The K^-pp system with a momentum-dependent type of Chiral SU(3)-based potential

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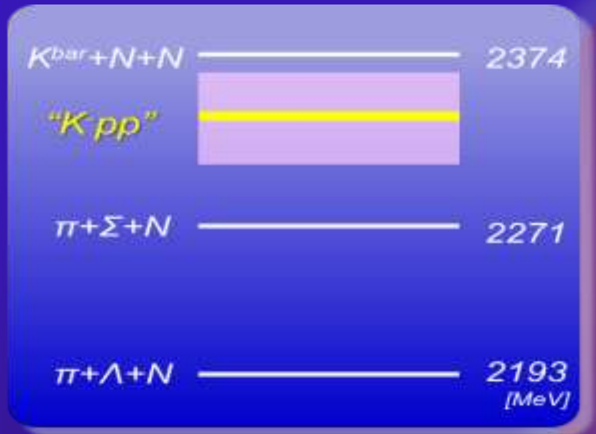
Workshop “Fundamental Physics at the Strangeness Frontier at DAΦNE”
February 26, 2021 (online)

1. *Introduction*

"K⁻pp" = the simplest kaonic nucleus

- $K^{bar}N$ potential is so attractive to generate a quasi-bound state, Λ(1405).
- K^{bar} meson can be bound in a nucleus: Kaonic nuclei.
- Among them, the three-body system "K^-pp" is a prototype of kaonic nuclei.

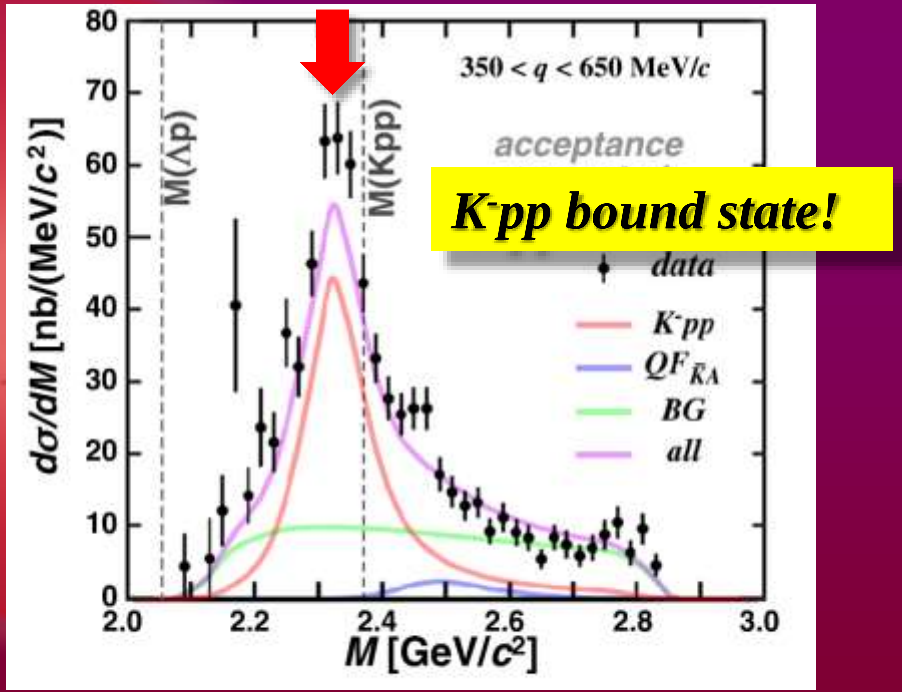
Theoretical studies:
"K^-pp" = $K^{bar}NN$-πΣN-πΛN ($J^\pi=0^-$, T=1/2)



- Akaishi, Yamazaki, PRC76, 045201 (2007)
- Shevchenko, Gal, Mares, PRC76, 044004 (2007)
- Wycech, Green, PRC79, 014001 (2009)
- Doté, Hyodo, Weise, PRC79, 014003 (2009)
- Ikeda, Kamano, Sato, PTP124, 533 (2010)
- Barnea, Gal, Liverts, PLB712, 132 (2012)
- Bayar, Oset, PRC88, 044003 (2013)
- Revai, Shevchenko, PRC90, 034004 (2014)
- Doté, Inoue, Myo, PLB784, 405 (2018)
- …

"K^-pp" should exist as a resonant state between $K^{bar}NN$ and πΣN thresholds.

J-PARC E15 (2nd run):
Exclusive exp. ^{3}He(K^-, Λp)$n_{missing}$



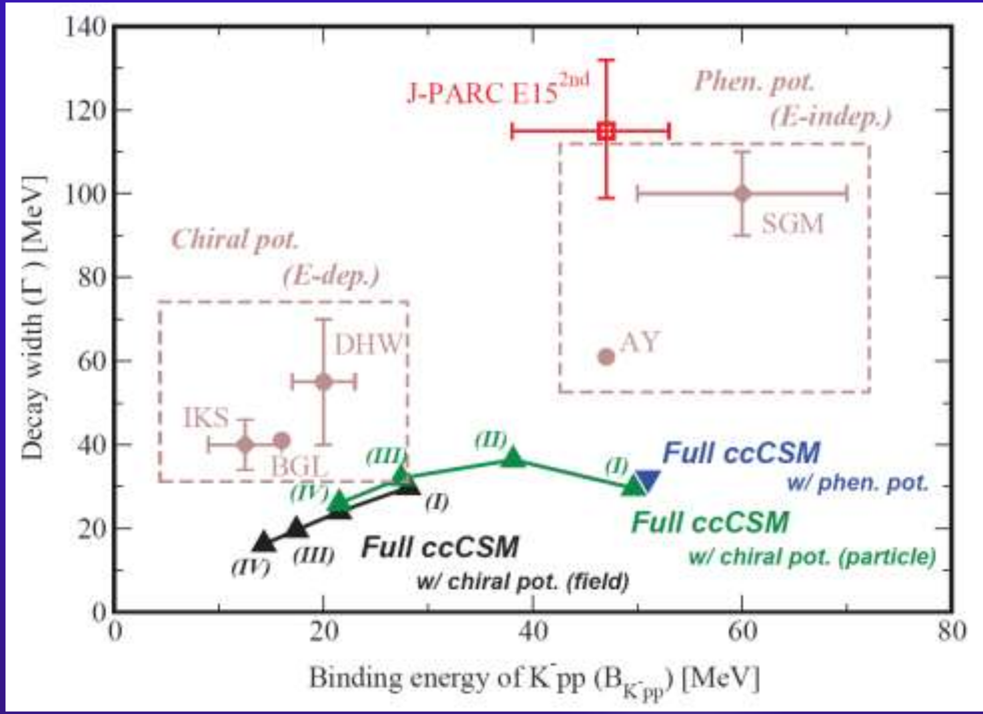
S. Ajimura et al., PLB 789, 620 (2019)

$K^{bar}N$ potential is crucial for the $K^{-}pp$ study.

Binding energy of $K^{-}pp$ depends strongly on the type of $K^{bar}N$ potential.

Theoretical studies:

"$K^{-}pp$" = $K^{bar}NN$-$\pi\Lambda N$-$\pi\Lambda N$ ($J^{\pi}=0^{-}$, $T=1/2$)



A. Doté, T. Inoue, T. Myo, AIP Conf. Proc. 2249, 030038 (2020)

- *Chiral potential (E-dep.)*
  → *Shallow binding*
- *Phenomenological potential (E-indep.)*
  → *Deep binding*

"Energy-independent" chiral SU(3) $K^{bar}N$ potential

Chiral potential that we have used so far is energy-dependent, but ...

- *Starting with Weinberg-Tomozawa term of the chiral Lagrangian, it can be treated in an energy-independent way under Non-Relativistic kinematics, by considering the two-term separable potential.*

J. Revai, Few-Body Syst 59, 49 (2018)
J. Revai, Few-Body Syst 61, 32 (2020)

- *Similarly, energy-independent treatments have been carried out under Relativistic kinematics.*

M. Mai, U.-G. Meißner, NPA 900, 51 (2013)
O. Morimatsu, K. Yamada, PRC 100, 025201 (2019)

Question: How will K^-pp be with this type of potential?

