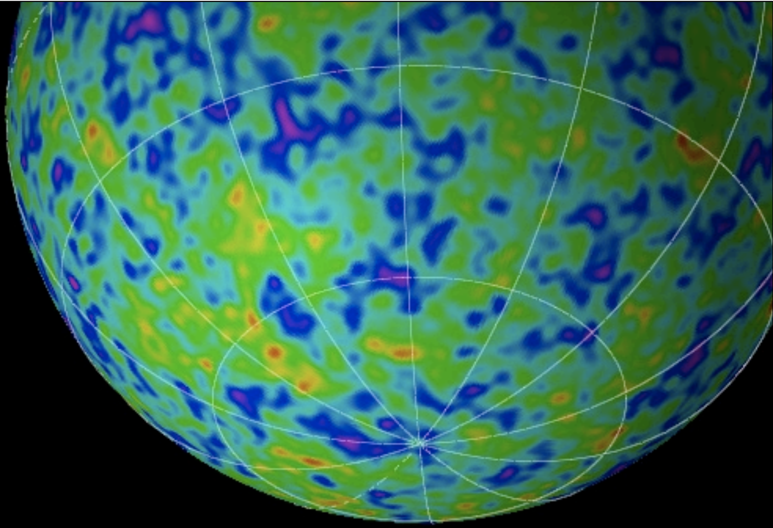
Neutrino Physics and Cosmology

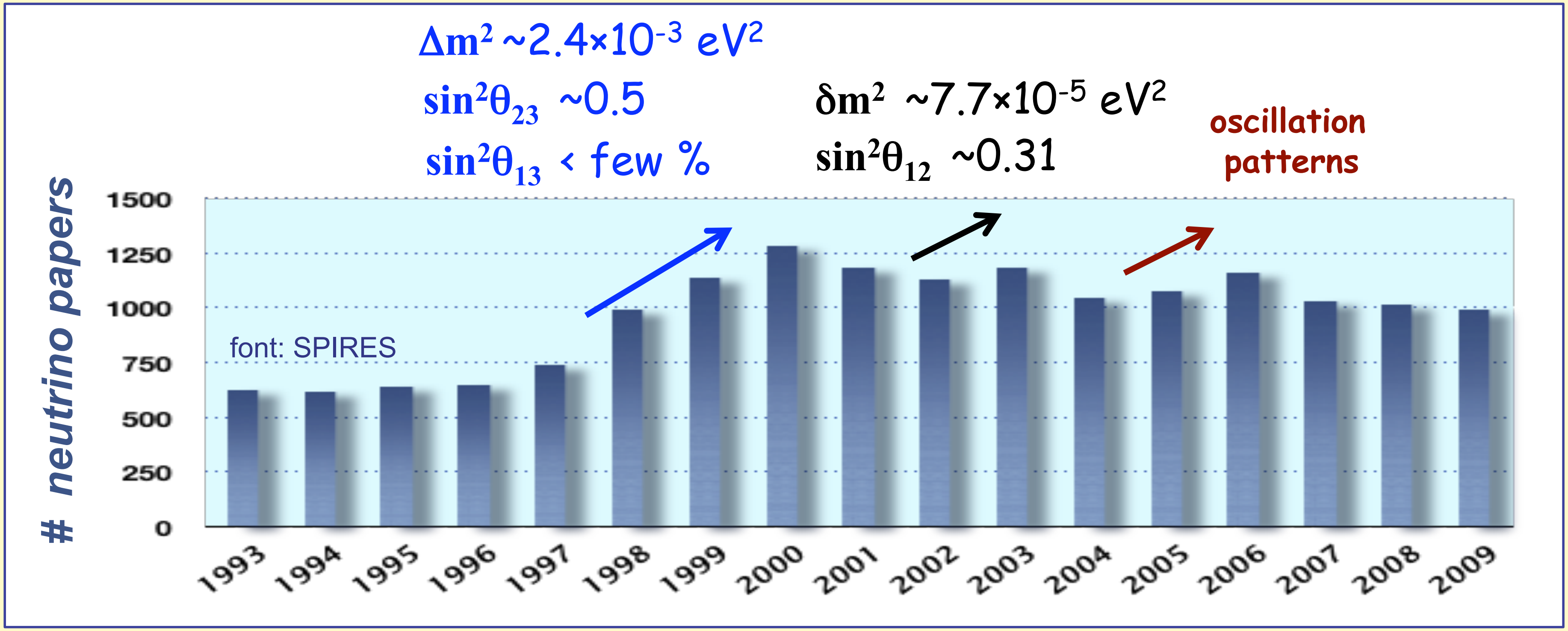


Eligio Lisi, INFN, Bari

UniverseNet School, Lecce, Sept. 2010

Bottom line of this lecture:

Peaks of interest in ν physics driven, so far, by oscillation discoveries



Absolute neutrino mass from cosmology might be a future peak!

Outline:

Introduction

Neutrino mass, mixing and oscillations*

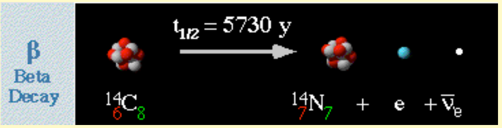
Absolute neutrino masses & Cosmology

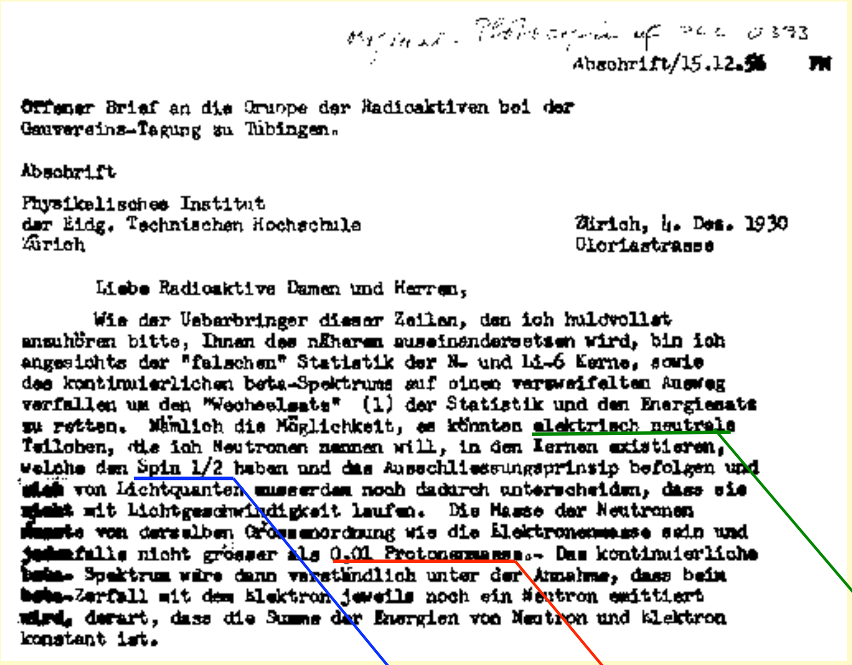
Conclusions

[*With simple exercises + solutions]

2010: the 80th Neutrino Birthday!

The neutrino was invented in 1930 by Wolfgang Pauli as a "desperate remedy" to explain the continuous β-ray spectrum via a 3-body decay, e.g.,





Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Kinematics: spin 1/2, tiny mass, zero electric harge

The name "neutrino" (="little neutral one", in Italian) was actually invented by Enrico Fermi, who first proposed in 1933-34 a theory for its **dynamics** (weak interactions)


ANNO IV · VOL. II · N. 12 QUINDICINALE 31 DICEMBRE 1933 · XII

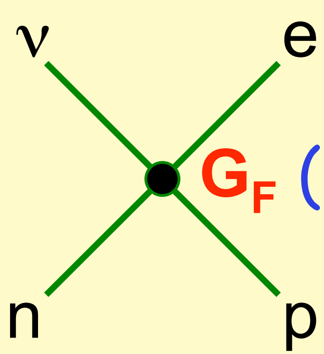
LA RICERCA SCIENTIFICA

ED IL PROGRESSO TECNICO NELL'ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi "beta"

Nota del prof. ENRICO FERMI

Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

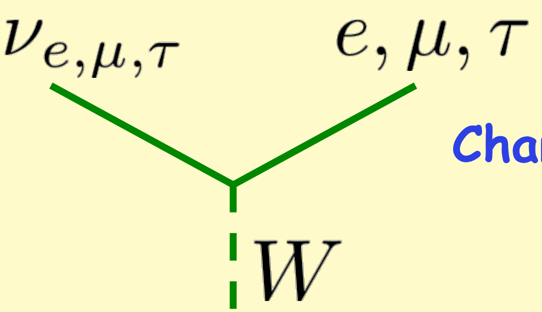


G_F (Fermi constant)

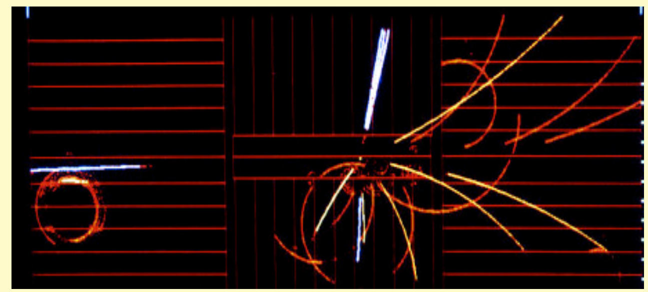
Many decades of research have revealed other properties of the neutrino. For instance, there are 3 different neutrino "flavors"

