Departamento de Física Teórica
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Dispersive study of πK scattering:
the κ/K_0*(700) and other strange resonances and threshold parameters

J. R. Peláez

arXiv:2001.08153. Phys.Rev.Lett. 124 (2020) 17, 172001 with A. Rodas
arXiv:2010.1122. To appear in Physics Reports with A. Rodas
arXive:2101.06506. tutorial review to appear in EJP-ST, with A. Rodas & J.Ruiz de Elvira

Laboratorio Nazionale di Frascati, General Seminar. 21/01/2021.

Supported by:

Motivation

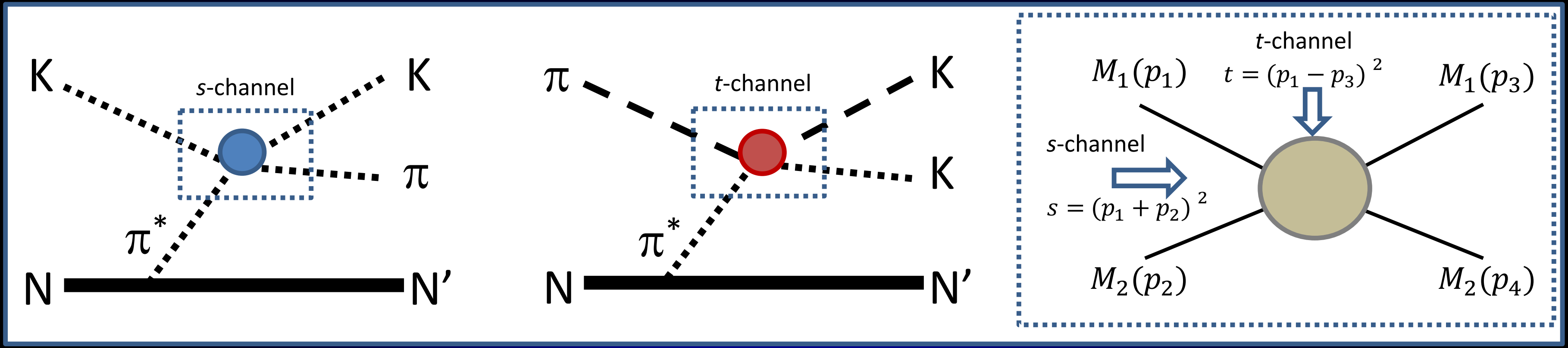
- π,K appear as final products of almost all hadronic strange processes:
  B,D, decays, CP violation studies…

- π,K are Goldstone Bosons of QCD:
  Threshold parameters test Chiral Symmetry Breaking
  In particular, the predictions of Chiral Perturbation Theory, which is the low-energy Effective Theory of QCD

- Main or relevant source for PDG parameters of:
  κ/$K_0^*(700)$, $K_0^*(1430)$, $K_1^*(892)$, $K_1^*(1410)$, $K_2^*(1410)$, $K_3^*(1780)$
- κ/$K_0^*(700)$:
  - existence and parameters controversial for 6 decades.
    Still *"Needs Confirmation"* on PDG
  - Needed to complete SU(3) classification of lightest scalars
  - Candidate for non-ordinary meson.

Problems

- Data: extracted from KN→πKN', assuming one pion exchange.
  Large systematic uncertainties and inconsistencies.

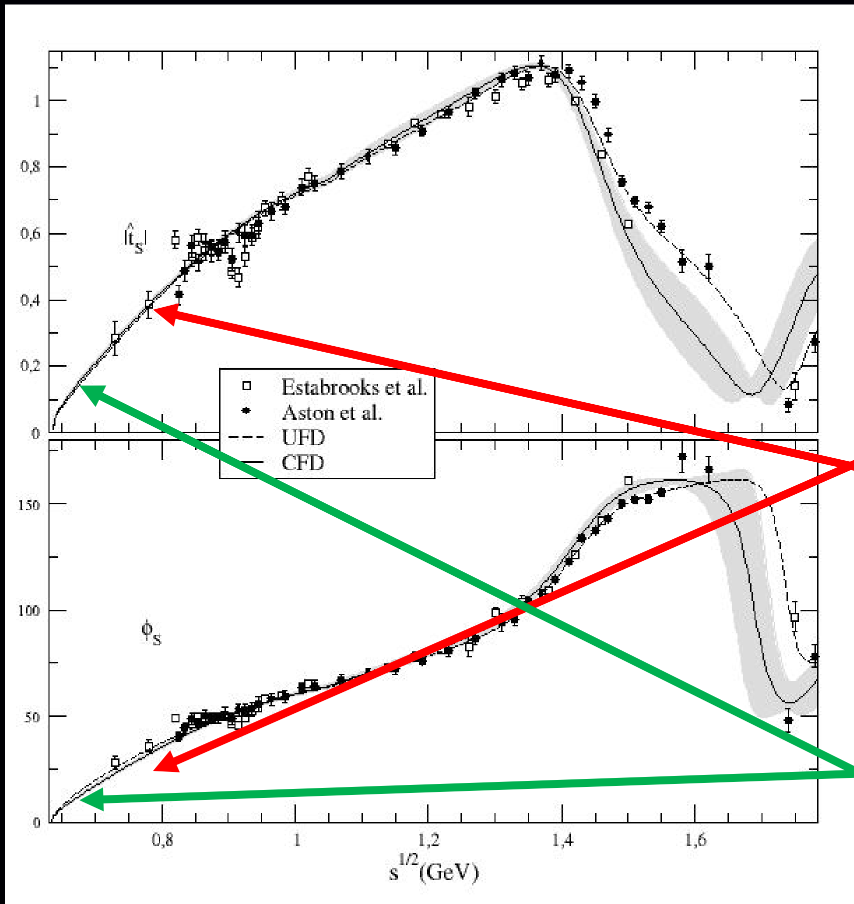


- Large model-dependences:
  naïve models often used for parameterizations and resonance poles

Dispersion Relations (This talk)

Model independent constraints,
precise threshold parameters and pole determinations.
Enhanced precision

Data on πK scattering: S-channel



Most reliable sets:
Estabrooks et al. 78 (SLAC)
Aston et al.88 (SLAC-LASS)

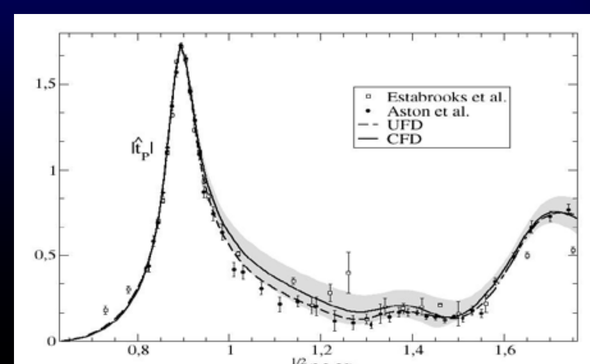
I=1/2 and 3/2 combination
MANY DATA IN CONFLICT

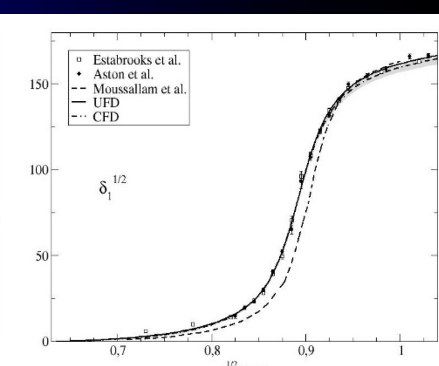
No clear “peak” or phase movement of $\kappa/K_0^*(800)$ resonance

Definitely NO BREIT-WIGNER shape

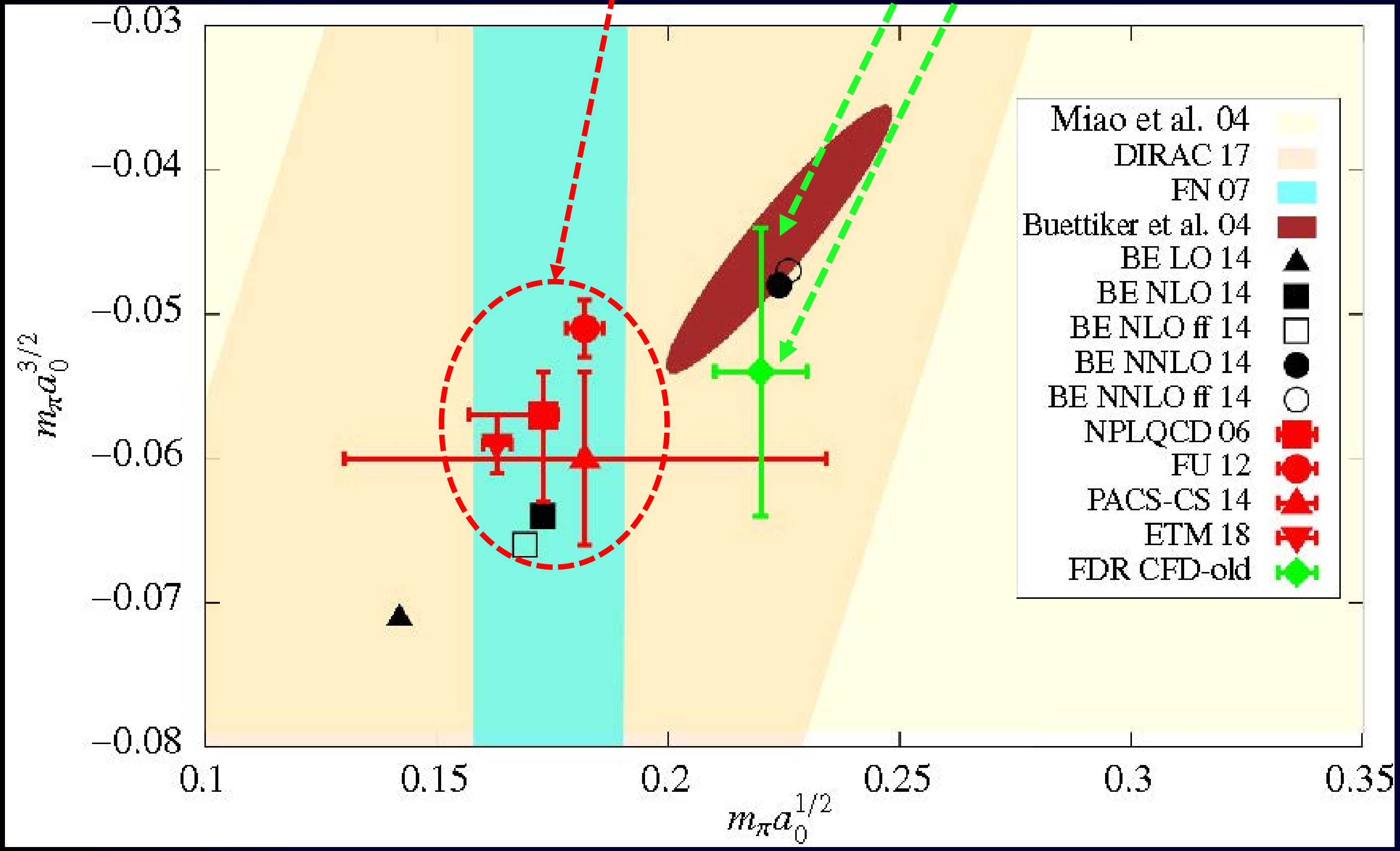
No data near threshold.
Models need dangerous extrapolations.
Dispersion relations →**sum-rules**

Compare to nice BW shape for $K_1*(892)$
(P-wave)





- Threshold parameters relevant to test ChPT (NNLO at present).
- Present tension between lattice and dispersive results



- Dalitz 1965: “Quite apart from the model discussed here,…such K* states are expected to exist simply on the basis of SU(3)” Procs. Oxford Int. Conf. on Elementary Particles 1965

- Many claims at different masses, narrow, wide… claims of absence. Confusion

- 1967 attitude

REVIEWS OF

MODERN PHYSIC

VOLUME 39, NUMBER 1

Data on Particles and Resonant States*

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MATTS ROOS
CERN, Geneva, Switzerland
WILLIAM J. WILLIS
Dept. of Physics, Yale University, New Haven, Connecticut

Data on the properties of leptons, mesons, and baryons are listed, referenced, averaged, and summarized in tables and wallet cards. This is an updating of the *Reviews of Modern Physics* article of October 1965.

1. The κ(725) (Lynch, Rittenberg, Rosenfeld, Söding, Dec. 1966)

We are beginning to think that κ should be classified along with flying saucers, the Loch Ness Monster, and the Abominable Snowman. We have heard of several experiments which were supposed to confirm it, and each one has either failed completely or failed to find it in the sought-for channel, but found instead a small Kπ peak near 725 MeV in some other channel.

- Removed from Review of Particle Physics in 1976 (with the σ)

- Back to RPP in 2004 as “controversial” $K_0^*(800)$. Omitted from summary tables

Strong support for κ/$K_0^*(800)$ from chiral theories and experimental decays of heavier mesons, but rigorous model-independent extractions absent. Often inadequate Breit-Wigner formalisms

- Omitttted from the 2018PDG summary table since, “needs confirmation”

Since the 70’s 90’s, all descriptions of data respecting unitarity and chiral symmetry find a pole at M=650-770 MeV and Γ~550 MeV or larger.

Best determination came from a **SOLUTION** (they did NOT use DATA on kappa region)

of a Roy-Steiner dispersive formalism, consistent with UChPT Decotes Genon et al 2006

2017PDG:

$\mathbf{K_0^*(800)}$ dominated by such a SOLUTION

M-i Γ/2=(682±29)-i(273±i12) MeV

PDG2018:

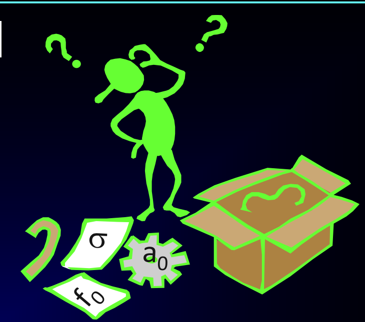
(630-730)-i(260-340) MeV
name changed to $\mathbf{K_0^*(700)}$

PDG2020:

$\mathbf{K_0^*(700)}$ Makes it to the summary tables**.**
Still “Needs Confirmation”

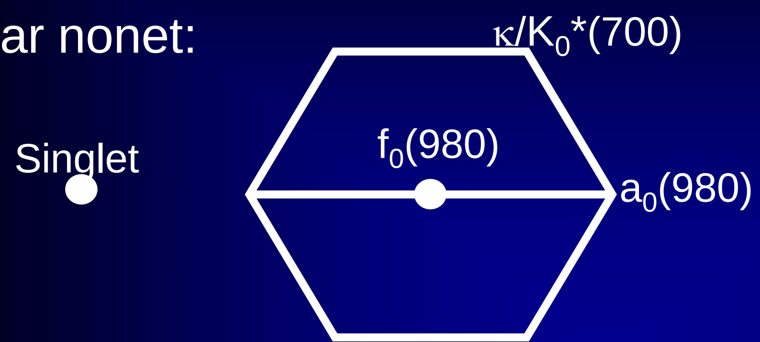
We were encouraged by PDG to confirm it with a dispersive DATA analysis (this talk)

- Scalar SU(3) multiplets identification controversial



- Too many or too few resonances for decades
  But there is an emerging picture

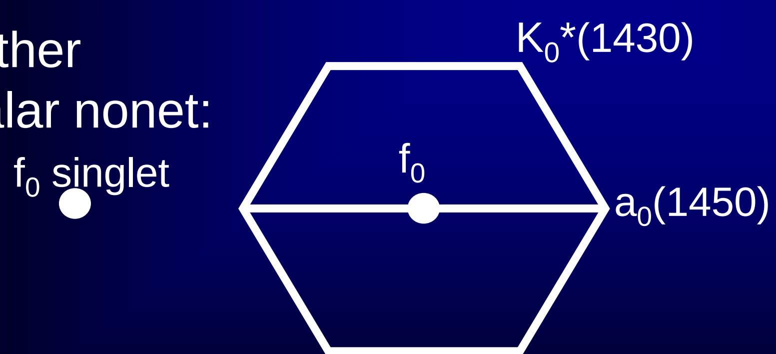
A Light scalar nonet:



Non-strange heavier!!
Inverted hierarchy problem
For quark-antiquark

$f_0(500)$ and $f_0(980)$ are
really octet/singlet mixtures

+ Another
heavier scalar nonet:



+ glueball ?

f_0

Enough f_0 states observed: $f_0(1370)$, $f_0(1500)$, $f_0(1700)$.
Picture complicated by mixture between them (lots of works here)
Note strange resonances “count” how many nonets exist.

