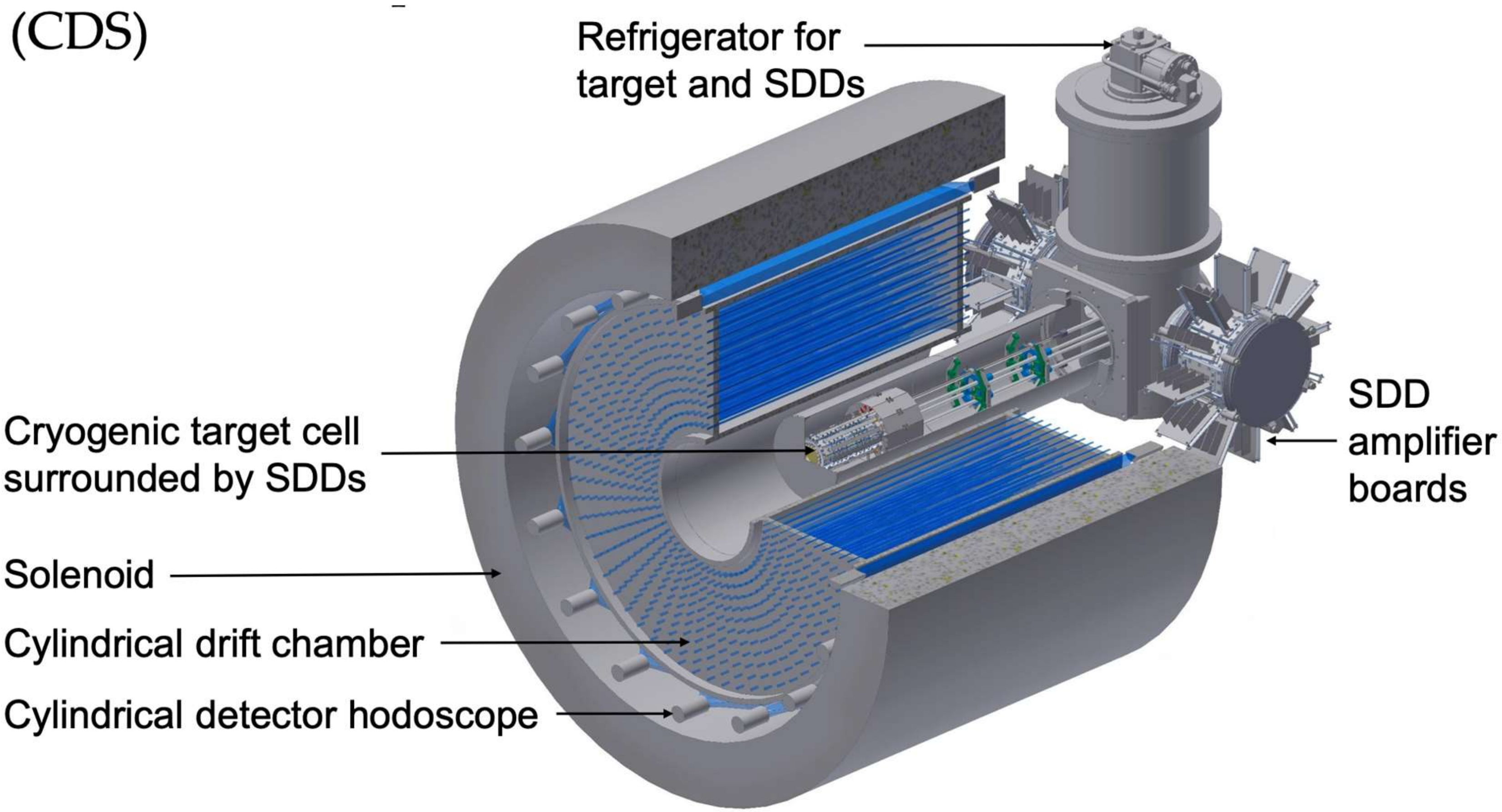
$$a_{K^-d} = \frac{k}{2}\left[a_{K^-p} + a_{K^-n}\right] + C = \frac{k}{4}\left[a_0 + 3a_1\right] + C$$

Theoretical calc.

$$k = \frac{4\left[m_n + m_K\right]}{\left[2m_n + m_K\right]}$$

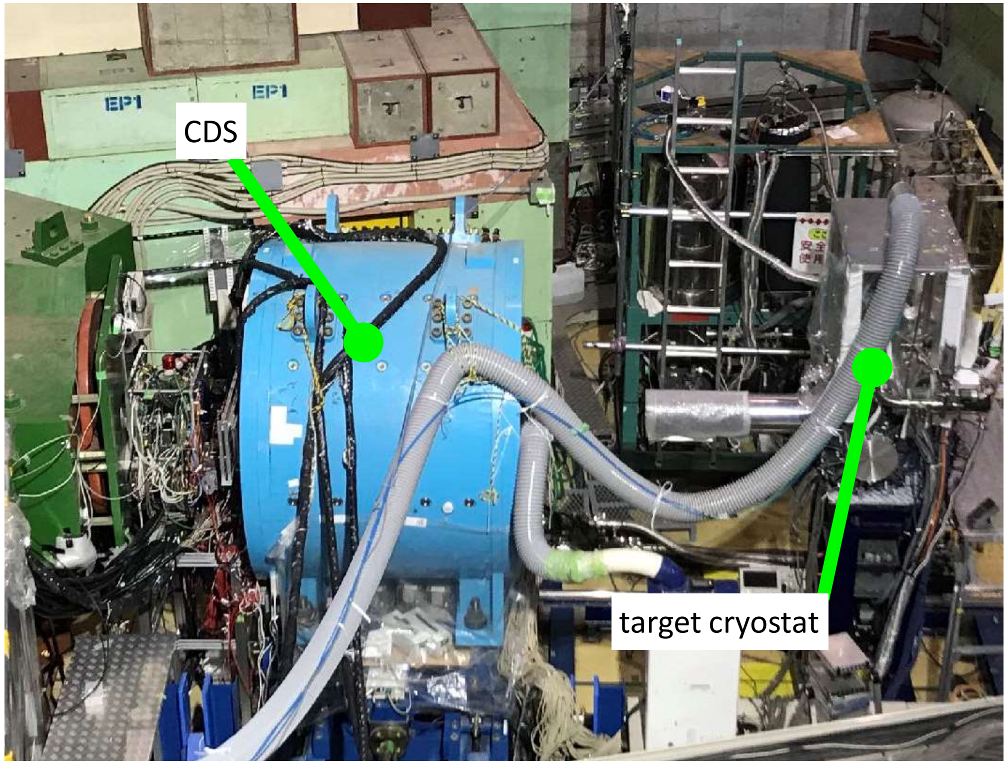
✓X-ray spectroscopy of K$^-$p and K$^-$d resolve
the isospin-dependent K^{bar}N scattering length

