



# Strangeness nuclear physics: Status and perspectives

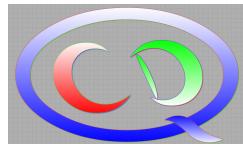
**Ulf-G. Meißner, Univ. Bonn & FZ Jülich**

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- Short introduction
- Baryon-baryon interactions in chiral EFT
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- Summary & outlook

Details in: UGM, Haidenbauer, *Int. J. Mod. Phys. E* **26** (2017) 1740019 [1603.06429 [nucl-th]]

Petschauer, Haidenbauer, Kaiser, UGM, Weise, *Front. in Phys.* **8** (2020) 12 [2002.00424 [nucl-th]]

# Short introduction

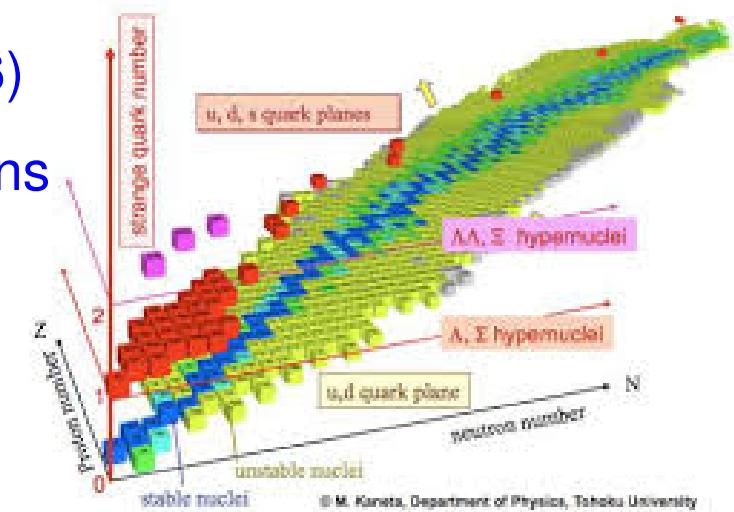
# FUNDAMENTAL QUESTIONS

- What are the baryon-baryon forces?

- NN interactions: thousands of data points, ultraprecise EFT at N4LO+  
Reinert, Krebs, Epelbaum (2018)
- YN & YY interactions: some 36 data points of modest quality
  - however, scattering lengths of natural size
  - little indication of exotic BB bound states

- How does the third dimension of the nuclear chart look?

- Hypernuclei: a new form of matter ( $\simeq 30 + 6$ )
- Properties determined by YN & YY interactions
  - hypernuclei as a tool to pin down these interactions
- Strange quarks modify the nuclear EoS
  - Properties of neutron stars



# Baryon-baryon interactions in chiral EFT

# Baryon-baryon interactions in chiral EFT

Polinder, Haidenbauer, UGM, Nucl. Phys. A779 (2006) 244, Haidenbauer, UGM, Phys. Lett. B653 (2007) 29, Phys. Lett. B684 (2010) 275

- consider BB interactions with  $S = -1, -2, -3, -4$  in chiral EFT

$$S = 0 : NN \rightarrow NN$$

$$S = -1 : \Lambda N \rightarrow \Lambda N, \Lambda N \rightarrow \Sigma N, \Sigma N \rightarrow \Sigma N$$

$$S = -2 : \Lambda\Lambda \rightarrow \Lambda\Lambda, \Lambda\Lambda \rightarrow \Xi N, \Lambda\Lambda \rightarrow \Sigma\Sigma, \Xi N \rightarrow \Xi N, \Xi N \rightarrow \Sigma\Sigma, \Sigma\Sigma \rightarrow \Sigma\Sigma, \Sigma\Lambda \rightarrow \Sigma\Lambda$$

$$S = -3 : \Xi\Lambda \rightarrow \Xi\Lambda, \Xi\Lambda \rightarrow \Xi\Sigma, \Xi\Sigma \rightarrow \Xi\Sigma$$

$$S = -4 : \Xi\Xi \rightarrow \Xi\Xi$$

- DOFs: octet baryons  $B$  coupled to octet of Goldstone bosons  $P$
- important channel coupling (e.g.  $\Lambda N \rightarrow \Lambda N, \Sigma N$ )
  - ↪ solve the LS equation in the particle basis
  - ↪ this induces SU(3) breaking even if the Lagrangian is SU(3) symmetric
  - ↪ LS equation requires regularization (cut-off regulator)
  - ↪ include the Coulomb interaction via the Vincent-Phatak method

Vincent, Phatak (1974)

# Lippmann-Schwinger equation

- Coupled channels partial wave Lippmann-Schwinger equation:

$$T_{\rho''\rho'}^{\nu''\nu',J}(p'', p'; \sqrt{s}) = V_{\rho''\rho'}^{\nu''\nu',J}(p'', p') + \sum_{\rho,\nu} \int_0^\infty \frac{dp p^2}{(2\pi)^3} V_{\rho''\rho}^{\nu''\nu,J}(p'', p) \frac{2\mu_\nu}{q_\nu^2 - p^2 + i\eta} T_{\rho\rho'}^{\nu\nu',J}(p, p'; \sqrt{s})$$

- $\nu$  denotes particle channels,  $\rho$  indicates the partial wave
- Intermediate state momentum:  $\sqrt{s} = \sqrt{M_{B_{1,\nu}}^2 + q_\nu^2} + \sqrt{M_{B_{2,\nu}}^2 + q_\nu^2}$
- partial wave projected potentials  $V_{\rho''\rho'}^J(p'', p')$  are subject to the power counting

Weinberg (1991)

- Regulator function for the potential:  $f_\Lambda(p', p) = e^{-(p'^4 + p^4)/\Lambda^4}$ 
  - cutoff varied in a certain range (see later)
  - more sophisticated regularization available (but ...)

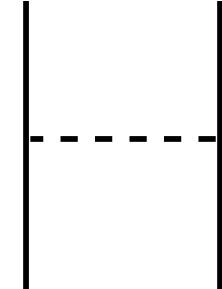
# Leading order interactions

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Polinder, Haidenbauer, UGM, Nucl. Phys. A779 (2006) 244, Haidenbauer, UGM, Phys. Lett. B653 (2007) 29, Phys. Lett. B684 (2010) 275

- LO potential: one-boson exchanges and four-baryon interactions w/o derivatives
- One-Boson Exchange contribution ( $P = \pi, K, \eta$ )

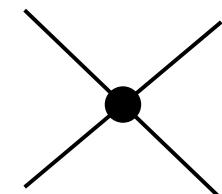
$$V^{B_1 B_2 \rightarrow B'_1 B'_2} = -f_{B_1 B'_1 P} f_{B_2 B'_2 P} \frac{(\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k})}{\vec{k}^2 + m_P^2}$$



with the coupling constants  $f_{BB'P} = \overbrace{\frac{g_A}{2F_\pi}}^{D+F} \times \underbrace{\text{Clebsch} - \text{Gordan}(D/F)}_{\text{SU(3) symmetry}}$

- LO potential from contact interactions w/o derivatives

$$V^{B_1 B_2 \rightarrow B'_1 B'_2} = \frac{1}{4}(1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2) C_{1S0} + \frac{1}{4}(3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2) C_{3S1}$$

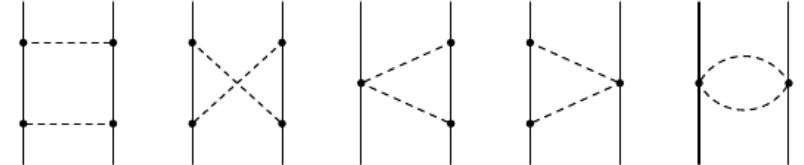


- SU(3) decomposition:  $8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10 \oplus \bar{10} \oplus 27$
- LECs are combinations of the six irreps  $C^1, C^{8_a}, C^{8_s}, C^{10}, C^{\bar{10}}, C^{27}$
- singlet only in  $S = -2$   $\Lambda\Lambda$ ,  $\Xi N$  and  $\Sigma\Sigma$

# Next-to-leading order interactions

Haidenbauer, Petschuer, Kaiser, UGM, Nogga, Nucl. Phys. A915 (2013) 24, Haidenbauer, UGM, Petschauer, Nucl. Phys. A954 (2016) 273  
 Haidenbauer, UGM, Nogga, EPJA 56 (2020) 3

- NLO potential: two-boson exchanges (TBE) and four-baryon interactions w/ 2 derivatives

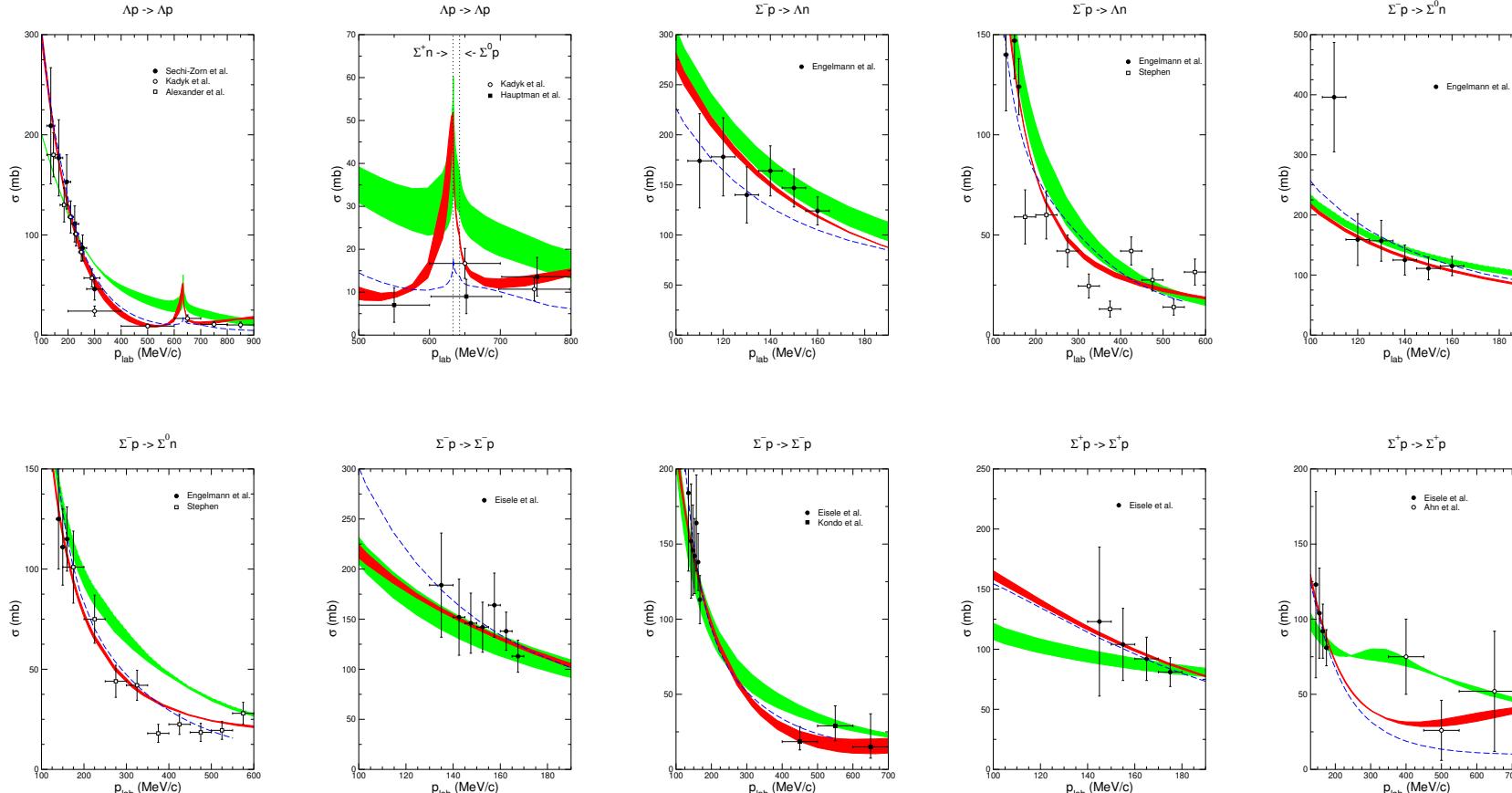


- TBE – some contributions lumped into the contact interactions (e.g. KK-loop)
- Contact interactions:  $V_{\text{NLO}}^{B_1 B_2 \rightarrow B'_1 B'_2} = C_\alpha (p'^2 + p^2) + C_\beta p'^2 + C_\gamma p' p$   
 $\alpha = {}^1S_0, {}^3S_1, \beta = {}^3S_1 - {}^3D_1, \gamma = {}^3P_0, {}^1P_1, {}^3P_1, {}^3P_2$
- Number of LECs limited by SU(3) symmetry:
  - 22 at NLO for all channels and waves
  - 5 at NLO for the S-waves (dominant for  $\Lambda N$  and  $\Sigma N$  at low energies)
- Two variants: **NLO13** = no recourse to NN interaction in the S-wave LECs  
**NLO19** = take 2 S-wave NLO LECs from NN phase shifts + ...