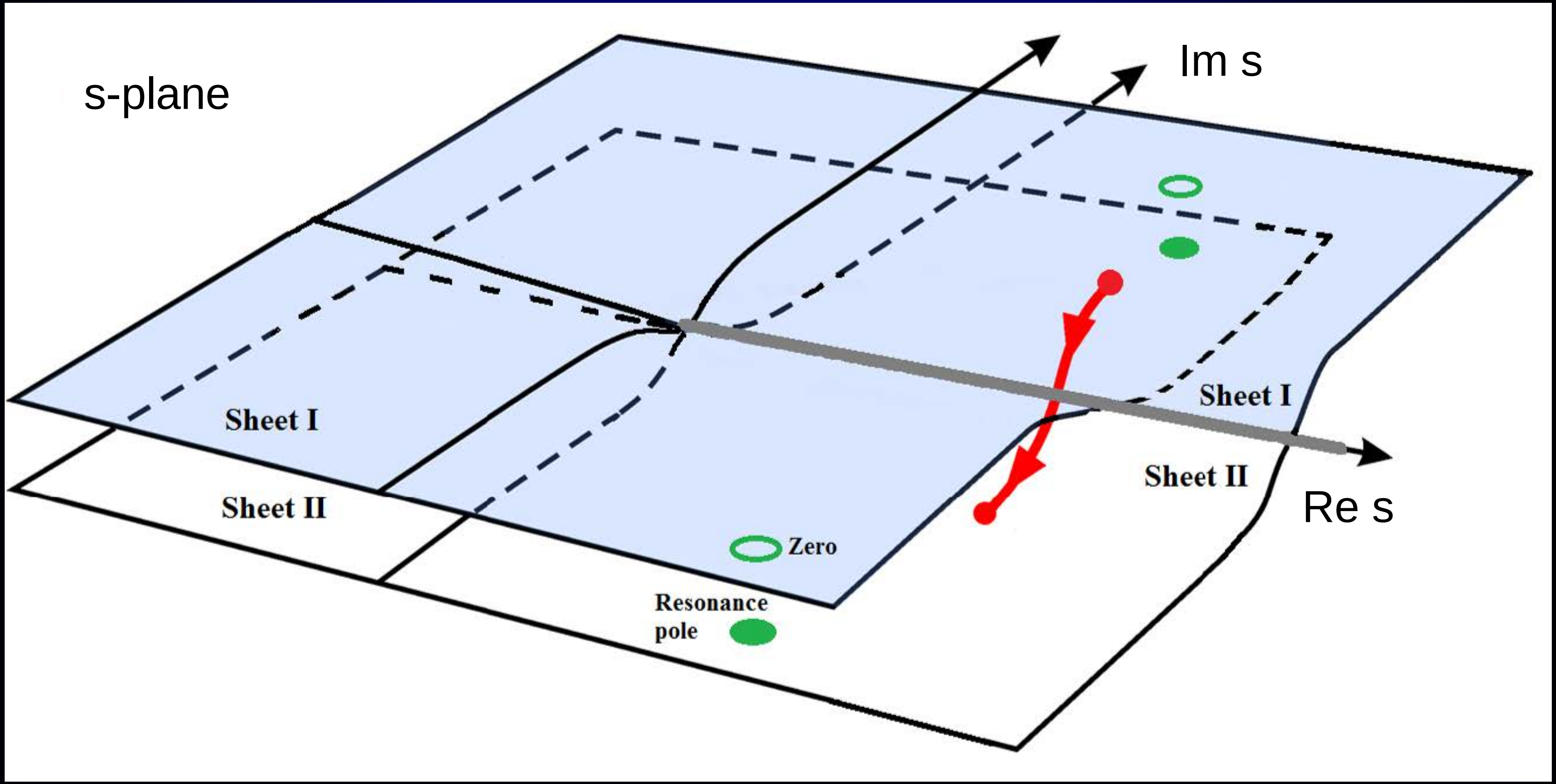
Only the light $\kappa(700)$ or $K_0^*(700)$ “Needs Confirmation” @ PDG2020

Resonances as poles

The Breit-Wigner shape is just an approximation for narrow and isolated resonances

The universal features of resonances are their
pole positions and residues *
$\sqrt{s_{pole}} \approx M - i\,\Gamma/2$

*in the Riemann sheet obtained from an analytic continuation through the physical cut



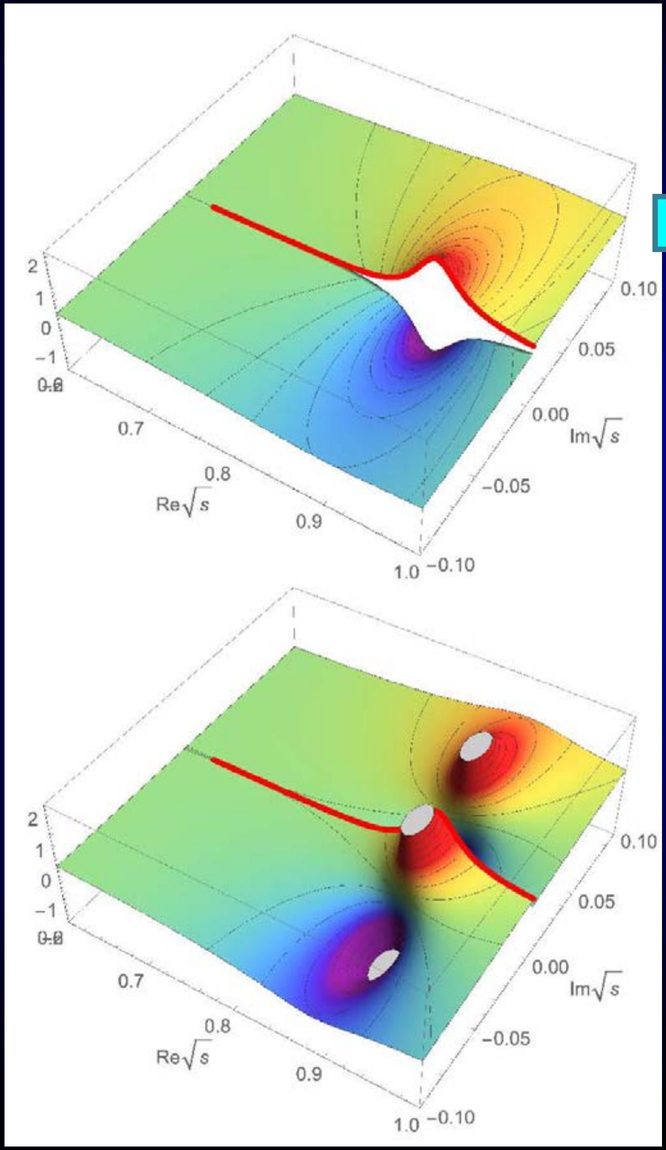
Amplitudes satisfy $f(s^*) = f^*(s)$. Thus, poles appear in conjugated pairs in the 2nd Riemann sheet.

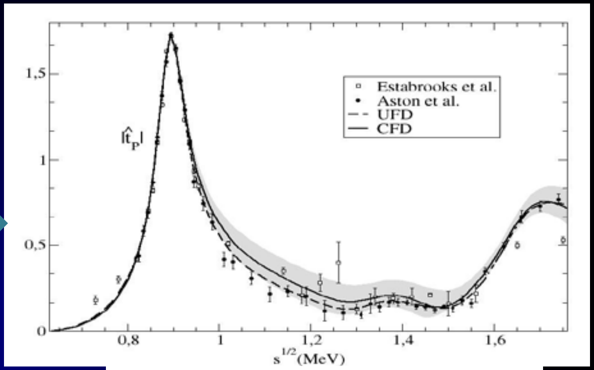
When poles are isolated from other singularities and “narrow”=near the real axis, the amplitude looks like usual BW

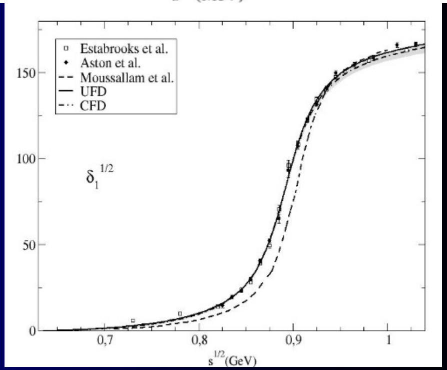
$K_1*(892)$

1st sheet

2nd sheet

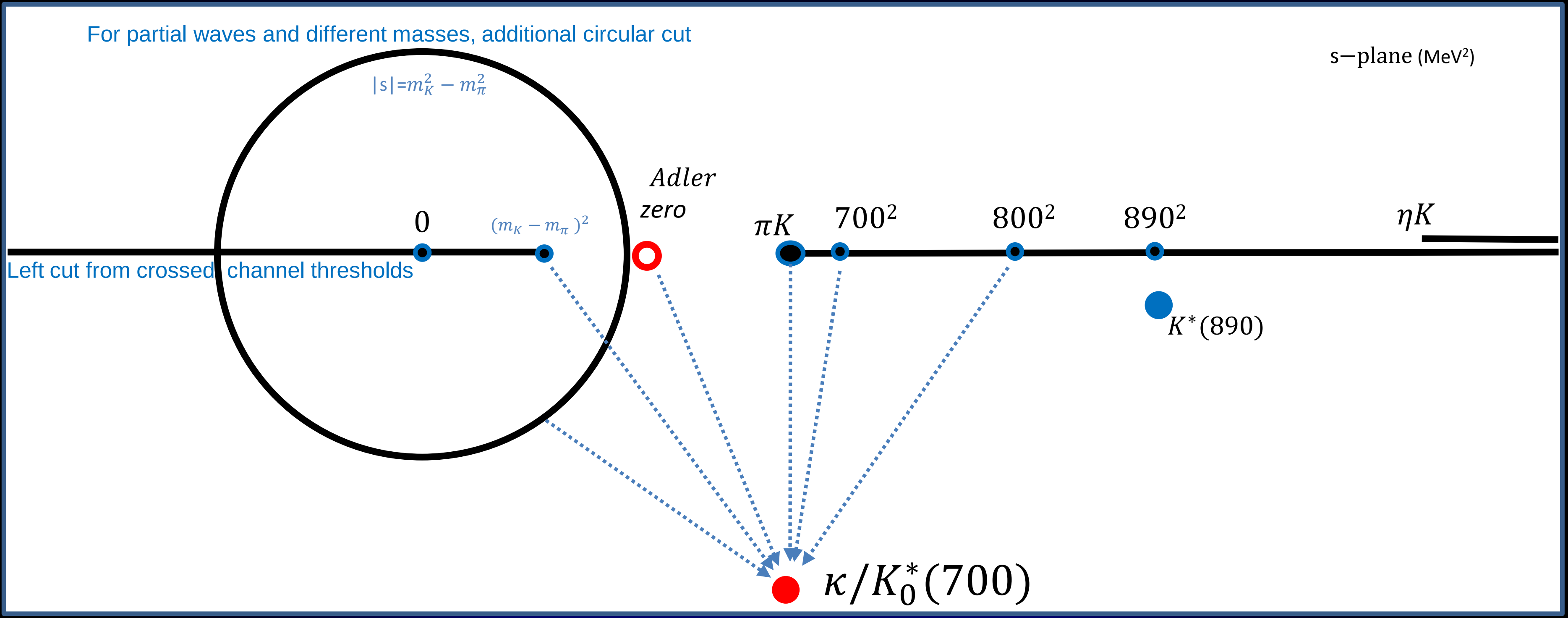






Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

Analyticity is expressed in the *s*-variable, not in Sqrt(s)



Important for the $\kappa/K_0^*(700)$ and threshold parameters

- Threshold behavior (chiral symmetry)
- Subthreshold behavior (chiral symmetry →Adler zeros)
- Other cuts (Left & circular)
- Avoid spurious singularities

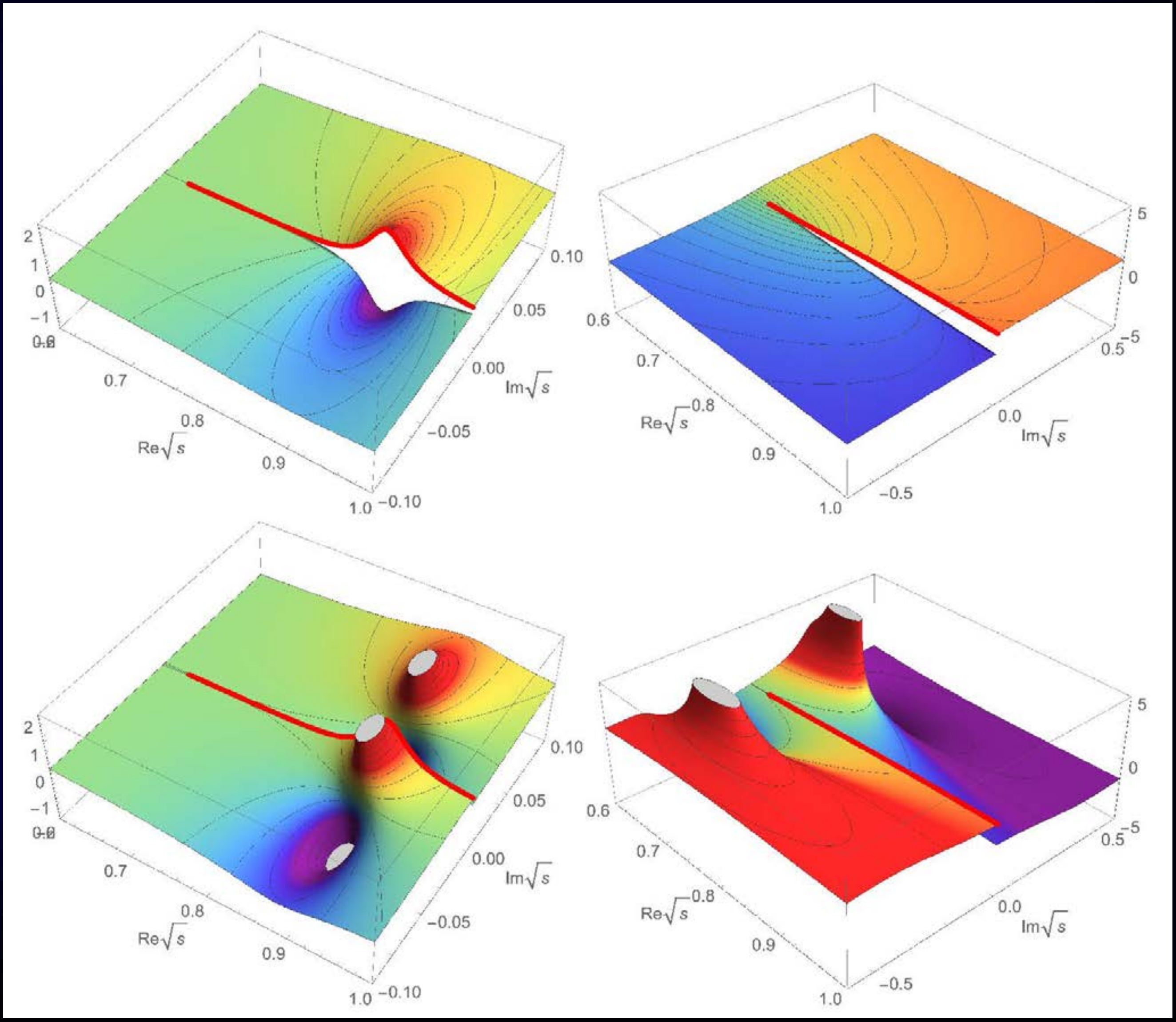
Less important for other resonances…

When poles are isolated from other singularities and “narrow”=near the real axis, the amplitude looks like usual BW

$K_1^*(892)$ $\kappa/K_0^*(700)$

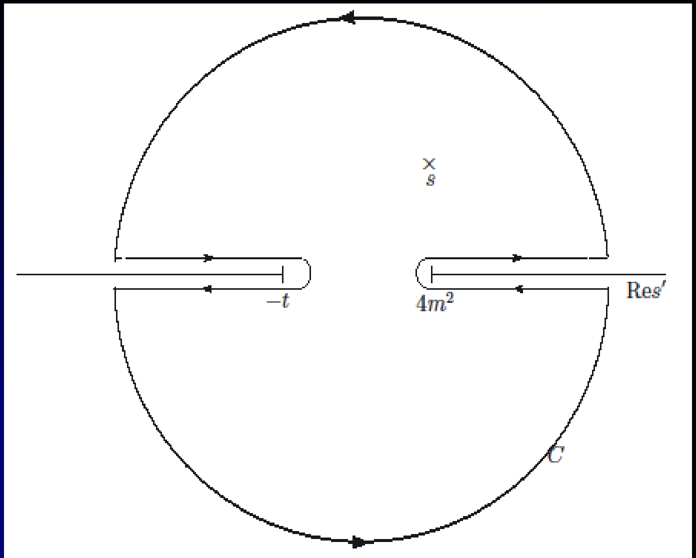
1st sheet

2nd sheet



Why use dispersion relations?

CAUSALITY: Amplitudes T(s,t) are ANALYTIC in complex s plane but for cuts for thresholds. Crossing implies left cut from u-channel threshold



Cauchy Theorem determines T(s,t) at ANY s, from an INTEGRAL on the contour

$$f(z) = \frac{1}{2\pi i}\oint_C dz' \frac{f(z')}{z'-z}.$$

EXAMPLE: Fixed t dispersión relation: recall $T(s^*) = T^*(s)$

If T->0 fast enough at high s, curved part vanishes

$$T(s,t,u) = \underbrace{\frac{1}{\pi}\int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\,T(s',t,u')}{s'-s}}_{\text{Right cut}} + \underbrace{\frac{1}{\pi}\int_{-\infty}^{-t} ds' \frac{\mathrm{Im}\,T(s',t,u')}{s'-s}}_{\text{Left cut}}$$

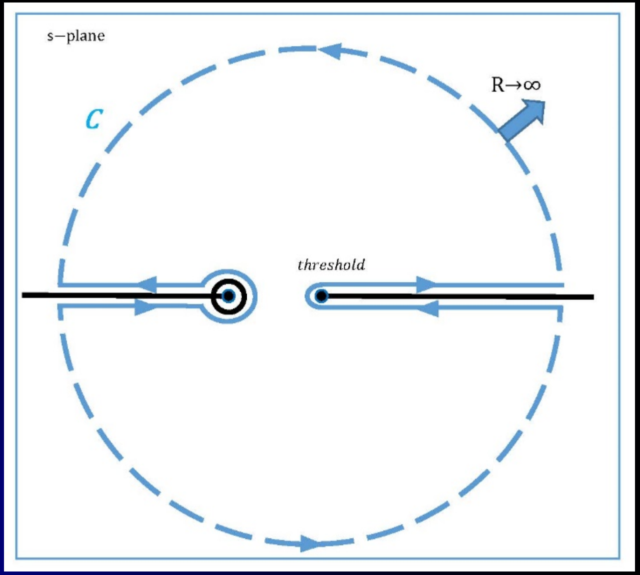
Otherwise, determined up to a polynomial (subtractions)

Left cut usually a problem

<u>EXAMPLE: For partial waves.</u>

We now integrate t, which is like integrating in z_s=cosθ:

$$f_\ell^I(s) = \frac{1}{32\pi N}\int_{-1}^{1} dz_s P_\ell(z_s) F^I(s,t(z_s)),$$



If T->0 fast enough at high s, curved part vanishes
Otherwise, determined up to a polynomial (subtractions)
Left and circular cuts usually a problem.
Example with 3 subtractions:

$$f(s) = f(0) + s f'(0) + \frac{s^2}{2} f''(0) + \frac{s^3}{\pi}\int_{RC} ds' \frac{\mathrm{Im} f(s')}{s'^3(s'-s)} + LC(f) + CC(f),$$

<u>Dispersion Relations are good for:</u>

1) Calculating T(s,t) where there is not data

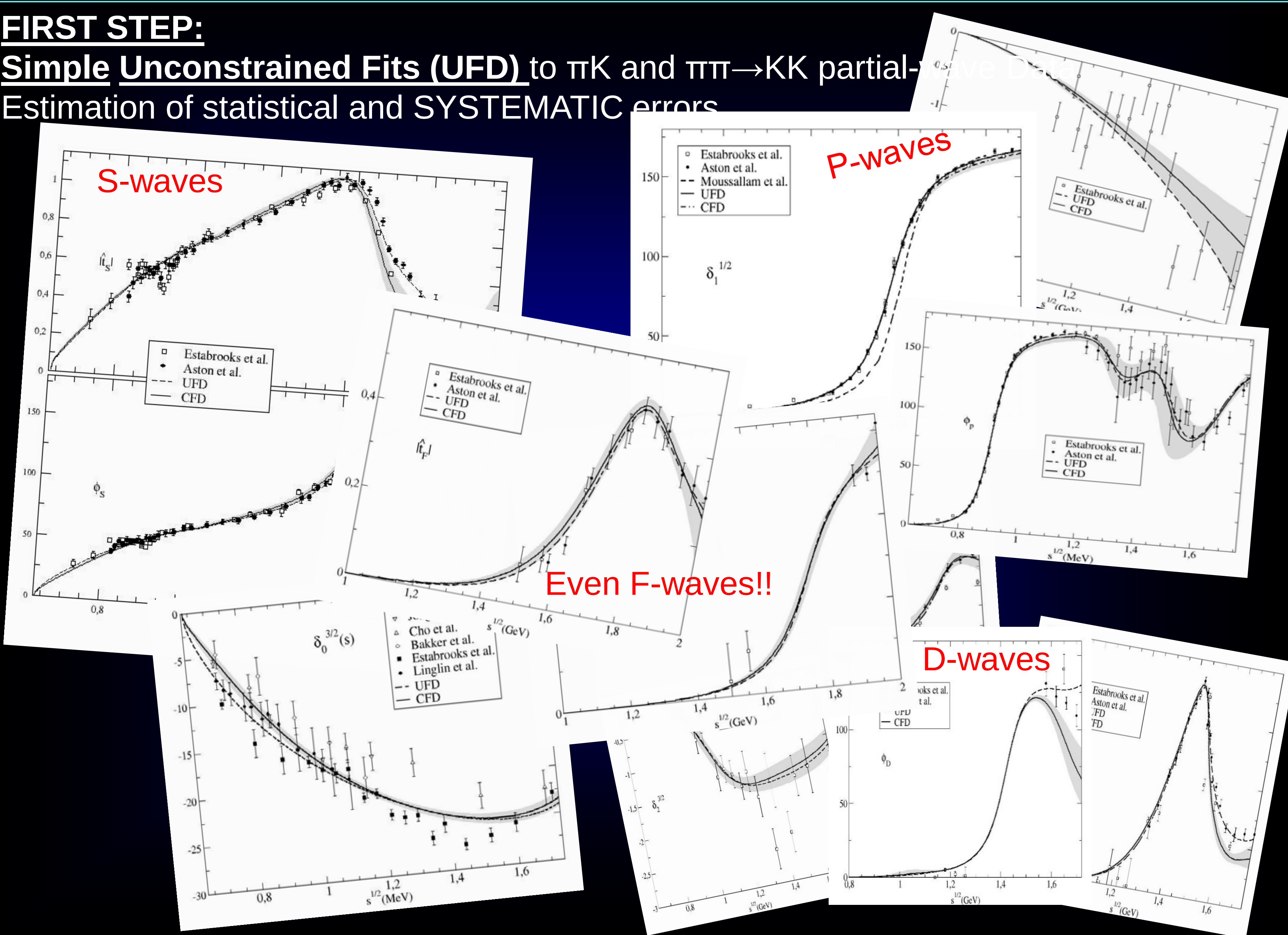
2) Constraining data analysis

3) ONLY MODEL INDEPENDENT extrapolation to complex s-plane
without extra assumptions

FIRST STEP:

Simple Unconstrained Fits (UFD) to πK and ππ→KK partial-wave Data

Estimation of statistical and SYSTEMATIC errors



<u>Simple</u> <u>Unconstrained Fits</u> to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

Forward dispersion relations for K π scattering.