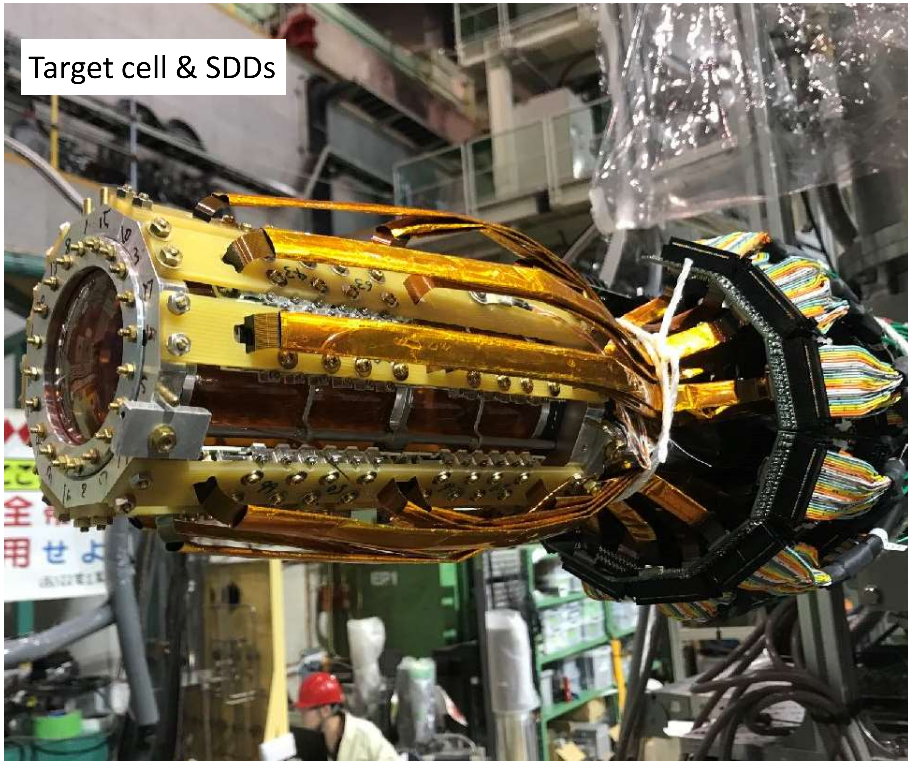
K^-d atom @J-PARC E57



- ✓ Large area Silicon Drift Detector arrays
- ✓ Target at 30K & 0.3 MPa to optimize stopping power & X-ray yield
- ✓ Vertex cut & charged particle veto by using CDC ←unique in J-PARC

K$^-$d atom @J-PARC E57

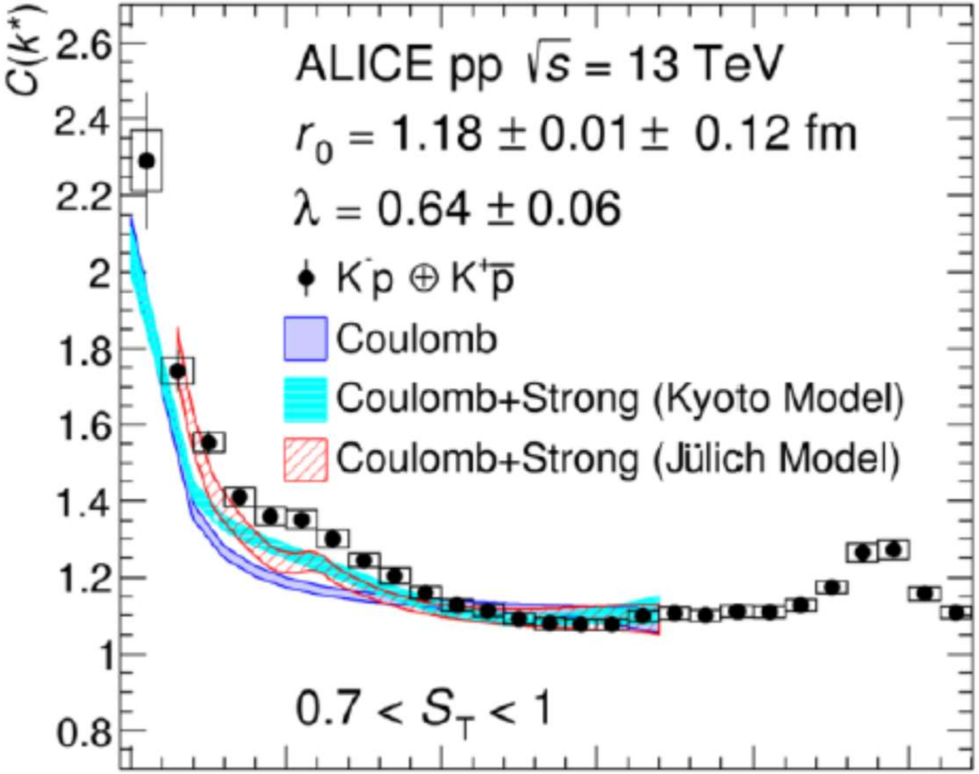


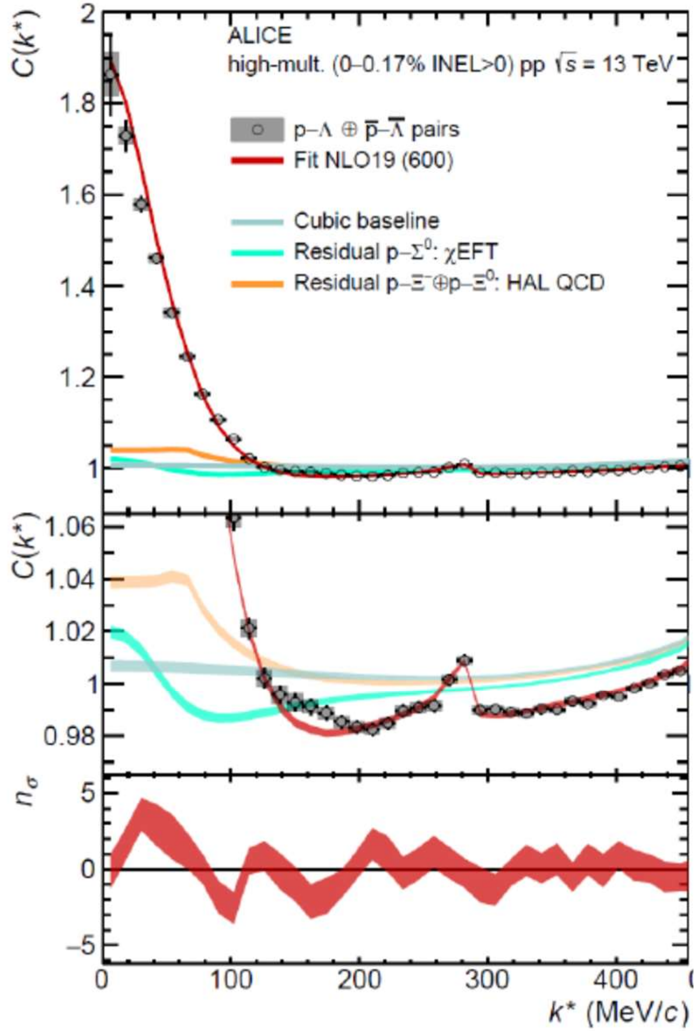


- Data taking in 2023 or later

Experimental methods (2)