# YN interaction at LO and NLO

- Total XS results (fit to 36 low-energy data points, only cut-off variations)
 

[compare LO with NLO13]



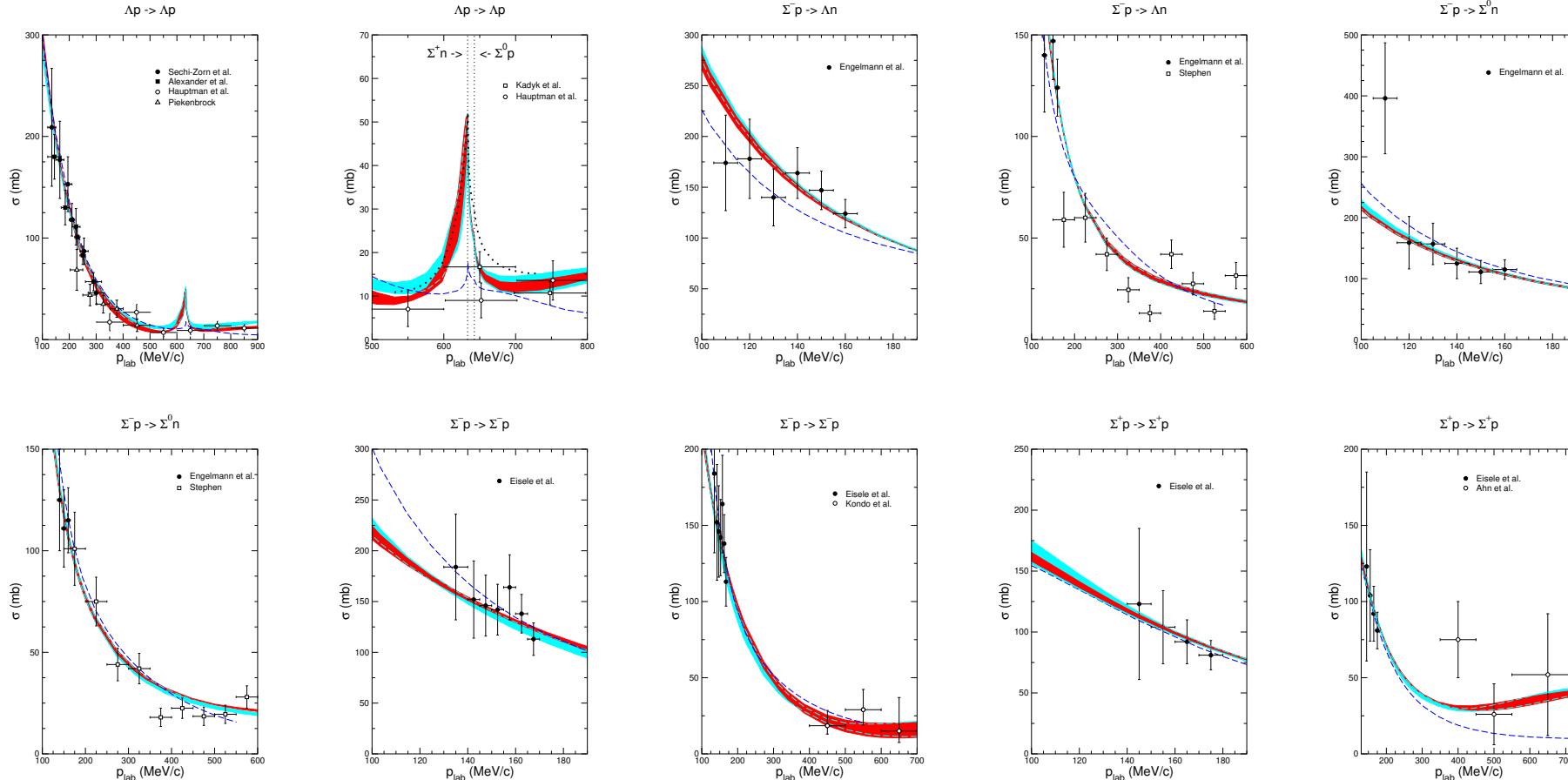
LO  
NLO13

closed symbols: fit  
open symbols: prediction

Jülich '04 potential: Haidenbauer and UGM, Phys. Rev. C 72 (2005) 044005

# YN interaction at NLO

- Total XS results (fit to 36 low-energy data points, only cut-off variations)  
 [better uncertainty estimate available for NLO19]



closed symbols: fit  
 open symbols: prediction

NLO13  
 NLO19  
 J'04

Jülich '04 potential: Haidenbauer and UGM, Phys. Rev. C 72 (2005) 044005

# YN at NLO w/ improved uncertainties

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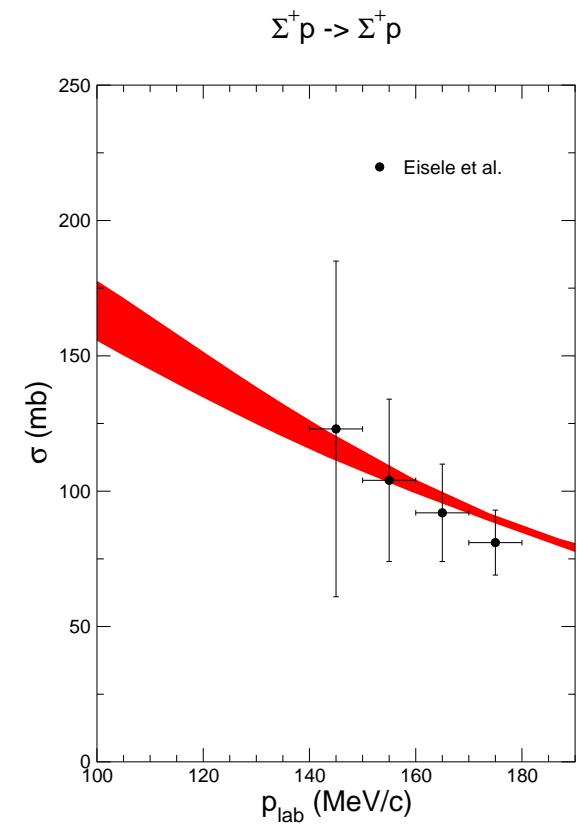
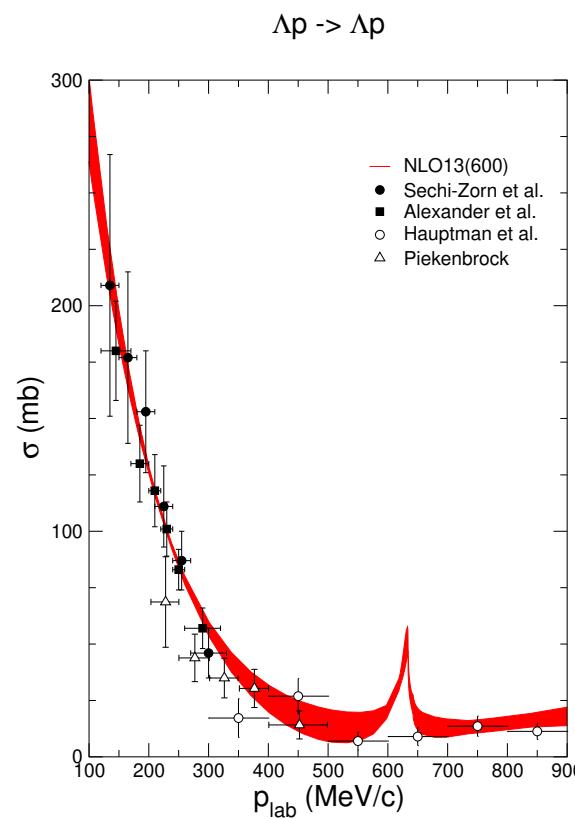
- Instead of varying the cut-off, use

Epelbaum, Krebs, UGM, Eur. Phys. J. A 51 (2015) 53

$$\Delta\sigma(p) = \max(Q^3|\sigma_{\text{LO}}(p)|, Q|\sigma_{\text{NLO}}(p) - \sigma_{\text{LO}}(p)|)$$

$$Q = \max\left(\frac{p}{\Lambda_{\text{hard}}}, \frac{M_\pi}{\Lambda_{\text{hard}}}\right)$$

$$\Lambda_{\text{hard}} = 600 \text{ MeV}$$



→ sizeably smaller bands than in the NN case (faster convergence)

# Hyperon-nucleon scattering lengths

- Varying the cut-off between 500 and 650 MeV

	NLO13	NLO19	Exp.*
$a_s^{\Lambda p}$ [fm]	$-2.91 \dots - 2.90$	$-2.91 \dots - 2.90$	$-1.8^{+2.3}_{-4.2}$
$a_t^{\Lambda p}$ [fm]	$-1.61 \dots - 1.51$	$-1.52 \dots - 1.40$	$-1.6^{+1.1}_{-0.8}$
$a_s^{\Sigma p}$ [fm]	$-3.60 \dots - 3.46$	$-3.90 \dots - 3.43$	
$a_t^{\Sigma p}$ [fm]	$0.49 \dots 0.48$	$0.48 \dots 0.42$	
$\chi^2$	$15.7 \dots 16.8$	$16.0 \dots 18.1$	
$E_B(^3\Lambda H)$ [MeV]	$-2.30 \dots - 2.33$	$-2.32 \dots - 2.32$	$-2.354(50)$

\* Alexander et al., Phys. Rev. **173** (1968) 1453 [not unique]

- $\Lambda p$  data do not allow to disentangle singlet ( ${}^1S_0$ ) and triplet ( ${}^3S_1$ ) contributions
  - ⇒ use the hypertriton BE as an additional constraint
  - ⇒ scattering lengths are of natural size (other than in NN)

# Light hypernuclei

# Three- and four-body systems

- S-shell hypernuclei: use a spin-averaged effective  $\Lambda N$  interaction:

$${}_{\Lambda}^3 \text{H} : \quad \tilde{V}_{\Lambda N} \simeq \frac{3}{4} V_{\Lambda N}^s + \frac{1}{4} V_{\Lambda N}^t$$

$${}_{\Lambda}^4 \text{He}(0^+) : \tilde{V}_{\Lambda N} \simeq \frac{1}{2} V_{\Lambda N}^s + \frac{1}{2} V_{\Lambda N}^t$$

$${}_{\Lambda}^4 \text{He}(1^+) : \tilde{V}_{\Lambda N} \simeq \frac{1}{6} V_{\Lambda N}^s + \frac{5}{6} V_{\Lambda N}^t$$

- Solve Faddeev-Yakubowsky equations with chiral N4LO+ NN potential and NLO YN:

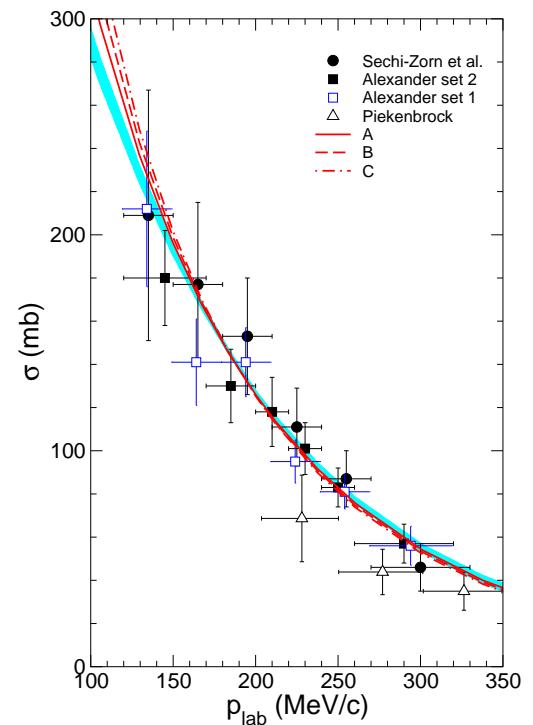
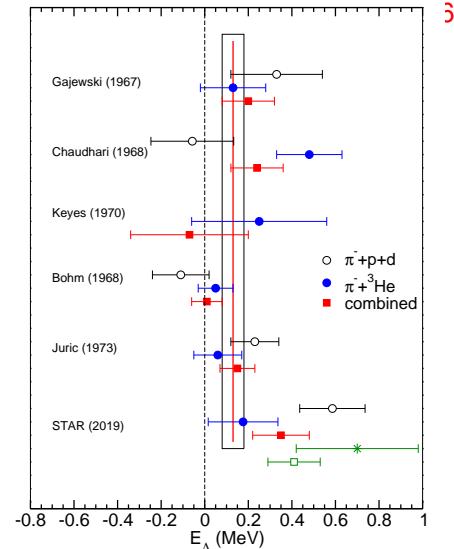
YY int.	$E_{\Lambda}({}_{\Lambda}^3 \text{H})$ [MeV]	$E_{\Lambda}({}_{\Lambda}^4 \text{He}(0^+))$ [MeV]	$E_{\Lambda}({}_{\Lambda}^4 \text{He}(1^+))$ [MeV]
NLO13	<b>0.087 … 0.135</b>	<b>1.490 … 1.705</b>	<b>0.615 … 0.790</b>
NLO19	<b>0.095 … 0.100</b>	<b>1.530 … 1.643</b>	<b>0.916 … 1.226</b>
Expt.	<b>0.13(5)</b>	<b>2.39(3)</b>	<b>0.98(3)</b>

$$E_{\Lambda} \equiv B_{\Lambda} - B_d$$

- Disclaimer:
  - no 3BFs (as seen from the cutoff dependence)  $\hookrightarrow$  see later
  - no CSB operators included (in the works), see also Gazda, Gal (2016)

# A stronger bound hypertriton?

- Benchmark:  $E_\Lambda = 0.13 \pm 0.05$  MeV  
Juric et al. (1973)
- New STAR result:  $E_\Lambda = 0.41 \pm 0.12 \pm 0.11$  MeV  
Adam et al. (STAR collab.) (2019)
- To increase  $^3\Lambda$ H binding and keep  $\sigma_{\Lambda p}$   
→ increase singlet and decrease triplet  $\Lambda p$  interaction
- Three fits with increased  $a_s^{\Lambda p} \rightsquigarrow$  little difference
  - $a_s = -4.0$  fm solid line
  - $-4.5$  fm dashed line
  - $-5.0$  fm dot-dashed line
- Requires SU(3) breaking in the LO  $\Lambda N$ ,  $\Sigma N$  LECs  
→ otherwise the  $\Sigma^+ p$  channel is no longer described  
Le, Haidenbauer, UGM, Nogga, Phys. Lett. B 801 (2020) 135189



# Three- and four-body systems revisited

- What are the consequences of a stronger bound hypertriton for light hypernuclei?
- Solve Faddeev-Yakubowsky equations with chiral N4LO+ NN potential and NLO YN:

YY int.	$E_\Lambda(^3\Lambda\text{H})$	$E_\Lambda(^4\Lambda\text{He}(0^+))$	$E_\Lambda(^4\Lambda\text{He}(1^+))$	$\Delta E(^4\Lambda\text{He})$
NLO13	<b>0.14</b> ... 0.09	<b>1.71</b> ... 1.49	<b>0.79</b> ... 0.62	<b>0.92</b> ... 0.88
NLO19	<b>0.10</b> ... 0.10	<b>1.64</b> ... 1.53	<b>1.23</b> ... 0.92	<b>0.42</b> ... 0.61
Fit A	<b>0.32</b> ... 0.29	<b>2.11</b> ... 1.83	<b>1.14</b> ... 0.67	<b>0.97</b> ... 1.16
Fit B	<b>0.39</b> ... 0.36	<b>2.14</b> ... 1.89	<b>0.95</b> ... 0.57	<b>1.18</b> ... 1.32
Fit C	<b>0.47</b> ... 0.44	<b>2.24</b> ... 1.96	<b>0.89</b> ... 0.49	<b>1.36</b> ... 1.47
Expt.	<b>0.13(5)/0.41(12)</b>	<b>2.39(3)</b>	<b>0.98(3)</b>	<b>1.406(2)(2)</b>

- All energies in MeV!
- Disclaimer:
  - no 3BFs (as seen from the cutoff dependence)  $\hookrightarrow$  see later
  - no CSB operators included (in the works), see also [Gazda, Gal \(2016\)](#)

# P-shell hypernuclei

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- Use the Jacobi no-core shell model:

started for s-shell nuclei in Navratil, Kamuntavicius, Barrett (2000)  
extended to p-shell nuclei in Liebig, UGM, Nogga (2016)

- exact antisymm. in Jacobi-coordinates: very difficult / time-consuming
- separation of the CM motion / basis states with definite  $J$  and  $I$
- once cfp's etc are calculated and stored, calculations are fast & easy
- use SRG-softened interactions to achieve convergence

Bogner, Kuo, Schwenk (2003), Bogner et al. (2007), ...

- Extension to p-shell hypernuclei: SRG-softened N4LO+ NN & NLO YN interactions

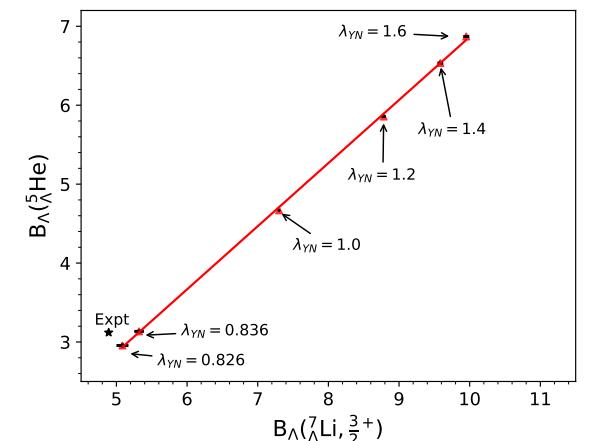
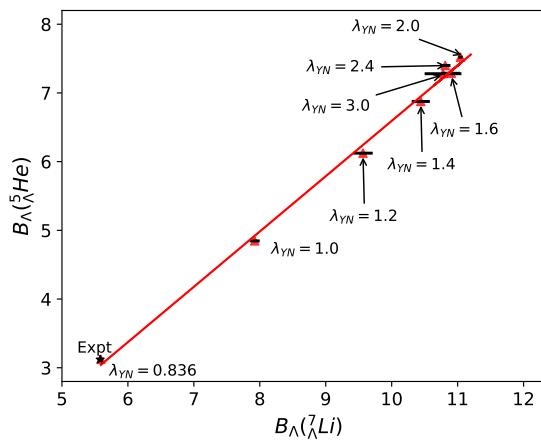
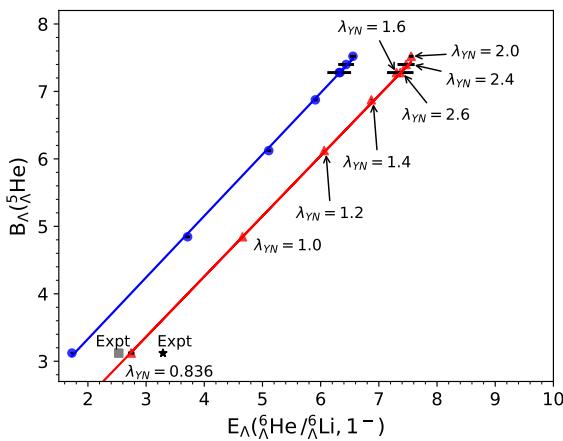
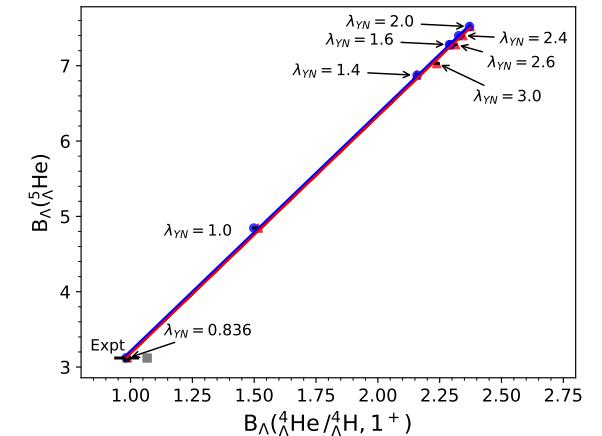
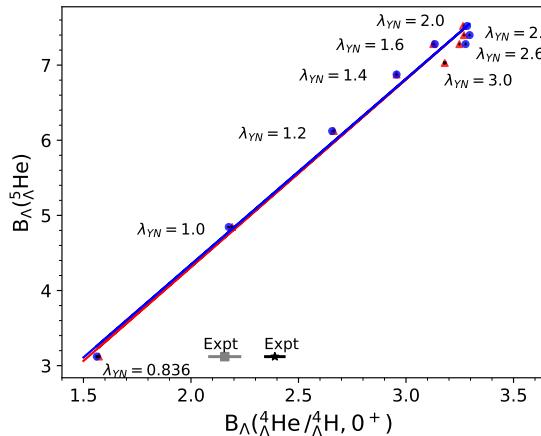
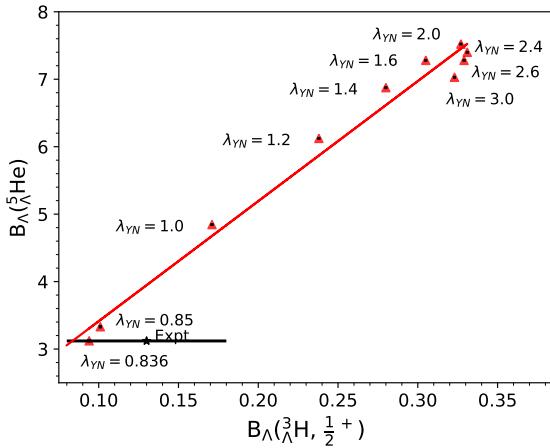
Le, Haidenbauer, UGM, Nogga, Eur. Phys. J. A **56** (2020) 301

- no induced and genuine 3NFs → dependence on the evolution parameter
- consider the ground states of  ${}^4_\Lambda\text{He}$ ,  ${}^4_\Lambda\text{H}$ ,  ${}^5_\Lambda\text{He}$ ,  ${}^6_\Lambda\text{He}$ ,  ${}^6_\Lambda\text{Li}$  and  ${}^7_\Lambda\text{Li}$   
and the first excited states of  ${}^4_\Lambda\text{He}$ ,  ${}^4_\Lambda\text{H}$  and  ${}^7_\Lambda\text{Li}$
- find strong correlations (Tjon-band like) → preferred SRG parameter

# P-shell hypernuclei continued

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- Fixed  $\lambda_{NN}$ , vary  $\lambda_{YN}$ : visible correlations & magical value at  $\lambda_{YN} = 0.836 \text{ fm}^{-1}$



→ this provides a good basis for hypernuclear calculations incl. 3BFs

# S-shell $\Lambda\Lambda$ hypernuclei

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Le, Haidenbauer, UGM, Nogga, in preparation

- Use the Jacobi no-core shell model:
  - ↪ based on realistic chiral NN, YN and YY interactions
- Goals:
  - ↪ study predictions of LO and NLO YY potentials for  $A = 4 - 6 \Lambda\Lambda$  hypernuclei

Haidenbauer, Polinder, UGM, Phys. Lett. B **653** (2007) 29; Haidenbauer, UGM, Petschauer, Nucl. Phys. A **954** (2016) 273

- ↪ provide useful constraints to improve YY interactions

$$H = T_{\text{rel}} + V_{NN}^{S=0} + V_{YN}^{S=-1} + V_{YY}^{S=-2} + \dots$$

- Hamiltonian:
- including all possible particle conversions:
  - ↪  $\Lambda N \leftrightarrow \Sigma N$  in the  $S = -1$  sector
  - ↪  $Y_1 Y_2 \leftrightarrow \Xi N$  and  $Y_1 Y_2 \leftrightarrow Y_1 Y_2$  ( $Y_1, Y_2 = \Lambda, \Sigma$ ) in the  $S = -2$  sector
- diagonalize this Hamiltonian in a finite A-particle HO basis