Since interested in the resonance region, we use minimal number of subtractions

Defining the s↔u symmetric and anti-symmetric amplitudes at t=0

$$T^{+}(s)=\frac{T^{1/2}(s)+2T^{3/2}(s)}{3}=\frac{T^{I_t=0}(s)}{\sqrt{6}},$$

$$T^{-}(s)=\frac{T^{1/2}(s)-T^{3/2}(s)}{3}=\frac{T^{I_t=1}(s)}{2}.$$

We need one subtraction for the symmetric amplitude

$$\mathrm{Re}T^{+}(s)=T^{+}(s_{\mathrm{th}})+\frac{(s-s_{\mathrm{th}})}{\pi}P\int_{s_{\mathrm{th}}}^{\infty}ds'\left[\frac{\mathrm{Im}T^{+}(s')}{(s'-s)(s'-s_{\mathrm{th}})}-\frac{\mathrm{Im}T^{+}(s')}{(s'+s-2\Sigma_{\pi K})(s'+s_{\mathrm{th}}-2\Sigma_{\pi K})}\right],$$

And none for the antisymmetric

$$\mathrm{Re}T^{-}(s)=\frac{(2s-2\Sigma_{\pi K})}{\pi}P\int_{s_{\mathrm{th}}}^{\infty}ds'\frac{\mathrm{Im}T^{-}(s')}{(s'-s)(s'+s-2\Sigma_{\pi K})}.$$

where $\Sigma_{\pi K}=m_\pi^2+m_K^2$

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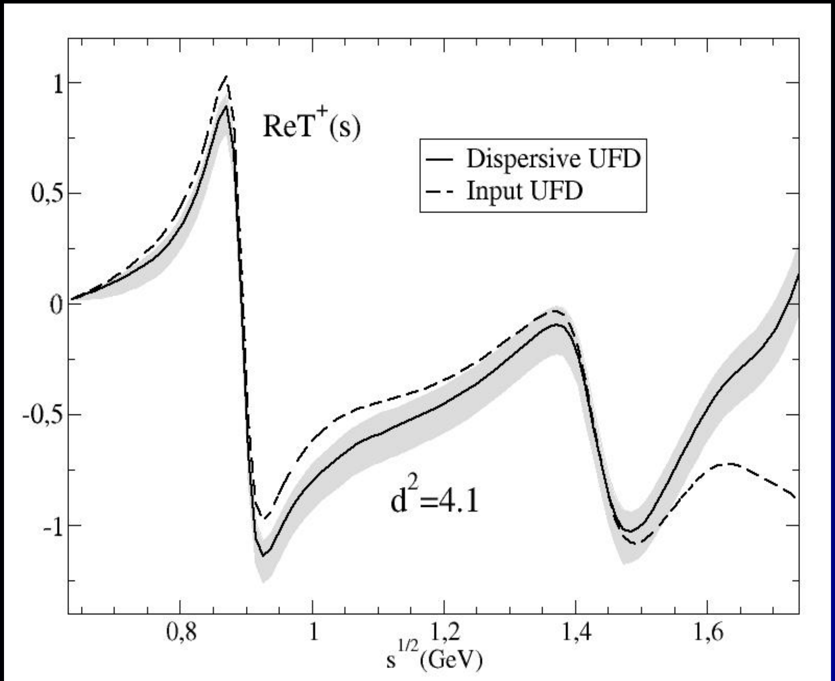
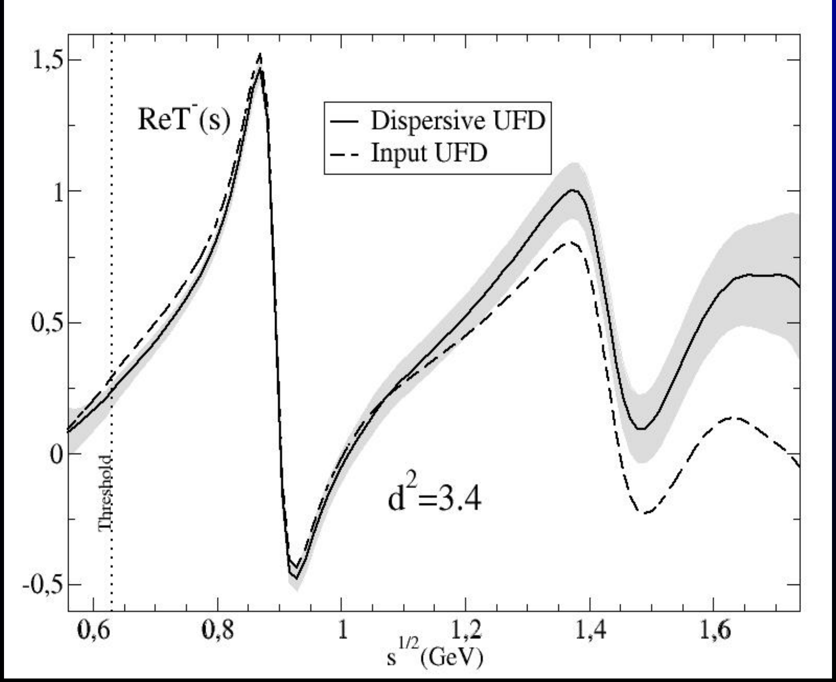
- As πK checks: Small inconsistencies.





Forward Dispersion Relation analysis of πK scattering DATA up to 1.6 GeV

(**not a solution** of dispersión relations, but a constrained fit)

A.Rodas & JRP, PRD93,074025 (2016)

First observation:
Forward Dispersion relations
Not well satisfied by data
Particularly at high energies

So we use
Forward Dispersion Relations
as CONSTRAINTS on fits

Our Dispersive/Analytic Approach for πK and strange resonances

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- As πK checks: Small inconsistencies.
- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)

Every 22 MeV calculate the difference between both sides of the DR /uncertainty

Define an averaged χ^2 over these points, that we call d^2

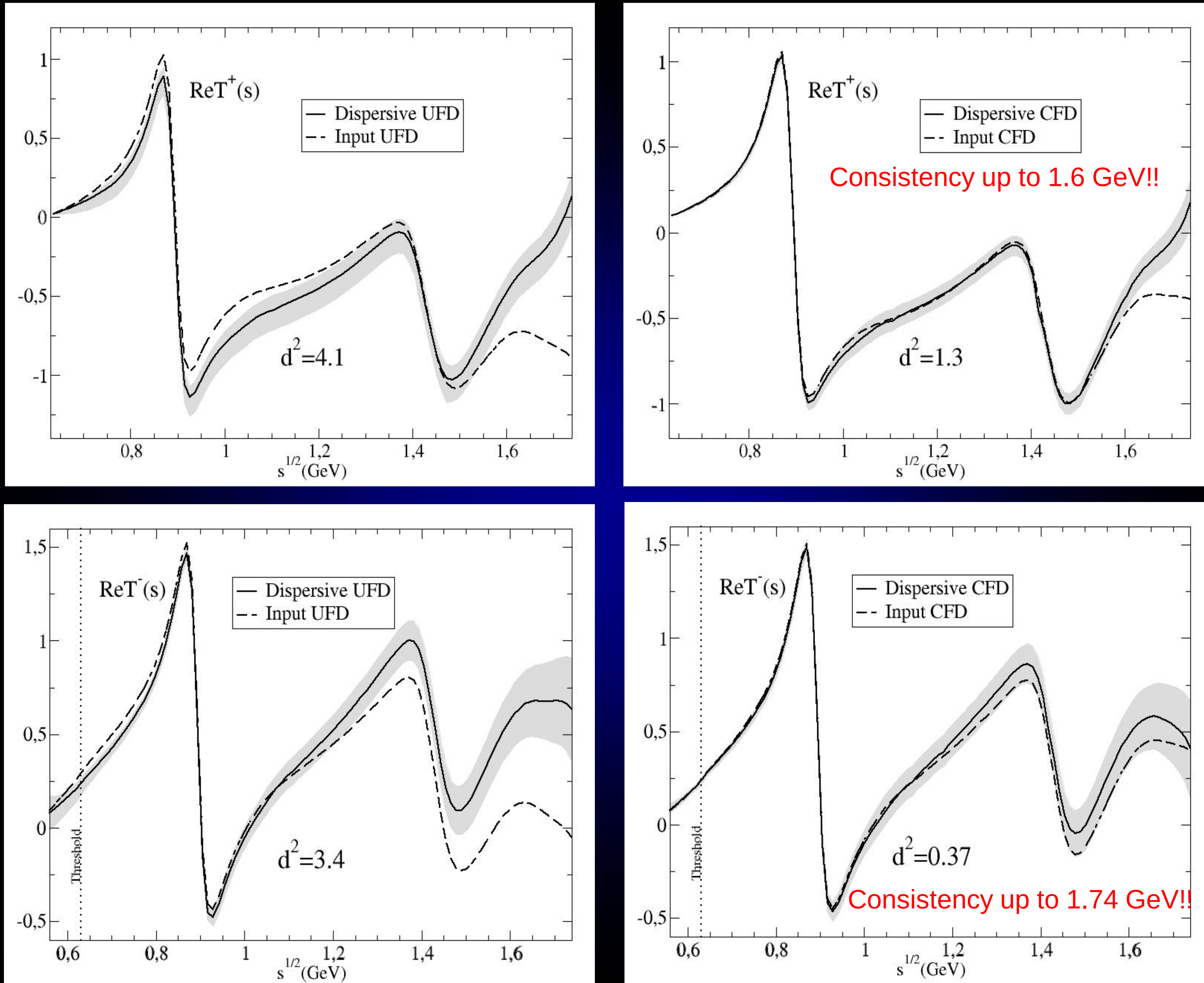
> d^2 close to 1 means that the relation is well satisfied
>
> d^2>> 1 means the data set is inconsistent with the relation.

This can be used to check DR

To obtain CONSTRAINED FITS TO DATA (CFD) we minimize:

$$\underbrace{W^2\left(d^2_{T+}+d^2_{T-}\right)}_{\text{2 FDR's}}+\underbrace{\sum_{I=\frac{1}{2},\frac{3}{2}}\left(\frac{\Delta_I}{\delta\Delta_I}\right)^2}_{\text{Sum Rules threshold}}+\underbrace{\sum_k\left(\frac{p_k^{UFD}-p_k}{\delta p_k^{UFD}}\right)^2}_{\text{Parameters of the unconstrained data fits}},$$

W roughly counts the number
of effective degrees of freedom
(sometimes we add weight on certain energy regions)



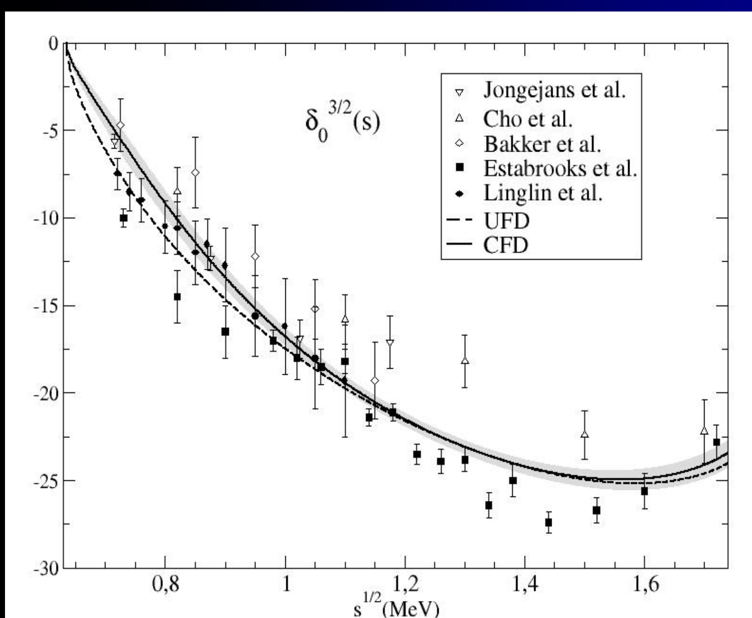
ReT+(s)
Dispersive UFD
Input UFD
d2=4.1
s1/2(GeV)
ReT+(s)
Dispersive CFD
Input CFD
Consistency up to 1.6 GeV!!
d2=1.3
s1/2(GeV)
ReT-(s)
Dispersive UFD
Input UFD
Threshold
d2=3.4
s1/2(GeV)
ReT-(s)
Dispersive CFD
Input CFD
Threshold
d2=0.37
Consistency up to 1.74 GeV!!
s1/2(GeV)

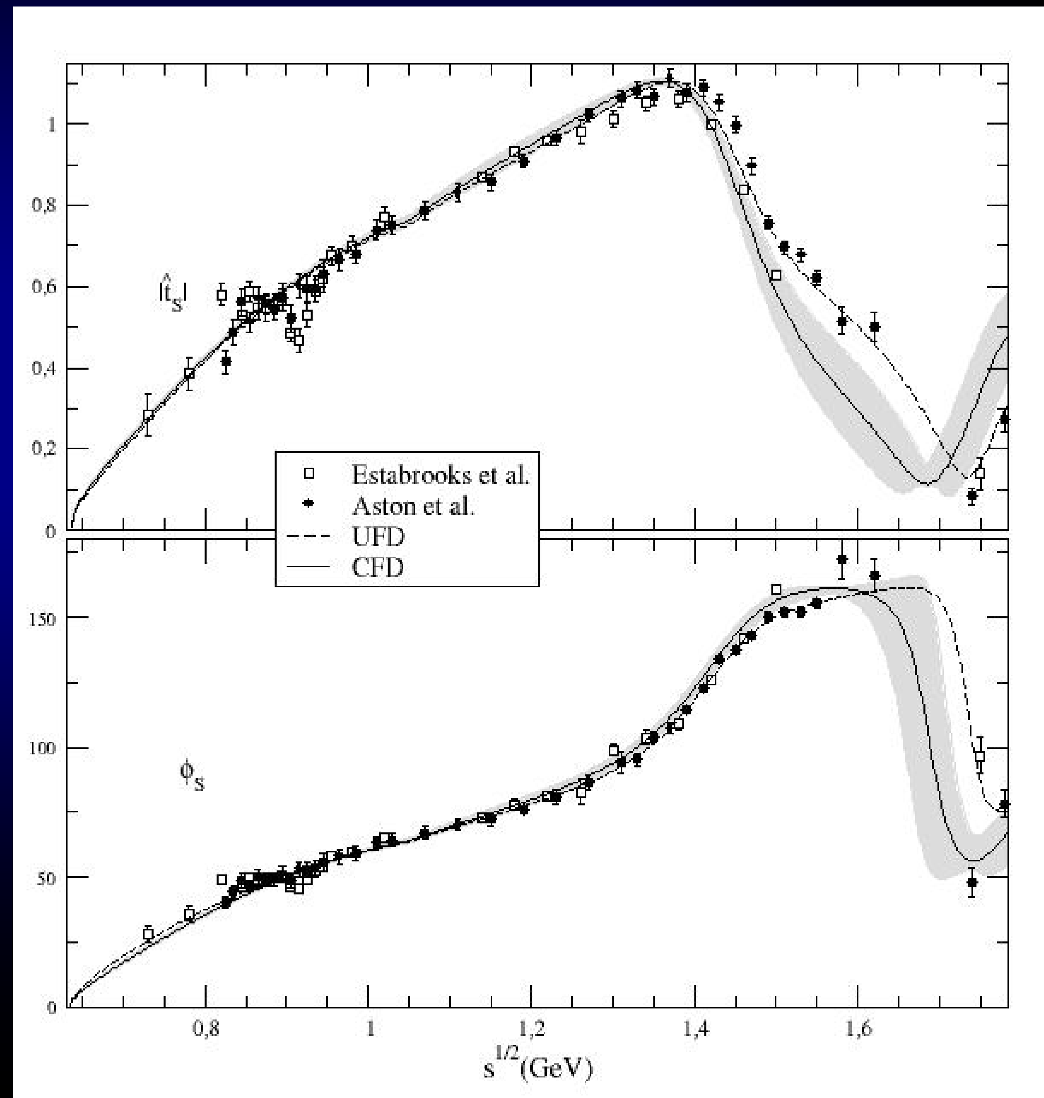
From Unconstrained (UFD) to Constrained Fits to data (CFD)

S-waves. The most interesting for the K_0^* resonances

Largest changes from UFD to CFD

at higher energies





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- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)
- Analytic methods to extract poles: reduced model dependence on strange resonances
  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

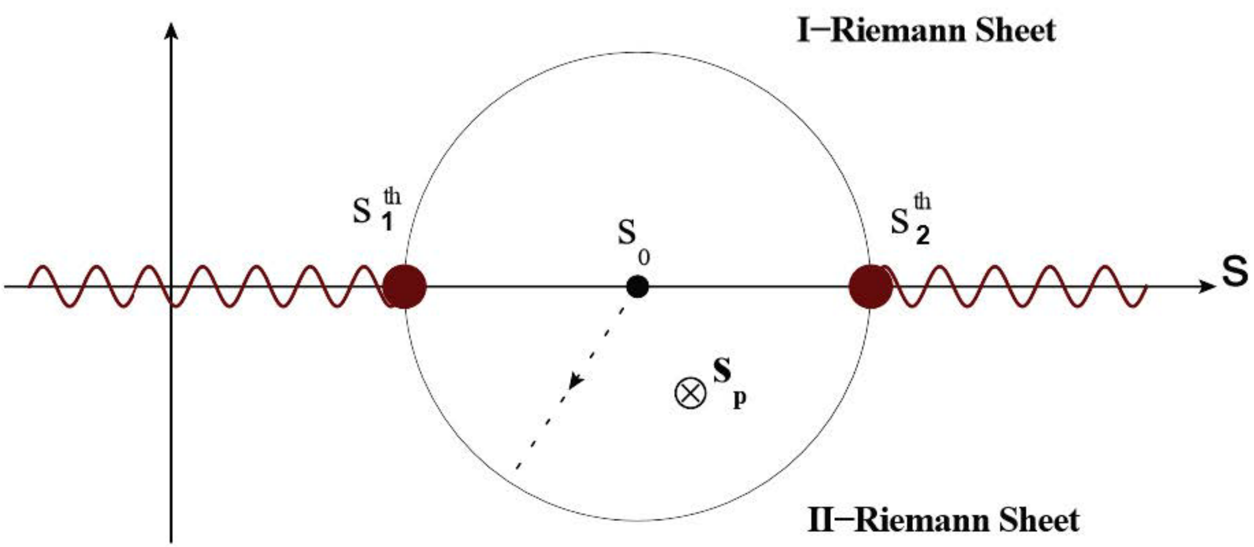
Strange resonance poles from CFD: Using Padé sequences

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

Almost model independent: Does not assume any particular functional form
(but local determination)

Based on previous works by P.Masjuan, J.J. Sanz Cillero, I. Caprini, J.Ruiz de ELvira

- The method is suitable for the calculation of both elastic and inelastic resonances.
- The Padé sequence gives us the continuation to the continuous Riemann Sheet.
- We take care of the calculation of the errors. Apart from the experimental and systematic errors of each parameterization we also include different fits.