- Low energy particle correlation
  - E.g., particle correlation at ALICE
  - Lower energy → lower statistics

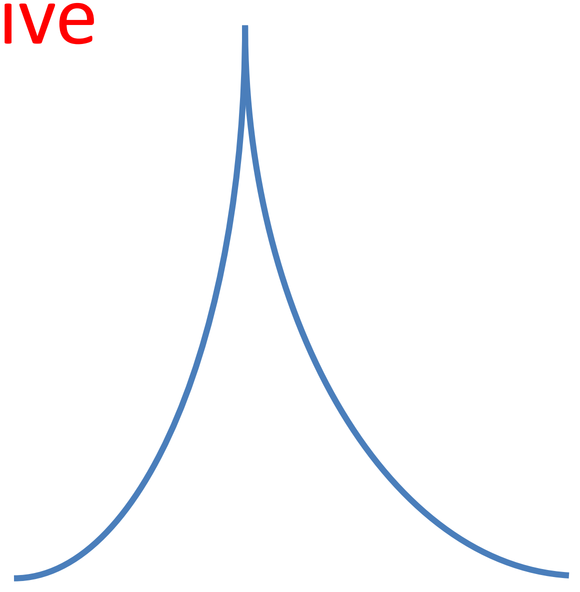




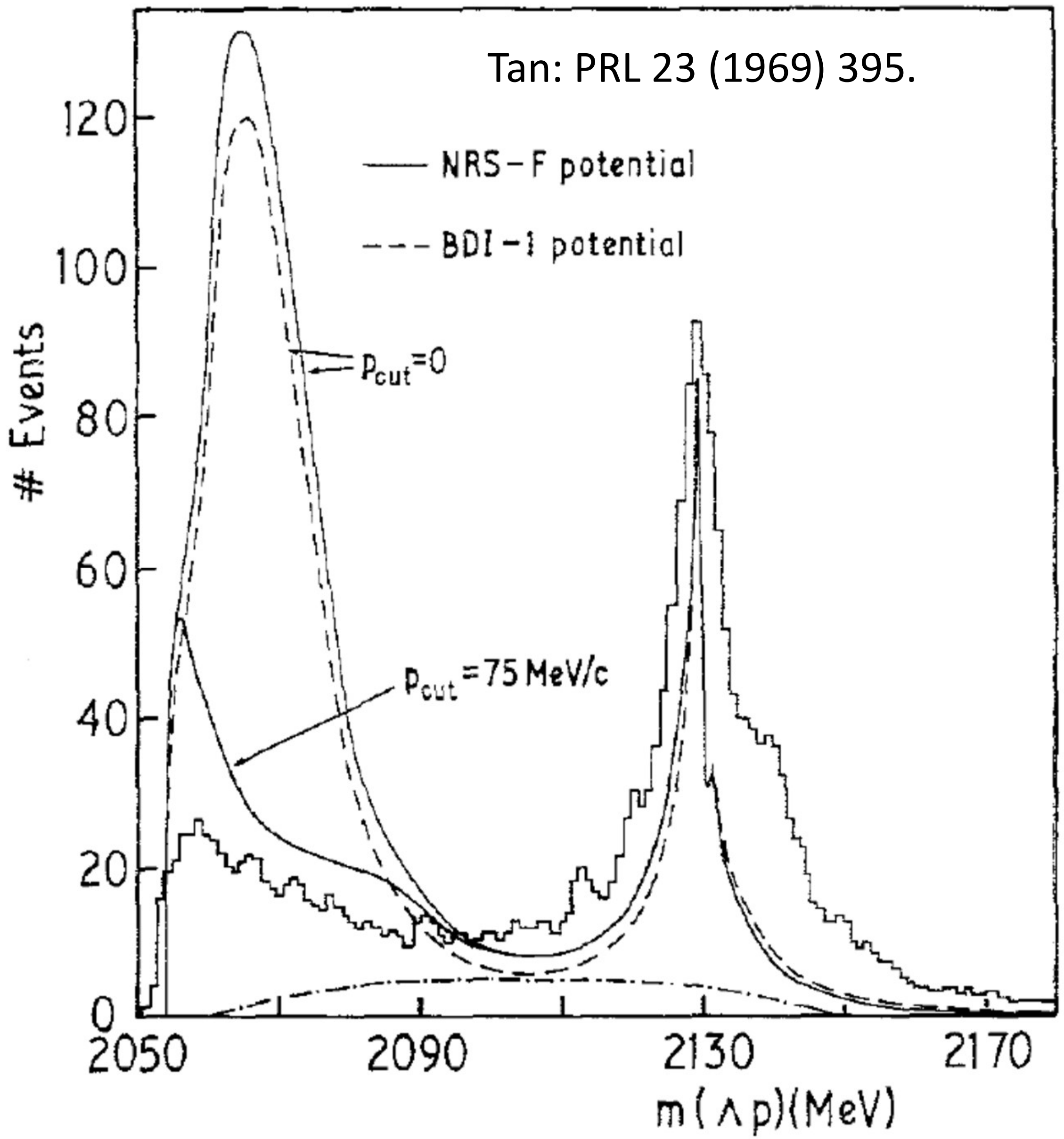
A new method: threshold cusp

Threshold cusp

- In the widest sense, cusp ALWAYS appears at thresholds.
  - Jump in strength (amp^2) in the (L+1)th derivative
- Practically, cusp appears only in S-wave
- Interesting case is the 1st derivative changes sign, especially from positive to negative
  - Cusp in the narrow sense.
  - In principle, can be distinguished from usual peak by the derivative at the top, but practically there is experimental resolution.



An example



- Rather few
- K^-(stopped)+d → $\Lambda p \pi^-$
  - Probably the cleanest cusp ever seen, but not confirmed.

Cusp & scattering length

- Cusp occurs at a threshold
  - The statistics is highest at the threshold
  - Behavior is determined by the complex scattering length of the threshold channel
- Specific form is given by Dalitz & Deloff [Czech. J. Phys. B 32 (1982)]
  - Slope to the right: Imaginery part (b)
  - Slope to the left: real part (a)
- Width approximately scales as $1/\mu|a|^2$

Cusp & scattering length

- Above the threshold:

$$S \propto \frac{b}{(1+kb)^2+(ka)^2} \sim b(1-2kb)$$

- Below the threshold:

$$S \propto \frac{b}{(1+|k|a)^2+(|k|b)^2} \sim b(1-2|k|a)$$

with $k = i\sqrt{2\mu|E|}$ is pure imaginary.