2. Formalism

- *Chiral SU(3)-based $K^{bar}N$ potential (Momentum-dependent type)*
- *Fully coupled-channel Complex Scaling Method*

"Energy-independent" chiral SU(3) $K^{bar}N$ potential

Prof. Revai's proposal (Non-relativistic)

1. *Lowest order Weinberg-Tomozawa term of the chiral Lagrangian*

$$\langle q_i | v_{ij} | q_j \rangle \sim -\frac{c_{ij}}{4 f_\pi^2}(q_i^0 + q_j^0)$$

2. *S-wave potential (non-rela.)*

$$\langle q_i | v_{ij} | q_j \rangle = -\frac{c_{ij}}{64\pi^3 F_i F_j}\sqrt{\frac{m_i + M_i}{m_i W}}\sqrt{\frac{m_j + M_j}{m_j W}}(q_i^{0\prime} + q_j^{0\prime})$$

$$q_i^{0\prime} = q_i^0 + \frac{q_i^{0\,2} - m_i^2}{2M_i} = q_i^0 + \frac{q_i^2}{2M_i} \underset{\text{nonrel}}{\approx} m_i + \frac{q_i^2}{2\mu_i}$$

3. *Introduce a form factor and treat as a two-term separable potential*

$$\langle q_i | V_{ij} | q_j \rangle = \lambda_{ij}(g_{iA}(q_i) g_{jB}(q_j) + g_{iB}(q_i) g_{jA}(q_j))$$

$$g_{iB}(q_i) = g_{iA}(q_i)\gamma_i(q_i) = g_{iA}(q_i)\left(m_i + \frac{q_i^2}{2\mu_i}\right)$$

$g_{iA}(q_i)$: dipole-type form factor

4. *Solve Lippmann-Schwinger eq. without any approximation*

$$\langle q_i | T_{ij}(W) | q_j \rangle = \langle q_i | V_{ij} | q_j \rangle + \sum_s \int \langle q_i | V_{is} | q_s \rangle G_s(q_s; W) \langle q_s | T_{sj}(W) | q_j \rangle d\vec{q}_s$$

Carry out the integral for the internal momentum explicitly

“Energy-independent” chiral SU(3) $K^{bar}N$ potential

Prof. Revai’s proposal (Non-relativistic)

1. Lowest order Weinberg-Tomozawa term of the chiral Lagrangian

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$$q_i^{0\prime} = q_i^0 + \frac{q_i^{0\,2}-m_i^2}{2M_i} = q_i^0 + \frac{q_i^2}{2M_i} \underset{\text{nonrel}}{\approx} m_i + \frac{q_i^2}{2\mu_i}$$

Meson’s energy is represented with the momentum.

3. Introduce a form factor and treat as a two-term separable potential

$$\langle q_i|V_{ij}|q_j\rangle = \lambda_{ij}(g_{iA}(q_i)g_{jB}(q_j)+g_{iB}(q_i)g_{jA}(q_j))$$

$$g_{iB}(q_i) = g_{iA}(q_i)\gamma_i(q_i) = g_{iA}(q_i)\left(m_i + \frac{q_i^2}{2\mu_i}\right)$$

$g_{iA}(q_i)$: dipole-type form factor

4. Solve Lippmann-Schwinger eq. without any approximation

It is treated as a ***Momentum-dependent*** potential.

$$\langle q_i|T_{ij}(W)|q_j\rangle = \langle q_i|V_{ij}|q_j\rangle + \sum_s \int \langle q_i|V_{is}|q_s\rangle G_s(q_s;W)\langle q_s|T_{sj}(W)|q_j\rangle d\vec{q}_s$$

Carry out the integral for the internal momentum explicitly

Chiral SU(3)-based K^{bar}N potential - Momentum-dependent type

- *Improve our chiral SU(3)-based K^{bar}N potential, referring Revai's proposal*

Original potential (Non-rela. ver.) :
$$V^{(NR)}_{MB,I} = \sum_{\alpha,\beta} -\frac{C^I_{\alpha\beta}}{8f_\pi^2}\left(\omega_\alpha + \omega_\beta\right)\sqrt{\frac{1}{m_\alpha m_\beta}}\, g_{\alpha\beta}(r)\,|\alpha\rangle\langle\beta|$$

A. Doté, T. Inoue, T. Myo, NPA 912 (2013) 66

Represent the meson-energy part with the momentum operator

$$\omega_\alpha \to m_\alpha + \frac{\mathbf{p}^2}{2\mu_\alpha}, \quad \omega_\beta \to m_\beta + \frac{\mathbf{p}^2}{2\mu_\beta}$$

$\mathbf{p} = -i\hbar\frac{\partial}{\partial\mathbf{r}}$, $\mathbf{r}$: meson-baryon relative coordinate

$\mu_\alpha = \frac{m_\alpha M_\alpha}{m_\alpha + M_\alpha}$: reduced mass in channel α

m_α : meson mass M_α : baryon masss

$$V^{(Mom-dep;NR)}_{MB,I} = \sum_{\alpha,\beta} -\frac{C^I_{\alpha\beta}}{8f_\pi^2}\sqrt{\frac{1}{m_\alpha m_\beta}}\left[\left(m_\alpha + \frac{\mathbf{p}^2}{2\mu_\alpha}\right)g_{\alpha\beta}(r) + g_{\alpha\beta}(r)\left(m_\beta + \frac{\mathbf{p}^2}{2\mu_\beta}\right)\right]|\alpha\rangle\langle\beta|$$

$$g_{\alpha\beta}(r) = \left(\sqrt{\pi}d_{\alpha\beta}\right)^{-3}\exp\left[-\left(r/d_{\alpha\beta}\right)^2\right]$$

... Normalized Gaussian

Chiral SU(3)-based $K^{bar}N$ potential - Momentum-dependent type

- *Improve our chiral SU(3)-based $K^{bar}N$ potential, referring Revai's proposal*

Momentum-dependent type Chiral potential (Non-rela. ver.)

$$V_{MB,I}^{(Mom-dep;\,NR)} = \sum_{\alpha,\beta} -\frac{C_{\alpha\beta}^{I}}{8f_\pi^2}\sqrt{\frac{1}{m_\alpha m_\beta}}\left[\left(m_\alpha + \frac{\mathbf{p}^2}{2\mu_\alpha}\right)g_{\alpha\beta}(r) + g_{\alpha\beta}(r)\left(m_\beta + \frac{\mathbf{p}^2}{2\mu_\beta}\right)\right]|\alpha\rangle\langle\beta|$$

- ***Meson's energy is not given before the calculation. It changes automatically during the calculation.***

→ Don't need to search for the self-consistent solution

... In case of the energy-dependent potential used in our previous studies, we needed to consider the self-consistent condition for the $K^{bar}N$ energy.

→ Don't worry about how to estimate the $K^{bar}N$ energy in many-body systems

.... In our studies of K^-pp system, we examined two ansatzes (Field picture / Particle picture) to estimate the energy of the $K^{bar}N$ pair in the three-body system.