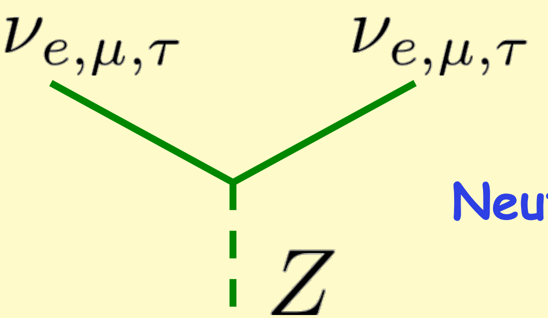
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \quad \begin{array}{l} \leftarrow \quad q = 0 \\ \leftarrow \quad q = -1 \end{array} \quad (\Delta q = 1)$$

and their Fermi interactions are mediated by a charged vector boson W, with a neutral counterpart, the Z boson

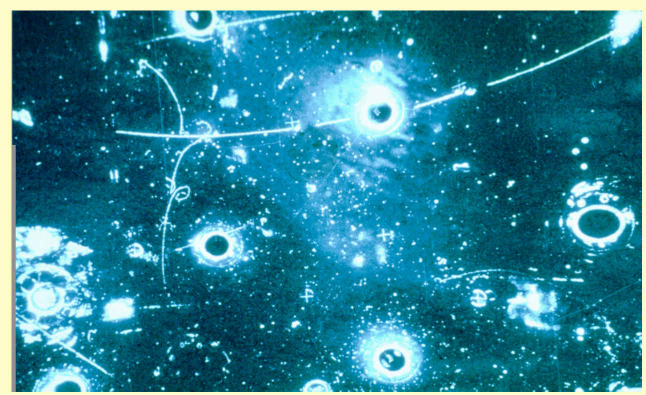


Charged Current (Δq=1)



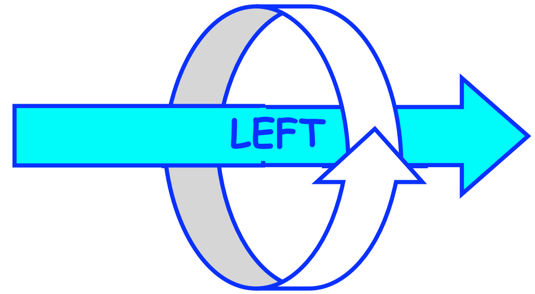


Neutral Current (Δq=0)

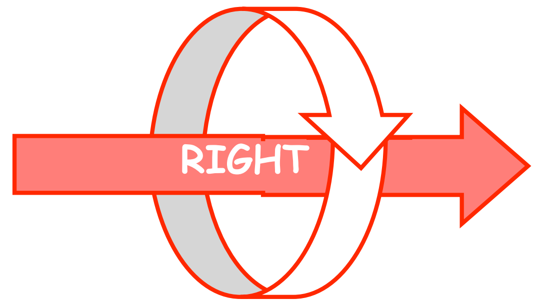


Such interactions are chiral (= not mirror-symmetric):

Neutrinos are created in a left-handed (LH) state ν



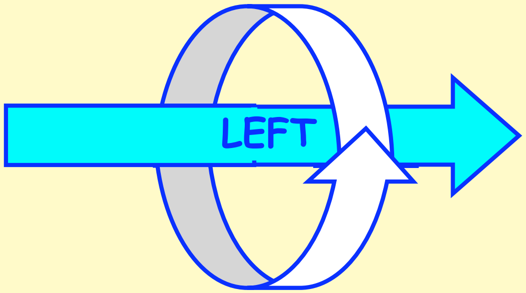
Anti-nus are created in a right-handed (RH) state $\bar{\nu}$



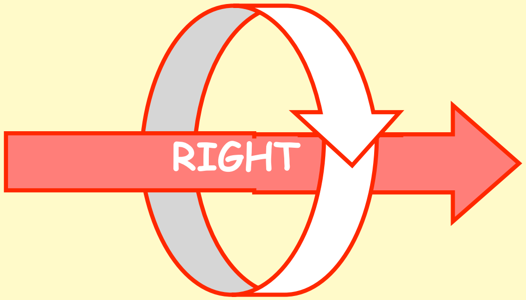
Neutrinos couldn't see themselves in a mirror - like vampires...

For massless neutrinos: handedness is a constant of motion

ν:

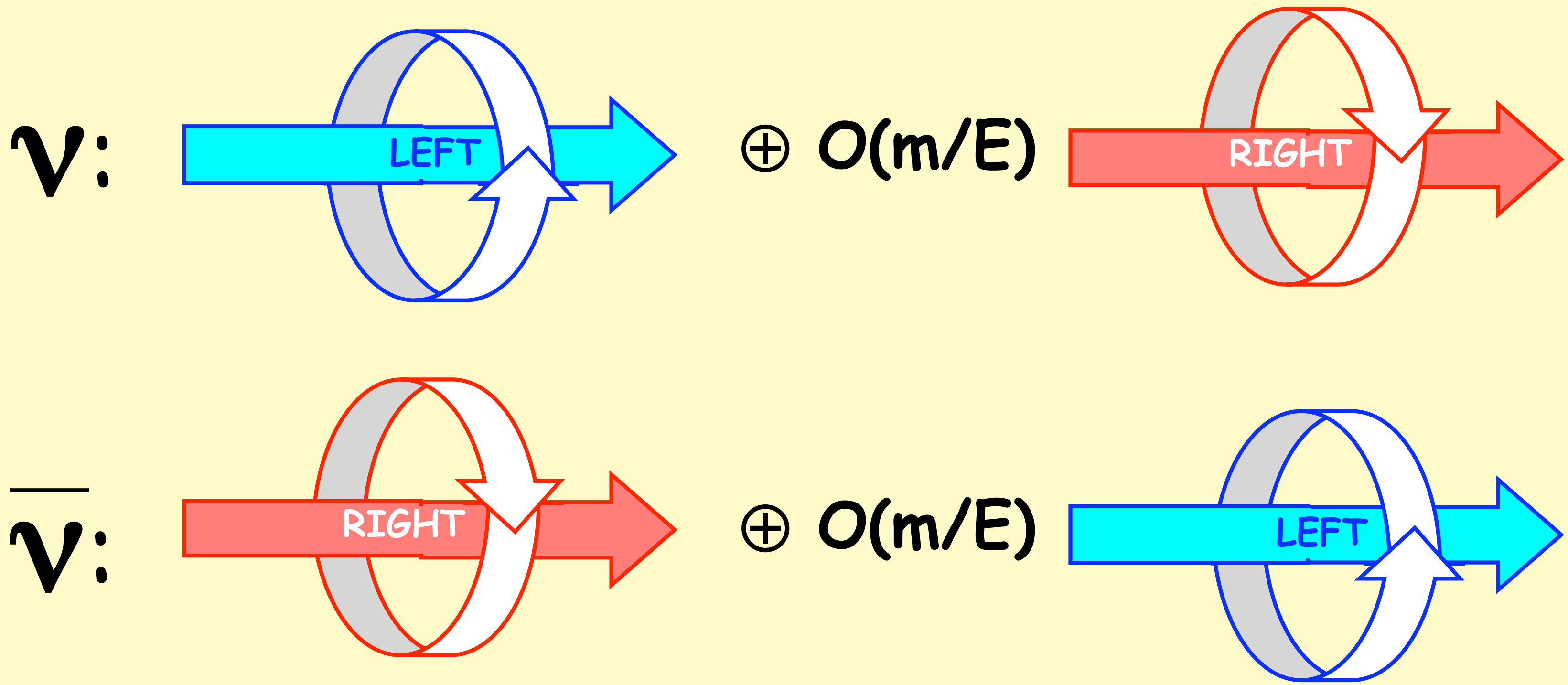


$\bar{\nu}$:



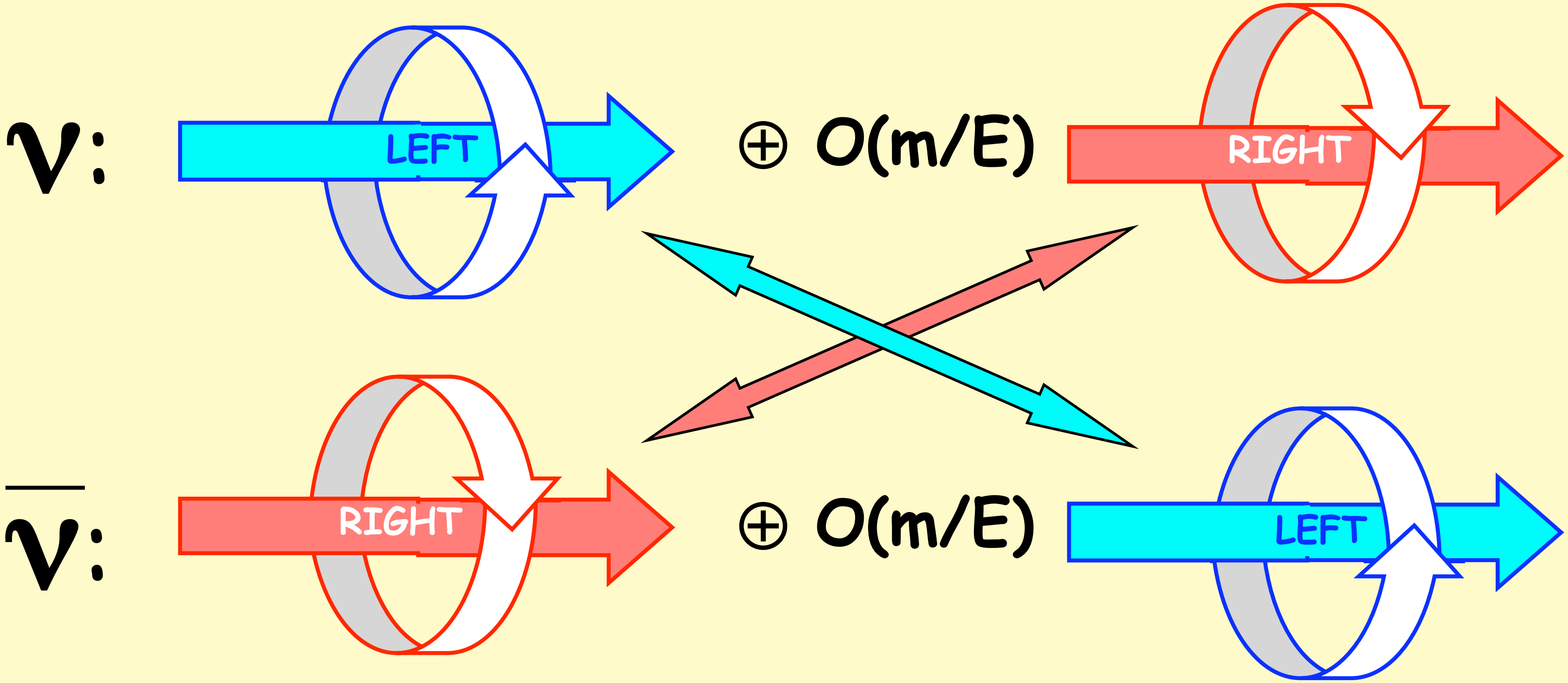
2 independent d.o.f.: massless ("Weyl") 2-spinor

But: massive ν can develop the "wrong" handedness at O(m/E)
(the Dirac equation mixes RH and LH states for $m_\nu \neq 0$):



If these 4 d.o.f. are independent: massive ("Dirac") 4-spinor
[→ Distinction between neutrinos and antineutrinos, as for electrically charged fermions. Can define a "lepton number"]

But, for neutral fermions, 2 components might be identical !



Massive ("Majorana") 4-spinor with 2 independent d.o.f.

[No distinction between neutrinos and antineutrinos, up to a phase:

A *very* neutral particle: no electric charge, no leptonic number…]

Exercise 1. Define the electron neutrino as the neutral particle emitted in β+ decay, and the electron antineutrino as the neutral particle emitted in β- decay. Reactions which have been observed:

$$\nu_e + n \to p + e^-$$

$$\overline{\nu}_e + p \to n + e^+$$

while the following reactions have not been observed:

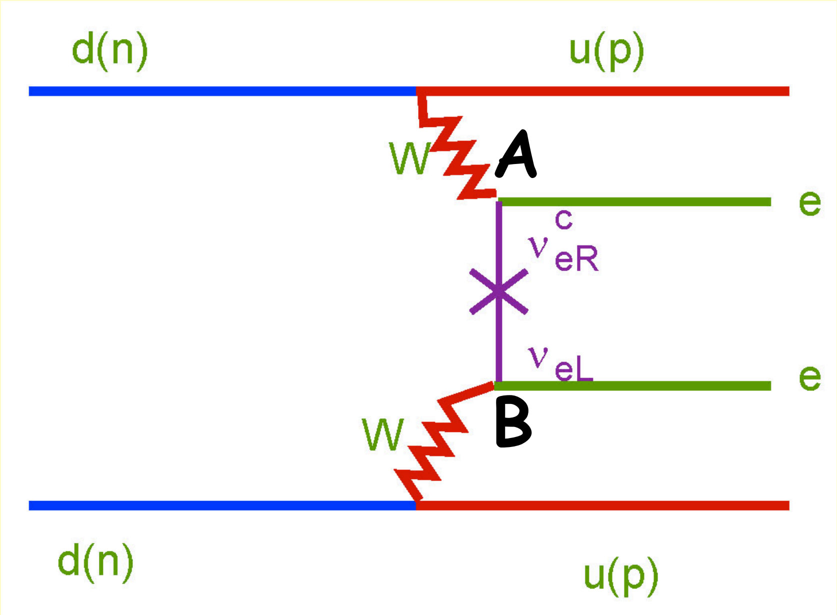
$$\overline{\nu}_e + n \to p + e^-$$

$$\nu_e + p \to n + e^+$$

If neutrinos and antineutrinos are different (Dirac case), that's easy to understand. Try to understand the same (non)observations in the case of Majorana neutrinos.

Only viable experimental handle to discriminate Dirac/Majorana:

Neutrinoless double beta decay: $(A,Z) \rightarrow (A,Z+2)+2e$



Can occur only for Majorana neutrinos. Intuitive picture:

1) A RH antineutrino is emitted at point "**A**" together with an electron
2) If it is massive, at O(m/E) it develops a LH component (not possible if Weyl)
3) If neutrino=antineutrino, this component is a LH neutrino (not possible if Dirac)
4) The LH (Majorana) neutrino is absorbed at "**B**" where a 2nd electron is emitted

Very rare to detect (if it occurs): doubly-weak and suppressed by m/E.

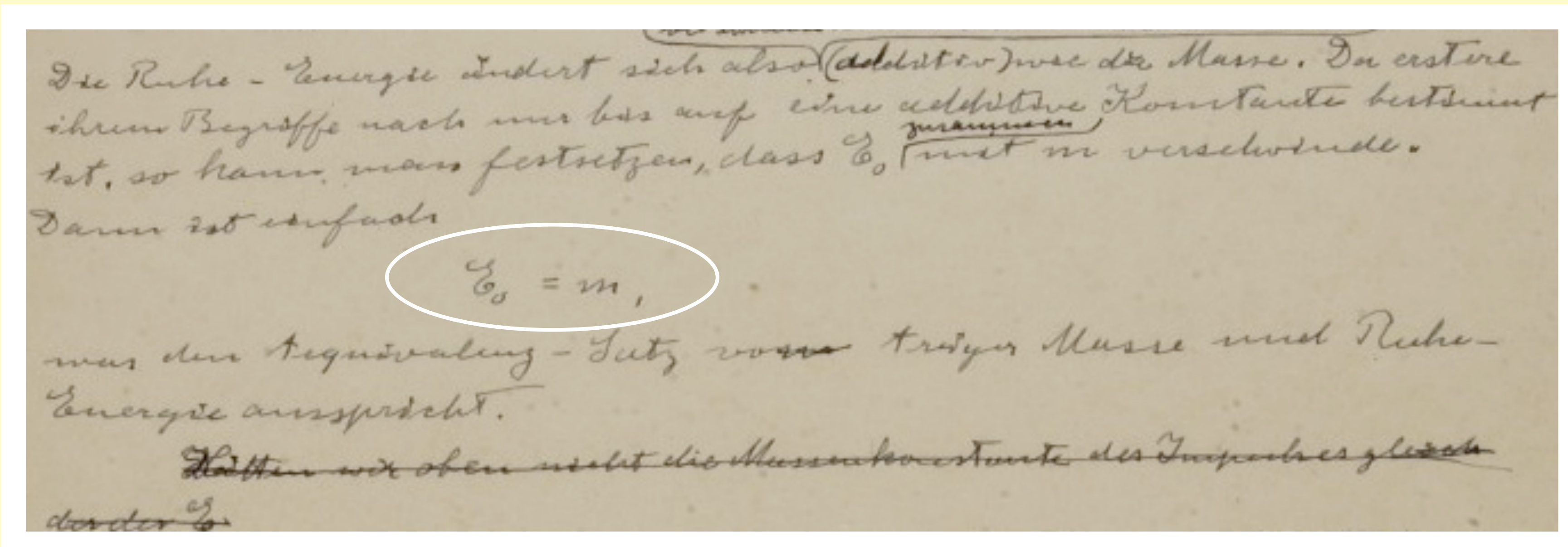
Recap: if neutrinos have mass, they can develop the **"wrong handedness"** with **amplitude of O(m$_{ass}$/E$_{nergy}$).** The only known chance to observe this tiny effect is **$0\nu\beta\beta$ decay.**

But, if neutrinos are not only massive but mixed, they can also develop in the **"wrong flavor"** as a major consequence (**"neutrino flavor oscillations"**). This effect, despite being only of **$O(m^2/E)$ in the phase**, can become observable over macroscopic distances (similar to optical interferometry).

We shall now discuss the phenomenon of flavor oscillations. Can forget about spinor properties in this case.

Neutrino flavor oscillations in vacuum (2ν)

The starting point is a century-old equation …



Die Ruhe-Energie ändert sich also (additiv) wie die Masse. Da erstere ihrem Begriffe nach nur bis auf eine additive Konstante bestimmt ist, so kann man festsetzen, dass E_0 zusammen mit m verschwinde. Dann ist einfach

$$E_0 = m,$$

was den Äquivalenz-Satz von träger Masse und Ruhe-Energie ausspricht.