# The nucleus $_{\Lambda\Lambda}^6\text{He}$

- $_{\Lambda\Lambda}^6\text{He}$  is unambiguously established (Nagara event):

$$B_{\Lambda\Lambda}(_{\Lambda\Lambda}^6\text{He}) = 6.91 \pm 0.16 \text{ MeV}$$

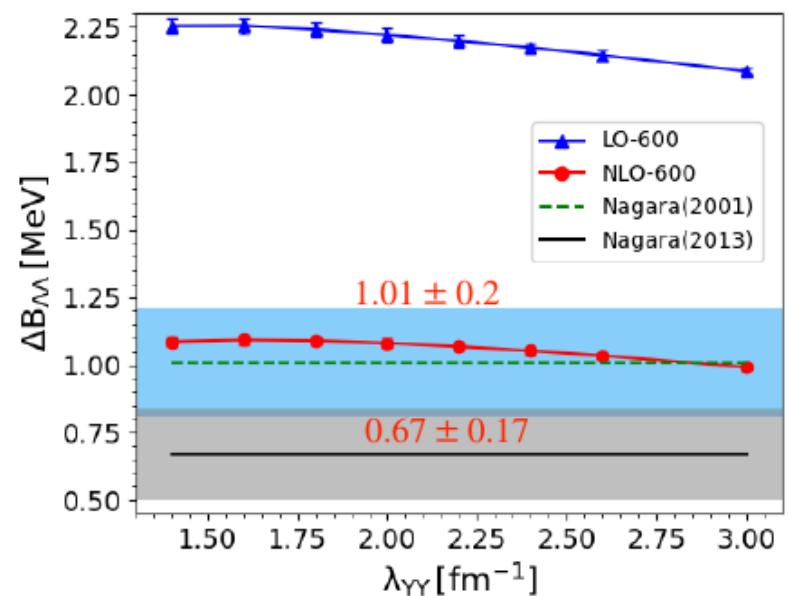
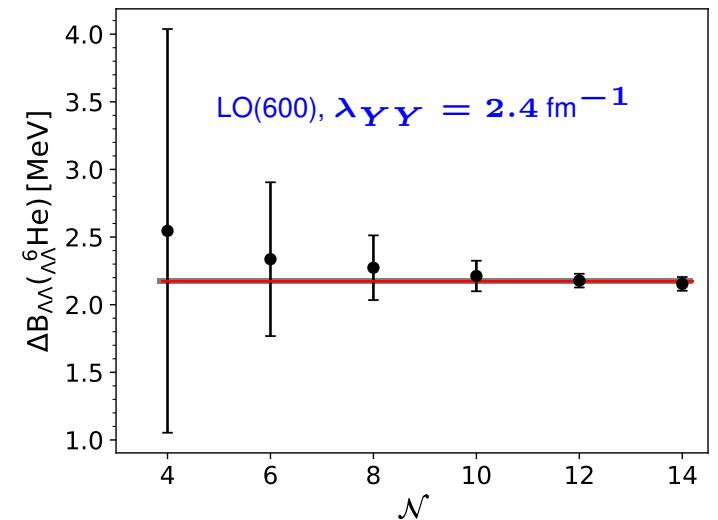
Takahashi et al. (2001), Nakazawa et al. (2010), Ahn et al. (2013)

- Use LO(600), NLO(600)  
with  $1.4 \leq \lambda_{YY} \leq 3.0 \text{ fm}^{-1}$

- Separation-energy difference:

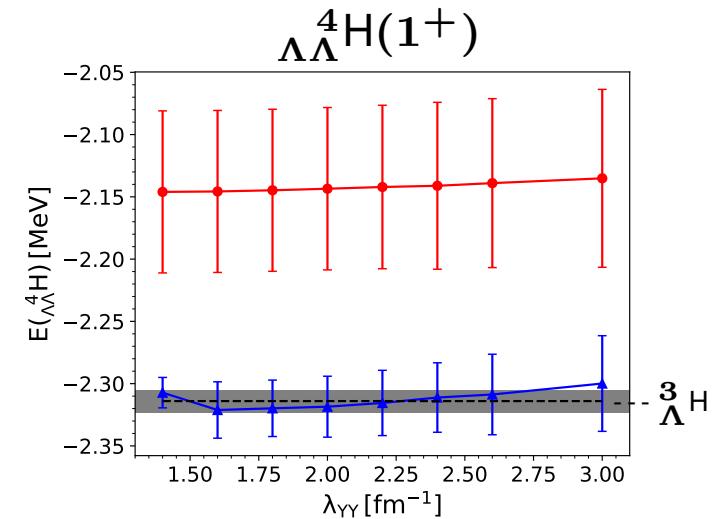
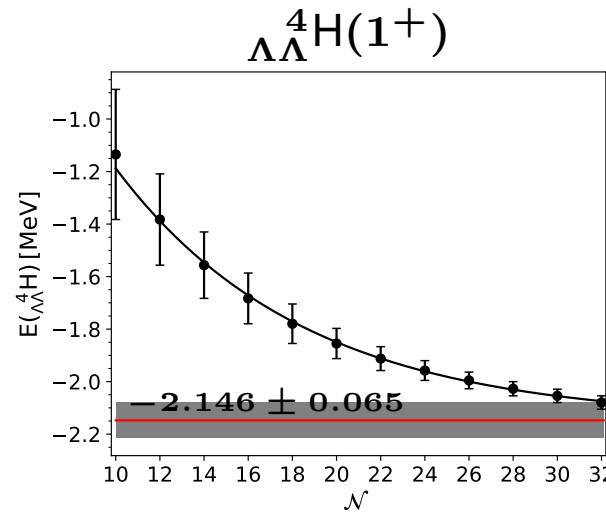
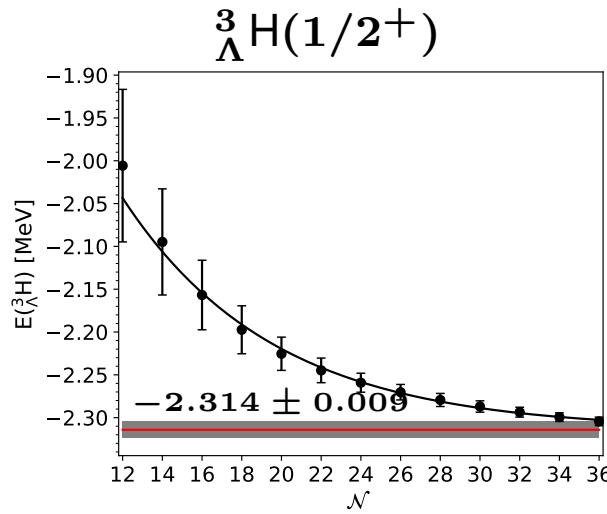
$$\Delta B_{\Lambda\Lambda}(_{\Lambda\Lambda}^6\text{He}) = B_{\Lambda\Lambda}(_{\Lambda\Lambda}^6\text{He}) - 2B_{\Lambda}(_{\Lambda}^5\text{He})$$

- ↪ Effect of SRG-induced YYN forces negligible
- ↪ LO results are incompatible with Nagara
- ↪ NLO results are comparable to Nagara



# The nucleus $\Lambda\Lambda^4H$

- Is  $\Lambda\Lambda^4H$  stable against breakup to  $^3\Lambda H + \Lambda$ ?



- No firm conclusion at LO
- NLO leads to a particle unstable  $\Lambda\Lambda^4H$
- Consistent with other studies

Filikhin, Gal (2002), Contessi et al. (2019)

# Three-baryon forces

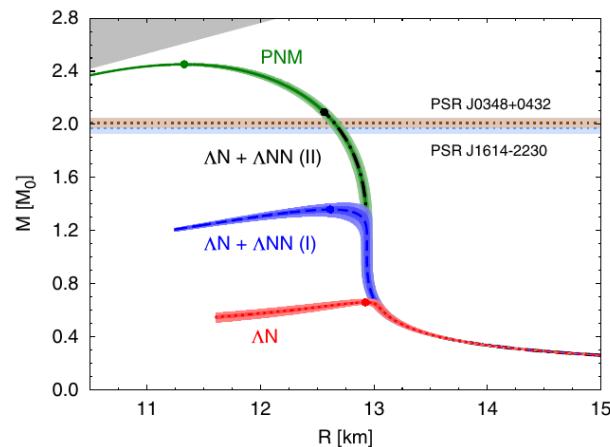
# Why three-baryon forces?

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- Motivation I: Deficiencies in the description of hyper-nuclei
- Motivation II: Such forces naturally arise in the EFT power counting
- Motivation III: The **hyperon puzzle**, i.e. hyperons soften the nuclear EoS too much
- Solutions to the hyperon puzzle

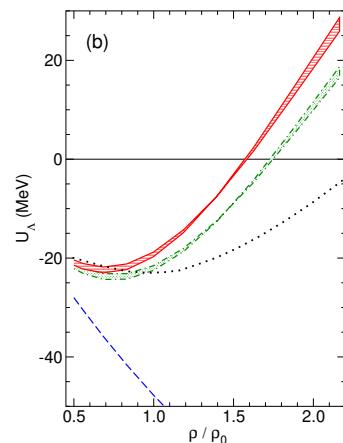
## $\Lambda NN$ force

Lonardoni et al., PRL **114** (2015) 092301



## $\Lambda N$ repulsion in matter

Haidenbauer et al., EPJA **53** (2017) 121



## Other proposals

Tolos et al., PPNP **112** (2020) 103770

- Y onset pushed by  $\Delta$  or meson condensates
- quark matter below Y onset
- dark matter
- modified gravity, . . .

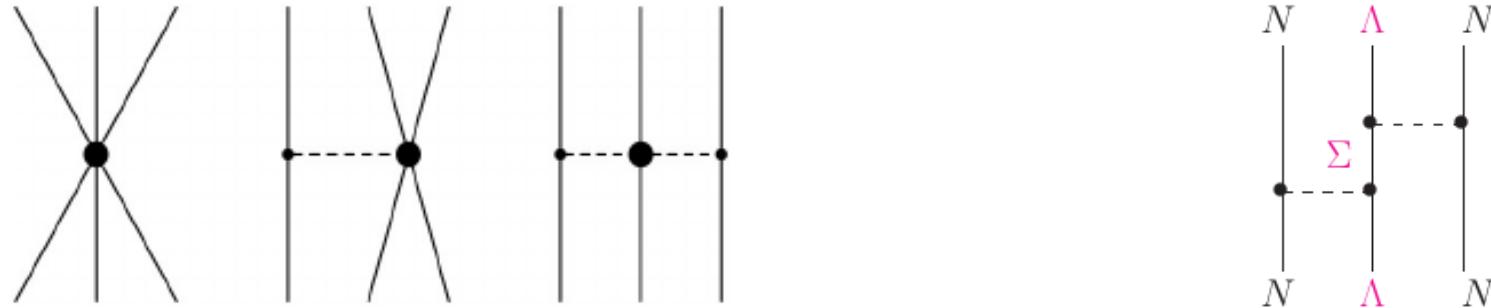
⇒ Three-body forces are most natural, so let us consider these

# Three-body forces

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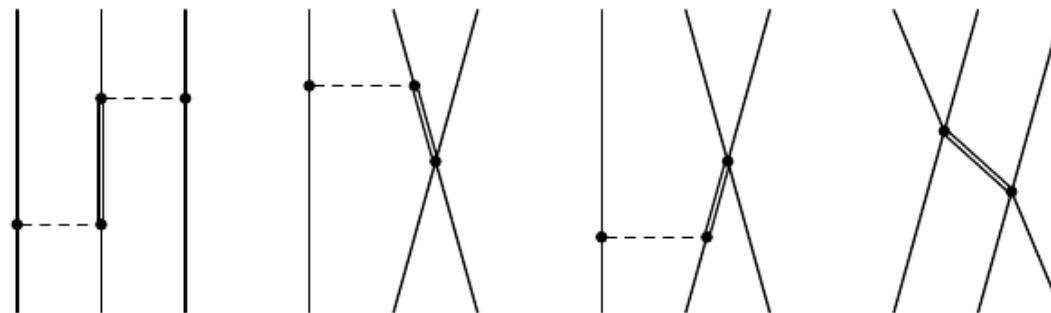
- 3BFs appear at NNLO, but channel couplings generates an reducible LO 3BF:

Petschauer, Kaiser, Haidenbauer, UGM, Weise, Phys. Rev. C **93** (2016) 014001



- solve coupled-channel FY eqs. →  $\Lambda NN$  3BF from coupling to the  $\Sigma$
- remaining 3BFs are expected to be small
- $\Lambda NN$  3BF can be estimated from resonance exchange ( $\Sigma^*$ -exchange)

Petschauer, Haidenbauer, Kaiser, UGM, Weise, Nucl. Phys. A **957** (2017) 347

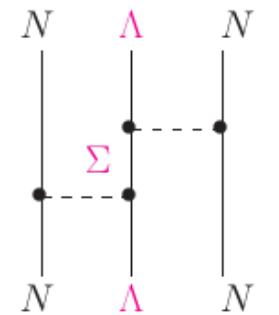


# Estimates of three-baryon forces

- 3BF in  $^3_\Lambda\text{H}$  expected to be very small
  - cutoff variation:  $\Delta E_\Lambda(3\text{BF}) \leq 50 \text{ keV}$
  - dimensional analysis:  $\Delta E_\Lambda(3\text{BF}) \simeq Q^3 |\langle V_{\Lambda N} \rangle|_{^3_\Lambda\text{H}} \simeq 40 \text{ keV}$
  - reason: large separation of the  $\Lambda$  and the  $d$  Hildenbrand, Hammer (2019)
  
- 3BF in  $^4_\Lambda\text{H}$ ,  $^4_\Lambda\text{He}$  more sizeable
  - cutoff variation:  $\Delta E_\Lambda(3\text{BF}) \simeq 200 \text{ keV}(0^+) \text{ & } \simeq 300 \text{ keV}(1^+)$
  - switch off  $\Sigma N$ - $\Lambda N$  coupling:
 
$$\Delta E_\Lambda(3\text{BF}) \simeq 230 \dots 340 \text{ keV}(0^+)$$

$$\Delta E_\Lambda(3\text{BF}) \simeq 150 \dots 180 \text{ keV}(1^+)$$

→ calculations with explicit inclusion are planned for the future



# Leading three-baryon forces

Petschauer, Kaiser, Haidenbauer, UGM, Weise, Phys. Rev. C **93** (2016) 014001

- Same topologies as before, but much more terms!  
→ just discuss the  $\Lambda NN$  force

- Two-pion exchange contribution:

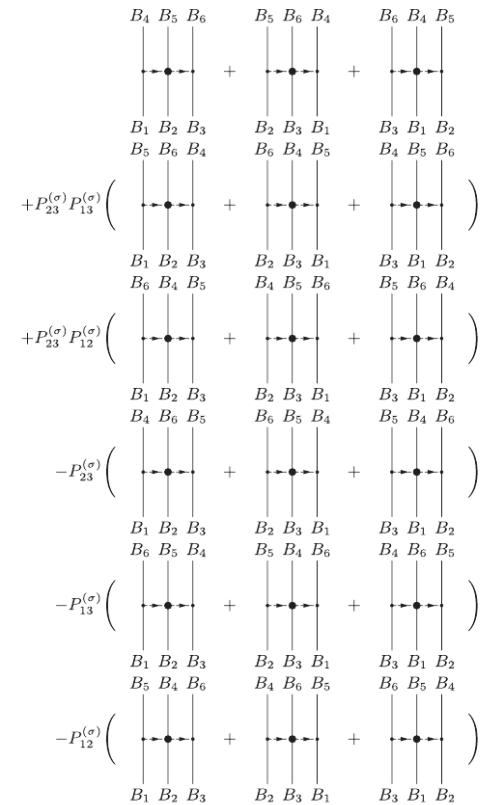
$$V_{\text{TPE}}^{\Lambda NN} = \frac{g_A^2}{3F_0^4} \frac{\vec{\sigma}_3 \cdot \vec{q}_{63} \vec{\sigma}_2 \cdot \vec{q}_{52}}{(\vec{q}_{63}^2 + M_\pi^2)(\vec{q}_{52}^2 + M_\pi^2)} \vec{\tau}_2 \cdot \vec{\tau}_3 \\ \times \left( - (3b_0 + b_D)M_\pi^2 + (2b_2 + 3b_4)\vec{q}_{63} \cdot \vec{q}_{52} \right) \\ - P_{23}^{(\sigma)} P_{23}^{(\tau)} \frac{g_A^2}{3F_0^4} \frac{\vec{\sigma}_3 \cdot \vec{q}_{53} \vec{\sigma}_2 \cdot \vec{q}_{62}}{(\vec{q}_{53}^2 + M_\pi^2)(\vec{q}_{62}^2 + M_\pi^2)} \vec{\tau}_2 \cdot \vec{\tau}_3 \\ \times \left( - (3b_0 + b_D)M_\pi^2 + (2b_2 + 3b_4)\vec{q}_{53} \cdot \vec{q}_{62} \right)$$

- Recover the standard 3NF: **matching**

$$c_1 = \frac{1}{2}(2b_0 + b_D + b_F)$$

$$c_3 = b_1 + b_2 + b_3 + 2b_4$$

$$c_4 = 4(d_1 + d_2)$$

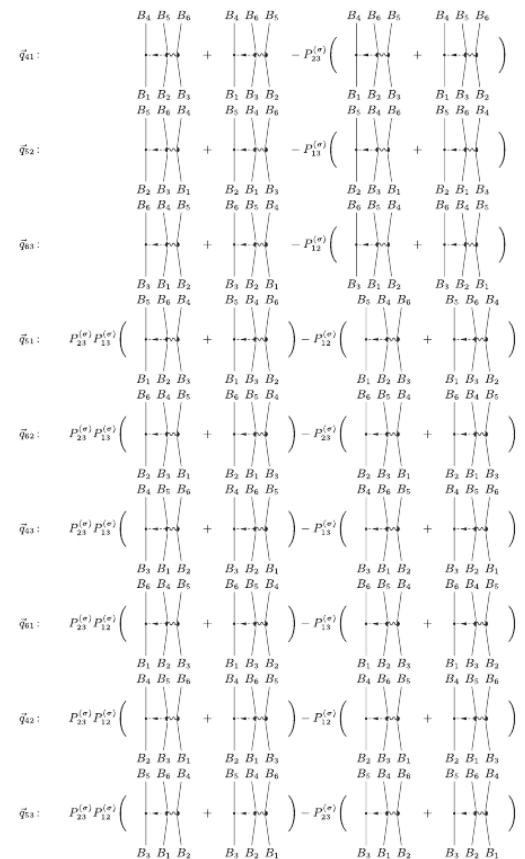


Frink, UGM, JHEP **07** (2004) 028; Mai, Bruns, Kubis, Phys. Rev. D **80** (2009) 094006

# Leading three-baryon forces continued

- One-pion exchange contribution:

$$\begin{aligned}
V_{\text{OPE}}^{\Lambda NN} = & -\frac{g_A}{2F_0^2} \left\{ \frac{\vec{\sigma}_2 \cdot \vec{q}_{52}}{\vec{q}_{52}^2 + M_\pi^2} \vec{\tau}_2 \cdot \vec{\tau}_3 \left[ (D'_1 \vec{\sigma}_1 + D'_2 \vec{\sigma}_3) \cdot \vec{q}_{52} \right] \right. \\
& + \frac{\vec{\sigma}_3 \cdot \vec{q}_{63}}{\vec{q}_{63}^2 + M_\pi^2} \vec{\tau}_2 \cdot \vec{\tau}_3 \left[ (D'_1 \vec{\sigma}_1 + D'_2 \vec{\sigma}_2) \cdot \vec{q}_{63} \right] \\
& + P_{23}^{(\sigma)} P_{23}^{(\tau)} P_{13}^{(\sigma)} \frac{\vec{\sigma}_2 \cdot \vec{q}_{62}}{\vec{q}_{62}^2 + M_\pi^2} \vec{\tau}_2 \cdot \vec{\tau}_3 \\
& \times \left[ -\frac{D'_1 + D'_2}{2} (\vec{\sigma}_1 + \vec{\sigma}_3) \cdot \vec{q}_{62} + \frac{D'_1 - D'_2}{2} i (\vec{\sigma}_3 \times \vec{\sigma}_1) \cdot \vec{q}_{62} \right] \\
& + P_{23}^{(\sigma)} P_{23}^{(\tau)} P_{13}^{(\sigma)} \frac{\vec{\sigma}_2 \cdot \vec{q}_{53}}{\vec{q}_{53}^2 + M_\pi^2} \vec{\tau}_2 \cdot \vec{\tau}_3 \\
& \times \left[ -\frac{D'_1 + D'_2}{2} (\vec{\sigma}_1 + \vec{\sigma}_3) \cdot \vec{q}_{53} + \frac{D'_1 - D'_2}{2} i (\vec{\sigma}_3 \times \vec{\sigma}_1) \cdot \vec{q}_{53} \right]
\end{aligned}$$



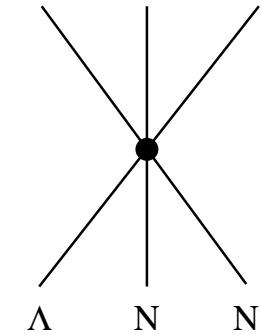
- 2 LECs instead of the  $D$ -term in in the 3NF

- Many contributions are zero due to the vanishing  $\Lambda\Lambda\pi$ -vertex (isospin)

# Leading three-baryon forces continued

- $\Lambda NN$  contact contribution:

$$\begin{aligned} V_{\text{ct}}^{\Lambda NN} = & C'_1 (1 - \vec{\sigma}_2 \cdot \vec{\sigma}_3)(3 + \vec{\tau}_2 \cdot \vec{\tau}_3) \\ & + C'_2 \vec{\sigma}_1 \cdot (\vec{\sigma}_2 + \vec{\sigma}_3) (1 - \vec{\tau}_2 \cdot \vec{\tau}_3) \\ & + C'_3 (3 + \vec{\sigma}_2 \cdot \vec{\sigma}_3)(1 - \vec{\tau}_2 \cdot \vec{\tau}_3) \end{aligned}$$



- 3 LECs instead of the  $E$ -term in in the 3NF
- $C'_1$  ( $C'_{2,3}$ ) parameterize(s) transitions with total isospin 1 (0)
- None of these three constants can be substituted by the constant  $E$  of the 3NF
- In total, there are 18 independent six-baryon contact terms, too many?

→ Method for estimating the LECs: Resonance saturation

→ Good starting point: **Decuplet** saturation

# Decuplet saturation

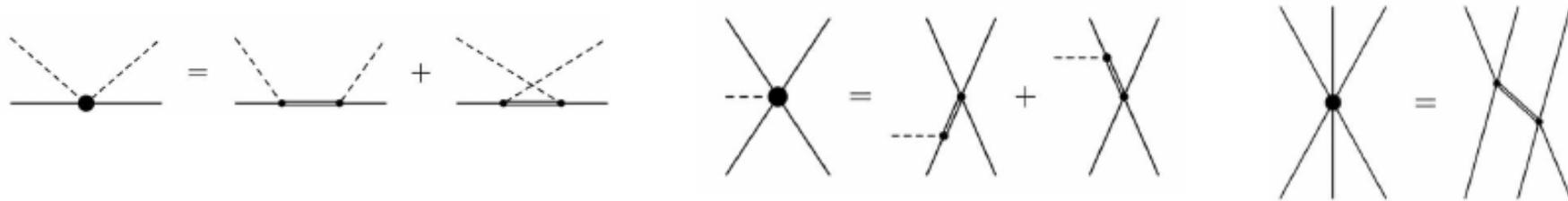
30

Petschauer, Haidenbauer, Kaiser, UGM, Weise, Nucl. Phys. A 957 (2017) 347

- Main idea: Low-lying decuplet gives largest contribution to most LECs

Bernard, Kaiser, UGM, NPA 615 (1997) 483

- Decuplet saturation for all topologies:



- Applied to the just discussed  $\Lambda NN$  forces:

$$3b_0 + b_D = 0, \quad 2b_2 + 3b_4 = -\frac{C^2}{\Delta}, \quad C = \frac{3}{4}g_A \simeq 0.95, \quad \Delta \simeq 300 \text{ MeV}$$

$$D'_1 = 0, \quad D'_2 = \frac{2CH'}{9\Delta}, \quad H' = H_1 + 3H_2$$

$$C'_1 = C'_3 = \frac{H'^2}{72\Delta}, \quad C'_2 = 0$$

- $H_{1,2}$  are LECs of the minimal  $B^* BBB$  Lagrangian

→ Only **one** LEC left to be determined → good starting point!