- *Local Gaussian form factor in r-space*
- *Range parameters in the Gaussian form factor are constrained with the $K^{bar}N$ scattering length. (SIDDHARTA data and a coupled-channel chiral dynamics)*

Complex Scaling Method

S. Aoyama, T. Myo, K. Kato, K. Ikeda, PTP116, 1 (2006)
T. Myo, Y. Kikuchi, H. Masui, K. Kato, PPNP79, 1 (2014)

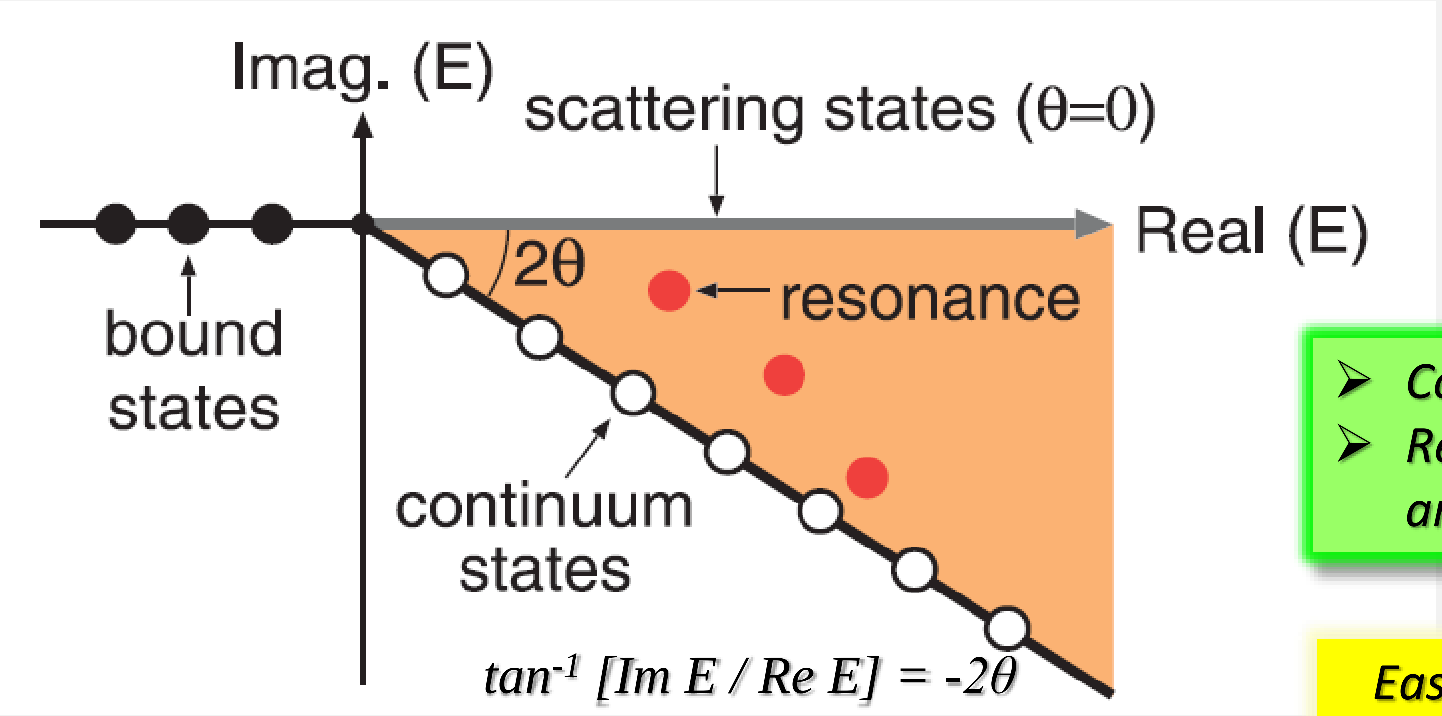
… Find resonance poles on complex energy plane!

Complex rotation (Complex scaling) of coordinate

Resonance wave function → L^2 integrable

$$U(\theta): \quad \mathbf{r} \to \mathbf{r}\, e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}\, e^{-i\theta}$$

Diagonalize $H_\theta = U(\theta)\, H\, U^{-1}(\theta)$ with Gaussian base,



- *Continuum state appears on 2ϑ line.*
- *Resonance pole is off from 2ϑ line, and independent of ϑ. (ABC theorem)*

Easily applicable to many-body systems!

Fully coupled-channel Complex Scaling Method

A. Dote, T. Inoue, T. Myo,
PRC 95, 062201(R) (2017)

… Treat all channels explicitly!

$$
\begin{aligned}
\left|"K^- pp"\right\rangle = & \sum_a C_a^{(K\{NN\}+)} G_a^{(K\{NN\}+)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \left|S_{NN}=0\right\rangle \left|\left[K\left[NN\right]_1\right]_{T=1/2,\,Tz=1/2}\right\rangle \\
& + \sum_a C_a^{(K\{NN\}-)} G_a^{(K\{NN\}-)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \left|S_{NN}=0\right\rangle \left|\left[K\left[NN\right]_0\right]_{T=1/2,\,Tz=1/2}\right\rangle \\
& + \sum_a C_a^{(\pi\{\Sigma N\}+)} G_a^{(\pi\{\Sigma N\}+)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \left|S_{\Sigma N}=0\right\rangle \left|\left[\left[\pi\Sigma\right]_0 N\right]_{T=1/2,\,Tz=1/2}, \{\Sigma N\}_S\right\rangle \\
& + \sum_a C_a^{(\pi\{\Sigma N\}-)} G_a^{(\pi\{\Sigma N\}-)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \left|S_{\Sigma N}=0\right\rangle \left|\left[\left[\pi\Sigma\right]_0 N\right]_{T=1/2,\,Tz=1/2}, \{\Sigma N\}_A\right\rangle \\
& + \sum_a C_a^{(\pi\{\Sigma N\}+)} G_a^{(\pi\{\Sigma N\}+)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \left|S_{\Sigma N}=0\right\rangle \left|\left[\left[\pi\Sigma\right]_1 N\right]_{T=1/2,\,Tz=1/2}, \{\Sigma N\}_S\right\rangle \\
& + \sum_a C_a^{(\pi\{\Sigma N\}-)} G_a^{(\pi\{\Sigma N\}-)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \left|S_{\Sigma N}=0\right\rangle \left|\left[\left[\pi\Sigma\right]_1 N\right]_{T=1/2,\,Tz=1/2}, \{\Sigma N\}_A\right\rangle \\
& + \sum_a C_a^{(\pi\{\Lambda N\}+)} G_a^{(\pi\{\Lambda N\}+)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \left|S_{\Lambda N}=0\right\rangle \left|\left[\left[\pi\Lambda\right]_1 N\right]_{T=1/2,\,Tz=1/2}, \{\Lambda N\}_S\right\rangle \\
& + \sum_a C_a^{(\pi\{\Lambda N\}-)} G_a^{(\pi\{\Lambda N\}-)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) \left|S_{\Lambda N}=0\right\rangle \left|\left[\left[\pi\Lambda\right]_1 N\right]_{T=1/2,\,Tz=1/2}, \{\Lambda N\}_A\right\rangle
\end{aligned}
$$

Ch. 1: $K^{bar}NN$, $NN={}^1E$

Ch. 2: $K^{bar}NN$, $NN={}^1O$

Ch. 3: $\pi\Sigma N$, $[\pi\Sigma]_{I=0}$, $\{\Sigma N\}_{Sym.}$

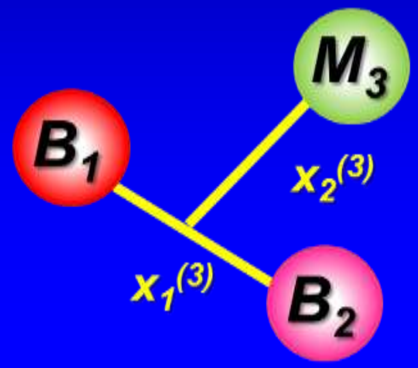
Ch. 4: $\pi\Sigma N$, $[\pi\Sigma]_{I=0}$, $\{\Sigma N\}_{Asym.}$

Ch. 5: $\pi\Sigma N$, $[\pi\Sigma]_{I=1}$, $\{\Sigma N\}_{Sym.}$

Ch. 6: $\pi\Sigma N$, $[\pi\Sigma]_{I=1}$, $\{\Sigma N\}_{Asym.}$

Ch. 7: $\pi\Lambda N$, $[\pi\Lambda]_{I=1}$, $\{\Lambda N\}_{Sym.}$

Ch. 8: $\pi\Lambda N$, $[\pi\Lambda]_{I=1}$, $\{\Lambda N\}_{Asym.}$



- *Spatial part*
  = Correlated Gaussian function
  … including 3-types of
  Jacobi coordinate