~~Hätten wir oben nicht die Massenkonstante des Impulses gleich~~

~~der der E_0~~

… namely, for p≠0:

$$E = \sqrt{m^2 + p^2}$$

(in natural units)

Our ordinary experience takes place in the limit: $p \ll m$

$$E \simeq m + \frac{p^2}{2m}$$

... while for neutrinos the proper limit is: $p \gg m$

$$E \simeq p + \frac{m^2}{2p}$$

Energy difference between two neutrinos ν_i e ν_j with mass m_i e m_j in the same beam $(p_i = p_j \simeq E)$:

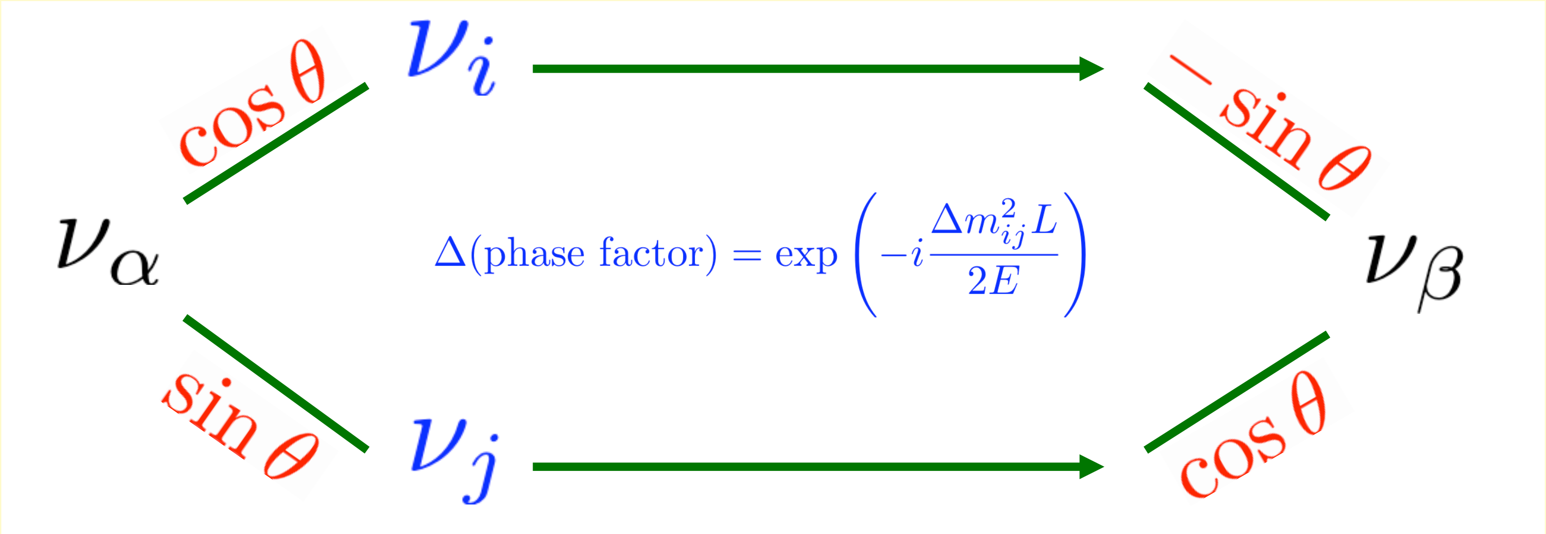
$$\Delta E \simeq \frac{\Delta m^2_{ij}}{2E}$$

PMNS*: neutrinos with definite mass (ν_i and ν_j) might have NO definite flavor (ν_α e ν_β), e.g.,

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_j \end{pmatrix}$$

*Pontecorvo; Maki, Nakagawa & Sakata

Analogy with a two-slit interference experiment in vacuum:



This is the simplest case (only 2 neutrinos involved, no interactions with matter). It shows that, if neutrinos are massive and mixed (like quarks), then flavor is not a good quantum number during propagation. Indeed, it changes ("oscillates") significantly over a distance L (≈Δt) dictated by the uncertainty relation:

$$1 \sim \Delta E \Delta t \simeq \frac{m_i^2 - m_j^2}{2E} L$$

Exercise 2. Prove that a neutrino created with flavor α can develop a different flavor β with a periodical oscillation probability in **L/E**:

$$P(\nu_\alpha \to \nu_\beta) = 4\sin^2\theta\cos^2\theta\sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right)$$ (B. Pontecorvo)

Amplitude (vanishes for $\theta=0$ or $\pi/2$)

Phase difference (vanishes for degenerate masses)

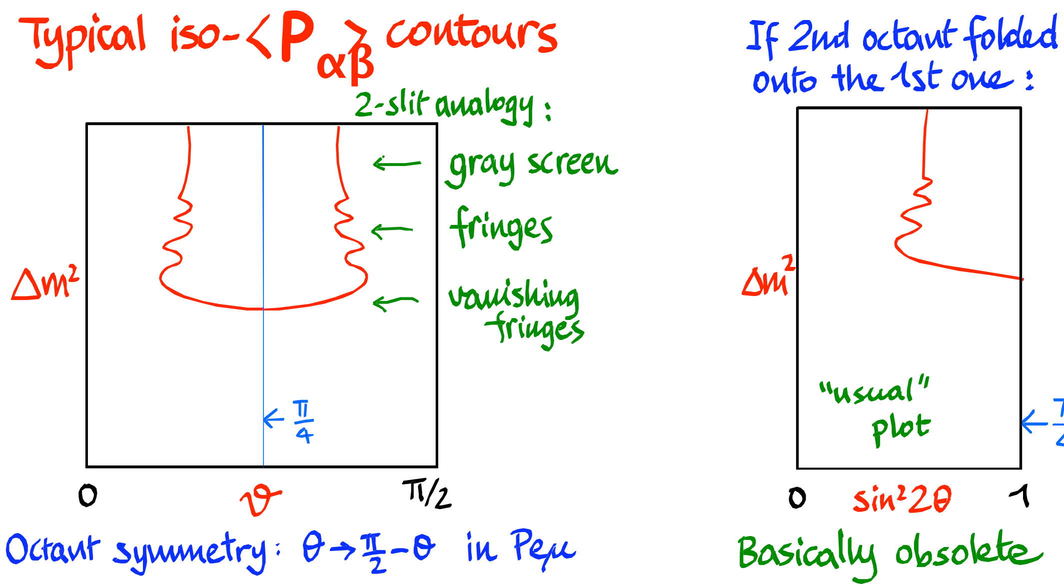
Note : This is the flavor "appearance" probability.
The flavor "disappearance" probability is the complement to 1.

Exercise 3. The oscillation effect depends on the **difference** of (squared) masses, not on the **absolute masses**. Why?

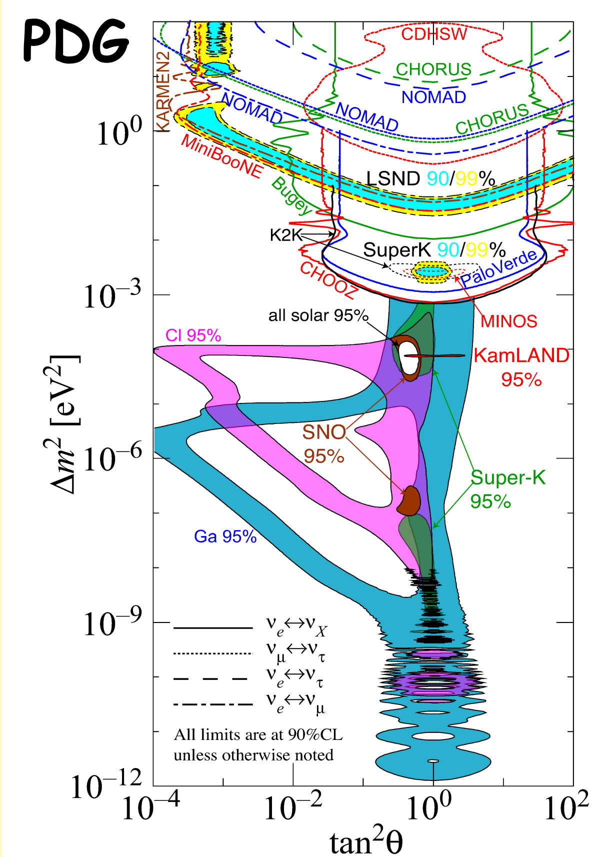
Exercise 4 . Show that:

$$\frac{\Delta m^2 L}{4E} = 1.267\left(\frac{\Delta m^2}{\mathrm{eV}^2}\right)\left(\frac{L}{\mathrm{km}}\right)\left(\frac{\mathrm{GeV}}{E}\right)$$

Associated contour plots:



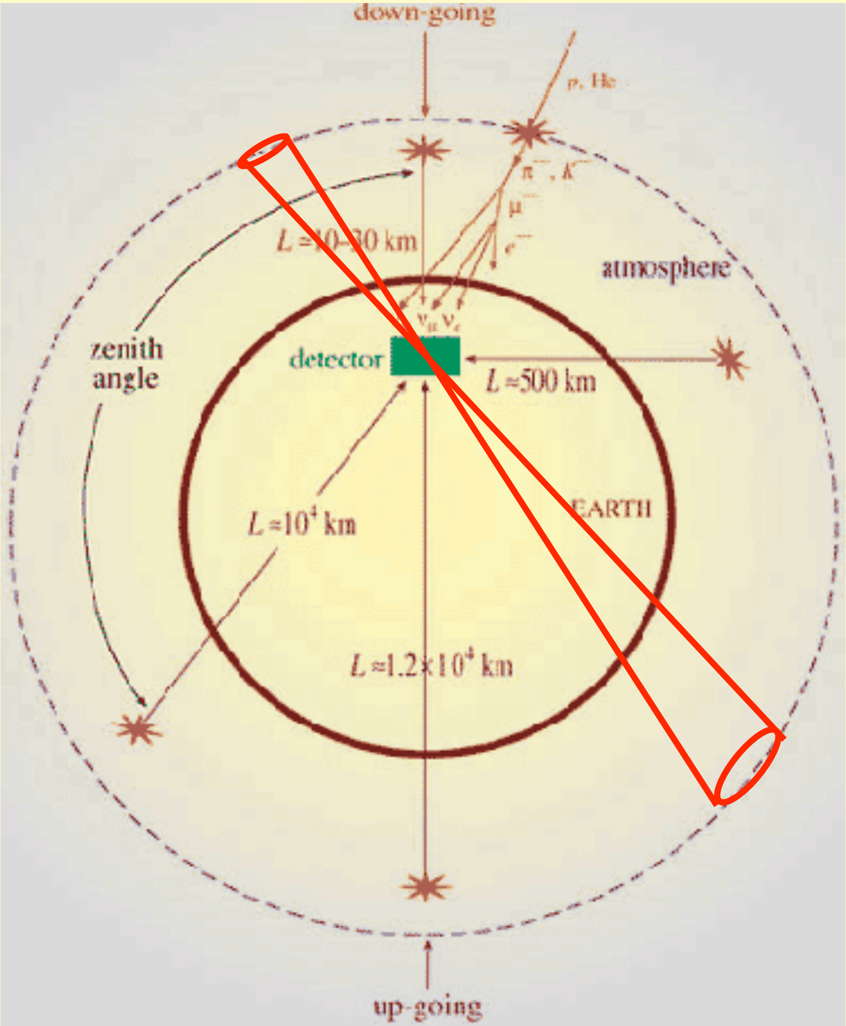
In general, better to use (preserve octant-symmetry) $\log \tan^2\theta$ or $\sin^2\theta$



(Note: Octant symmetry broken by 3ν and/or matter effects)

Observation of "effective 2ν" oscillations of atmospheric ν's

Cosmic rays hitting the atmosphere can generate secondary (anti)neutrinos with electron and muon flavor via meson decays.
Energies: E~ 0.1 - 100 GeV. Pathlengths: L~ 10 - 10000 km



Same ν flux expected from opposite solid angles (up-down symmetry)

[Flux dilution (~1/r^2) is compensated by larger production surface (~r^2)]

Should be reflected in symmetry of event zenith spectra, if energy & angle can be reconstructed well enough

Super-K observations over several decades in L/E:

ν_e induced events: ~ as expected

ν_μ induced events: disappearance from below!

Interpretation in terms of oscillations:

Channel $\nu_\mu \rightarrow \nu_e$? No (or subdominant)

Channel $\nu_\mu \rightarrow \nu_\tau$? Yes (dominant)

2ν-like approximation works very well...

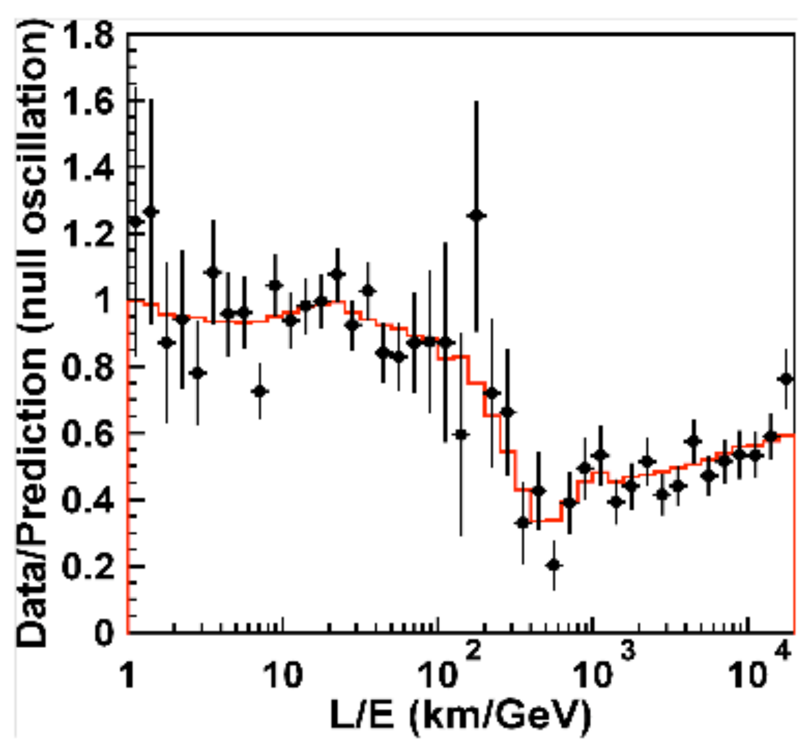
$$P_{\mu\tau} = \sin^2(2\theta)\,\sin^2(\Delta m^2 L/4E_\nu)$$

[In this channel, oscillations are ~vacuum-like, despite the presence of Earth matter]

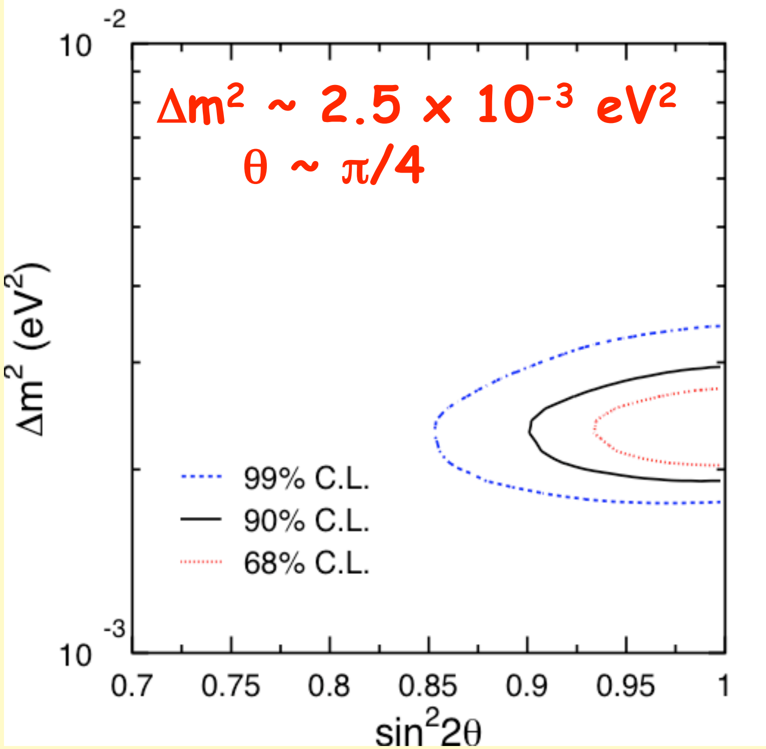
... but where are the "oscillations" ?

Dedicated L/E analysis to "see" half-period of oscillations…

1st oscillation dip still visible despite large L & E smearing



Strong constraints on the parameters (Δm^2, θ)



Same mass/mixing parameters confirmed in disappearance mode ($\nu_\mu \to \nu_\mu$) by other atmospheric expts (MACRO, Soudan2) and by expts with accelerator beams (K2K, MINOS: which also see dip)

Open questions for Δm^2-driven ν_μ oscillations:

The quest for hierarchy and octant: Is the sign of Δm^2 positive ("normal hierarchy") or negative ("inverted hierarchy")? Is θ > or < $\pi/4$?

The quest for ν_τ appearance: We expect dominant $\nu_\mu \to \nu_\tau$ transitions, but we should see the τ flavor directly – the hunt is going on with the CERN-to-Gran Sasso beam. **FIRST CANDIDATE THIS YEAR!**

The quest for ν_e appearance: We haven't seen $\nu_\mu \to \nu_e$ transitions; are they absent or just suppressed? This is a crucial problem for its implications on leptonic CP violation.

The quest for sterile neutrinos: Besides the known neutrinos $\nu_{e\mu\tau,L}$ (LH, gauge doublets) there might be new "sterile" states $\nu_{s,R}$ (RH, gauge singlets) leading to further disappearance $\nu_{\mu L} \to (\nu_{s,R})^c$

Useful to rephrase some of these questions in 3ν language →

Neutrino flavor oscillations in vacuum (~3ν)

- 3 flavor and mass states:

$$(\nu_e,\, \nu_\mu,\, \nu_\tau)^T = U\,(\nu_1,\, \nu_2,\, \nu_3)^T$$

Unitary matrix U depends on: 3 rotation angles θ_{ij} + 1 complex CP phase.
Conventionally, same ordering of the CKM quark matrix used for neutrinos:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij}=\cos(\theta_{ij})$ etc. [For antineutrinos: U→U*]