- Determine a+ib with a fit to above.

Flatte distribution

- Or, we can use Flatte(-like) distribution for fitting.
  - The simplest model that can describe cusp (& usual peak)
- It is almost the same as usual BW distribution
  - Many versions reflecting variations of BW.
  - In case of two channels (P & K), it looks like

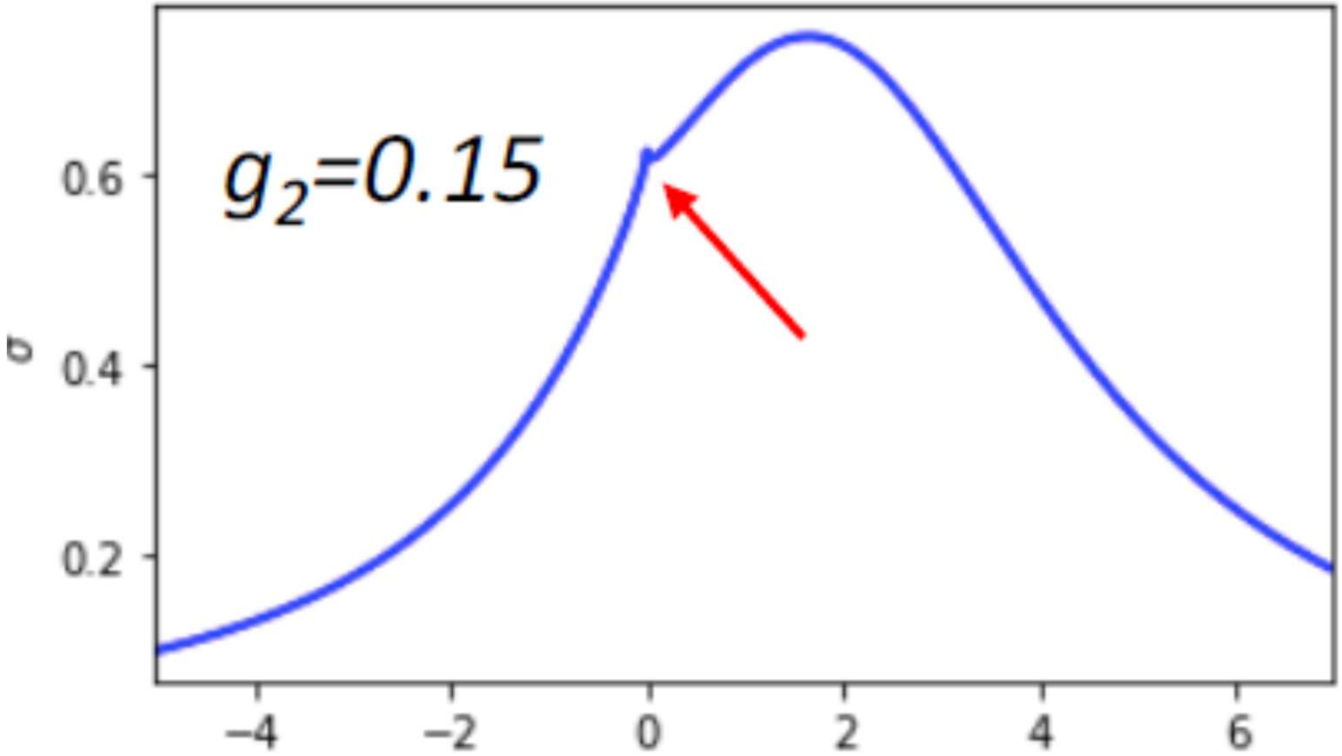
$$\boxed{f_{el} = -\frac{1}{2q}\frac{\Gamma_P}{E - E_{BW} + i\frac{\Gamma_P}{2} + i\bar{g}_K\frac{k}{2}} \ .}$$

(K: threshold channel. P: threshold is far below.)

  - k becomes imaginary below threshold (analytic cont.)

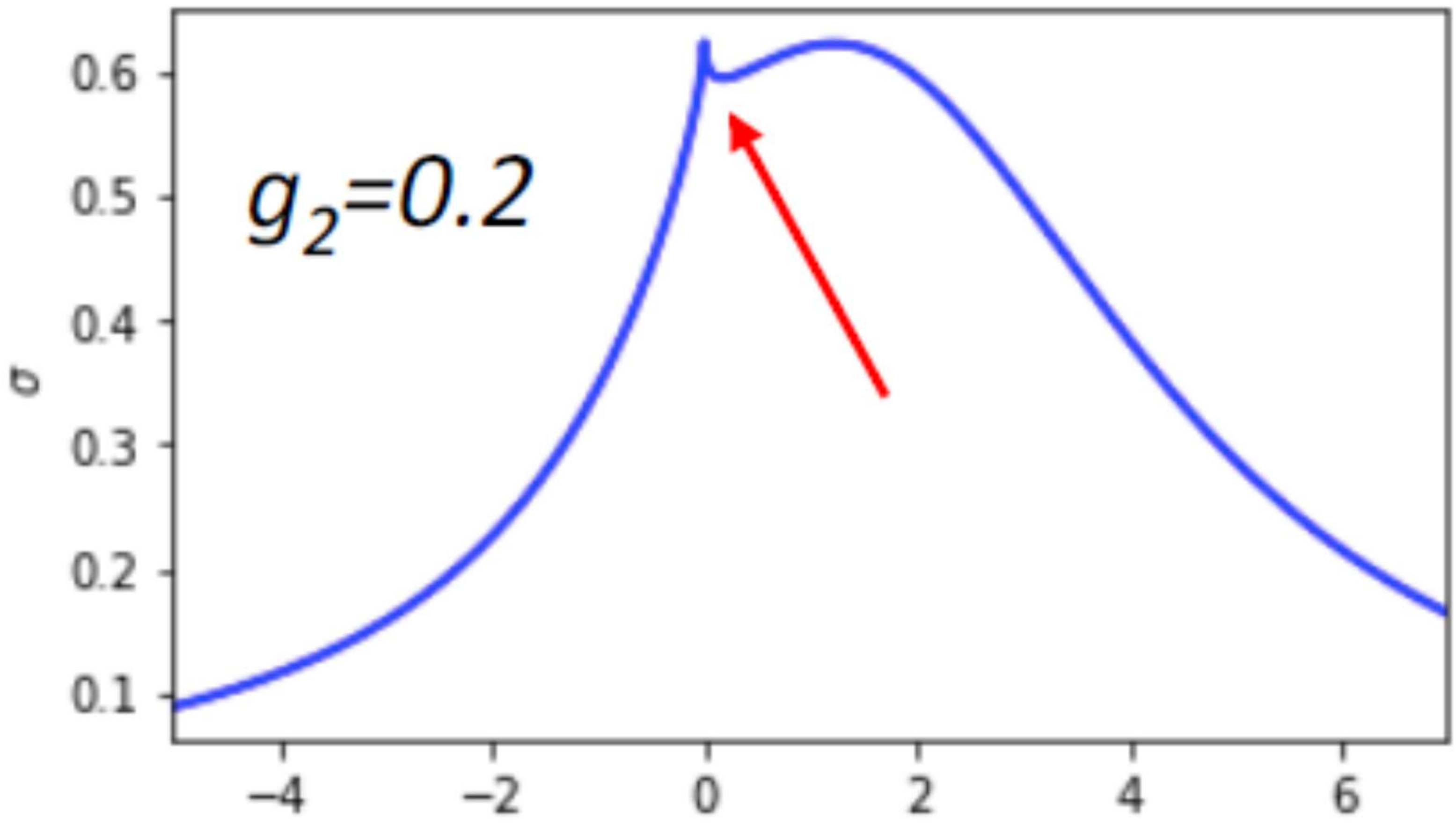
From peak to cusp (1)