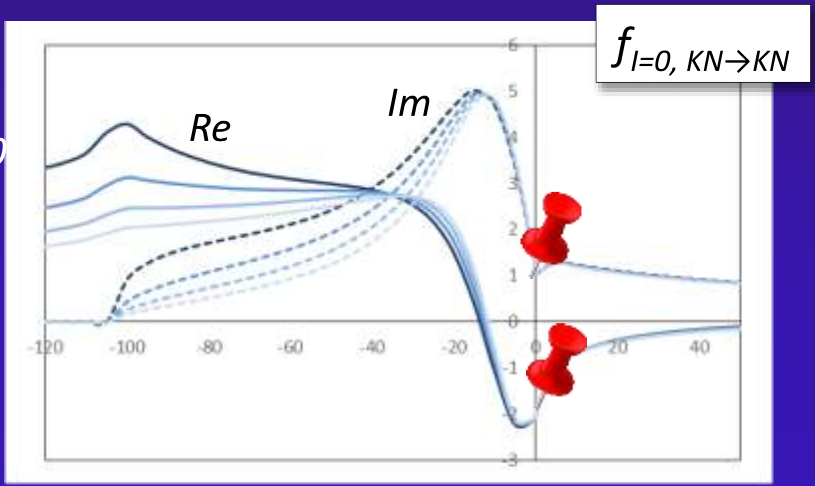
$$\text{“}K^-pp\text{”} = K^{bar}NN - \pi\Sigma N - \pi\Lambda N \;(J^\pi=0^-,\, T=1/2)$$

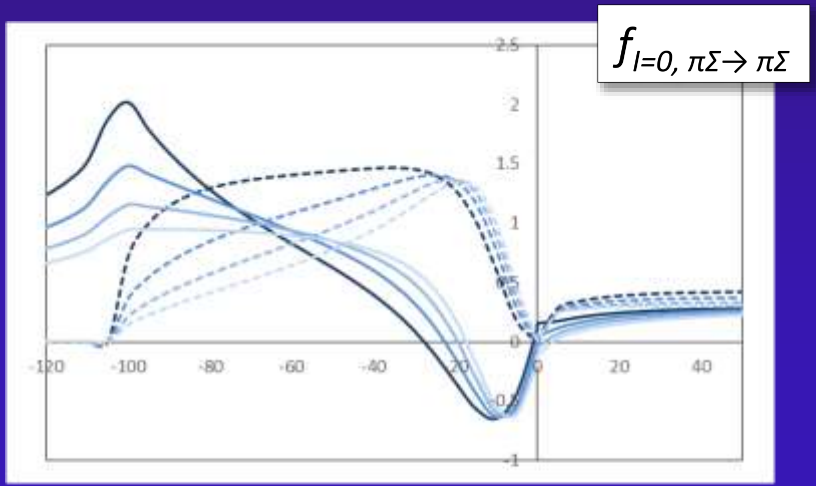
3. Result and Discussion

2-body system: Scattering amplitude & Pole position

- I=0 channel ($K^{bar}N$-$\pi\Sigma$)

f_π=90
100
110
120





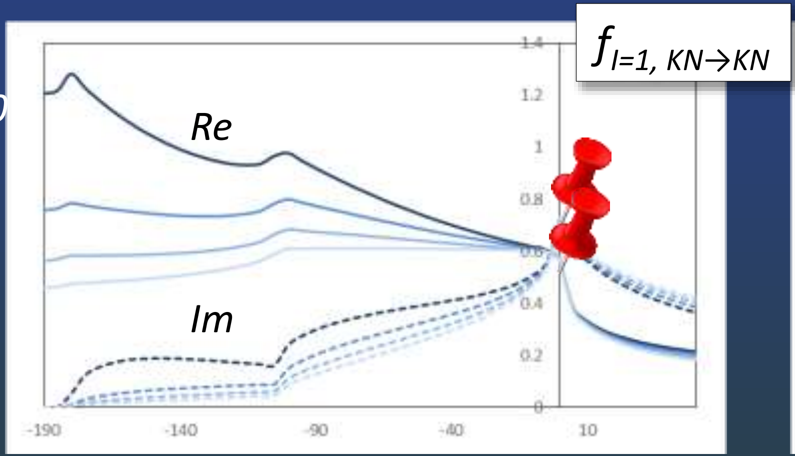
Λ* pole position

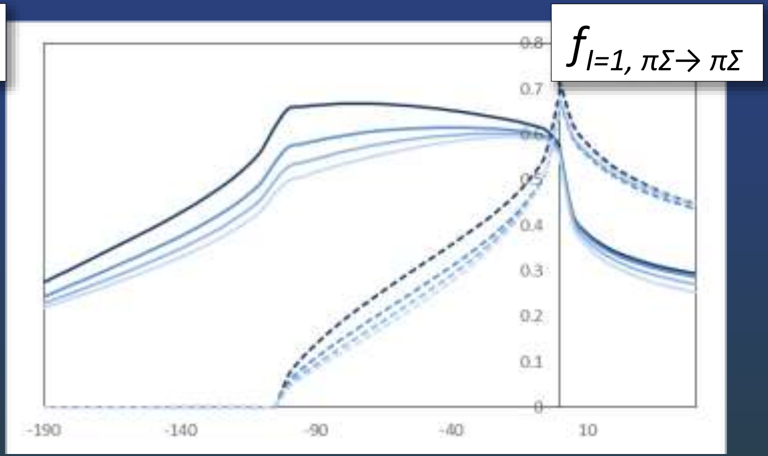
fpi	E(KN)	−Γ /2
90	−6.4	−16.0
100	−6.6	−15.1
110	−7.0	−14.2
120	−7.3	−13.5

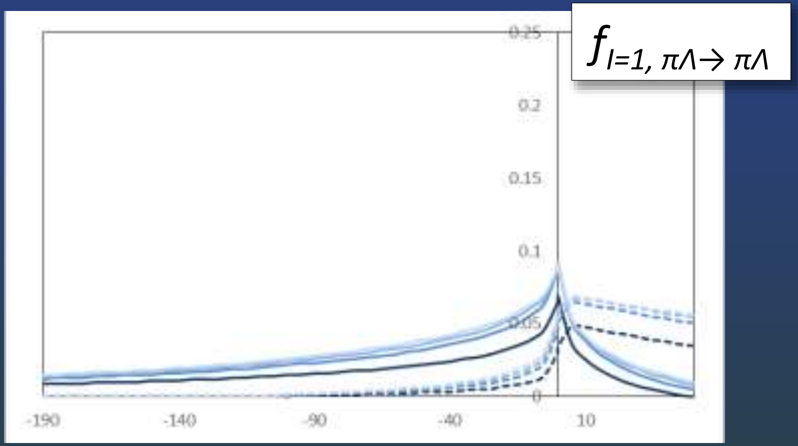
[MeV]

- I=1 channel ($K^{bar}N$-$\pi\Sigma$-$\pi\Lambda$)