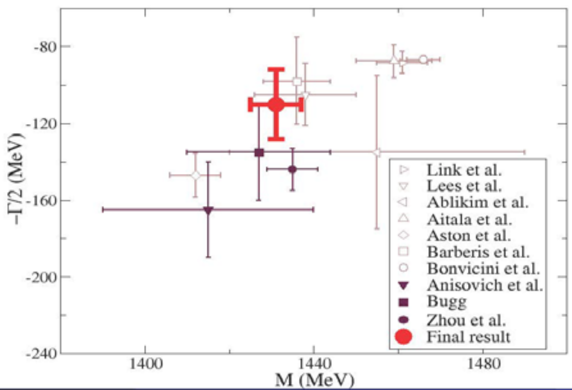
Strange resonance poles from CFD: Using Padé sequences

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

The method can be used for inelastic resonances too. Provides resonance parameters WITHOUT ASSUMING SPECIFIC FUNCTIONAL FORM

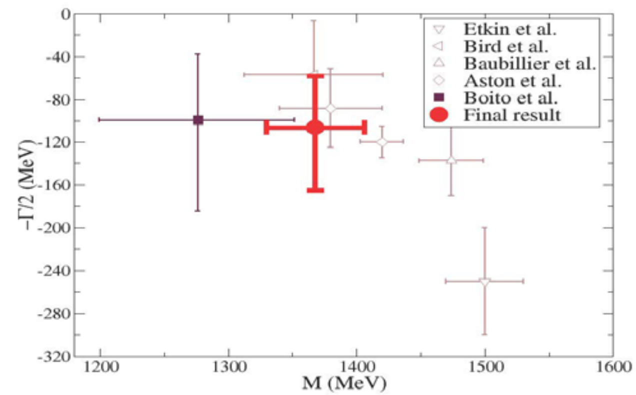
- For the $K_0^*(1430)$ we find

$$\sqrt{s_p} = (1431 \pm 6) - i(110 \pm 19)MeV$$
$$\sqrt{s_p} = (1425 \pm 50) - i(135 \pm 40)MeV(PDG)$$



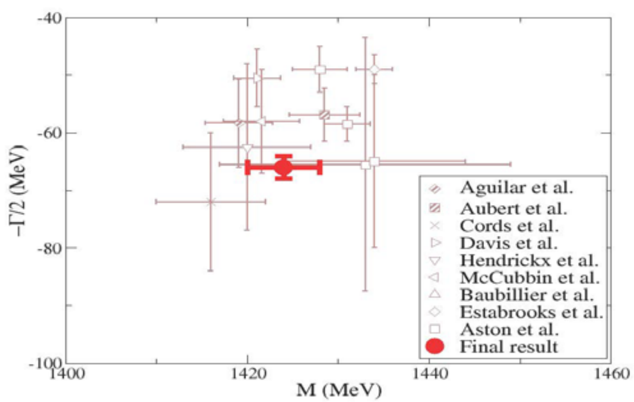
- For the $K_1^*(1410)$ we find

$$\sqrt{s_p} = (1368 \pm 38) - i(106^{+48}_{-59})MeV$$
$$\sqrt{s_p} = (1414 \pm 15) - i(116 \pm 10)MeV(PDG)$$



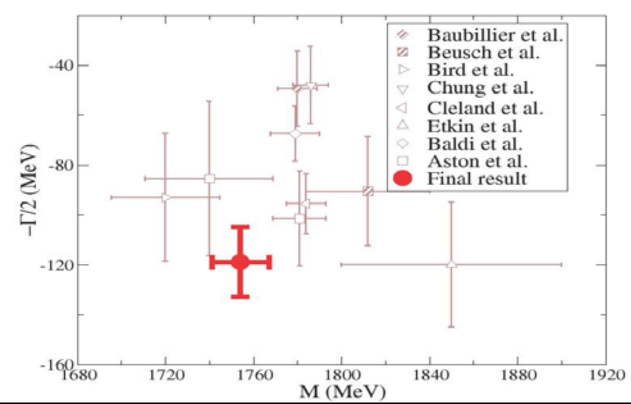
- For the $K_2^*(1430)$ we find

$$\sqrt{s_p} = (1424 \pm 4) - i(66 \pm 2)MeV$$
$$\sqrt{s_p} = (1432.4 \pm 1.3) - i(55 \pm 3)MeV(PDG)$$



- For the $K_3^*(1780)$ we find

$$\sqrt{s_p} = (1754 \pm 13) - i(119 \pm 14)MeV$$
$$\sqrt{s_p} = (1776 \pm 7) - i(80 \pm 11)MeV(PDG)$$



In 2021, the PDG willstart giving pole positions for some of these besides BW parameters

Kappa pole from CFD

1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

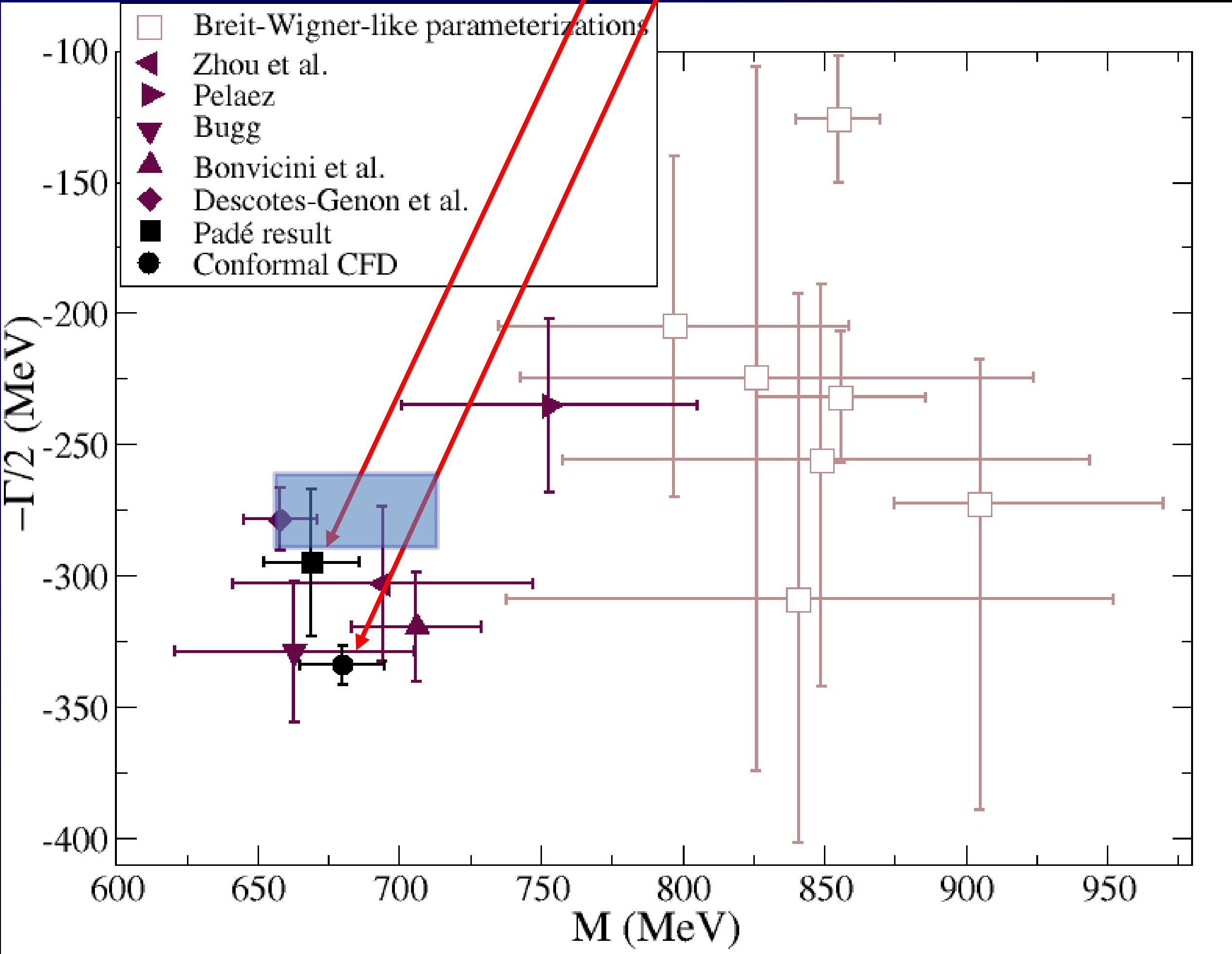
Fantastic analyticity properties,
but not model independent

(680±15)-i(334±7.5) MeV

2) Using Padé Sequences…
JRP, A. Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017) 77:91

(670±18)-i(295± 28) MeV

Compare to PDG2017:
(682±29)-i(273±12) MeV



The resonance is NO LONGER the κ nor the $K_0{*}(800)$

But Still "Needs Confirmation" !

Best analysis so far:
Roy-Steiner
dispersion relations

Plenty of room for improvement on parameters

Our
Pade sequences

Kappa pole analytic determinations from constrained fits

1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

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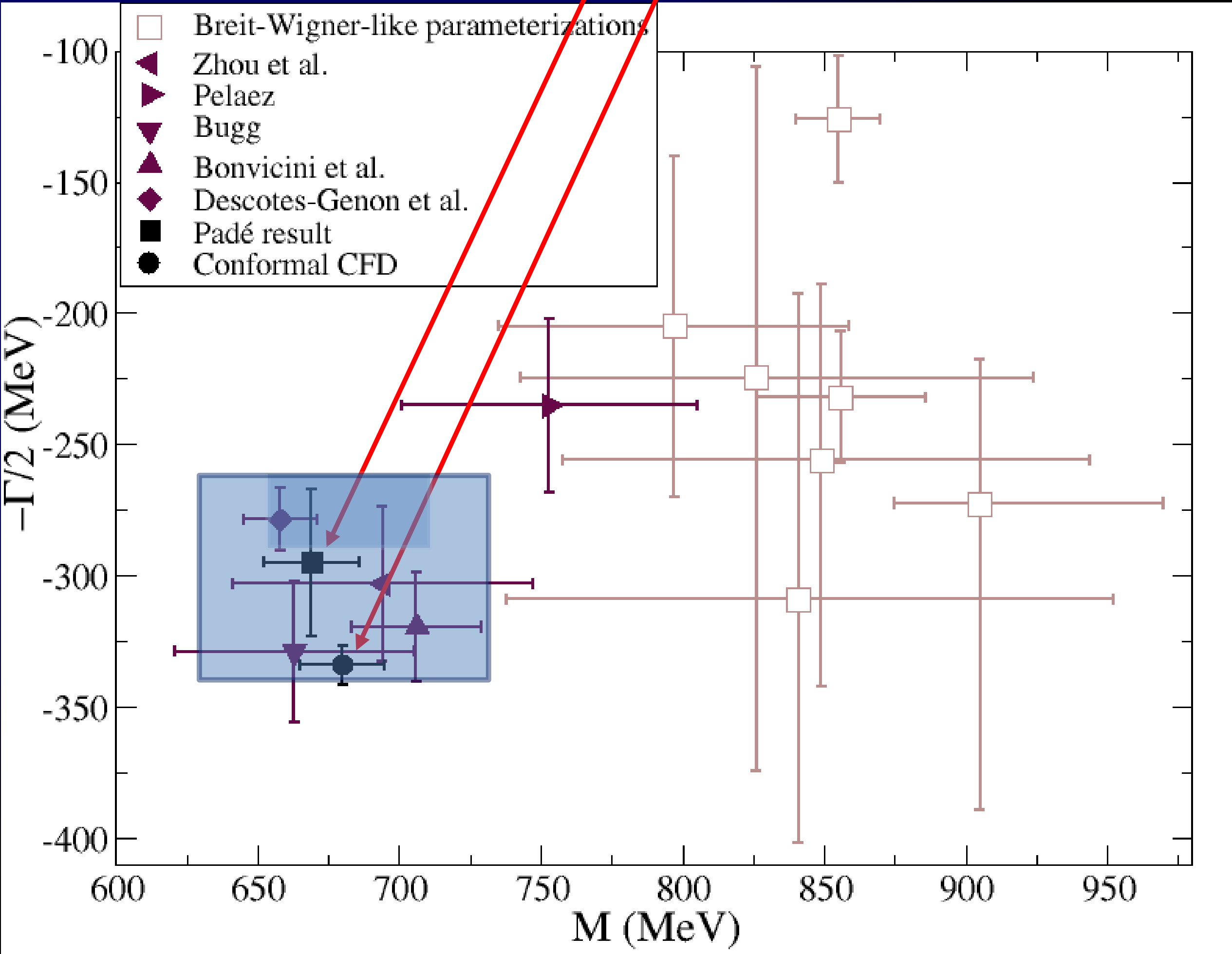

Compare to PDG2017:
(682±29)-i(273±12) MeV

New PDG2018:
(630-730)-i(260-340) MeV
And name changed
K_0^*(700)
Still “Needs Confirmation”

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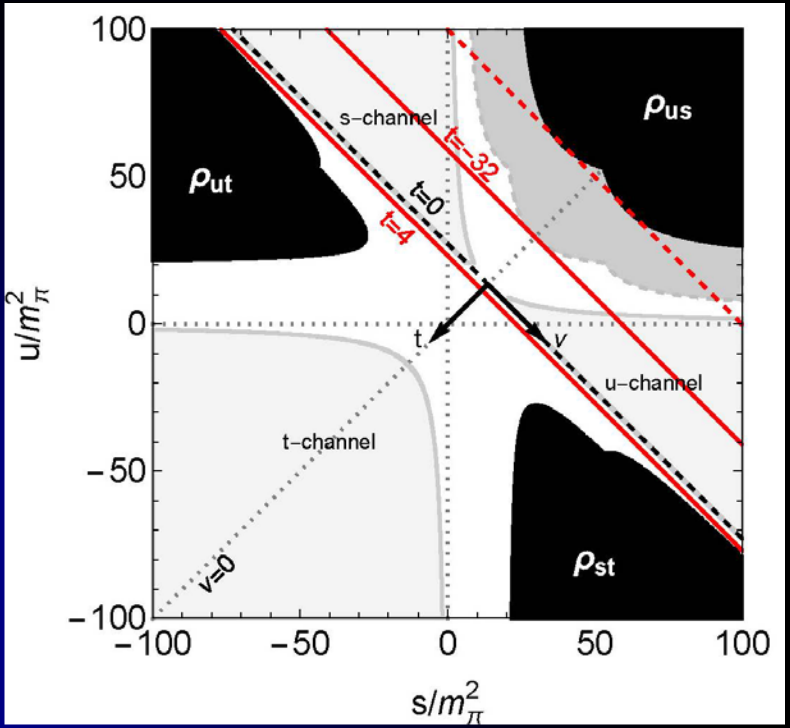
- Padé Sequences to extract poles: reduced model dependence on strange resonances
  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

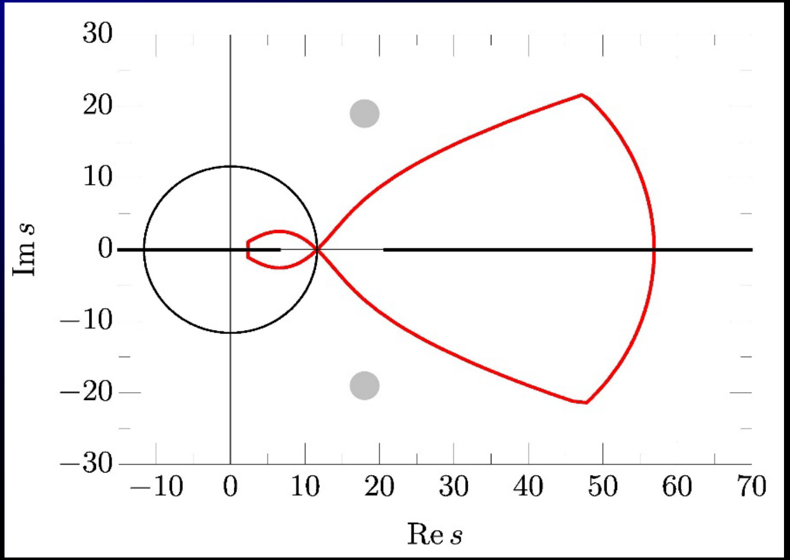
Partial Wave πK→πK and ππ→KK Dispersion Relations (Roy-Steiner eqs.)

To get a resonance pole we need PARTIAL-WAVE dispersion relations.