- Jump in derivative:
  → Always with $E_{BW} > 0$ and $\bar{g}_K > 0$
- When $\bar{g}_K$ is small, cusp is hardly visible



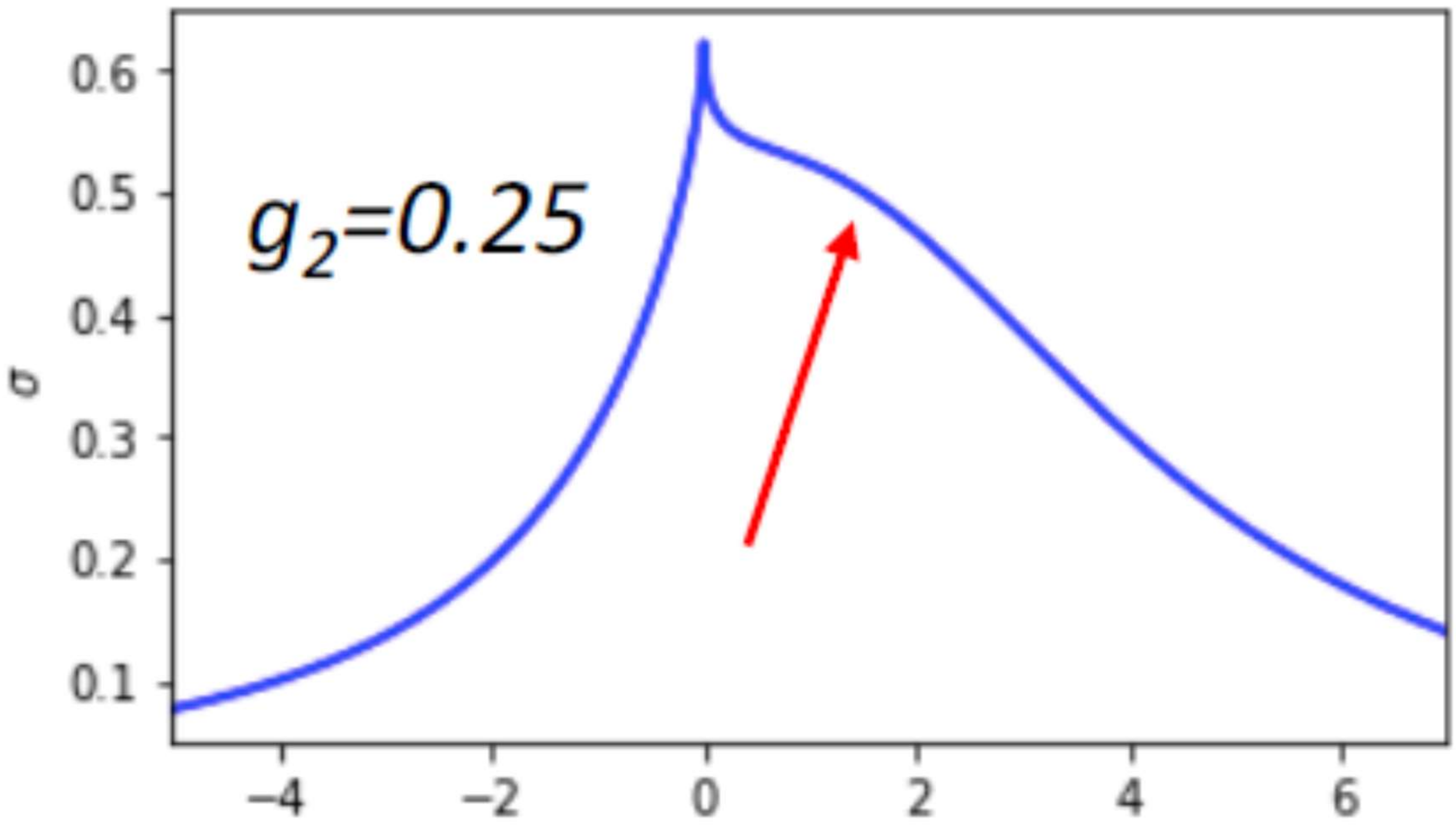
From peak to cusp (2)

- For slightly larger $\bar{g}_K$, cusp & peak heights are equal.