f_π=90
100
110
120



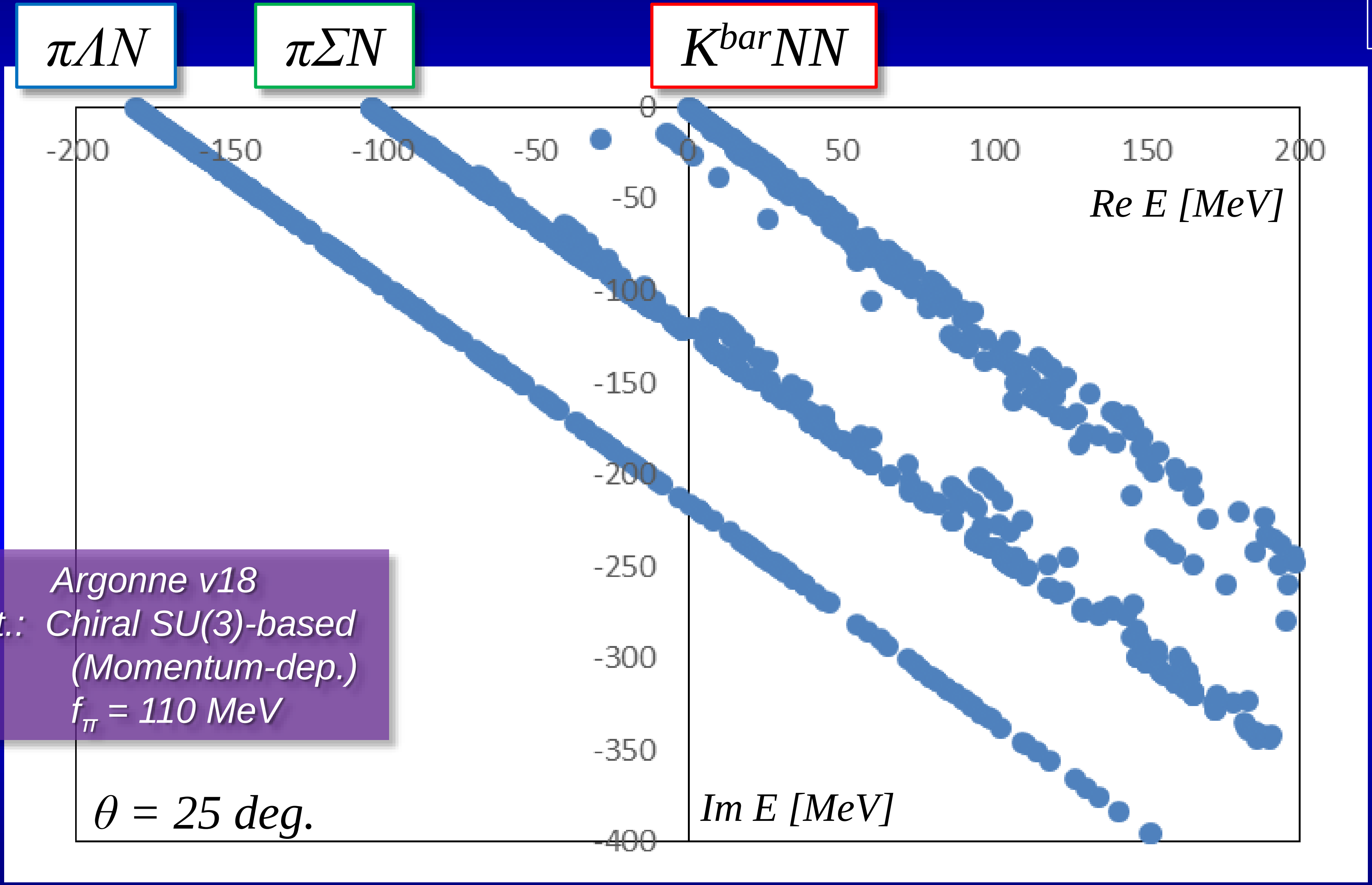




- *SIDDHARTA K^-p data: M. Bazzi et al., NPA 881, 88 (2012)*
- *Coupled-channel chiral dynamics: Y. Ikeda, T. Hyodo, W. Weise, NPA 881, 98 (2012)*

3-body system "K⁻pp": Eigenvalue distribution

Gaussian basis:
- *Range = 0.1~20 fm*
- *#base / coordinate = 20*

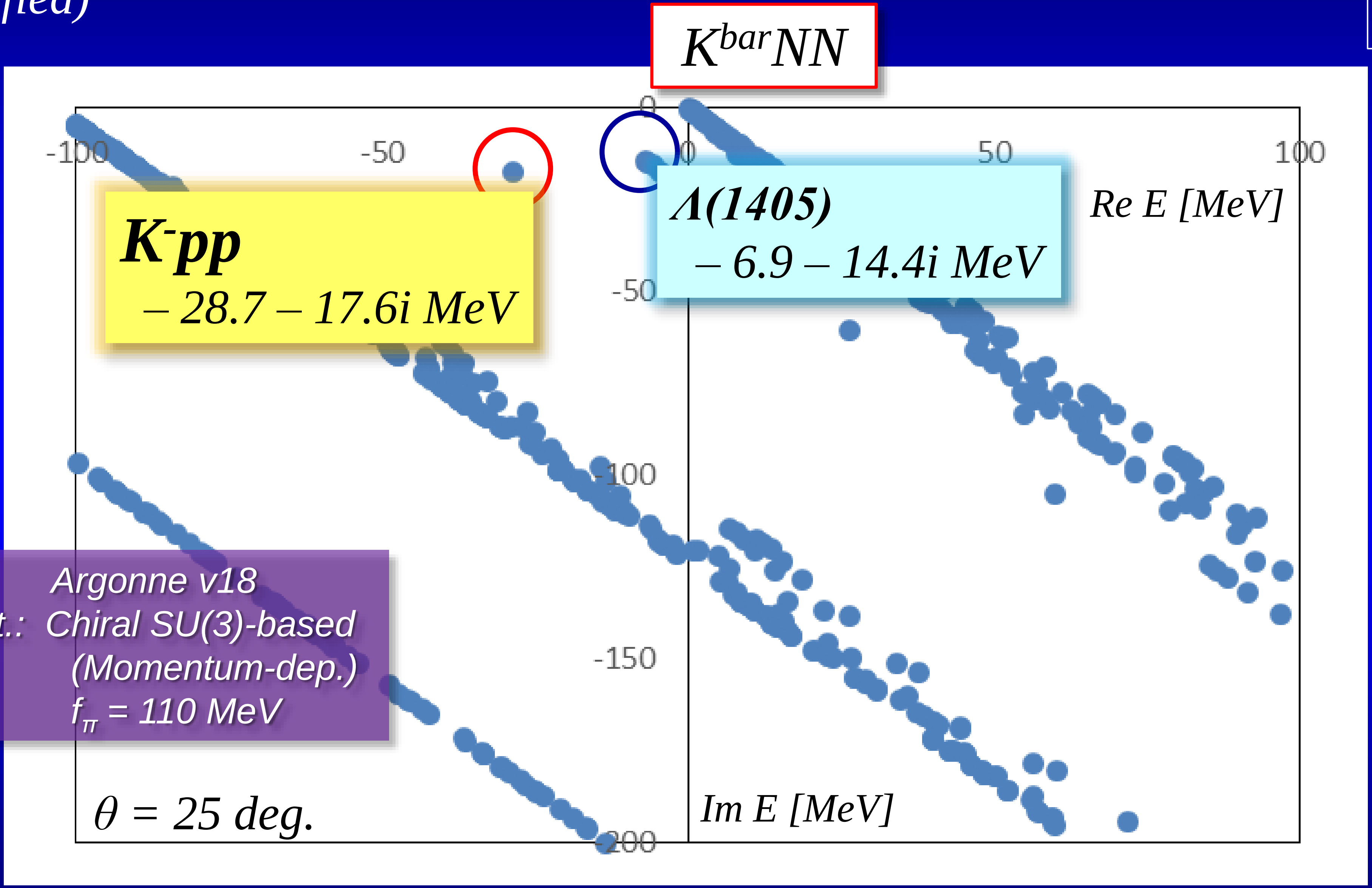


- NN pot.: Argonne v18
- K^{bar}N pot.: Chiral SU(3)-based (Momentum-dep.) f_π = 110 MeV