# Summary & outlook

- YN and YY interactions at NLO in chiral EFT
  - ↪ better convergence than in the NN case
  - ↪ more data needed, hypernuclei play a crucial role!
- Light hypernuclei using *ab initio* methods
  - ↪ influence of the hypertriton BE → modest
  - ↪ p-shell  $S = -1$  hypernuclei using the J-NCSM → 3-body forces
  - ↪ s-shell  $S = -2$  hypernuclei: first promising results
  - ↪ nuclear lattice EFT extended to hypernuclei: scaling with  $A$  ✓

Frame, Lähde, Lee, UGM, EPJA **56** (2020) 248

- Status of three-baryon forces
  - ↪ three-nucleon forces beyond N2LO presently at the forefront
  - ↪ 3BFs worked out, applications forthcoming (nuclei, neutron stars)

**Strangeness nuclear physics is a vibrant field w/ a huge discovery potential**

# SPARES

# QCD LAGRANGIAN

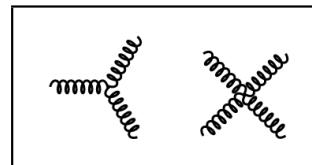
33

- $$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G_{\mu\nu}^a G^{\mu\nu,a} + \sum_f \bar{q}_f (iD - \mathcal{M}) q_f + \dots$$

$$D_\mu = \partial_\mu - ig A_\mu^a \lambda^a / 2$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g[A_\mu^b, A_\nu^c]$$

$$f = (u, d, s, c, b, t)$$



- running of  $\alpha_s = \frac{g^2}{4\pi} \Rightarrow \Lambda_{\text{QCD}} = 210 \pm 14 \text{ MeV}$  ( $N_f = 5$ ,  $\overline{MS}$ ,  $\mu = 2 \text{ GeV}$ )
- light (u,d,s) and heavy (c,b,t) quark flavors:

$m_{\text{light}} \ll \Lambda_{\text{QCD}}$

$$m_u = 2.2^{+0.6}_{-0.4} \text{ MeV}$$

$$m_d = 4.7^{+0.5}_{-0.4} \text{ MeV}$$

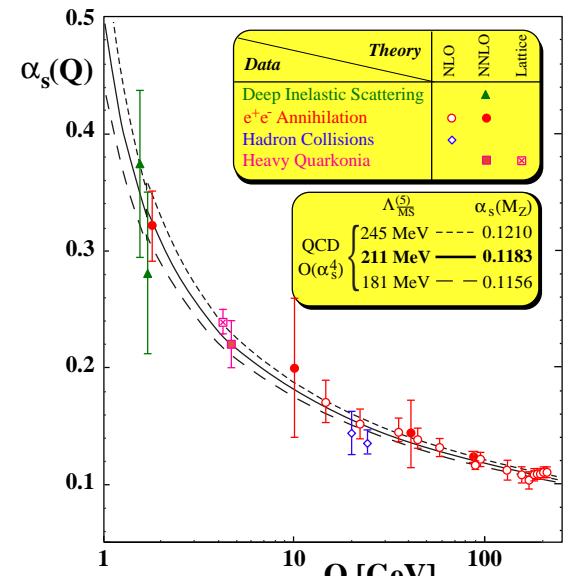
$$m_s = 96^{+8}_{-4} \text{ MeV}$$

$m_{\text{heavy}} \gg \Lambda_{\text{QCD}}$

$$m_c = 1.28 \pm 0.03 \text{ GeV}$$

$$m_b = 4.18^{+0.04}_{-0.03} \text{ GeV}$$

$$m_t = 173.1 \pm 0.6 \text{ GeV}$$



# CHIRAL LIMIT of QCD

- **light quarks:**

$$\mathcal{L}_{\text{QCD}}^{\text{light}} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R + \mathcal{O}(m_f/\Lambda_{\text{QCD}})$$

- L and R quarks decouple  $\Rightarrow$  chiral symmetry
- spontaneous chiral symmetry breaking  $\Rightarrow$  pseudo-Goldstone bosons
- surviving symmetry: SU(2) isospin and SU(3) flavor
- pertinent EFT  $\Rightarrow$  chiral perturbation theory (CHPT)

- **how light is light?**

- chiral corrections in the up/down quark sector:  $M_\pi^2/(4\pi F_\pi)^2 \simeq 0.015$
  - chiral corrections in the strange quark sector:  $M_K^2/(4\pi F_\pi)^2 \simeq 0.18$
  - inclusion of baryons: odd powers of  $M_{\text{GB}}/(4\pi F_\pi)$  enter  
 $\qquad\qquad\qquad \hookrightarrow$  further complications
- $\Rightarrow$  SU(2) isospin much more exact than SU(3) flavor

# Power counting

Polinder, Haidenbauer, UGM, Nucl. Phys. A779 (2006) 244

- Following Weinberg: chiral expansion of the **effective potential**

Weinberg (1991)

- Basic formula:

$$V_{\text{eff}} = V_{\text{eff}}(Q, g, \mu) = \sum_{\nu} Q^{\nu} \mathcal{V}_{\nu}(Q/\mu, g)$$

$$\nu = 2 - B + 2L + \sum_i \nu_i \Delta_i, \quad \Delta_i = d_i + \frac{1}{2} b_i - 2 \geq 0$$

$Q$  = small momentum / mass [in units of the hard scale]

$g$  = generic coupling constants (LECs)

$\mu$  = regularization scale [mostly cutoff]

$\mathcal{V}_{\nu}$  = function of  $\mathcal{O}(1)$

$B, b_i$  = number of external baryons, baryons at vertex  $i$

$L, d_i$  = number of GB loops, derivatives at vertex  $i$

# Improved error estimates

Epelbaum, Krebs, UGM, Eur. Phys. J. A **51** (2015) 53

- Various sources of uncertainties, dominated by the orders neglected
- small parameter  $Q$ , must deal with the double expansion (momenta/masses):

$$Q = \max \left( \frac{p}{\Lambda_{\text{hard}}}, \frac{M_\pi}{\Lambda_{\text{hard}}} \right), \quad \Lambda_{\text{hard}} = \text{breakdown scale}$$

- at low momenta ( $p < M_\pi$ ) the error is dominated by the pion mass corrections
- conservative way of estimating the uncertainty: take the maximum of all the differences of the lower orders one has considered for a given observable  $X(p)$  at order  $Q^N$  [note particular pattern for BB int.]

$$\Delta X^N(p) = \max (Q^{N+1} \cdot |X^{\text{LO}}(p)|, Q^{N-1} \cdot |X^{\text{NLO}}(p) - X^{\text{LO}}(p)|, \\ Q^{N-2} \cdot |X^{\text{N}^2\text{LO}}(p) - X^{\text{NLO}}(p)|, \dots, Q \cdot |X^{\text{N}^N\text{LO}}(p) - X^{\text{N}^{N-1}\text{LO}}(p)|)$$

# Exotic bound states ?

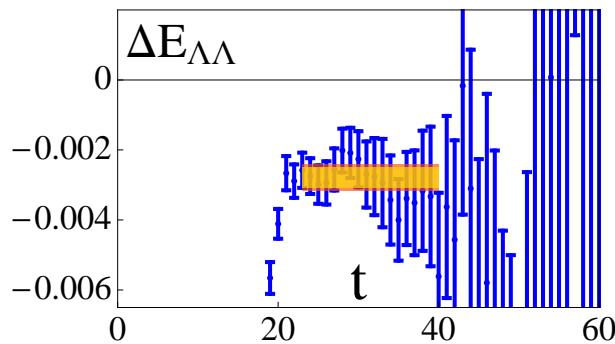
# The H-dibaryon on the lattice

38

- Jaffe's prediction: deeply bound 6-quarks state with  $S = -2, I = 0, J^P = 0^+$

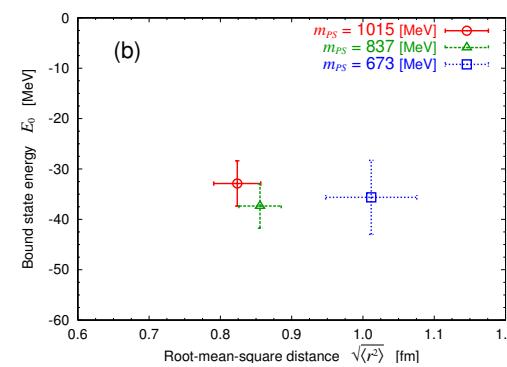
R.L. Jaffe, PRL 38 (1977) 195

- many unsuccessful experimental searches since then
- new interest: signals on the lattice for large pion masses



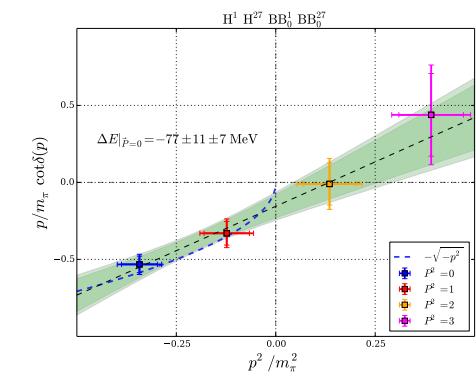
$$B_H = 16.6 \pm 2.1 \pm 4.6 \text{ MeV}$$
$$M_\pi = 389 \text{ MeV}$$

NPLQCD, PRL 106 (2011) 1621001



$$B_H = 30 - 40 \text{ MeV}$$
$$M_\pi = 673 - 1015 \text{ MeV}$$

HALQCD, PRL 106 (2011) 1621002



$$B_H = 77 - 92 \text{ MeV}$$
$$M_\pi = 450 - 1000 \text{ MeV}$$

Junnarkar et al., PoS CD15 (2015) 07

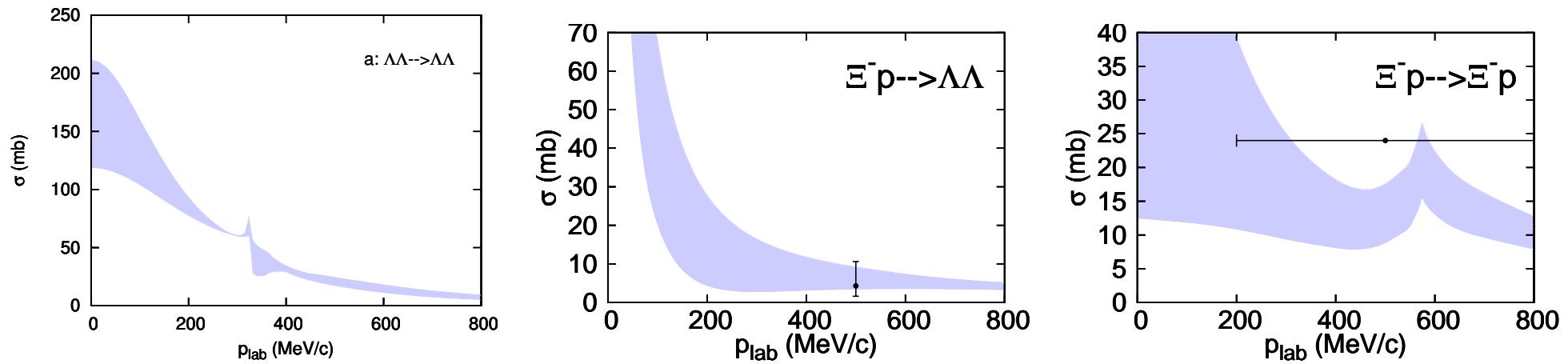
⇒ what does this mean for physical quark masses?