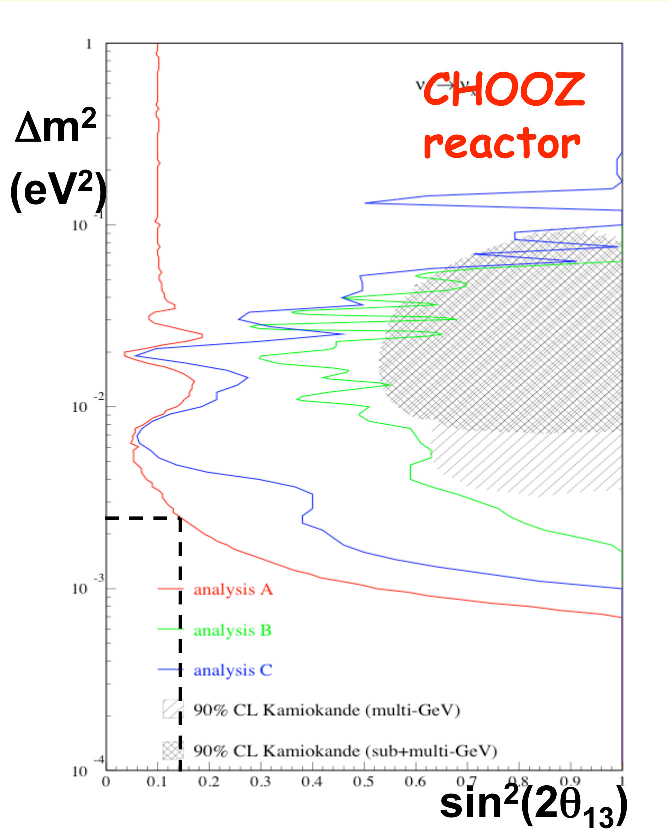
In first approximation, one may assume (~3ν)

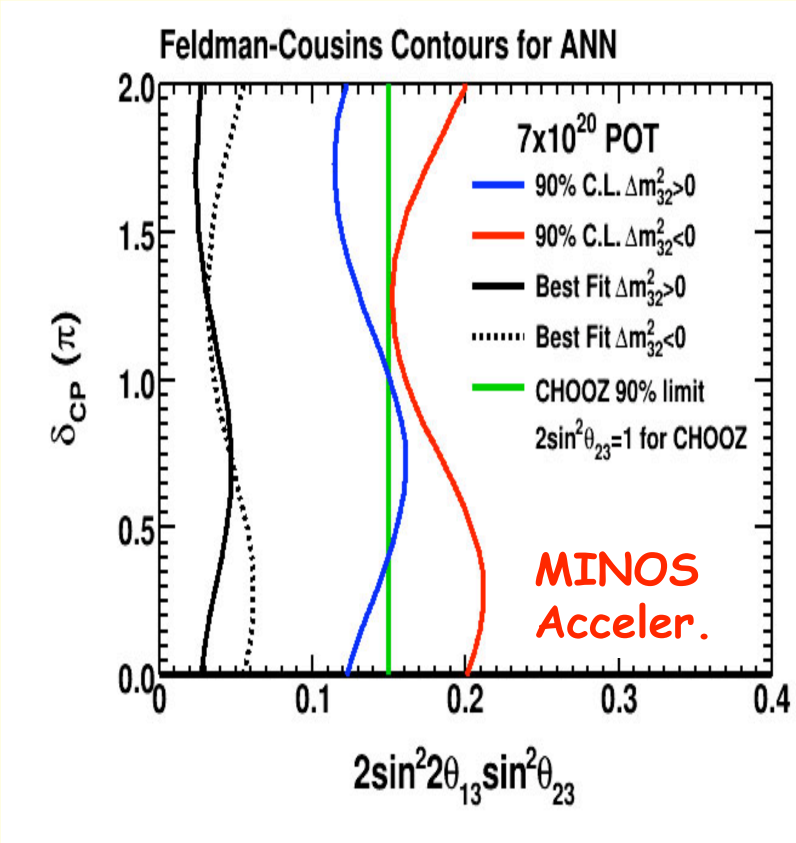
$$m_1 \simeq m_2 \quad \text{and} \quad \Delta m^2 = |m_3^2 - m_{1,2}^2|$$

In such notation, the previous "$\nu_\mu \rightarrow \nu_\tau$" mixing angle is $\theta_{23} \sim \pi/4$, while θ_{13} modulates the oscillation amplitude in the $\nu_e \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_e$ channels where, unfortunately, no signal has been found so far…

$P_{ee} = 1-\sin^2(2\theta_{13})\sin^2(\Delta m^2 L/4E_\nu)$

$P_{\mu e}=\sin^2\theta_{23}\sin^2(2\theta_{13})\sin^2(\Delta m^2 L/4E_\nu)$





World data consistent with $\sin^2\theta_{13} <$ few %.

Neutrino flavor oscillations in vacuum (full 3ν)

We have seen that atmospheric (and long-baseline accelerator) experiments have established the leading mass splitting of ν_3 with respect to $\nu_{1,2}$, with oscillation parameters:

$$\Delta m^2 = |m_3^2 - m_{1,2}^2| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2$$
$$\sin^2\theta_{23} \simeq 0.5$$

We shall now see how solar and long-baseline reactors, sensitive to much larger L/E, have also established the splitting between ν_1 and ν_2 , with oscillation parameters:

$$\delta m^2 = m_2^2 - m_1^2 \simeq 8 \times 10^{-5}\ \mathrm{eV}^2$$
$$\sin^2\theta_{12} \simeq 0.3$$

This opens the door to leptonic CP violation, iff $\theta_{13}>0$!

In a full 3ν scenario, a CP violating difference may arise between neutrino and antineutrino oscillation probabilities:

$$P_{\alpha\beta}(\nu) - P_{\alpha\beta}(\bar{\nu}) = 2 \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$
$$\times \sin\left(\frac{\Delta m^2 - \frac{\delta m^2}{2}}{4E} L\right) \sin\left(\frac{\Delta m^2 + \frac{\delta m^2}{2}}{4E} L\right) \sin\left(\frac{\delta m^2}{4E} L\right)$$

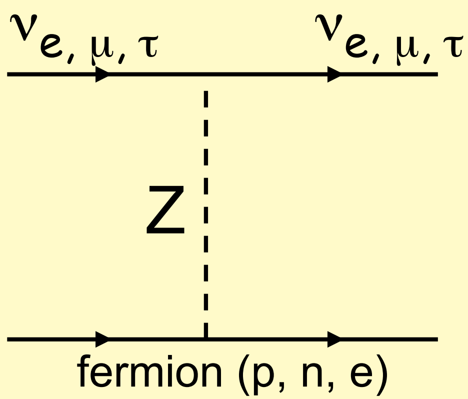
The above CPV difference is nonzero if:

- θ_{13} is nonzero
- $\sin\delta$ is nonzero
- the oscillation phases are neither too small nor too large

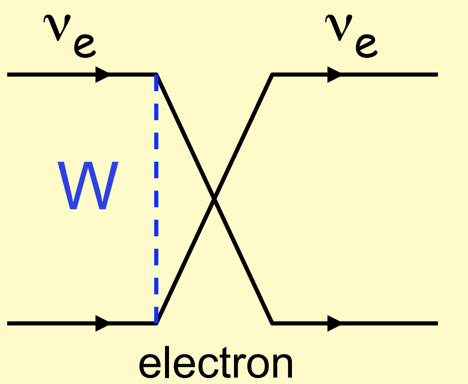
θ_{13} : main issue in current oscillation searches

Neutrino flavor oscillations in matter

Neutrinos of all flavors ($\nu_{e,\ \mu,\ \tau}$) have the same amplitude for coherent forward scattering in matter via NC. However, only ν_e can further scatter via CC, since ordinary matter contains e, not μ or τ. This fact implies a difference in the relative propagation of ν_e versus $\nu_{\mu,\ \tau}$, (but not between ν_μ and ν_τ): the Mikheyev-Smirnov-Wolfenstein (MSW) effect.

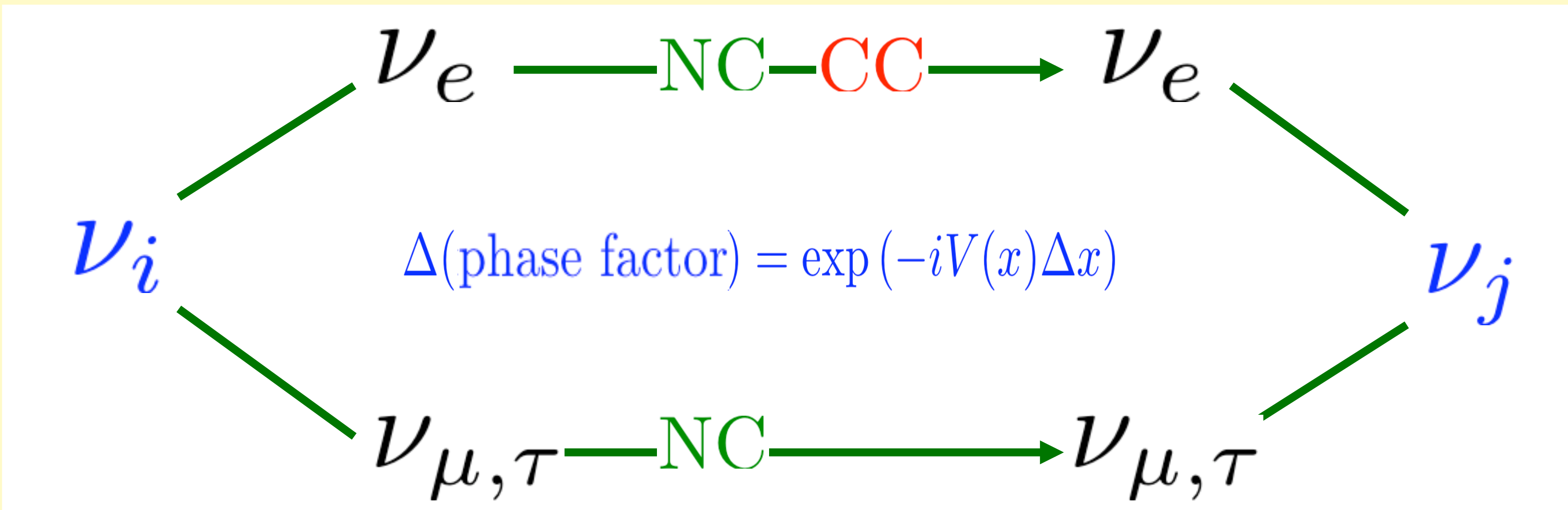


ν_μ & ν_τ (e.g., atmospheric) feel background fermions in the same way (through NC); no relative phase change while propagating (~ vacuum-like propagation, as anticipated)