3-body system "K⁻pp": Eigenvalue distribution

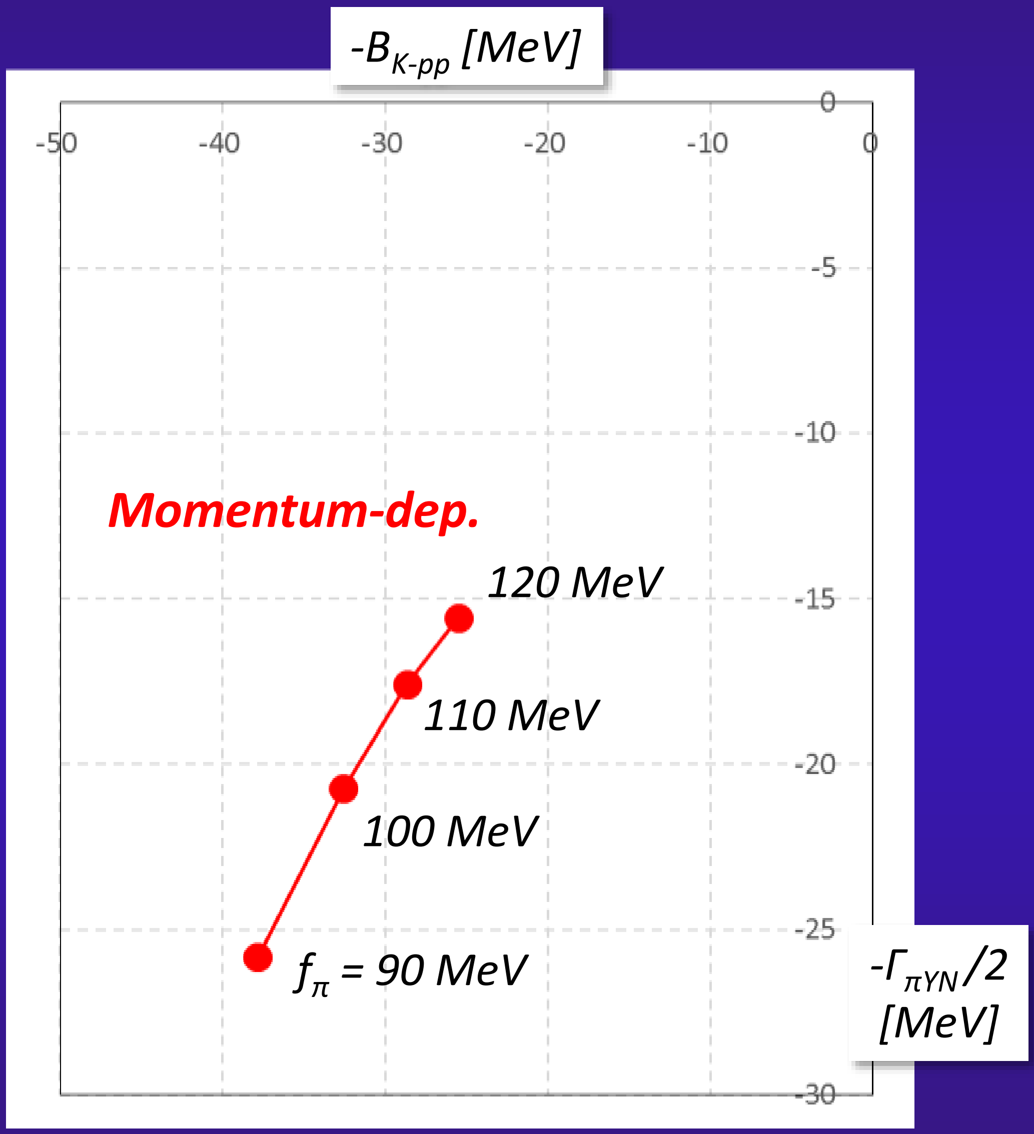
(Magnified)

Gaussian basis:
- Range = 0.1~20 fm
- #base / coordinate = 20



- NN pot.: Argonne v18
- $K^{bar}N$ pot.: Chiral SU(3)-based (Momentum-dep.) f_π = 110 MeV

3-body system "K^-pp": Property of the resonance pole



fπ	110	
	Re	Im
E(K−pp) [MeV]	**−28.7**	**−17.6**
B(M) [MeV]	86.0	6.0
Distance [fm]		
NN	1.80	−0.02
K−(NN)	1.23	−0.11
KN	1.53	−0.10
KN (I=0)	1.41	−0.12
KN (I=1)	1.82	−0.05
Distance [fm]		
Σ N	1.42	0.41
Λ N	1.18	0.13
Norm		
KbarNN	1.119	−0.189
π Σ N	−0.119	0.184
π Λ N	0.000	0.004

cf) Deuteron NN distance = 3.9 fm

cf) Size of Λ = 1.43 -0.72i fm*

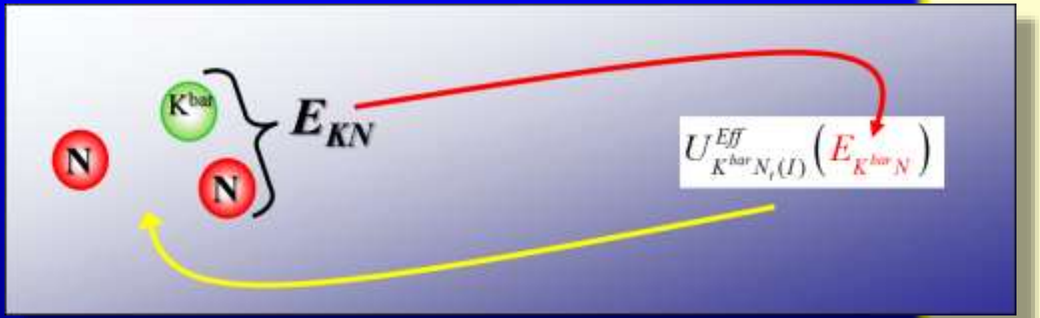
- *Dominant component = $K^{bar}NN$*

3. Result and Discussion

Comparison with Energy-dependent case

- *How different is the current result from the previous result obtained with Energy-dependent potential?*
- *In the study of K^-pp, we have examined two ansatzes (Field picture / Particle picture) to evaluate the K^{bar}N-pair energy in the three-body system. Which ansatz is better?*

Self-consistency for K^{bar}N energy

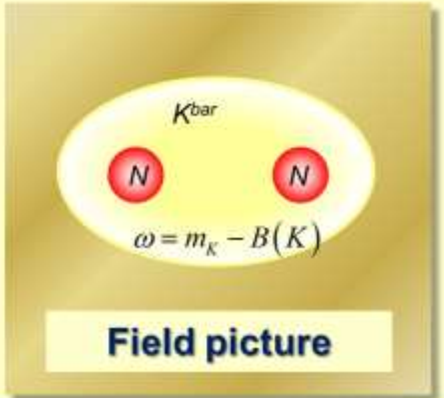


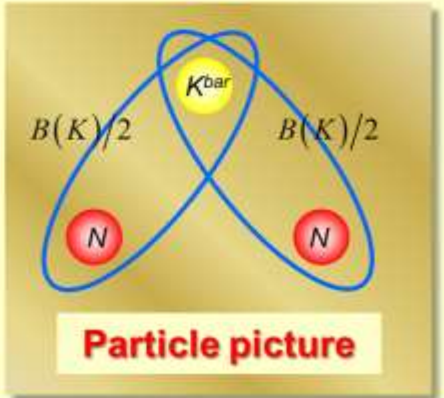
How to estimate the two-body energy in the three-body system?

A. D., T. Hyodo, W. Weise, PRC79, 014003 (2009)

1. Kaon's binding energy: $B(K) \equiv -\left\{\langle \hat{H} \rangle - \langle \hat{H}_{NN} \rangle\right\}$ $\hat{H}_{NN}$: Hamiltonian of two nucleons
2. Define a K^{bar}N-bond energy in two ways

$$E_{KN} = M_N + \omega = \begin{cases} M_N + m_K - B(K) & \text{: Field picture} \\ M_N + m_K - B(K)/2 & \text{: Particle picture} \end{cases}$$





Comparison with Energy-dependent case

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- *In the study of K^-pp, we have examined two ansatzes (Field picture / Particle picture) to evaluate the $K^{bar}N$-pair energy in the three-body system. Which ansatz is better?*

Momentum−dependent Chiral potential

$$V_{MB,I}^{(Mom-dep;\,NR)} = \sum_{\alpha,\beta} -\frac{C_{\alpha\beta}^{I}}{8f_\pi^2}\sqrt{\frac{1}{m_\alpha m_\beta}}\left[\left(m_\alpha + \frac{\mathbf{p}^2}{2\mu_\alpha}\right) g_{\alpha\beta}(r) \;+\; g_{\alpha\beta}(r)\left(m_\beta + \frac{\mathbf{p}^2}{2\mu_\beta}\right)\right]|\alpha\rangle\langle\beta|$$