# Predictions for YY interactions

39

Haidenbauer, Polinder, M., Phys. Lett. B 653 (2007) 29

- set singlet LEC to zero, vary cut-off  $\rightarrow$  predictions

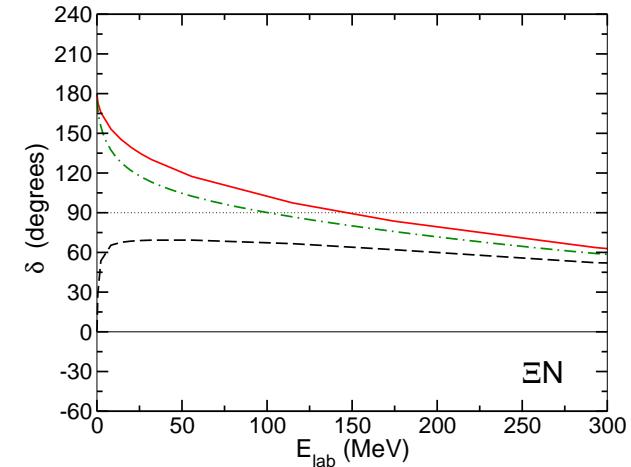
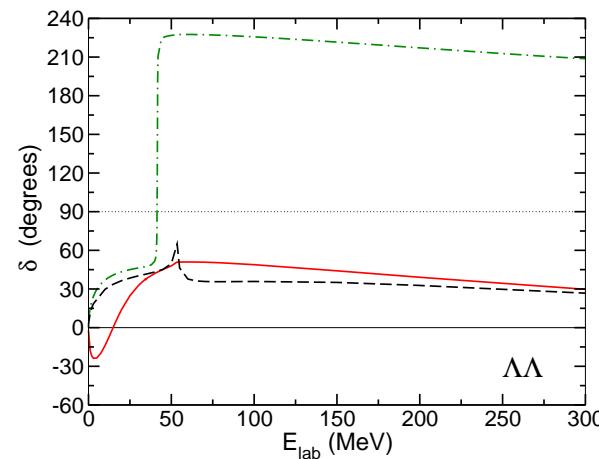
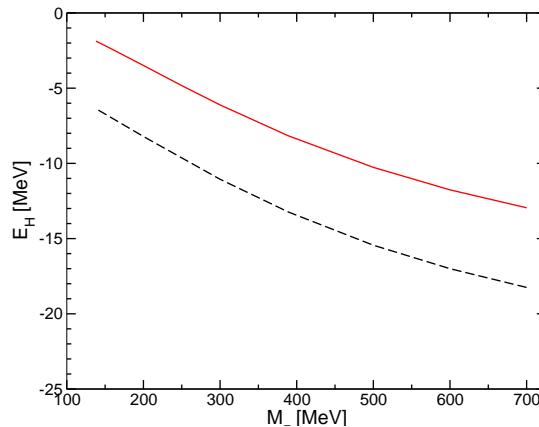
Ahn *et al.*, Phys. Lett. B 633 (2006) 214 [arXiv:nucl-ex/0502010]

- assume a b.s. at physical masses with the same  $\gamma$  as the deuteron (tune  $C^1$ )  
 $\rightarrow E_H = -1.87 \text{ MeV}$
- we are now in the position to study the quark mass dependence of the  $H$ 
  - linearity of the quark mass dependence of BE  $E_H$  ?
  - effect of SU(3) breaking through threshold effects ( $\Lambda\Lambda$ ,  $\Sigma\Sigma$ ,  $\Xi N$ )

# Quark mass dependence & nature of the H-dibaryon <sup>40</sup>

Haidenbauer, UGM Phys. Lett. B **706** (2011) 100; Nucl. Phys. A **881** (2012) 44

- curves: solid = “physical” dashed = HALQCD dot-dashed = NPLQCD



- Pion mass dependence approx. linear (as used by NPLQCD and HALQCD)
- However, important channel coupling effect:
  - more flavor singlet attraction in the  $\Xi N$  than the  $\Lambda\Lambda$  channel
    - if at all, the  $H$  is predominantly a  $\Xi N$  bound state
    - no b.s. for HALQCD, quasi-bound  $\Xi N$  for NPLQCD
    - Mainz group also finds no b.s. for physical pion masses

# Other exotic dibaryons with $S = -2, -3, -4$ ?

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- Indications of such bound states from a) quark models  
b) meson-exchange models  
c) lattice QCD
- Intuitive argument: assume maximal isospin & SU(3) symmetry

Miller (2013)

$$V_{NN} \simeq V_{\Sigma\Sigma} \simeq V_{\Xi\Xi}$$

- Schrödinger equation for the two-baryon system:

$$-\frac{d^2u}{dr^2} - \underbrace{2\mu_{B_1B_2}V_{B_1B_2}}_{\text{eff. pot.}} u = k^2 u$$

⇒ increasing strength of the eff. potential with increasing baryon masses

$$\mu_{\Sigma\Sigma}/\mu_{NN} \simeq 1.27, \quad \mu_{\Xi\Xi}/\mu_{NN} \simeq 1.40,$$

⇒ drastic effects for attractive potentials,  
i.e. bound state with increasing baryon mass (reduced mass in the Schrödinger eq.)

# Chiral EFT for systems with $S = -2, -3, -4$

Haidenbauer, UGM, Phys. Lett. B **684** (2010) 275; Haidenbauer, UGM, Petschauer, Eur. Phys. J. A **51** (2015) 17

- A similar pattern is found in chiral EFT in the SU(3) limit for the interactions (consider states with maximal isospin, i.e. no mixing):

	LO	NLO
$\Lambda$ [MeV]	550 ... 700	500 ... 650
$\Sigma\Sigma$ ( $I = 2$ )	—	0 ... -0.01
$\Xi\Sigma$ ( $I = 3/2$ )	-2.23 ... -6.18	-0.58 ... -0.19
$\Xi\Xi$ ( $I = 1$ )	-2.56 ... -7.27	-0.40 ... -1.00

- pure QCD, no Coulomb included
- weakly bound systems showing the expected pattern
- cutoff dependence reduced when going from LO to NLO ✓
- but is this realistic? → study SU(3) breaking

# SU(3) breaking in two-baryon systems

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- SU(3) breaking at NLO, general analysis of possible terms exists, 12 terms

Petschauer, Kaiser, NPA **916** (2013) 1

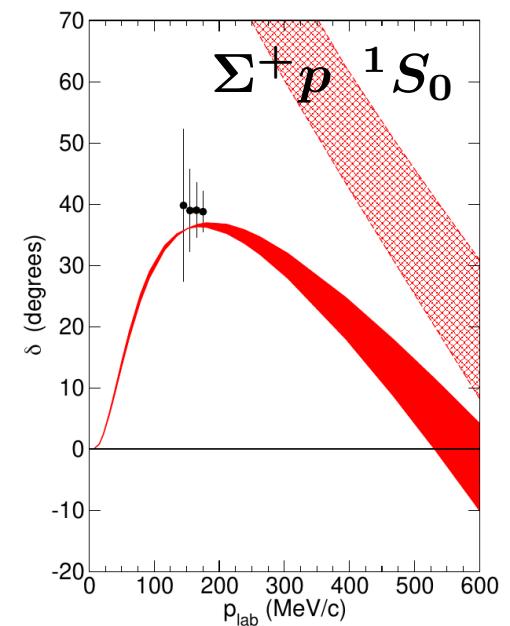
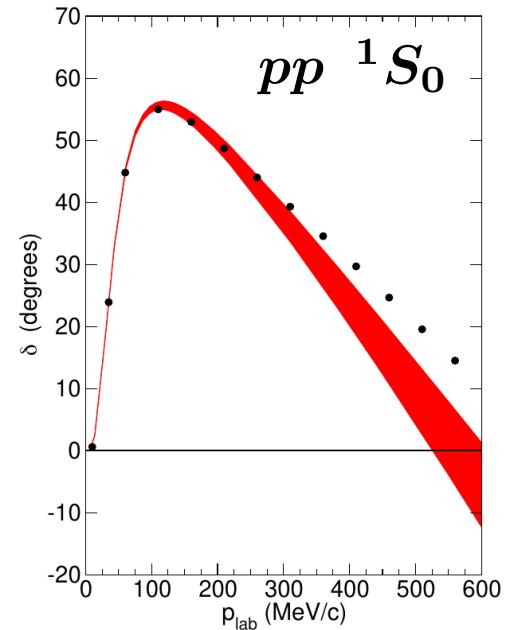
- Concentrate on  $^1S_0$  partial wave w/ maximal isospin: 2 terms left

$$\begin{aligned} V_{NN}^{(I=1)} &= \tilde{C}_{1S_0}^{27} + C_{1S_0}^{27}(p^2 + p'^2) + \frac{1}{2}C_1^\chi(M_K^2 - M_\pi^2), \\ V_{\Sigma N}^{(I=3/2)} &= \tilde{C}_{1S_0}^{27} + C_{1S_0}^{27}(p^2 + p'^2) + \frac{1}{4}C_1^\chi(M_K^2 - M_\pi^2), \\ V_{\Sigma\Sigma}^{(I=2)} &= \tilde{C}_{1S_0}^{27} + C_{1S_0}^{27}(p^2 + p'^2), \\ V_{\Xi\Sigma}^{(I=3/2)} &= \tilde{C}_{1S_0}^{27} + C_{1S_0}^{27}(p^2 + p'^2) + \frac{1}{4}C_2^\chi(M_K^2 - M_\pi^2), \\ V_{\Xi\Xi}^{(I=1)} &= \tilde{C}_{1S_0}^{27} + C_{1S_0}^{27}(p^2 + p'^2) + \frac{1}{2}C_2^\chi(M_K^2 - M_\pi^2). \end{aligned}$$

- Three LECs ( $\tilde{C}^{27}, C^{27}, C_1^\chi$ ) from a combined fit to  $pp$  and  $\Sigma^+p$
- One LEC ( $C_2^\chi$ ) varied within a reasonable range
- Note: no coupled channels  $\rightarrow$  particularly suitable to investigate SU(3) breaking

# SU(3) breaking in two-baryon systems

- Use upper limit from:  $\sigma_{\Sigma^+ p} = \frac{\pi}{k^2} \sin^2 \delta_1 S_0$
- ⇒ more repulsion in  $\Sigma^+ p$  than in  $pp$  needed, so  $C_1^\chi < 0$
- assume that this trend continue when going to  $S = -3, -4$
- ↪  $C_2^\chi \leq 0 \rightarrow$  choose  $C_2^\chi = -C_1^\chi \dots 0$
- ⇒ no more bound states  $\Xi\Sigma(I = 3/2)$  and  $\Xi\Xi(I = 1)$
- consistent with HALQCD for  $\Xi\Xi$ : no bound state  
for  $M_\pi = 510$  MeV,  $m_\Xi = 1465$  MeV Sasaki et al. (2014)
- inconsistent with NPLQCD:  $E_B = -14.0 \pm 1.4 \pm 6.7$  MeV  
for  $M_\pi = 389$  MeV,  $m_\Xi = 1350$  MeV Beane et al. (2012)
- Need data! Difficult but could be produced at JLab and J-PARC  
or from correlation functions measured in heavy-ion collisions



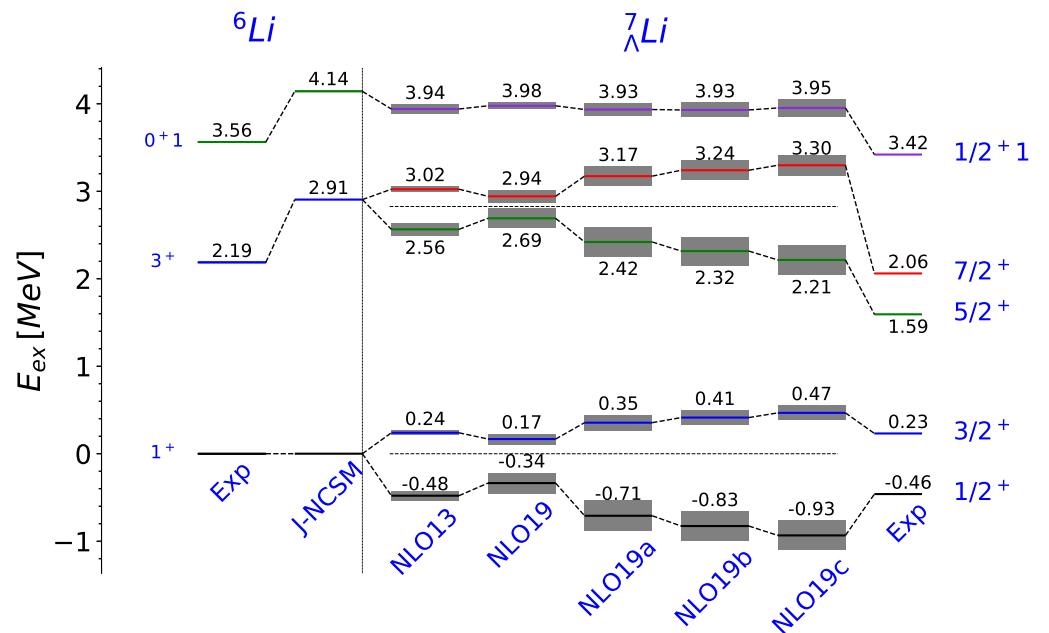
# Energy spectrum of ${}^7\text{Li}$

- Since no 3BFs are included, concentrate on relative positions of the levels
- Centroid energies of  $J_N \pm \frac{1}{2}$  doublets for the magical parameter

$$\bar{E} = \frac{J_N + 1}{2J_N + 1} E_+ + \frac{J_N}{2J_N + 1} E_-$$

Gal, Soper, Dalitz (1978)