But ν_e , in addition to NC, have CC interac. with background electrons (density N_e). Energy difference: $\mathbf{V = +\sqrt{2}\ G_F\ N_e}$ leads to a phase difference in matter

Again, analogy with the two-slit experiment:
one "arm" (flavor) feels a different "refraction index"



governed by the local (electron) density:

$$V(x) = V_e - V_{\mu,\tau} = \sqrt{2}\, G_F\, N_e(x) \quad [N_e = \text{electron density}]$$

(-V for antineutrinos)

Exercise 5. Prove that oscillations between ν_e and ν_x (=ν_μ,ν_τ) in matter with **constant** density lead to Pontecorvo's formula

$$P(\nu_e \to \nu_x) = \sin^2 2\tilde{\theta} \sin^2\left(\frac{\Delta\tilde{m}^2 L}{4E}\right)$$

with effective (tilde) parameters defined as

$$\frac{\Delta\tilde{m}^2}{\Delta m^2} = \frac{\sin 2\theta}{\sin 2\tilde{\theta}} = \sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + (\sin 2\theta)^2}$$

where $$A = 2VE = 2\sqrt{2} G_F N_e E$$

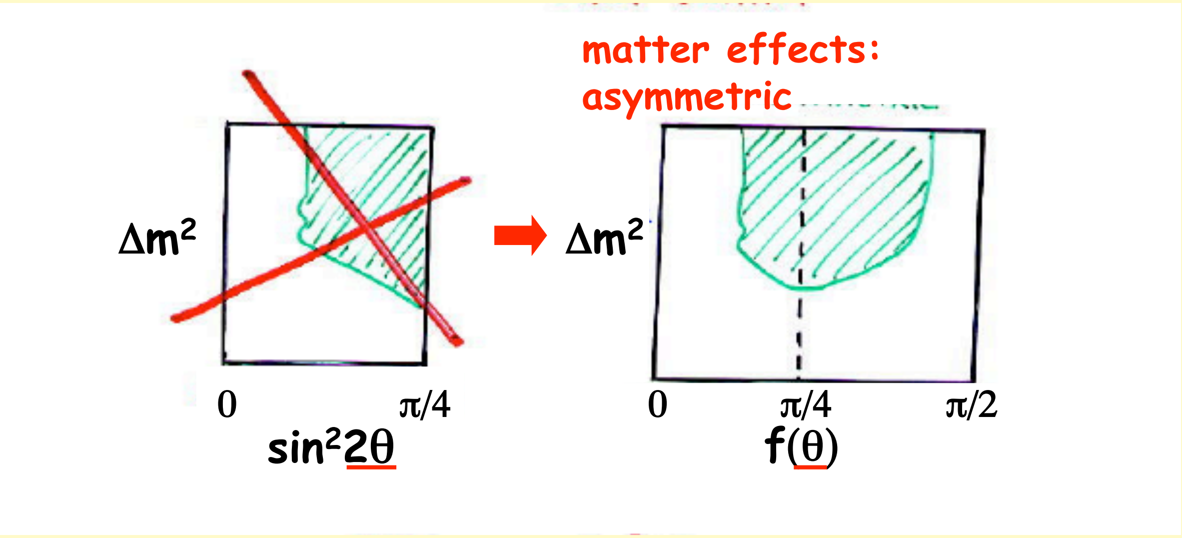
Exercise 6 (Conversion factors). Prove that

$$\frac{A}{\Delta m^2} = 1.526 \times 10^{-7} \left(\frac{N_e}{\mathrm{mol/cm^3}}\right)\left(\frac{E}{\mathrm{MeV}}\right)\left(\frac{\mathrm{eV}^2}{\Delta m^2}\right)$$

Rule of thumb (**~valid also for non-constant density**):

Expect strong matter effects when **$A/\Delta m^2 \sim O(1)$.**

Note: matter effects are octant-asymmetric;
need to unfold second octant.

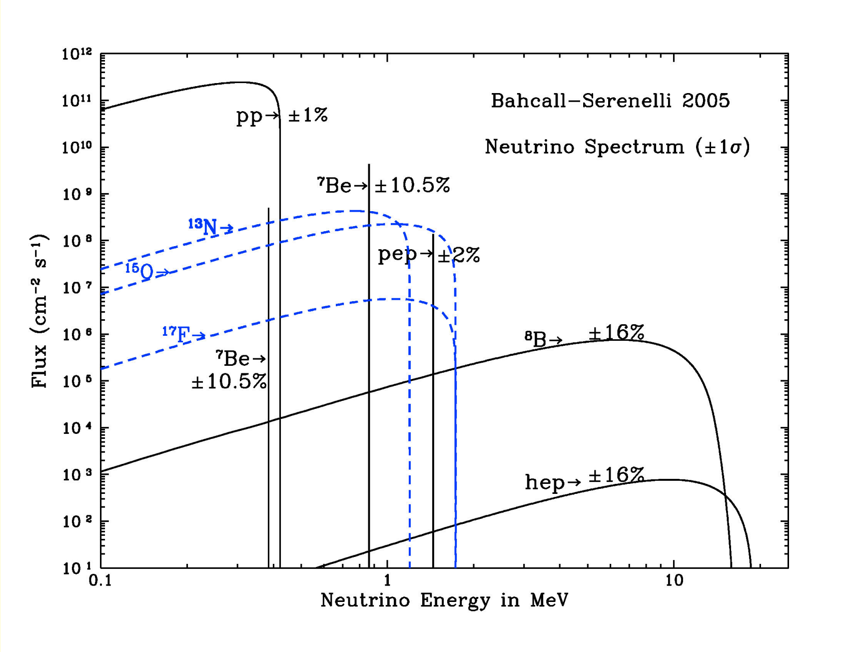


Asymmetry is particular pronounced for solar neutrinos, with mass-mixing parameters (δm^2, θ_{12})

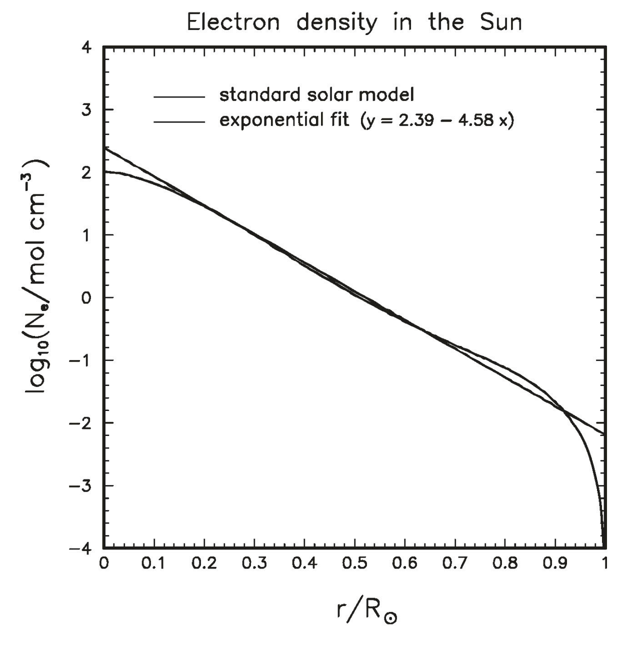
[N.B.: Effects also depend on sign of squared mass difference: Handle to hierarchy discrimination.]