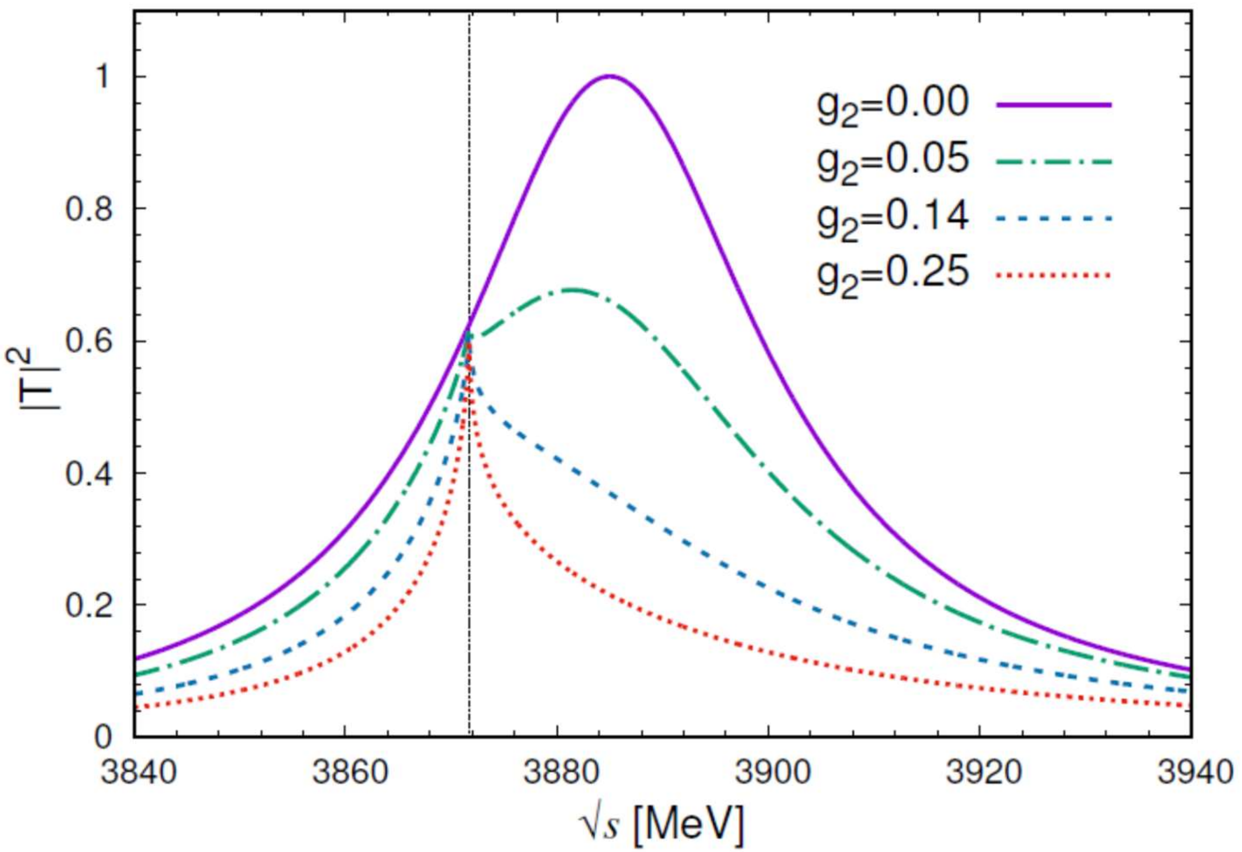
From peak to cusp (3)

- For even larger $\bar{g}_K$, the spectrum becomes monotonically decreasing above the threshold



From peak to cusp (4)

- With $g_K > \sqrt{4E_{BW}/\mu}$, pole moves to Re(E) < 0
  $\rightarrow$ virtual pole
  - Peak is no longer seen



Flatte & scattering length

- From Flatte distribution, scattering length can be derived as:

$$a + ib = \frac{\bar{g}_K}{2E_{BW} - i\Gamma_P}$$

  and the spectrum shape is consistent with Dalitz & Deloff up to the first order of k.
- Cusp occurs if $E_{BW} > 0$ and $\bar{g}_K > 0$
  – scattering length is “attractive”

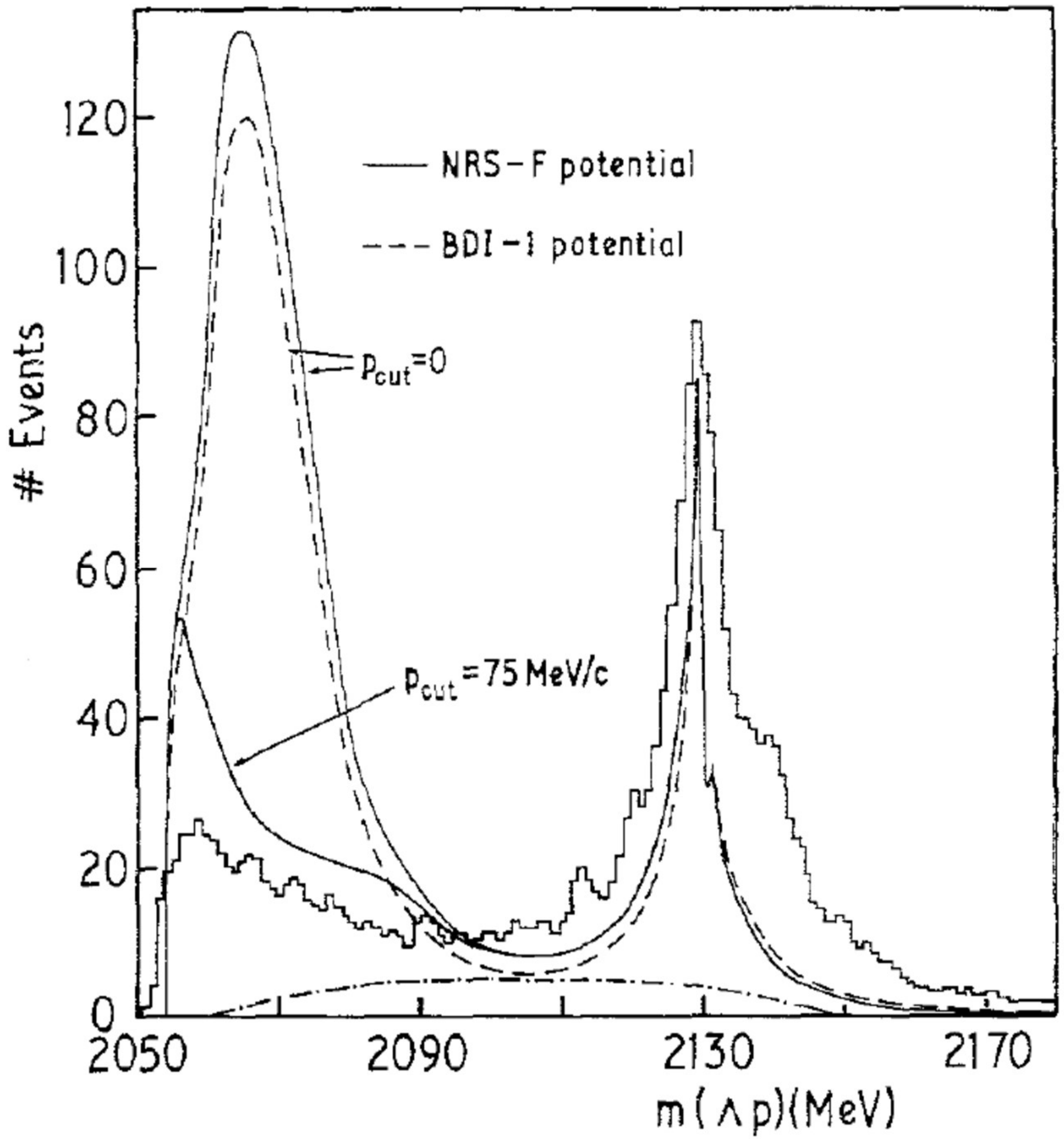
Note on width and resolution

- Width $\propto 1/\mu|a|^2$ can be arbitrary narrow near the unitarity limit
  - ~ 18 MeV for $\mu = 0.3\ GeV$ & a=b=1 fm
    ~ 2 MeV for $\mu = 0.6\ GeV$ & a=b=2 fm
- The expression is model independent only up to the first order of k
  - High resolution is necessary
  - Important to distinguish cusp from usual peak