Their applicability is limited
-by the double spectral regions
-by the Lehmann ellipses
(way too technical. See our apendices)

Two possibilities in the literature:

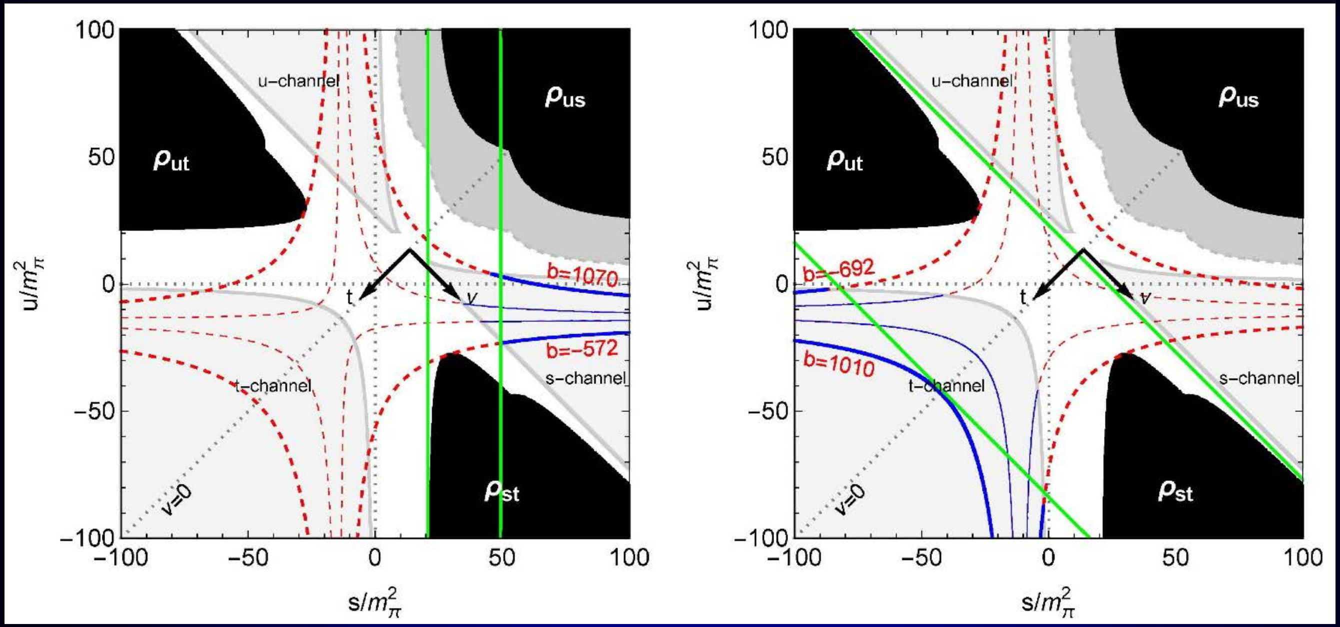


1) Integrate "t" for fixed-t dispersion relations.
Fine for the real axis (1.1 GeV)
Very mild dependence on ππ→KK
but bad to reach the pole.
Were used to obtain solutions by the Paris Group
We will only used them as constraints on data

πK→πK and ππ→KK **Hyperbolic Dispersion Relations (HDR)**

2) Integrate along (s-a)(u-a)=b hyperbolae in the Mandelstam plane

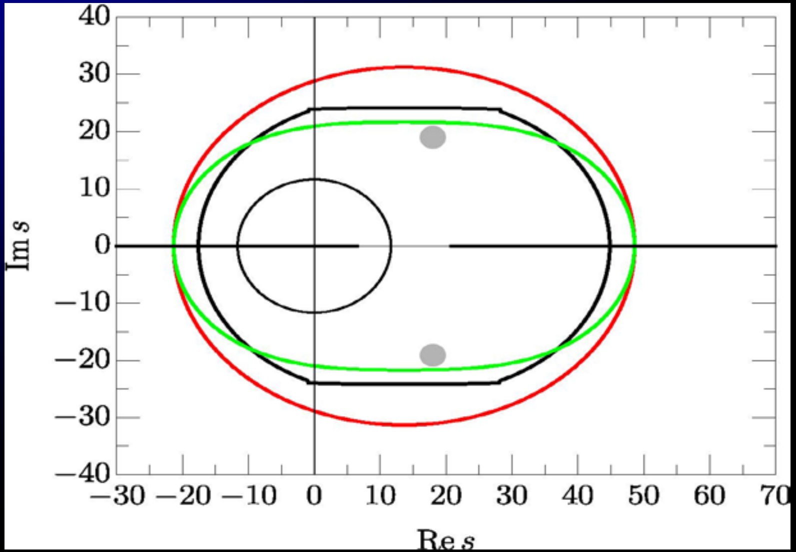
We tuned a=-13m_π^2 to maximize applicability for ππ→KK up to 1.47 GeV.



Applicability range slightly smaller in real axis for πK, but covers the kappa pole if a chosen appropriately

We will use them as constraints and to get the pole.

a=-10m_π^2 chosen to include also error bars inside applicability region



πK→πK and ππ→KK Hyperbolic Dispersion Relations (HDR)

g^I_J = $\pi\pi \to K\bar{K}$ partial waves. We study (I,J)=(0,0),(1,1),(0,2)

f^I_J = $K\pi \to K\pi$ partial waves. Taken from previous dispersive study   JRP, A. Rodas PRD 2018

$$
\begin{aligned}
g_0^0(t) &= \frac{\sqrt{3}}{2} m_+ a_0^+ + \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, g_0^0(t')}{t'(t'-t)} dt' + \frac{t}{\pi}\sum_{\ell\geq 2}\int_{4m_\pi^2}^{\infty} \frac{dt'}{t'} G^0_{0,2\ell-2}(t,t')\mathrm{Im}\, g^0_{2\ell-2}(t') + \sum_{\ell}\int_{m_+^2}^{\infty} ds' G^+_{0,\ell}(t,s')\mathrm{Im}\, f^+_\ell(s'),\\
g_1^1(t) &= \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, g_1^1(t')}{t'-t} dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^{\infty} dt' G^1_{1,2\ell-1}(t,t')\mathrm{Im}\, g^1_{2\ell-1}(t') + \sum_{\ell}\int_{m_+^2}^{\infty} ds' G^-_{1,\ell}(t,s')\mathrm{Im}\, f^-_\ell(s'),\\
g_2^0(t) &= \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, g_2^0(t')}{t'(t'-t)} dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^{\infty} \frac{dt'}{t'} G'^0_{2,4\ell-2}(t,t')\mathrm{Im}\, g^0_{4\ell-2}(t') + \sum_{\ell}\int_{m_+^2}^{\infty} ds' G'^+_{2,\ell}(t,s')\mathrm{Im}\, f^+_\ell(s'). \qquad (39)
\end{aligned}
$$

$G^I_{J,J'}$(t,t') =integral kernels, depend on a parameter
Lowest # of subtractions. Odd pw decouple from even pw.

$$
\begin{aligned}
g_\ell^0(t) &= \Delta_\ell^0(t) + \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{dt'}{t'}\frac{\mathrm{Im}\, g_\ell^0(t)}{t'-t}, \qquad \ell = 0, 2,\\
g_1^1(t) &= \Delta_1^1(t) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} dt' \frac{\mathrm{Im}\, g_1^1(t)}{t'-t}, \qquad (40)
\end{aligned}
$$

Δ(t) depend on higher waves or on Kπ→Kπ.

Integrals from 2π threshold !

Solve in descending J order

We have used models for higher waves, but give very small contributions

For unphysical region below KK threshold, we used Omnés function

$$\Omega_\ell^I(t) = \exp\left(\frac{t}{\pi}\int_{4m_\pi^2}^{t_m} \frac{\phi_\ell^I(t')dt'}{t'(t'-t)}\right),$$

$$\Omega_\ell^I(t) \equiv \Omega_{l,R}^I(t) e^{i\phi_\ell^I(t)\theta(t-4m_\pi^2)\theta(t_m-t)},$$

This is the form of our HDR: Roy-Steiner+Omnés formalism

$$\begin{aligned}
g_0^0(t) &= \Delta_0^0(t) + \frac{t\Omega_0^0(t)}{t_m - t}\left[\alpha + \frac{t}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{(t_m - t')\Delta_0^0(t')\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)} + \frac{t}{\pi}\int_{t_m}^{\infty} dt' \frac{(t_m - t')|g_0^0(t')|\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)}\right] \\
g_1^1(t) &= \Delta_1^1(t) + \Omega_1^1(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_1^1(t')\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_1^1(t')|\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)}\right], \\
g_2^0(t) &= \Delta_2^0(t) + t\Omega_2^0(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_2^0(t')\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_2^0(t')|\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)}\right].
\end{aligned}$$

We can now check how well these HDR are satisfied

Our Dispersive/Analytic Approach for πK and strange resonances

Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

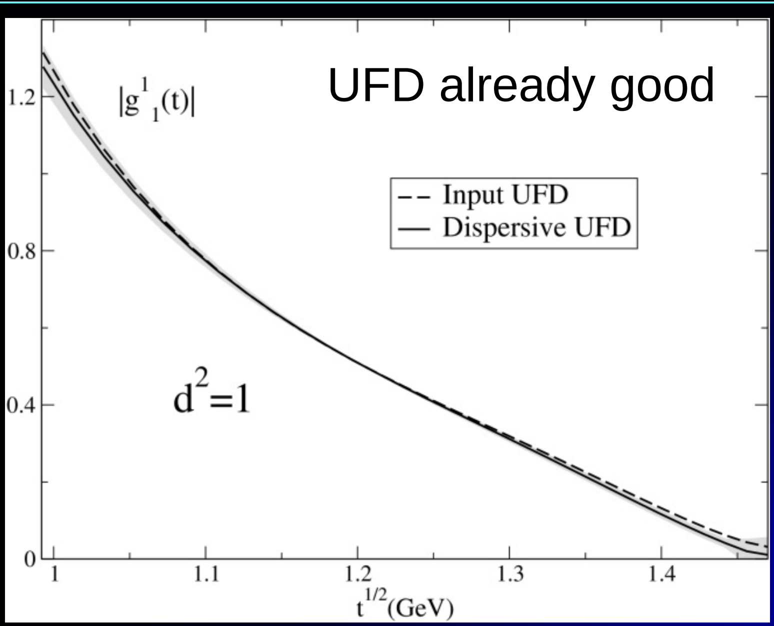
- As πK checks: Small inconsistencies.
- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)
- Analytic methods to extract poles: reduced model dependence on strange resonances
  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

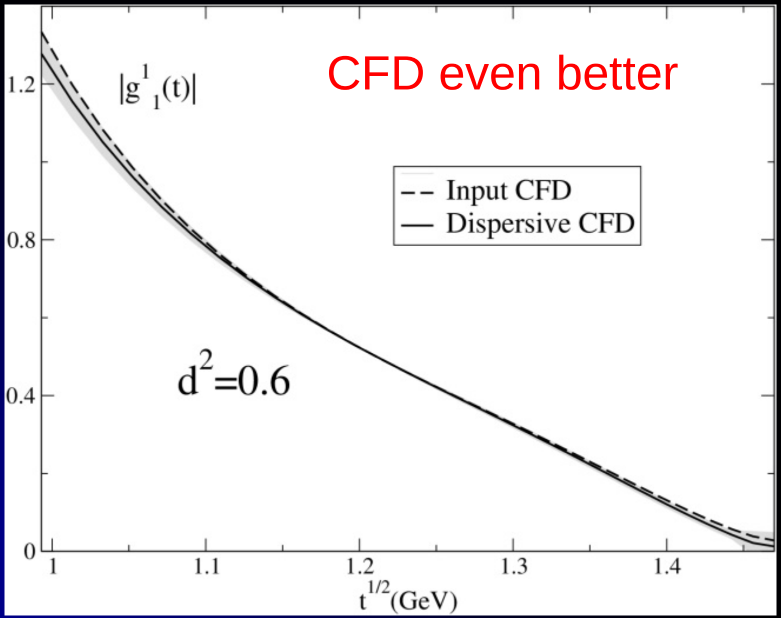
Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits up to 1.5 GeV
  JRP, A.Rodas, Eur.Phys.J. C78 (2018)

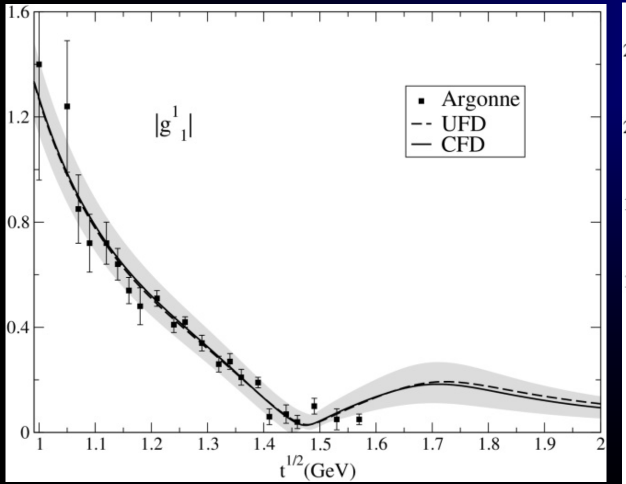
ππ→KK Hiperbolic Dispersion Relations   I=1,J=1, UFD vs.CFD

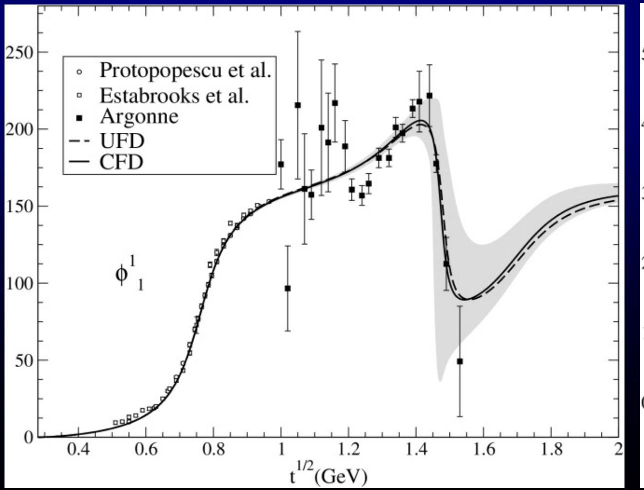
JRP, A.Rodas, Eur.Phys.J. C78 (2018)

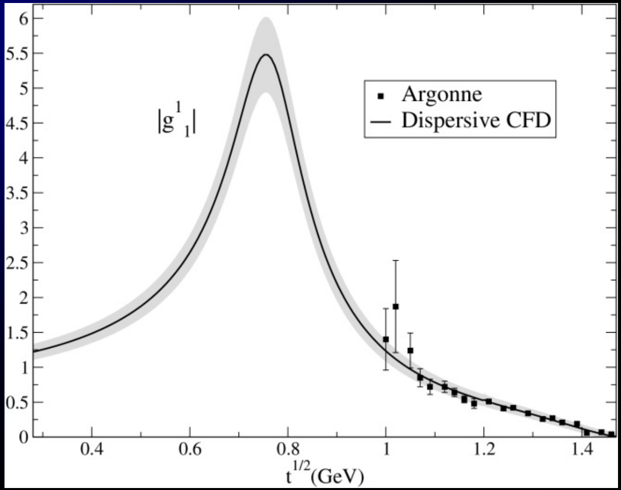




Requires almost imperceptible change from UFD to CFD

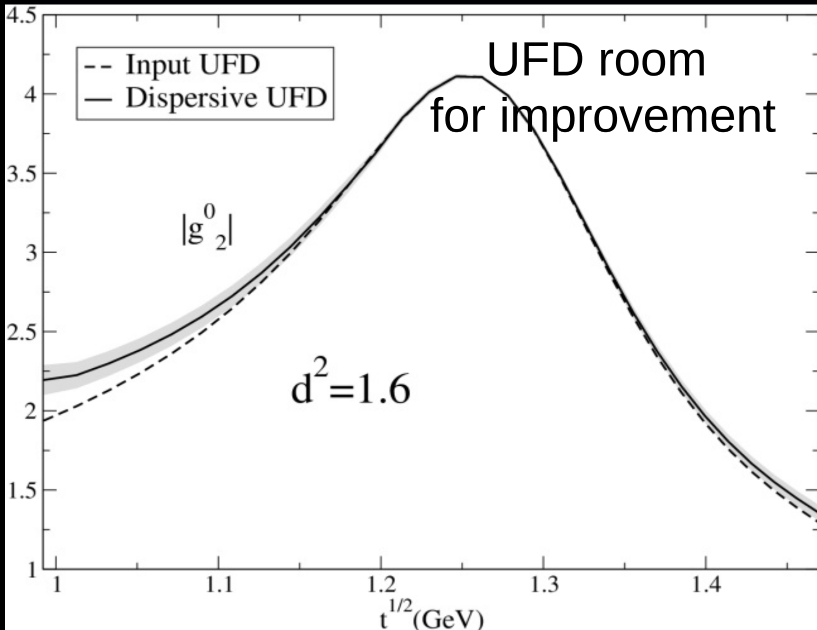


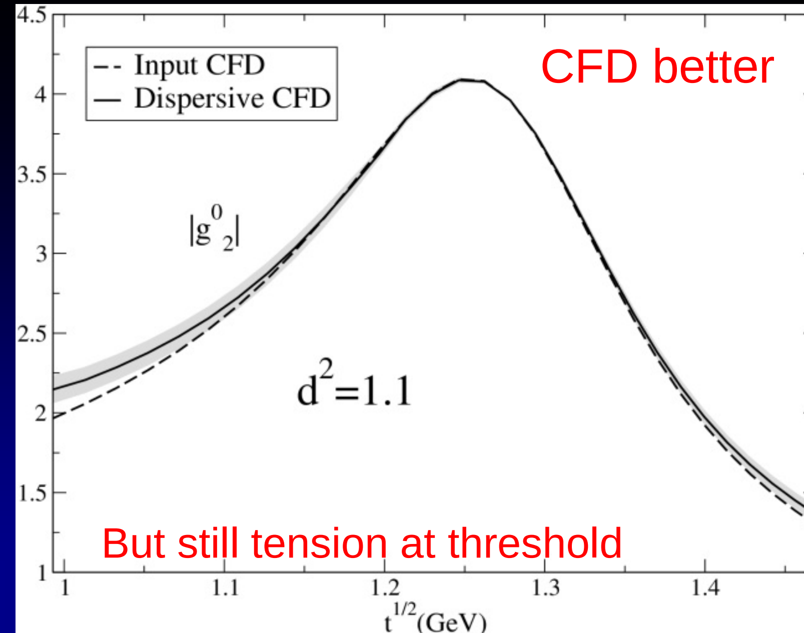




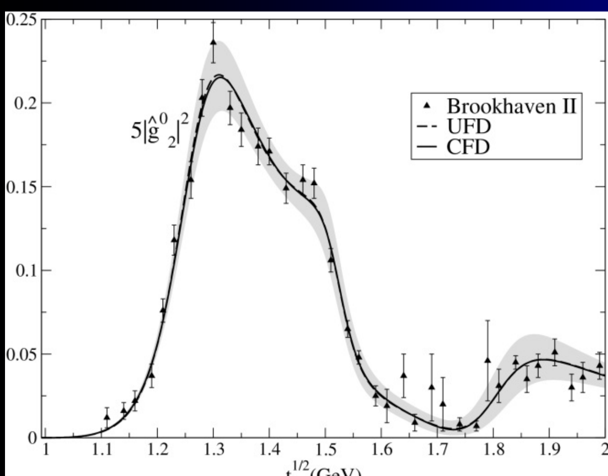
ππ→KK Hiperbolic Dispersion Relations I=2,J=2, UFD vs. CFD