Energy−dependent Chiral potential

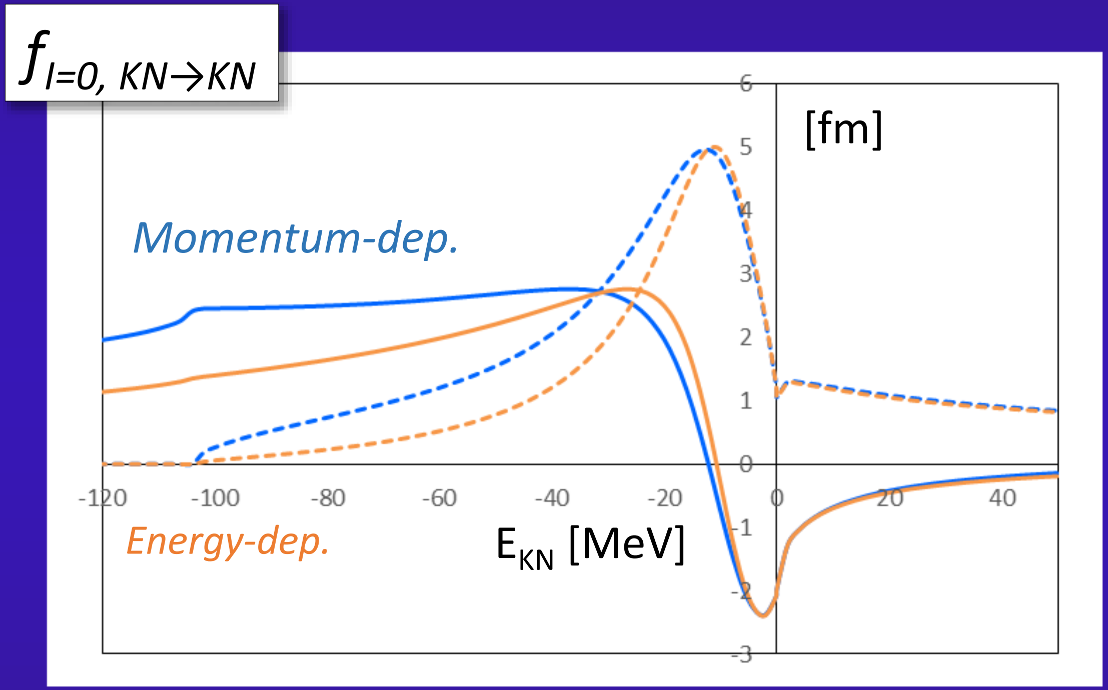
$$V_{MB,I}^{(E-dep;\,NR)} = \sum_{\alpha,\beta} -\frac{C_{\alpha\beta}^{I}}{8f_\pi^2}\sqrt{\frac{1}{m_\alpha m_\beta}}\left(2\sqrt{s} - M_\alpha - M_\beta\right) g_{\alpha\beta}(r)\,|\alpha\rangle\langle\beta|$$

$\sqrt{s}$: Meson−Baryon CM energy

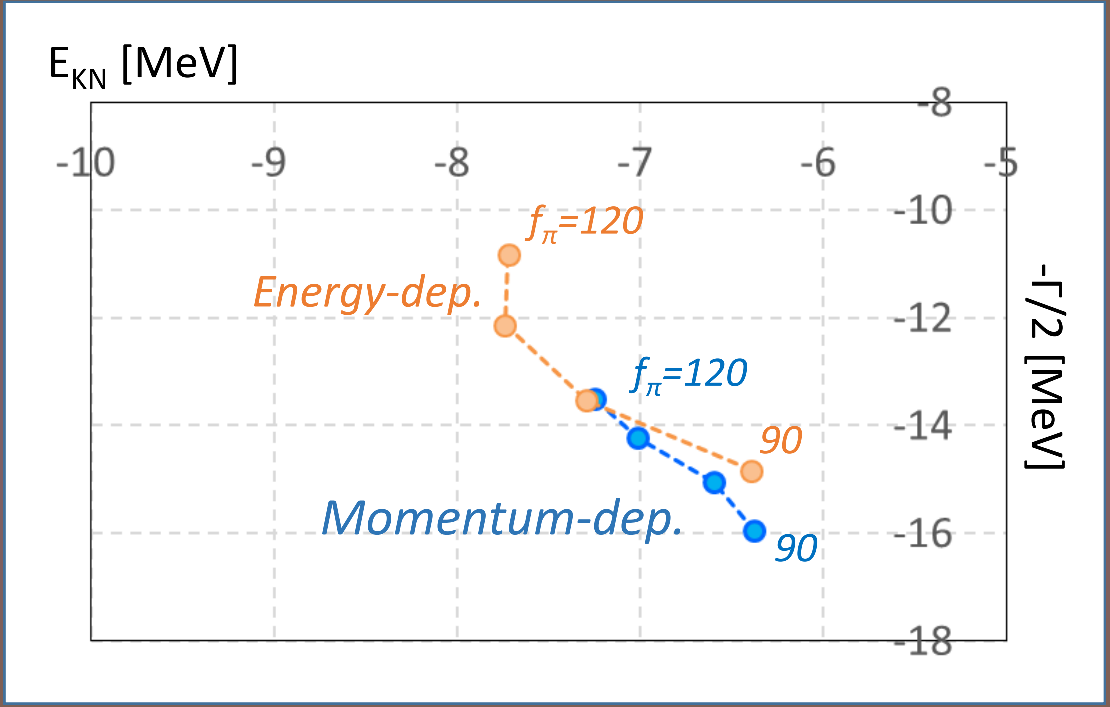
$M_{\alpha(\beta)}$: Baryon mass in channel $\alpha(\beta)$