Some cases:

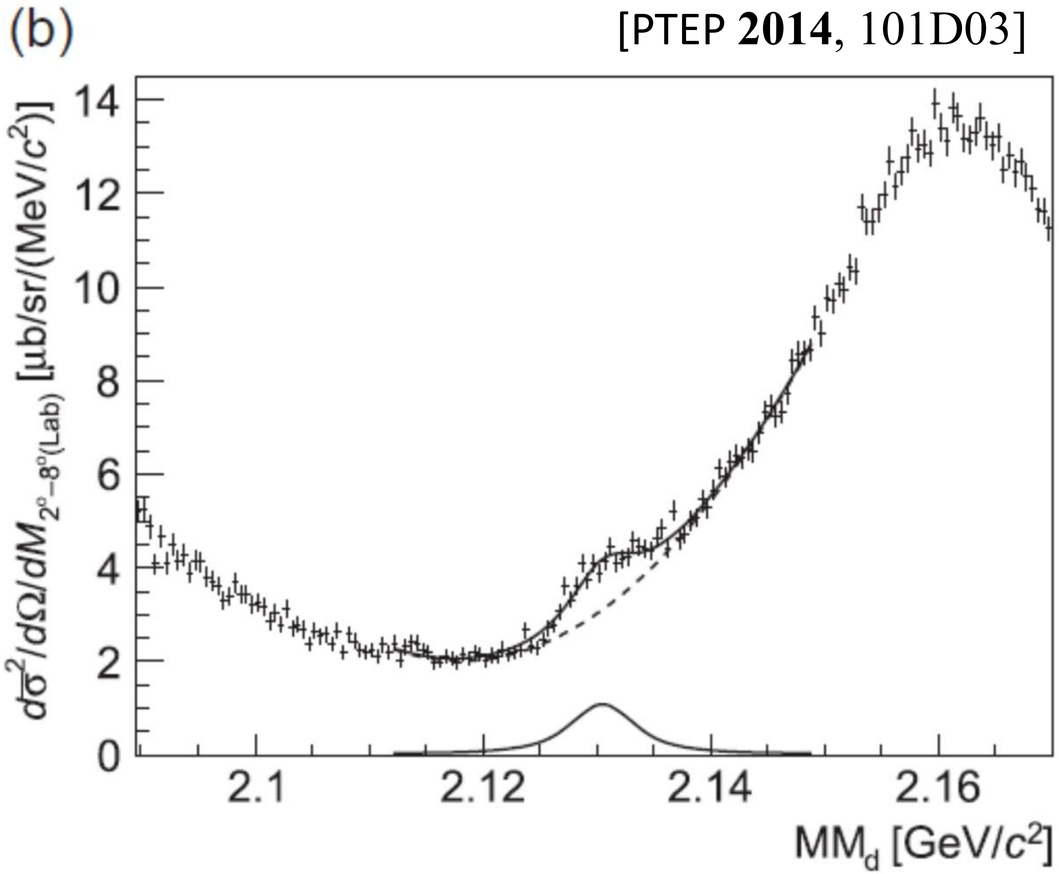
1. ΣN
2. $\bar{K}N(I = 1)$

1. ΣN cusp



- K^-(stopped)+d → $\Lambda p \pi^-$
- Probably the cleanest cusp ever seen, but not confirmed.
  - Because the resolution is not enough

J-PARC E27 result



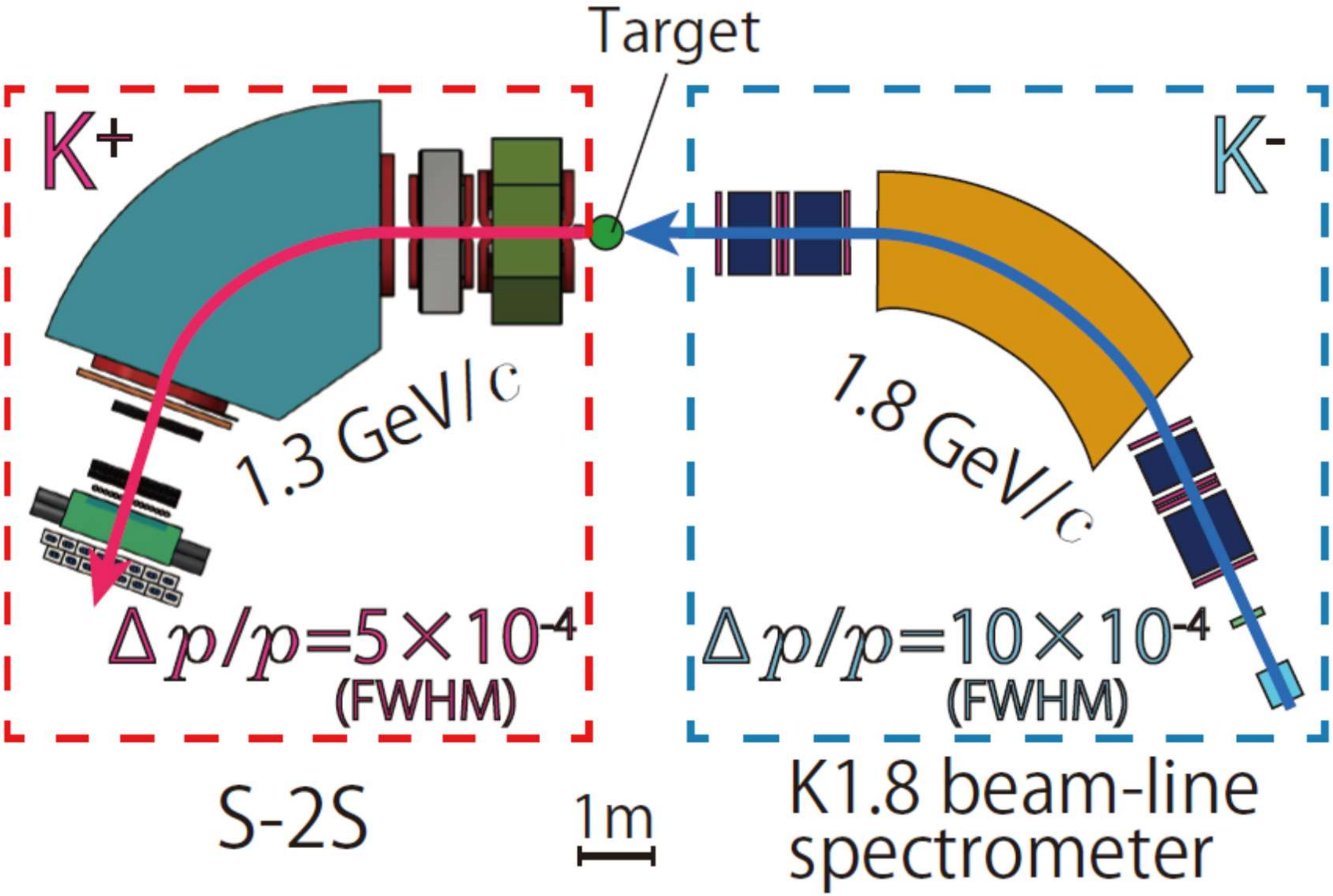
- d(π^+,K^+) reaction at 1.69 GeV/c
- Cannot distinguish "cusp" from usual peak

- Fit with Breit-Wigner (Resolution: σ =1.4 MeV)
  $\Gamma = 5.3^{+1.4+0.6}_{-1.2-0.3}$ MeV

What should we do?