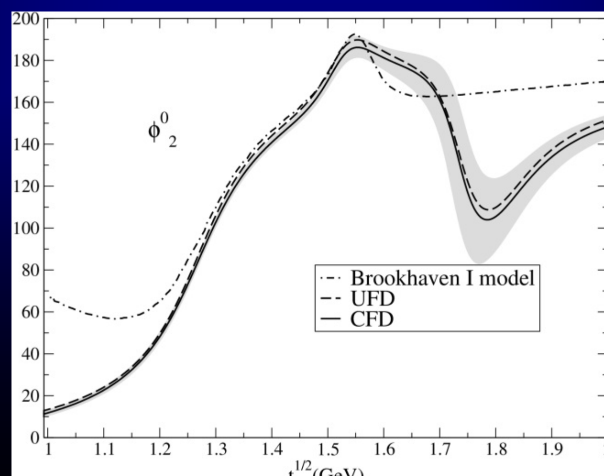
JRP, A.Rodas, Eur.Phys.J. C78 (2018)





Very small change from UFD to CFD. Only significant at threshold and high energies



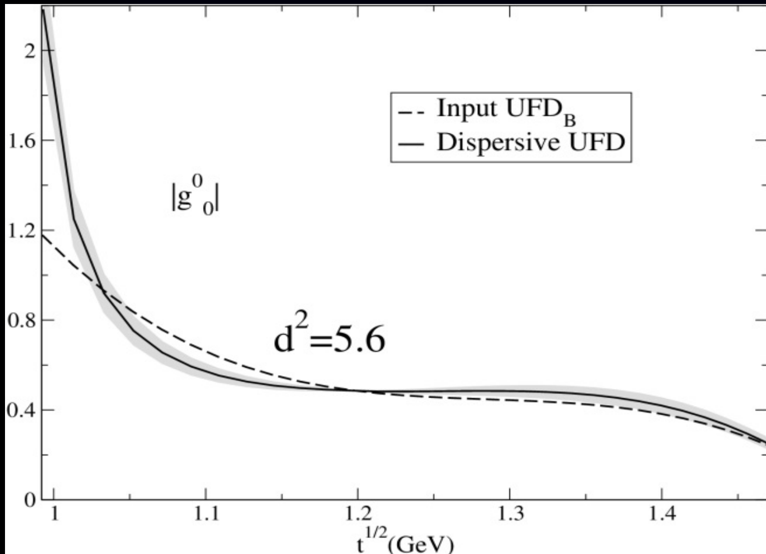


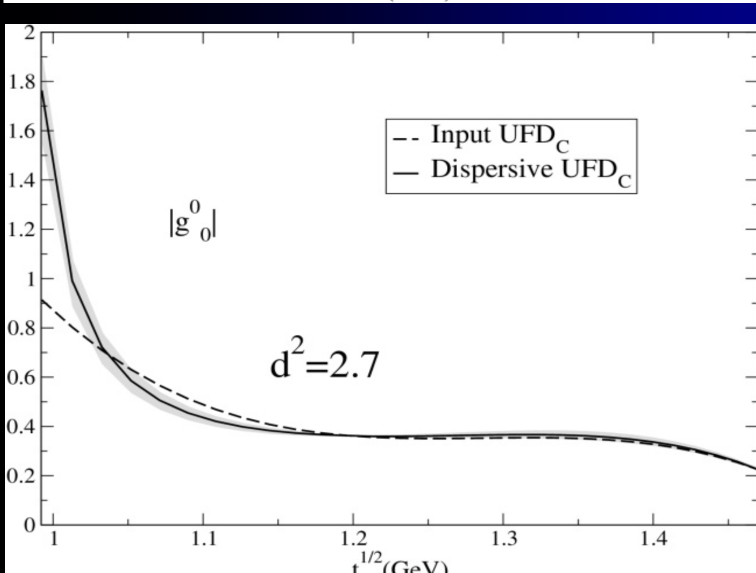
Other parameterizations (BW…), worse.

ππ→KK Hiperbolic Dispersion Relations I=0,J=0, UFD vs. CFD

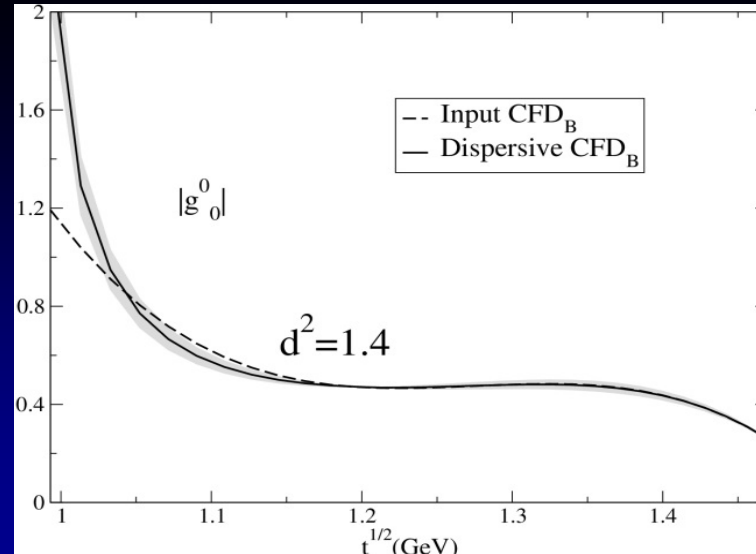
JRP, A.Rodas, Eur.Phys.J. C78 (2018)

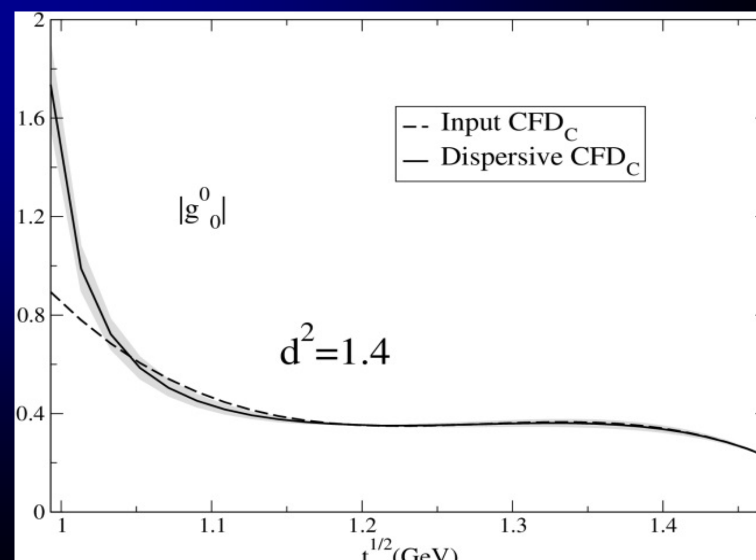
Two possible sets of data





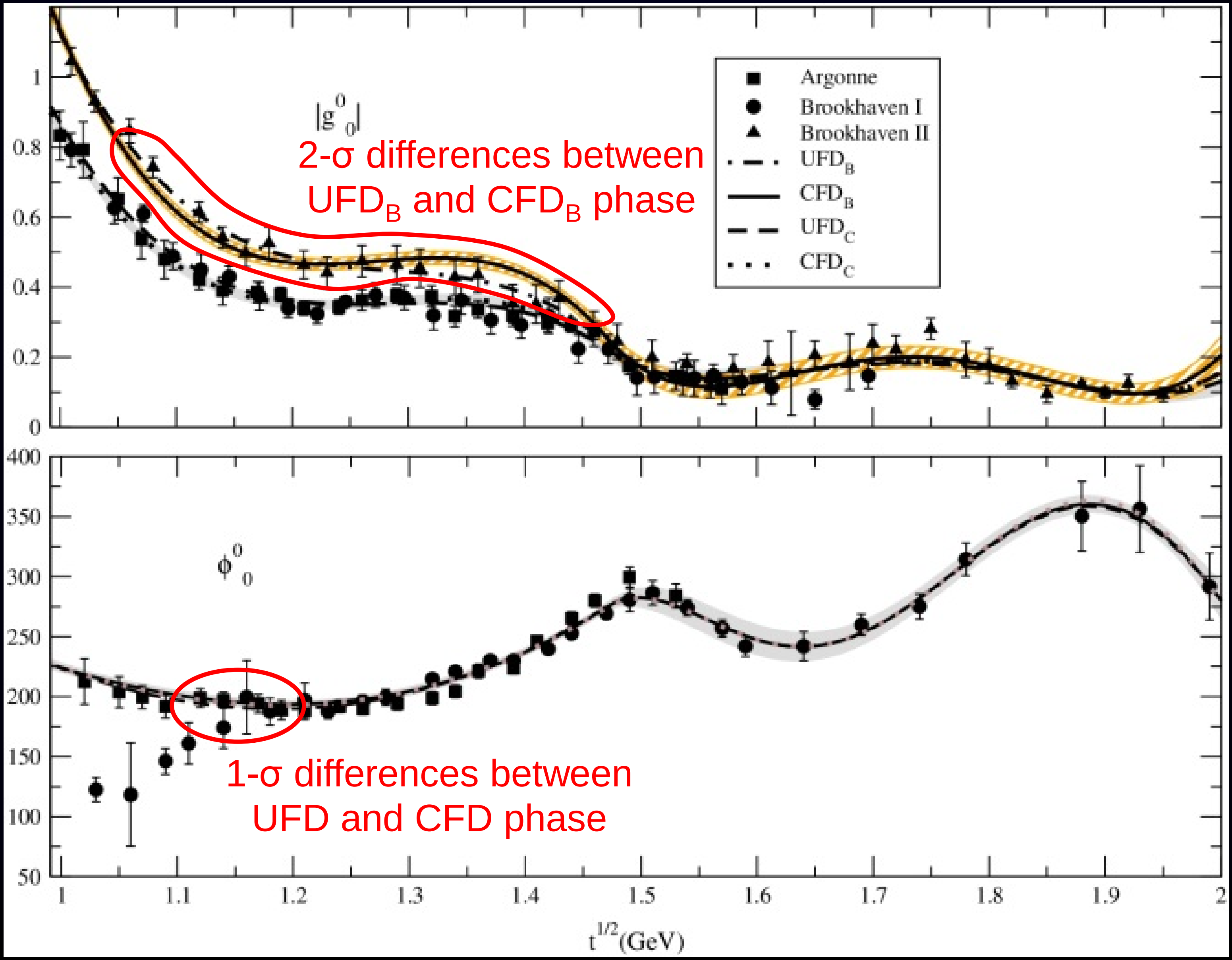
We use I=0,J=2 CFD as input.





Remarkable improvement from UFD to CFD, except at threshold.
Both data sets equally acceptable now.

Some 2-σ level differences between UFD_B and CFD_B between 1.05 and 1.45 GeV
CFD_C consistent within 1-σ band of UFD_C



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- From Hyperbolic DR:
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  JRP, A.Rodas, arXiv:2010.1122. To appear in Physics Reports

- As πK Checks: Large inconsistencies.

$\pi K \to \pi K$ Hyperbolic Dispersion Relations (HDR)

$g^I_J = \pi\pi \to KK$ partial waves. We study (I,J)=(0,0),(1,1),(0,2)
$f^I_J = K\pi \to K\pi$ partial waves. We study (I,J)= (1/2,0),(3/2,0),(1/2,1),(3/2,1)

But now Simultaneously!

I show you here the Eqs. for $K\pi$

$$f_l^+(s) = \frac{m_+ a_0^+}{2} + \frac{1}{\pi}\sum_{\ell\geq 0}\int_{4m_\pi^2}^{\infty} dt' K^0_{l,2\ell}(s,t')\,\mathrm{Im}\, g^0_{2\ell}(t') + \frac{1}{\pi}\sum_{\ell}\int_{m_+^2}^{\infty} ds' K^+_{l,\ell}(s,s')\,\mathrm{Im}\, f^+_\ell(s'),$$

Coupled to $\pi\pi \to KK$

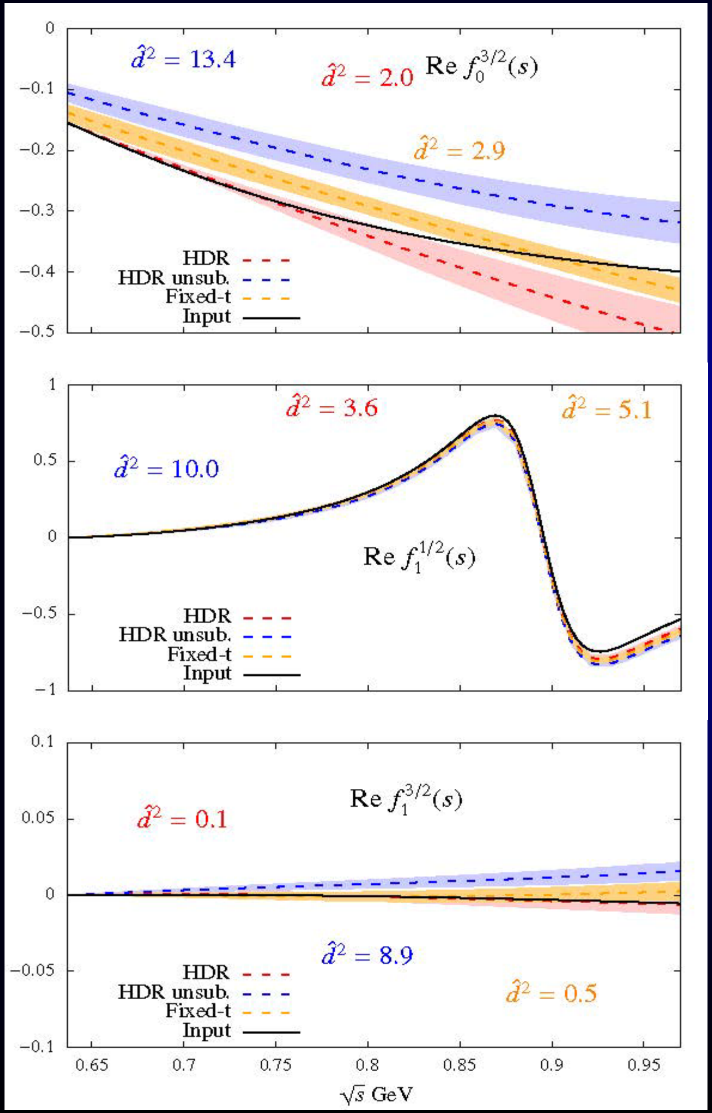
For the antisymmetric ones we study both one- and no-subtractions

$$f_l^-(s) = \frac{1}{\pi}\sum_{\ell}\int_{m_+^2}^{\infty} ds' K^-_{l,\ell}(s,s')\,\mathrm{Im}\, f^-_\ell(s') + \frac{1}{\pi}\sum_{\ell\geq 1}\int_{4m_\pi^2}^{\infty} dt' K^1_{l,2\ell-1}(s,t')\,\mathrm{Im}\, g^1_{2\ell-1}(t'),$$

$$f_l^-(s) = \delta_{l,0}\frac{m_+ a_0^-}{2}\frac{3s^2 - 2s\Sigma - \Delta^2}{8 s m_\pi m_K} + \delta_{l,1}\frac{m_+ a_0^-}{2}\frac{m_\pi^4 + (m_K^2 - s)^2 - 2m_\pi^2(m_K^2 + s)}{24 s m_\pi m_K}$$
$$+ \frac{1}{\pi}\sum_{\ell}\int_{m_+^2}^{\infty} ds' \hat{K}^-_{l,\ell}(s,s')\,\mathrm{Im}\, f^-_\ell(s') + \frac{1}{\pi}\sum_{\ell\geq 1}\int_{4m_\pi^2}^{\infty} dt' \hat{K}^1_{l,2\ell-1}(s,t')\,\mathrm{Im}\, g^1_{2\ell-1}(t'),$$

LARGE inconsistencies IF UNCONSTRAINED

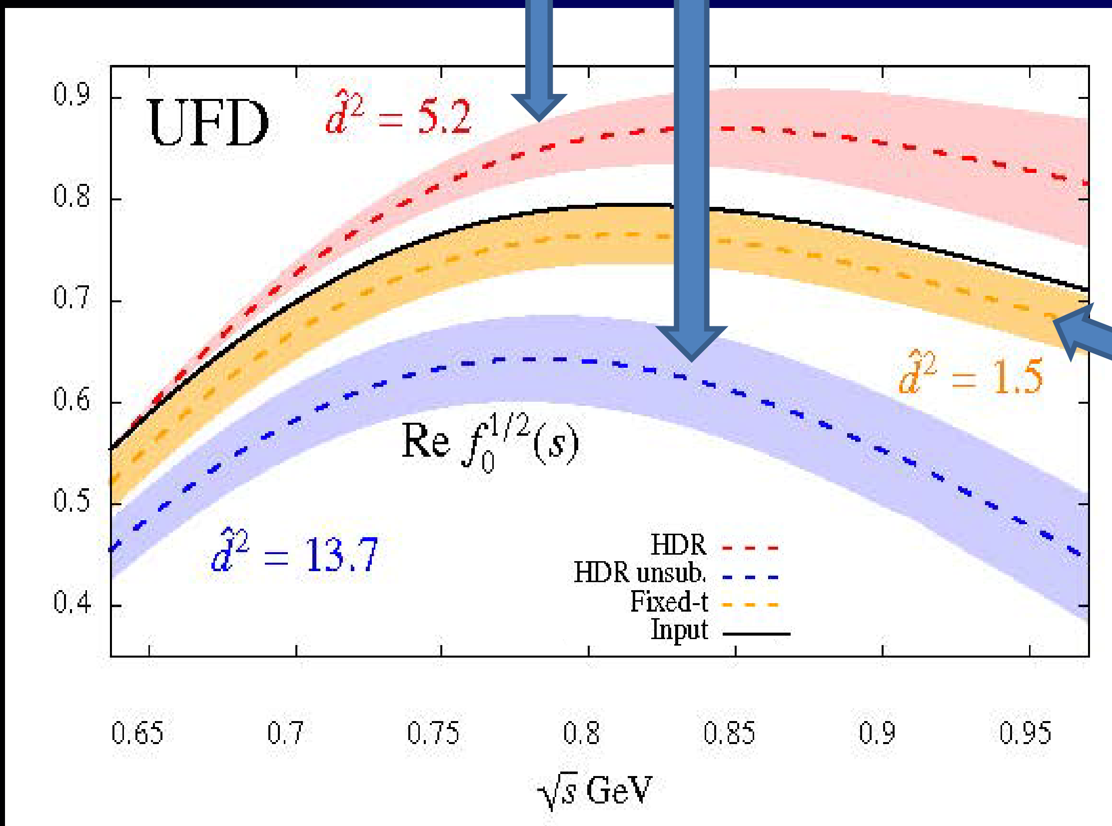
Unconstrained Fit to Data



The most relevant wave for the kappa resonance.