Comparison with Energy-dependent case: 2-body system

- *I=0 $K^{bar}N$ scattering amplitude (f_π=110 MeV)*



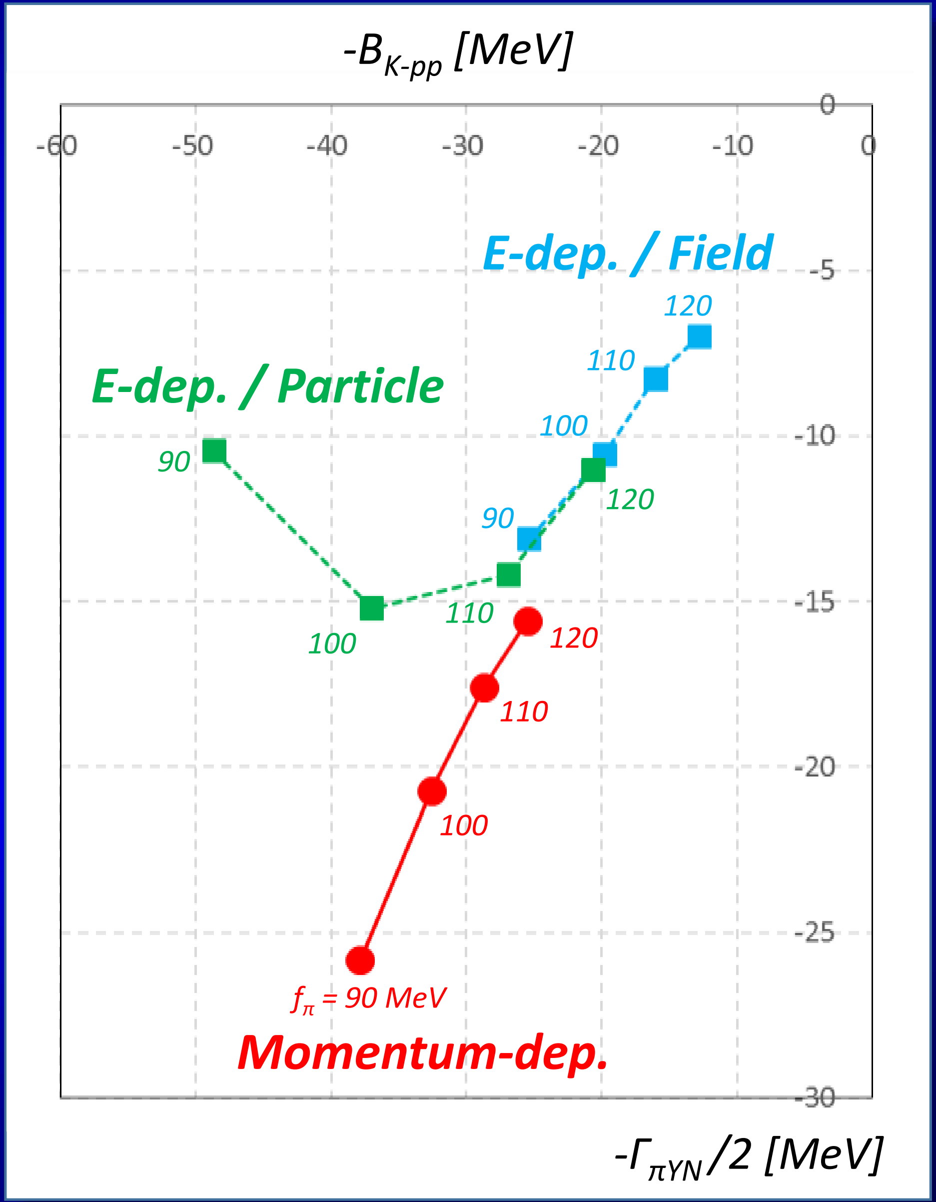
- *Λ* resonance pole (f_π=90~120 MeV)*



- Near the $K^{bar}N$ threshold, both results are almost the same.
- Far below the $K^{bar}N$ threshold, Momentum-dep. becomes more attractive than Energy-dep.

- Binding energy ($-E_{KN}$) is almost the same in both cases.
- Momentum-dep. potential gives slightly larger decay with than Energy-dep. potential.

Comparison with Energy-dependent case: 3-body system



- **Momentum-dependent potential**

fπ	−B(K−pp)	−Γ (π YN)/2
90	−37.9	−25.8
100	−32.6	−20.7
110	−28.7	−17.6
120	−25.5	−15.6

- **Energy-dependent potential**

Field picture

fπ	−B(K−pp)	−Γ (π YN)/2
90	−25.4	−13.1
100	−19.8	−10.5
110	−16.1	−8.3
120	−12.8	−7.0

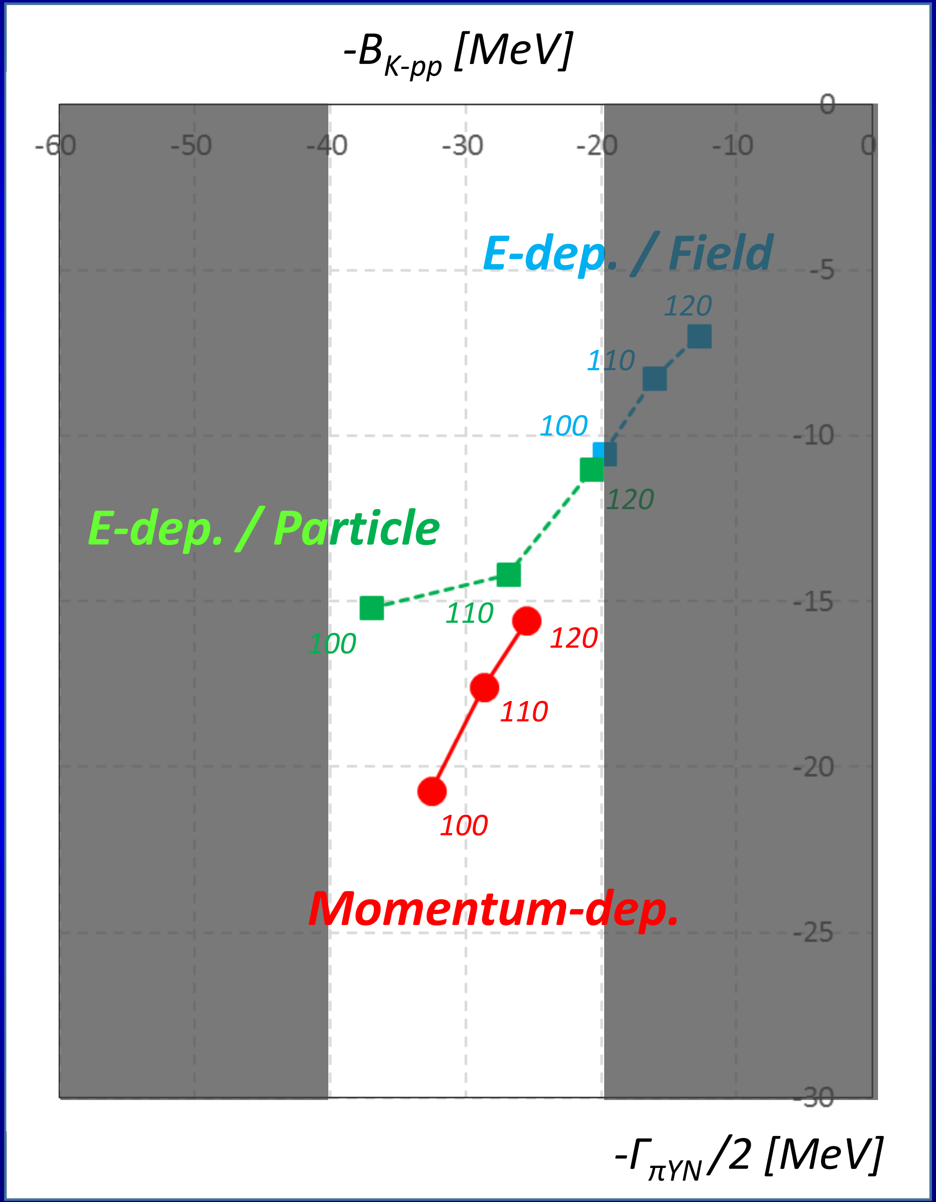
- **Energy-dependent potential**

Particle picture

fπ	−B(K−pp)	−Γ (π YN)/2
90	−48.7	−10.4
100	−37.0	−15.2
110	−26.9	−14.2
120	−20.7	−11.0

□ *In Momentum-dependent case,*
decay width is nearly twice as large as Energy-dependent case.

Comparison with Energy-dependent case: 3-body system



- Momentum-dependent potential

fπ	−B(K−pp)	−Γ (π YN)/2
90	−37.9	−25.8
100	−32.6	−20.7
110	−28.7	−17.6
120	−25.5	−15.6

- Energy-dependent potential

Field picture

fπ	−B(K−pp)	−Γ (π YN)/2
90	−25.4	−13.1
100	−19.8	−10.5
110	−16.1	−8.3
120	−12.8	−7.0

- Energy-dependent potential

Particle picture

fπ	−B(K−pp)	−Γ (π YN)/2
90	−48.7	−10.4
100	−37.0	−15.2
110	−26.9	−14.2
120	−20.7	−11.0

- ☐ *In Momentum-dependent case, decay width is nearly twice as large as Energy-dependent case.*
- ☐ *Particle picture in Energy-dependent case gives the binding energy close to the Momentum-dependent case.*

4. Summary

Summary

- ***Motivated by Prof. Revai's proposal, we have improved our chiral SU(3)-based potential to be a Momentum-dependent potential.***
  - *Meson's energy involved in the potential is represented with the momentum operator.*
  - *Unlike the energy-dependent potential,*
    we are free from searching for self-consistent solution.

- ***Confirmed that such a Momentum-dependent chiral potential works well in Fully coupled-channel Complex Scaling Method.***
  - *Binding energy of $K^{-}pp$ = 25~38 MeV,   Half decay width (mesonic) = 15~26 MeV*

- ***Compared with the Energy-dependent potential,***
  - ☐ ***Decay width is obtained to be nearly twice larger.***
  - ☐ ***Binding energy is close to that obtained with Particle picture.***