- Try even higher resolution
  - High resolution pion spectrometer + stopped K at DAΦNE: $d(K^-_{stopped},\pi^-)$
    → Good S/N, yield
  - J-PARC: S-2S spectrometer
    & High-Intensity High-Resolution beamline at extended Hadron Hall
- Tagging of the final state is necessary
  - Must be ΛN to derive ΣN (I=1/2) scattering length
  - ΣN (I=3/2) contaminate if not tagged

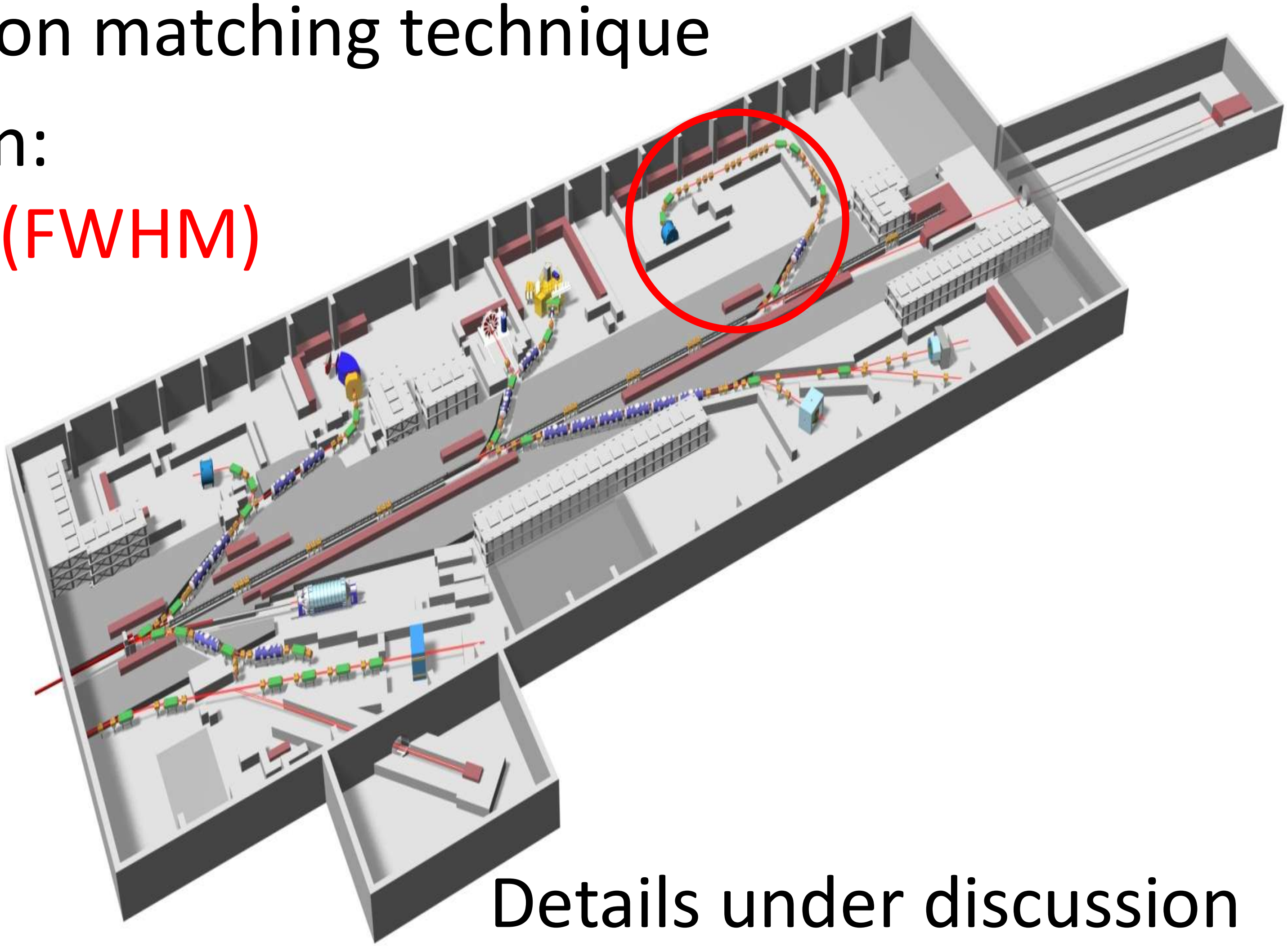
S-2S Spectrometer



- 1 MeV (FWHM) resolution possible
- Designed for spectroscopy of Ξ hypernuclei

HIHR @J-PARC Extension

- Beamline with large dispersion
  + dispersion matching technique
- Resolution:
  ~0.1 MeV(FWHM)
  possible



Details under discussion

2. $\bar{K}N(I = 1)$

- Target of J-PARC E57 and SIDDHARTA-2 @DAΦNE
- Cusp candidates are observed in $\Lambda\pi^{\pm}$ invariant mass spectra, especially from Λ_c decay

More experiments?

- x50 more statistics will be accumulated in Belle II.
- J-PARC:
  S-2S & HIHR can be used for this study, too.
  - $p(K^-,\pi^\pm)\Lambda\pi^\mp$ or $p(\pi^\pm,K^+)\Lambda\pi^\pm$
- Difficult at DAΦNE with low-energy kaons.

- Cusps at other thresholds can be searched for:
  - $N\eta$, $\Lambda\eta$, $\Sigma\eta$, ΛK, ……
  - Charmed or even bottom particles
    (cf. X(3872) vs D^0D^{0*})

Summary

- Scattering length
  - Fundamental parameter to characterize low energy interaction
  - Measured with several methods
- Threshold cusp
  - A peak-like structure on a threshold
  - Can be used to derive scattering length
- Some examples
  - ΣN(I=1/2), $\bar{K}N(\mathrm{I}=1)$, …
  - Interesting possibility with the present data, and future experiments at J-PARC & DAΦNE

Backup