LARGE inconsistencies with HDR Roy-Steiner from unconstrained fits (UFD)

One or no subtraction for F^- lie on opposite sides of input



Fixed-t Roy-Steiner is fair but kappa pole outside their applicability region

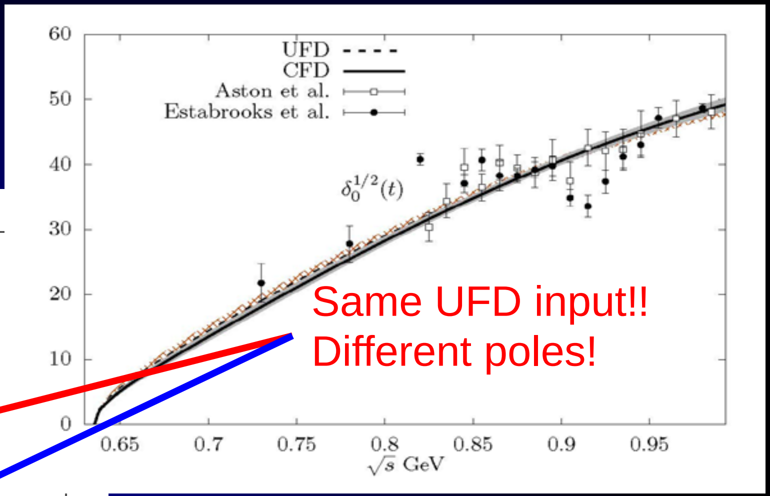
We have chosen the hyperbolae family so that the kappa pole and its uncertainties lie within their applicability region

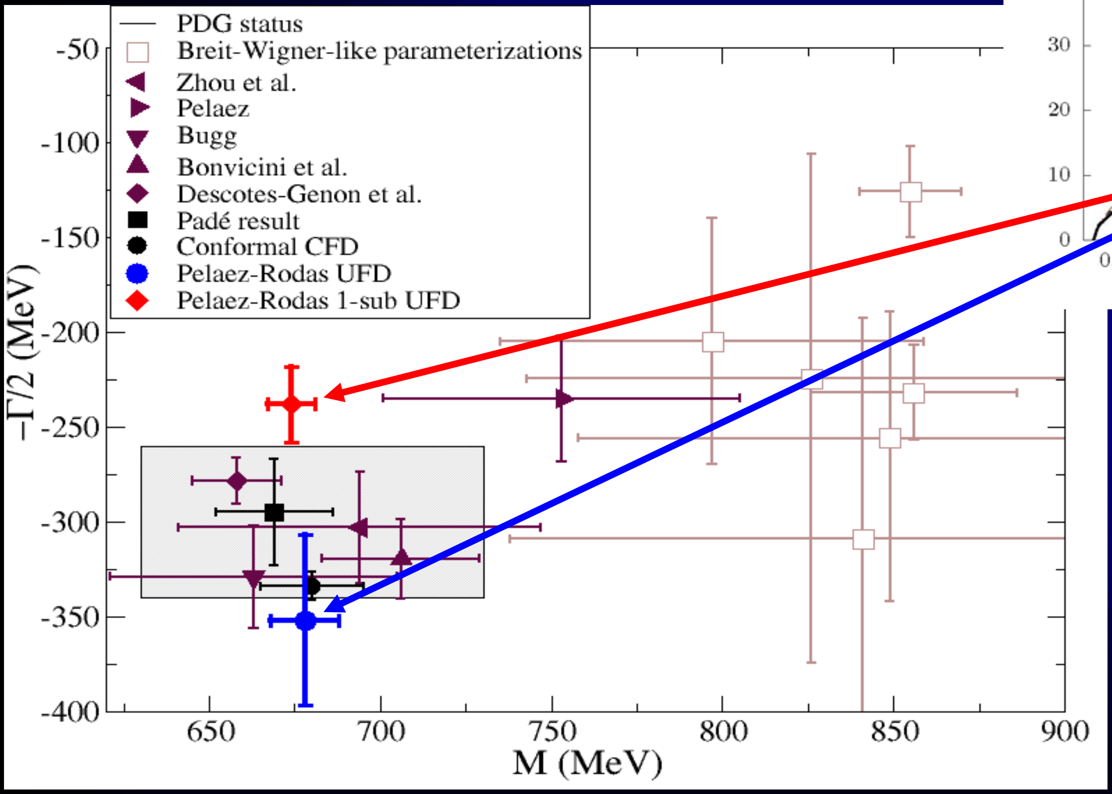
WARNING ABOUT THE PRECISION OF UNCONSTRAINED FITS

Before imposing Roy Eqs. incompatible results with different # of subtractions !!

This is part ly due to left/circular cuts.

(Crossed Channel)





Nice-looking fits are NOT enough to get an stable and precise continuation to the complex plane

You can imagine what precision you get if you use simple models only of πK, without left cut or without dispersion relations…

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- Analytic methods to extract poles: reduced model dependence on strange resonances
  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

- As ππ→KK checks: Small inconsistencies.
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- From Hyperbolic DR:
  ππ→KK influence important.

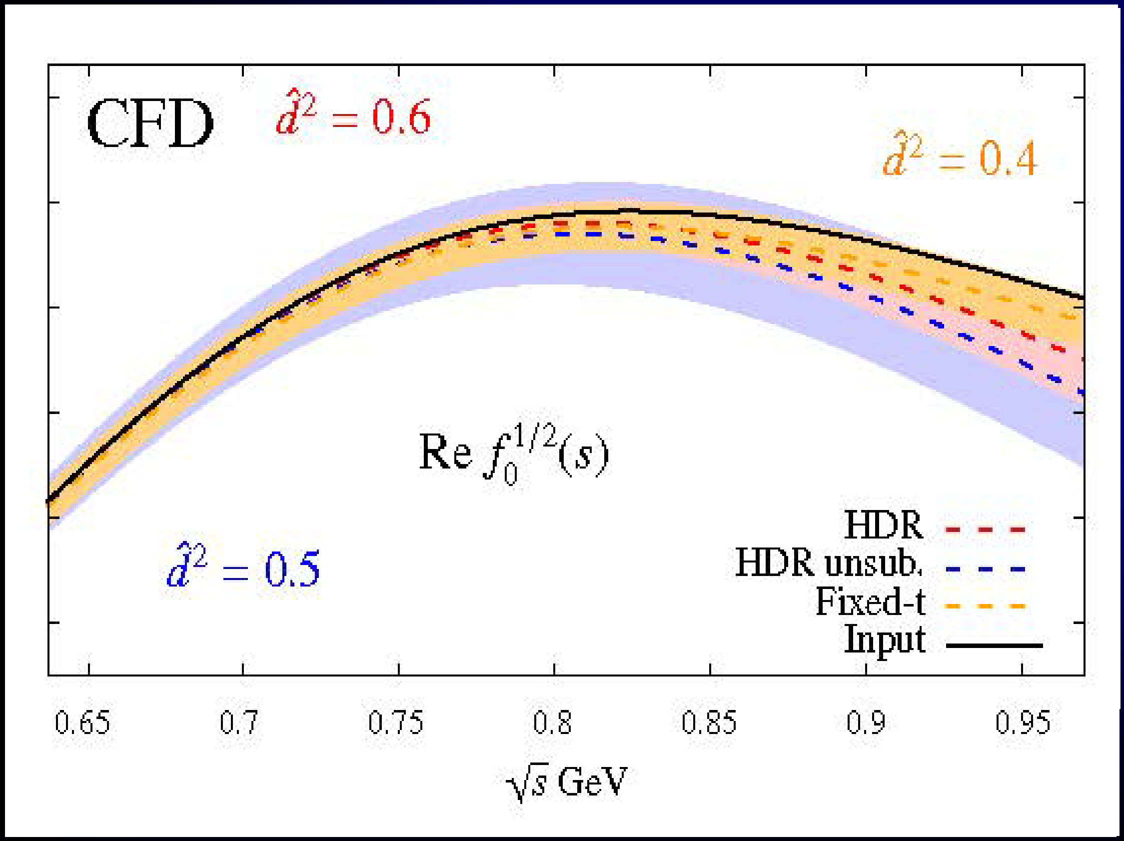
JRP, A.Rodas,
arXiv:2010.1122.
To appear in Physics
Reports

- As πK Checks: Large inconsistencies.
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV

πK Hiperbolic Dispersion Relations I=1/2, J=0

JRP, A.Rodas, arXiv:2010.1122. To appear in Physics Reports

We provide a constrained fit to data (CFD) satisfying 16 Dispersion relations
(FDRs, fixed-t, HDR, different # subtractions)
Fairly simple and ready to use parameterizations

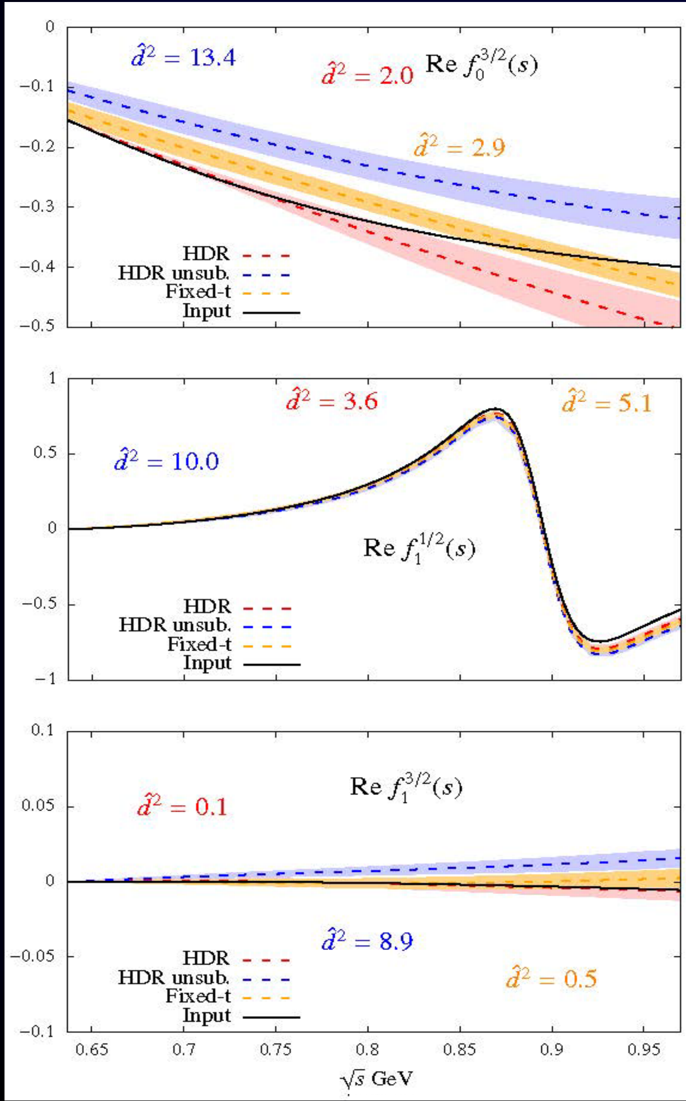


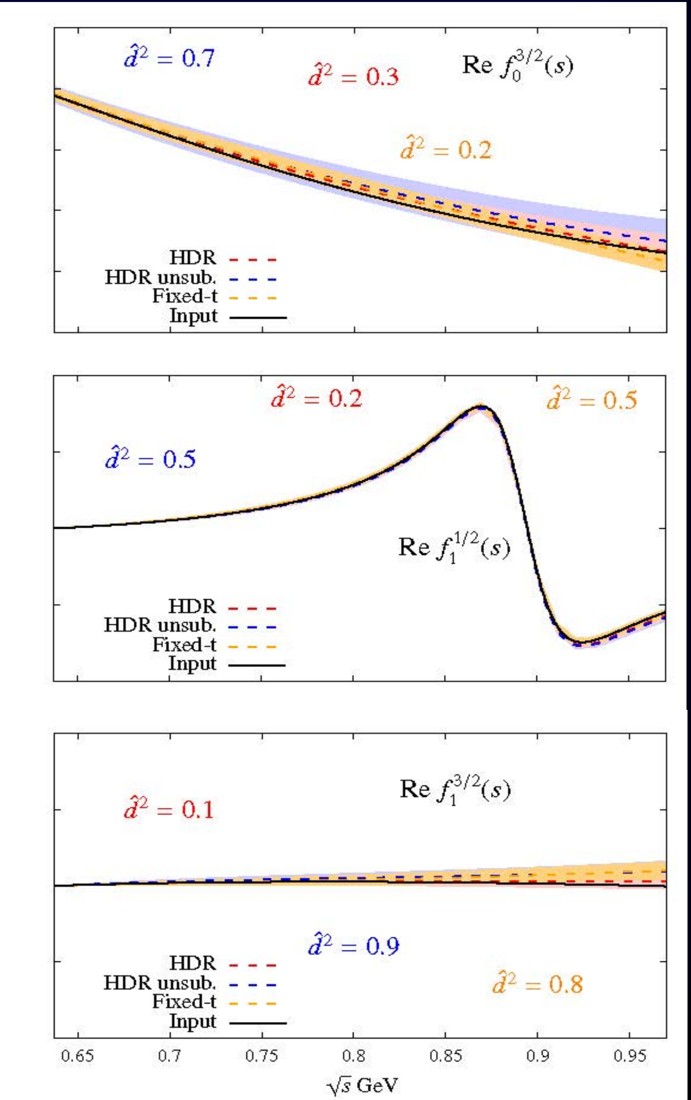
Our Constrained parameterization now yields consistent output for all Dispersion Relations

LARGE inconsistencies FOR THE OTHER WAVES IF UNCONSTRAINED

Made consistent within uncertainties for the CFD

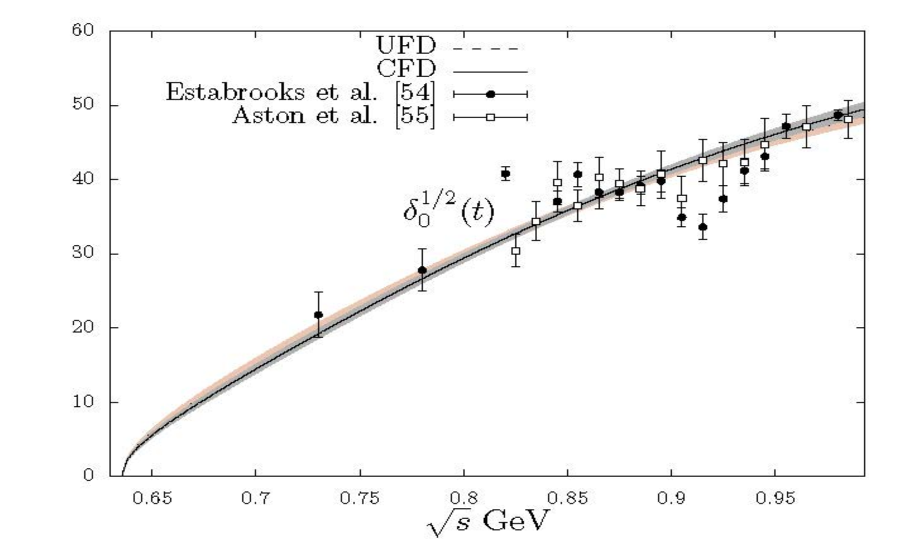
Unconstrained Fit to Data | Constrained Fit to Data

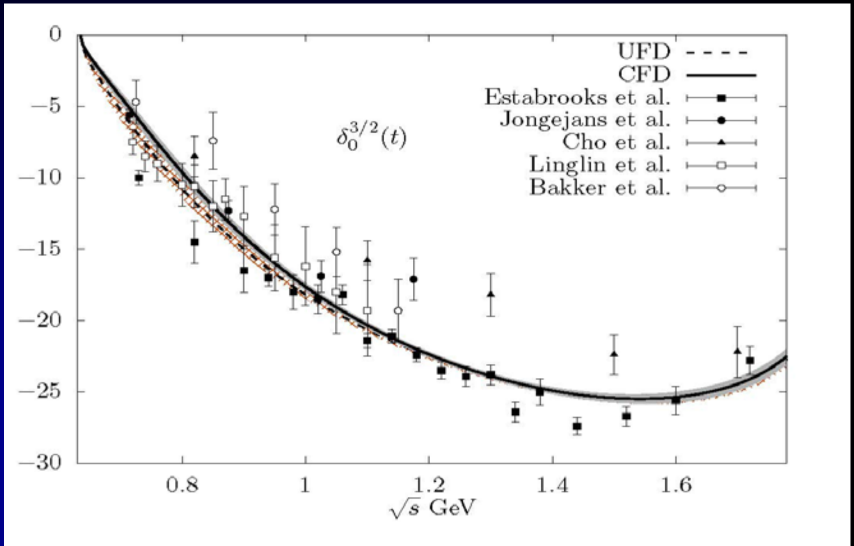


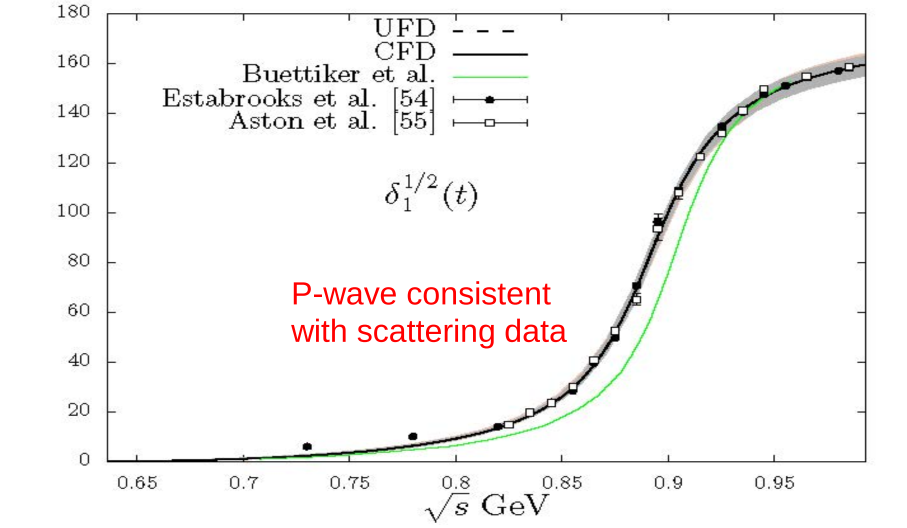


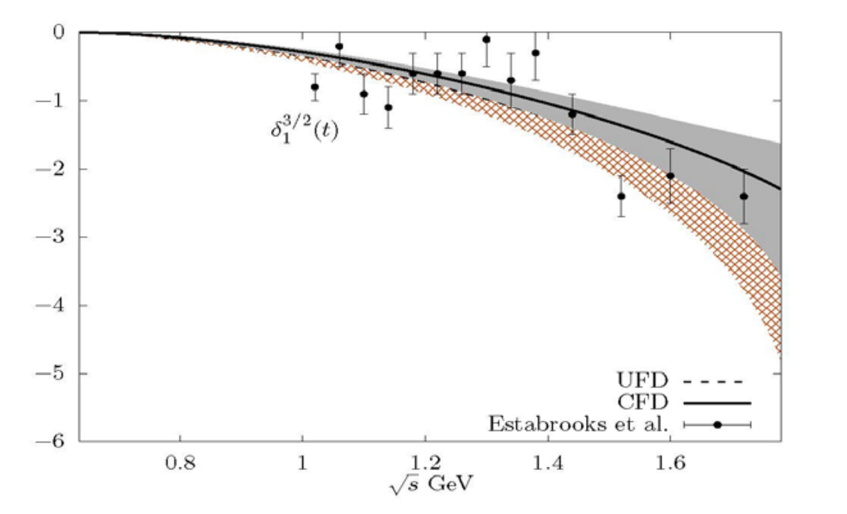
πK CFD vs. UFD

Constrained parameterizations suffer minor changes but still describe πK data fairly well. Here we compare the unconstrained fits (UFD) versus the constrained ones (CFD)









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- Padé sequences to extract poles from local information: reduced model dependence on strange resonances JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

JRP, A.Rodas, Phys.Rev. D93 (2016)

Partial-wave πK Dispersion Relations (PWDR)
Need ππ→KK to rewrite left cut. Range optimized.

- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits
  from KK threshold to 1.5 GeV

JRP, A.Rodas, Eur.Phys.J. C78 (2018)

- From fixed-t DR:
  ππ→KK influence small.
  $\kappa/K_0^*(700)$ pole out of reach
- From Hyperbolic DR:
  ππ→KK influence important.
  As πK Checks:
  Large inconsistencies

JRP, A.Rodas, arXiv:2010.1122. To appear in Physics Reports

- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV for PWDR, up to 1.6 for FDRs,
  ππ→KK up to 1.5 GeV and unphysical region
- **Precise** πK **threshold parameters**

- Threshold parameters relevant to test ChPT (NNLO at present).
- Present tension between lattice and dispersive results

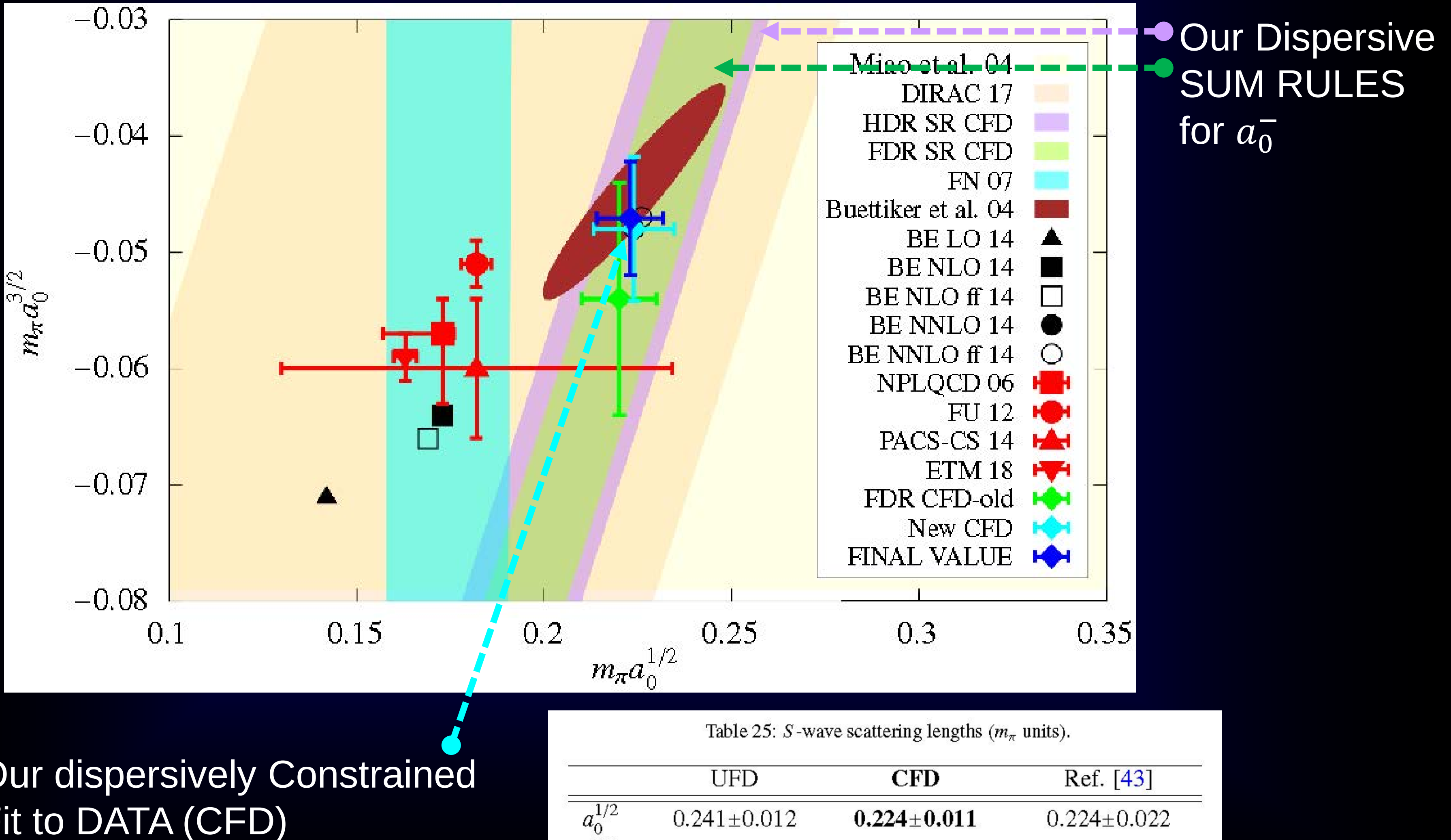


Table 25: S-wave scattering lengths (m_π units).

	UFD	**CFD**	Ref. [43]
$a_0^{1/2}$	0.241±0.012	**0.224±0.011**	0.224±0.022
$a_0^{3/2}$	-0.067±0.012	**-0.048±0.006**	-0.0448±0.0077

JRP, A.Rodas, arXiv:2010.1122. To appear in Physics Reports

πK scattering lengths. All waves

- We provide sum rule values for scattering lengths and slopes up to D-waves.
- Good consistency with CFD for S,P waves (constrained) and D-wave lengths

	This work sum rules with CFD input				This work direct	Sum rules [43]	NNLO ChPT
	Fixed-t	HDR	HDR$_{sub}$	**Final Value**	CFD	Fixed-t	[85] and [86]*
$m_\pi a_0^{1/2}$	0.224±0.009	0.221± 0.012	like CFD	**0.223±0.009**	0.224±0.011	0.224±0.022	0.224*
$m_\pi^3 b_0^{1/2} \times 10$	1.04± 0.04	1.05±0.07	1.15± 0.04	**1.08±0.08**	0.95±0.04	0.85±0.04	1.278
$m_\pi a_0^{3/2} \times 10$	-0.478± 0.052	-0.460±0.064	like CFD	**-0.471±0.049**	-0.48±0.06	-0.448±0.077	-0.471*
$m_\pi^3 b_0^{3/2} \times 10$	-0.42±0.02	-0.41±0.03	-0.44±0.02	**-0.43±0.03**	-0.36±0.04	-0.37±0.03	-0.326
$m_\pi^3 a_1^{1/2} \times 10$	0.228±0.010	0.218±0.008	0.222±0.006	**0.222±0.009**	0.20±0.04	0.19±0.01	0.152
$m_\pi^5 b_1^{1/2} \times 10^2$	0.58±0.03	0.59±0.03	0.60±0.03	**0.59±0.02**	0.5±0.2	0.18±0.02	0.032
$m_\pi^3 a_1^{3/2} \times 10^2$	0.15±0.05	0.19±0.05	0.17±0.04	**0.17±0.05**	0.15±0.11	0.065±0.044	0.293
$m_\pi^5 b_1^{3/2} \times 10^3$	-0.94±0.09	-0.97±0.08	-1.03±0.07	**-0.99±0.09**	-1.04±0.8	-0.92±0.17	0.544
$m_\pi^5 a_2^{1/2} \times 10^3$	0.60±0.13	0.54±0.03	0.55±0.02	**0.55±0.05**	0.53±0.05	0.47±0.03	0.142
$m_\pi^7 b_2^{1/2} \times 10^4$	-0.89±0.10	-0.96±0.09	-0.95±0.09	**-0.94±0.09**	0.20±0.02	-1.4±0.3	-1.98
$m_\pi^5 a_2^{3/2} \times 10^4$	-0.05±0.60	-0.11±0.16	-0.18±0.15	**-0.14±0.17**	-0.09±0.03	-0.11±0.27	-0.45
$m_\pi^7 b_2^{3/2} \times 10^4$	-1.12±0.10	-1.13±0.09	-1.14±0.09	**-1.13±0.06**	-0.03±0.01	-0.96±0.26	0.61

JRP, A.Rodas, arXiv:2010.1122. To appear in Physics Reports

Our Dispersive/Analytic Approach for πK and strange resonances

Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

- As πK checks: Small inconsistencies.
- As constraints:
  πK consistent fits up to 1.6 GeV

JRP, A.Rodas, Phys.Rev. D93 (2016)

- Padé sequences to extract poles from local information: reduced model dependence on strange resonances JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Partial-wave πK Dispersion Relations (PWDR)
Need ππ→KK to rewrite left cut. Range optimized.

- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits
  from KK threshold to 1.5 GeV

JRP, A.Rodas, Eur.Phys.J. C78 (2018)

- From fixed-t DR:
  ππ→KK influence small.
  $\kappa/K_0^*(700)$ pole out of reach
- From Hyperbolic DR:
  ππ→KK influence important.
  As πK Checks:
  Large inconsistencies

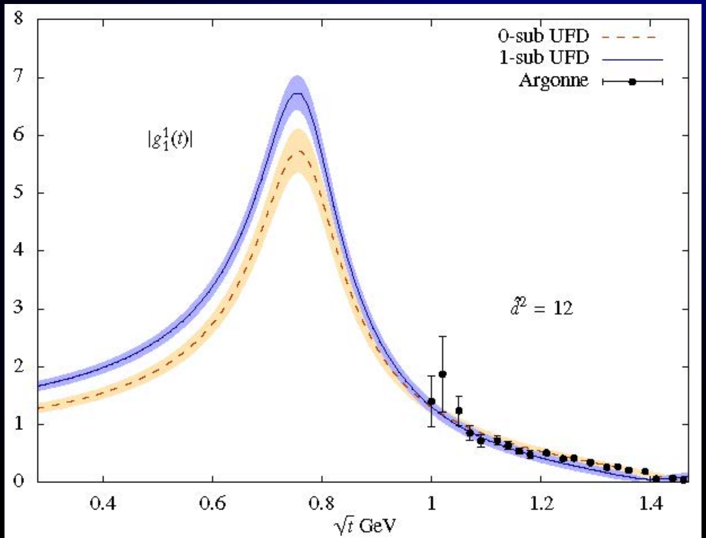
JRP, A.Rodas, arXiv:2010.1122. To appear in Physics Reports

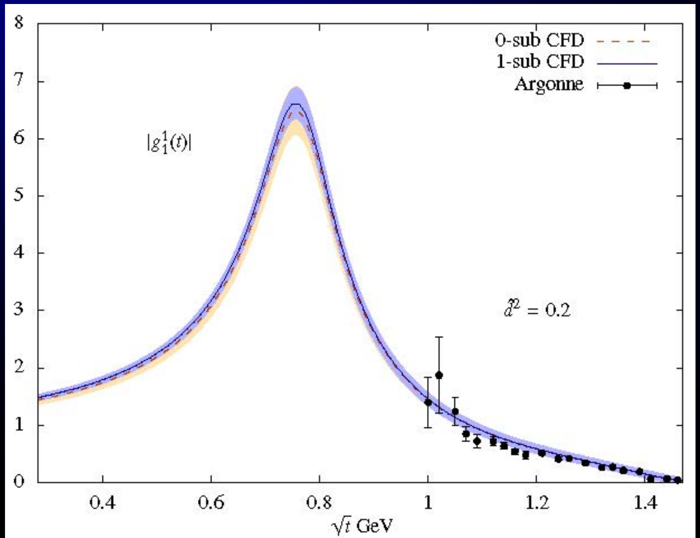
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV for PWDR, up to 1.6 for FDRs,
  ππ→KK up to 1.5 GeV and unphysical region
- **Precise** πK **threshold parameters**
- **Rigorous $\kappa/K_0^*(700)$ pole** JRP, A.Rodas,. PRL. 124 (2020) 17, 172001

Dispersive πK analysis from constrained fit to data JRP, A.Rodas, arXiv:2010.1122. To appear in Physics Reports

Now we have:

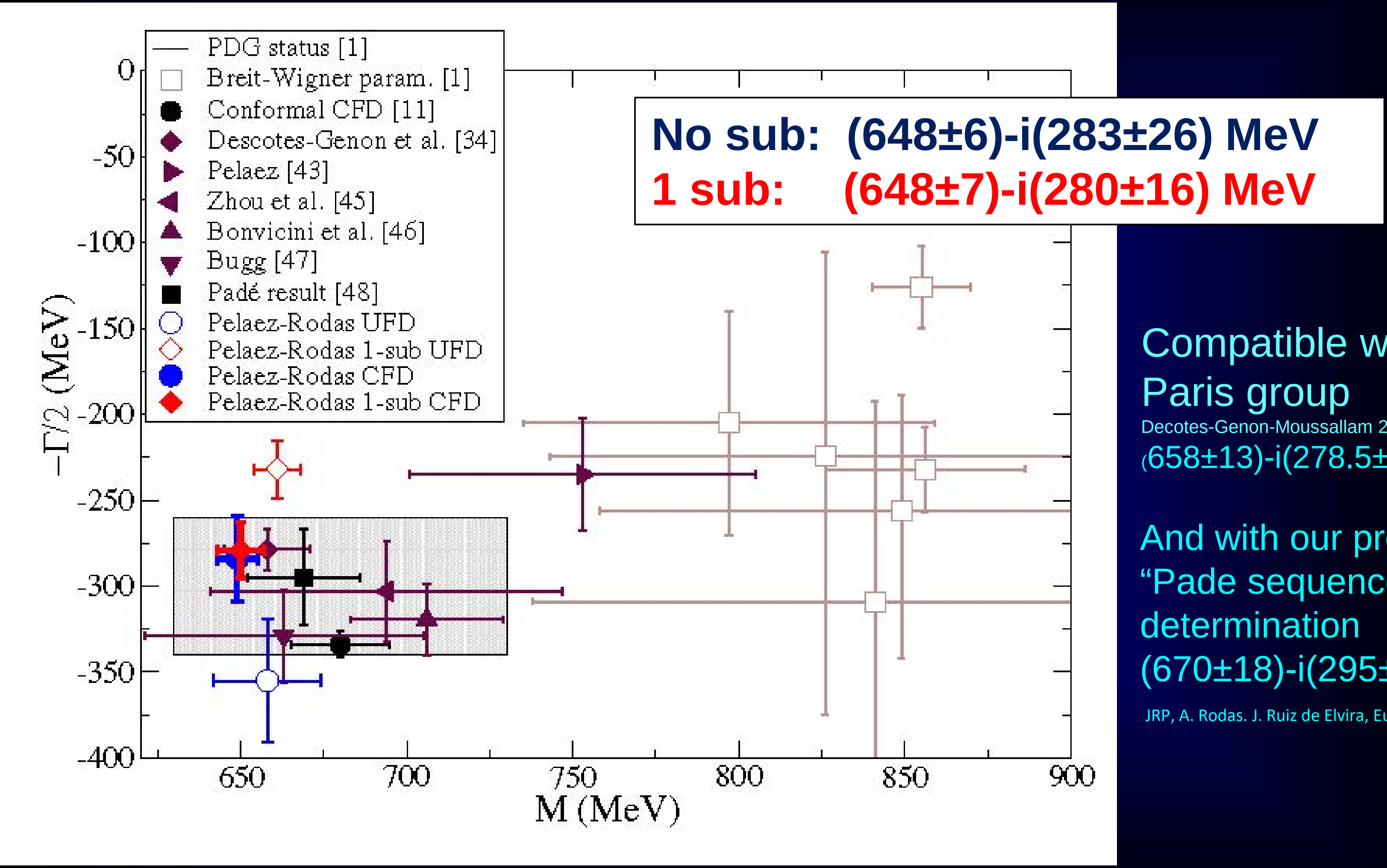
- **FIT TO DATA** (not solution but fit) **CONSTRAINED WITH 16 DR**
- Improved $P^{1/2}$-wave (consistent with data) and $P^{3/2}$
- Improved Pomeron
- Realistic ππ→KK uncertainties (none before)
- Constrained ππ→KK input with DR
- FDR up to 1.6 GeV
- Fixed-t Roy-Steiner Eqs.
- Hyperbolic Roy Steiner Eqs.
  - Both one and no-subtractions for F^- HDR (only the subtracted one before)
  - both in real axis (not HDR before) and complex plane
  - Unphysical P-wave ππ→KK region VERY RELEVANT





Dispersive pole analysis from constrained fit to data **JRP, A. Rodas,** arXiv:2001.08153

When using the constrained fit to data both poles come out nicely compatible



Compatible with Paris group

Decotes-Genon-Moussallam 2006

(658±13)-i(278.5±12) MeV

And with our previous "Pade sequence" determination

(670±18)-i(295±28) MeV

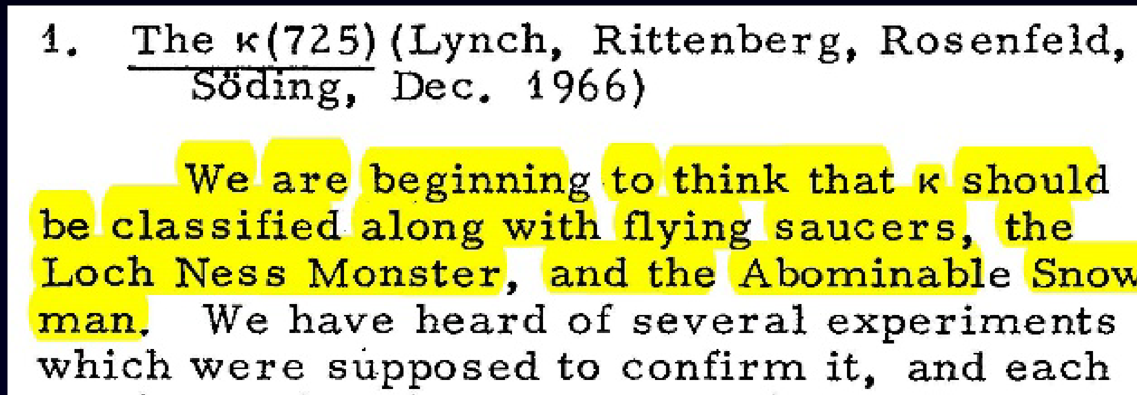
JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

We also provide pole positions for the $K_1*(892)$,

Summary

- πK and ππ →KK data do not satisfy well basic dispersive constraints
- Using dispersion relations as constraints we provide **simple and ready to use** consistent data parameterizations.
- We have implemented partial-wave dispersion relations whose applicability range reaches the kappa pole.
- We have also derived and used SUM RULES to obtain precise threshold parameters
- We confirm previous studies and provide a precise determination of the $\kappa/K_0^*(700)$ parameters **FROM DATA. A good control on the left/circular cuts is needed to claim this precision.**
- This resonance will be considered "well-established" in next RPP, completing the nonet of lightest scalars.

EPILOGUE:
Long way since 1966 TO DO LIST



1. The κ(725) (Lynch, Rittenberg, Rosenfeld, Söding, Dec. 1966)

We are beginning to think that κ should be classified along with flying saucers, the Loch Ness Monster, and the Abominable Snow-man. We have heard of several experiments which were supposed to confirm it, and each

Confirm the $\kappa/K_0^*(700)$ **At last @PDG 2021* !!**

* C. Hanhart, private communication

OUTLOOK

Confirm flying saucers

Confirm Nessie

Abominable Snowman

HOLD OUR BEER!!

Work in progress.... stay tuned!

Thank you!