- Levels not well described,  
dependence on the interaction
- Second doublet at  $\bar{E} = 2.83$  MeV
  - ↪ independent of the int.
  - ↪ overall strength of the int.'s similar
  - ↪ differ in their spin dependence



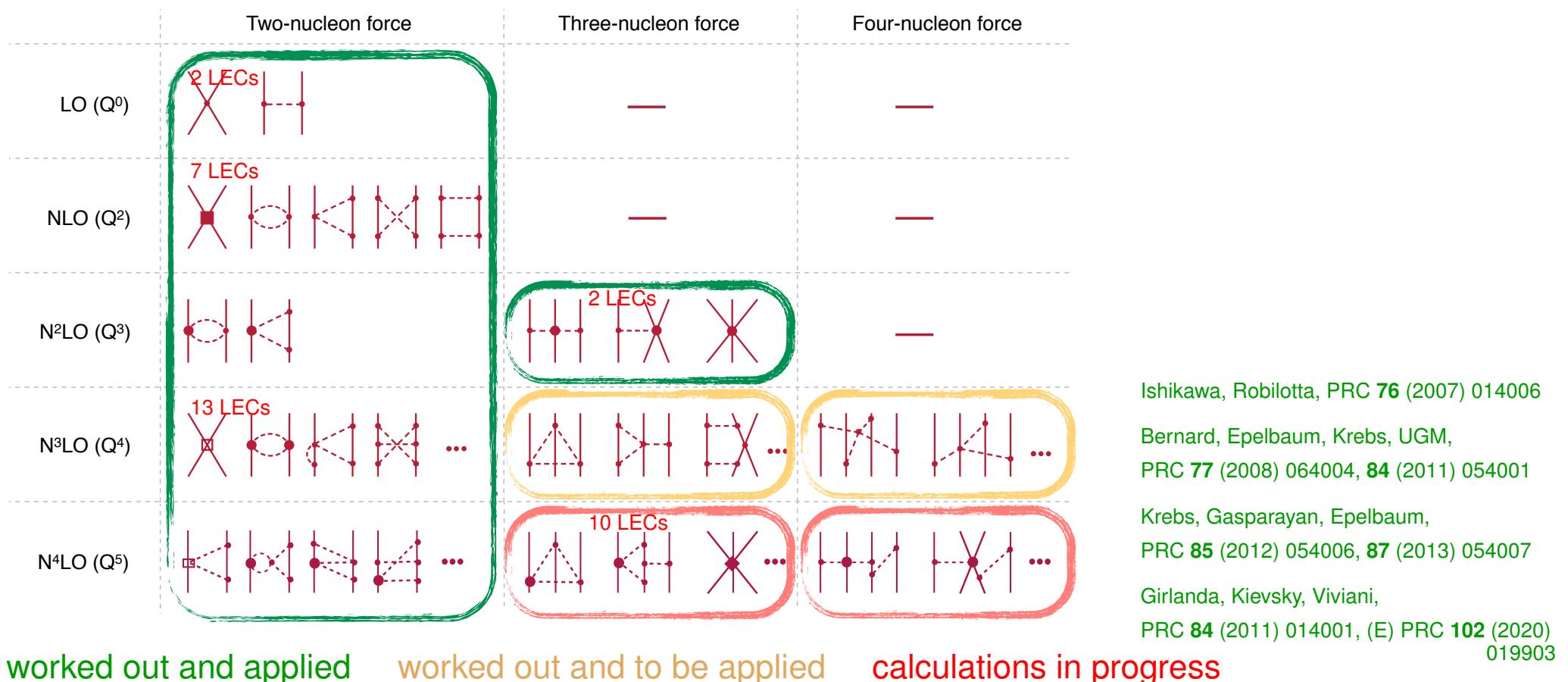
⇒ Need to include three-baryon forces!

# Nuclear forces in chiral EFT

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- expansion of the potential in powers of  $Q$  [small parameter]:  $\{p/\Lambda_b, M_\pi/\Lambda_b\}$
- explains observed hierarchy of the nuclear forces
- extremely successful in few-nucleon systems

Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773



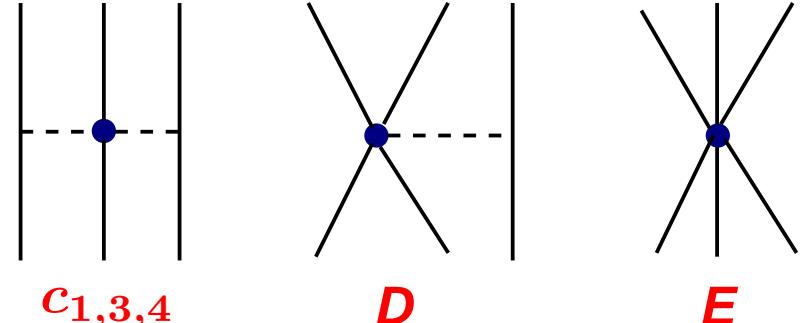
# Status of the three-nucleon force

- 3N forces at NNLO:

- Three topologies
- LECs  $c_i$  determined from Roy-Steiner analysis of pion-nucleon scattering

Hoferichter, Ruiz de Elvira, Kubis, UGM, PRL **115** (2015) 192301

- Two parameters  $D$  and  $E$



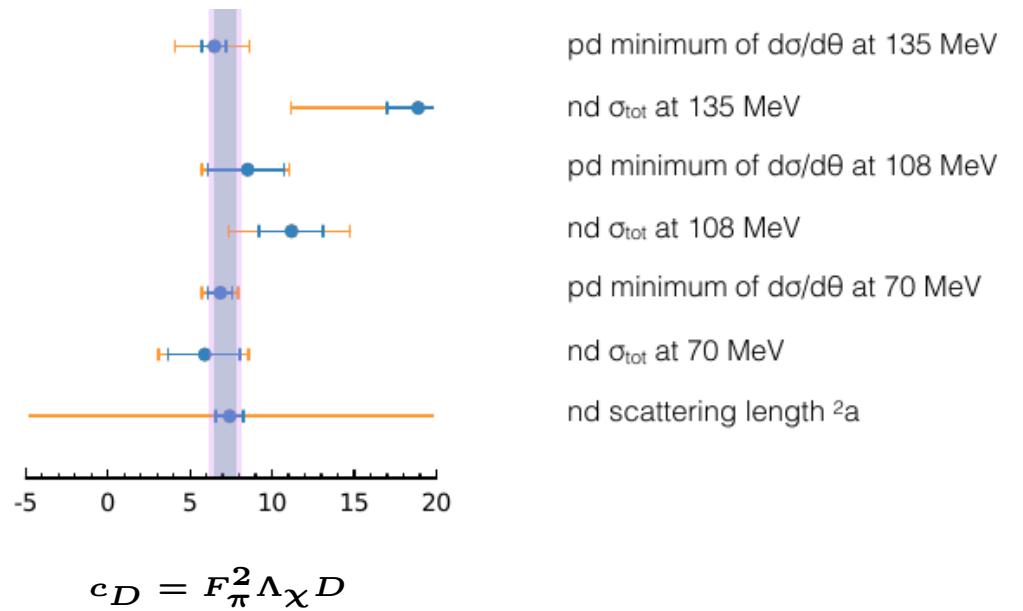
- Determine  $D$  and  $E$  from fits to the  $E_B(^3\text{H})$  and  $Nd$ -scattering

Epelbaum et al. [LENPIC], PRC **99** (2019) 024313

⇒ make predictions

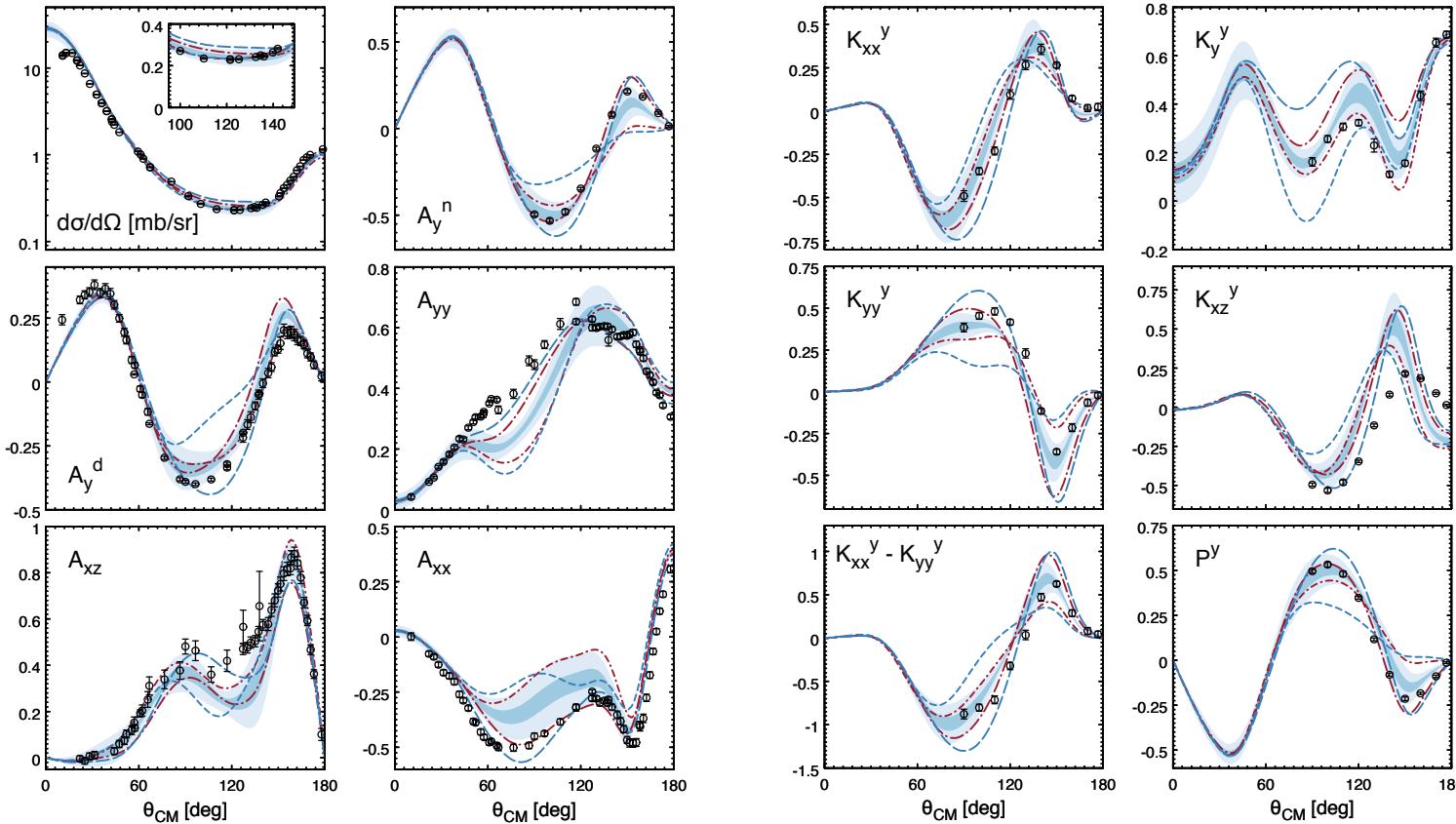
- $D$  can also be determined in pion production experiments or from electroweak processes

→ power of EFT



# Three-nucleon force: State of the art

- Many calculations world-wide with these N2LO 3NFs → they are needed!
- Beyond N2LO 3NFs: on-going efforts by the **LENPIC** collaboration (and others)  
[www.lenpic.org](http://www.lenpic.org)
- State of the art: Including short-distance N4LO operators (**central**, spin-orbit)



$Nd$  scattering  
 $E_N^{\text{lab}} = 135 \text{ MeV}$

$c_{E_1} = \pm 2$

$c_{E_7} = \pm 2$

Epelbaum et al.,  
EPJA **56** (2020) 92

see also

Girlanda et al.,  
PRC **99** (2020) 054003