Solar neutrinos

The Sun is an intense source of ν_e with $E \sim O(10^{\pm 1})$ MeV …



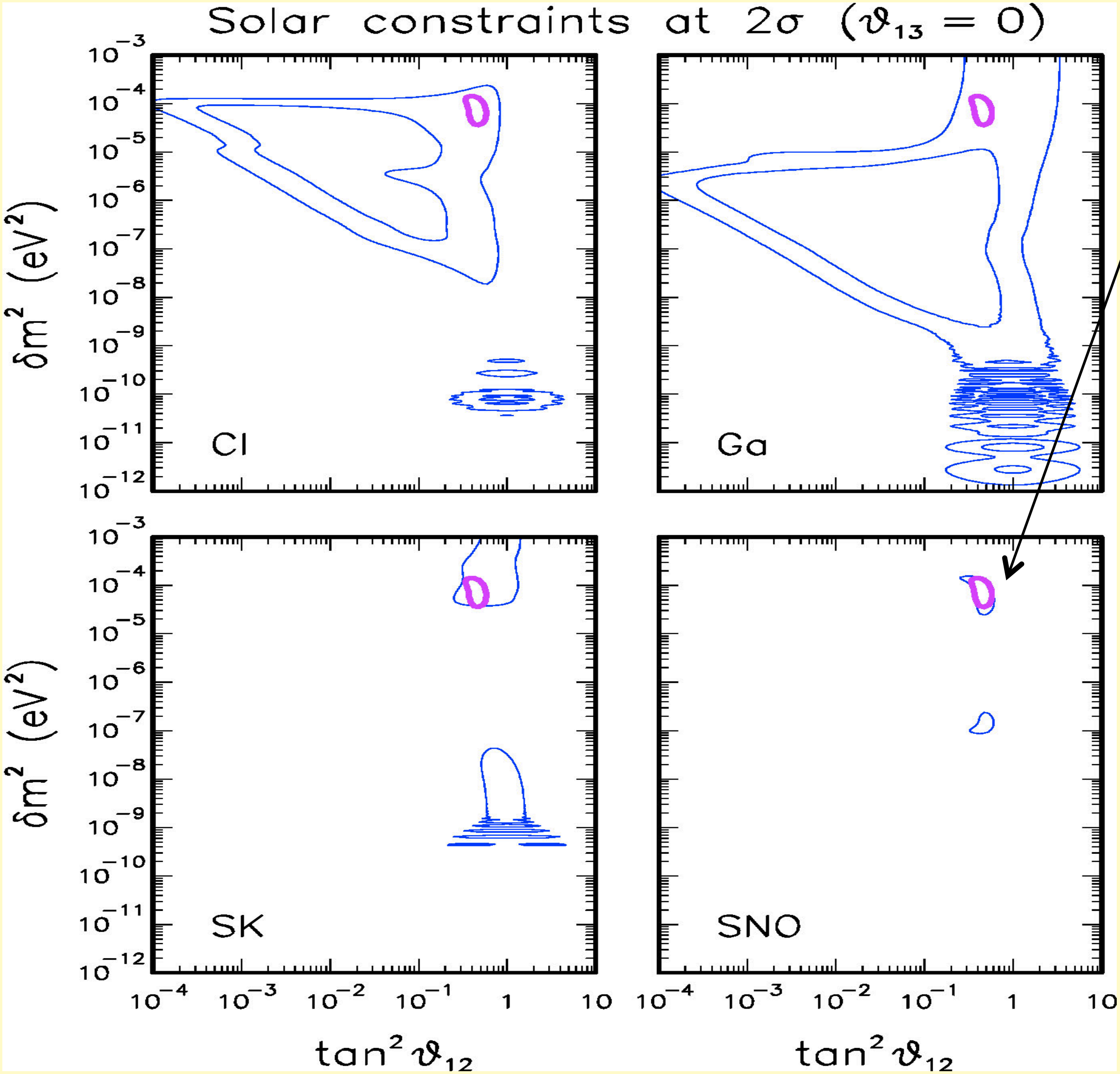
… and its electron density range is $\sim O(10^{\pm 2})$ mol/cm^3



$$\ldots\text{therefore, } A/\delta m^2 \sim O(1) \text{ if } \delta m^2 \sim O(10^{-10} - 10^{-3})\ \text{eV}^2$$

The Sun is an ideal place to look for oscillations in matter, driven the "**small" squared mass difference** δm^2 (not the "large" Δm^2), and Nature has been kind enough to fulfill these expectations! The corresponding (solar) mixing angle is θ_{12}

In 2002 ("annus mirabilis"), one global solution was finally singled out by combination of solar data (so-called "large mixing angle" or **LMA** solution).



Also in 2002… KamLAND: Detector surrounded by many nuclear reactors producing anti-ν_e with "lucky" parameters:

$A/\delta m^2 \ll 1$ in Earth crust
(vacuum approxim. OK)
L~100-200 km
E_ν~ few MeV

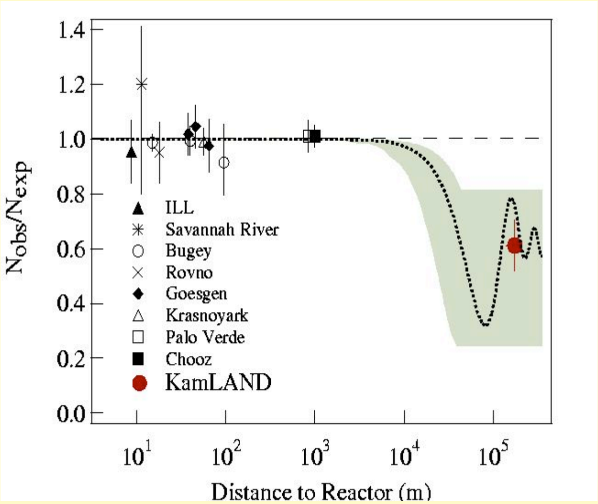
⇨

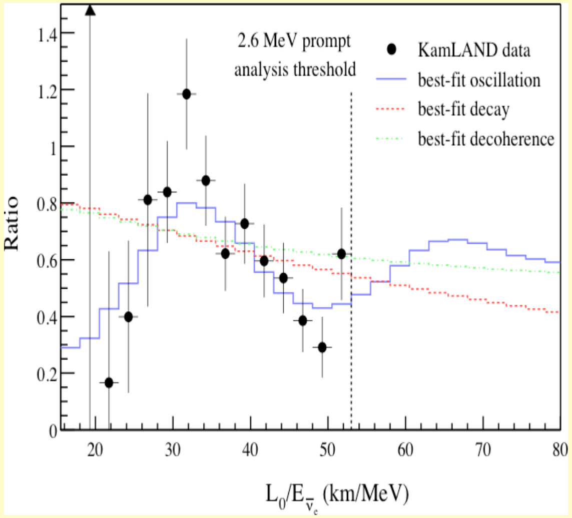
With LMA $(\delta m^2, \theta_{12})$ parameters it is $(\delta m^2 L/4E)$~$O(1)$ and reactor neutrinos should oscillate with large amplitude (large θ_{12})

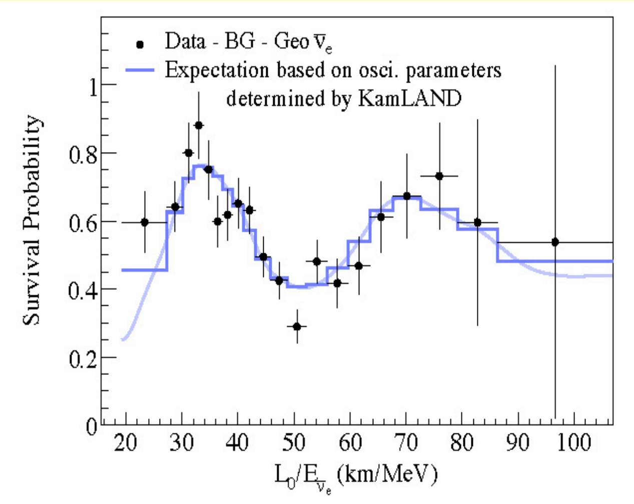
2002: electron flavor disappearance observed

2004: half-period of oscillation observed

2007: one period of oscillation observed



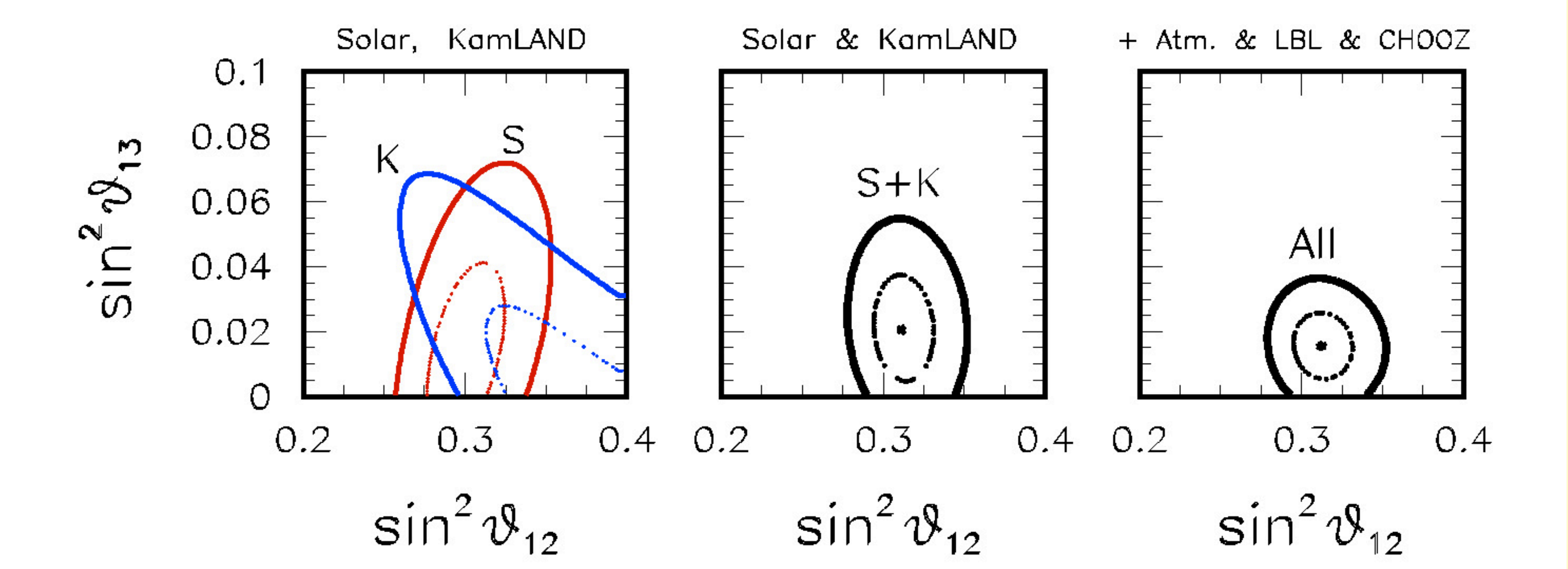




More refined (3ν) interpretation

Go beyond leading 3ν oscillation effects. Include all subleading effects due to θ_{13} and averaged Δm^2 oscillations in vacuum/matter.

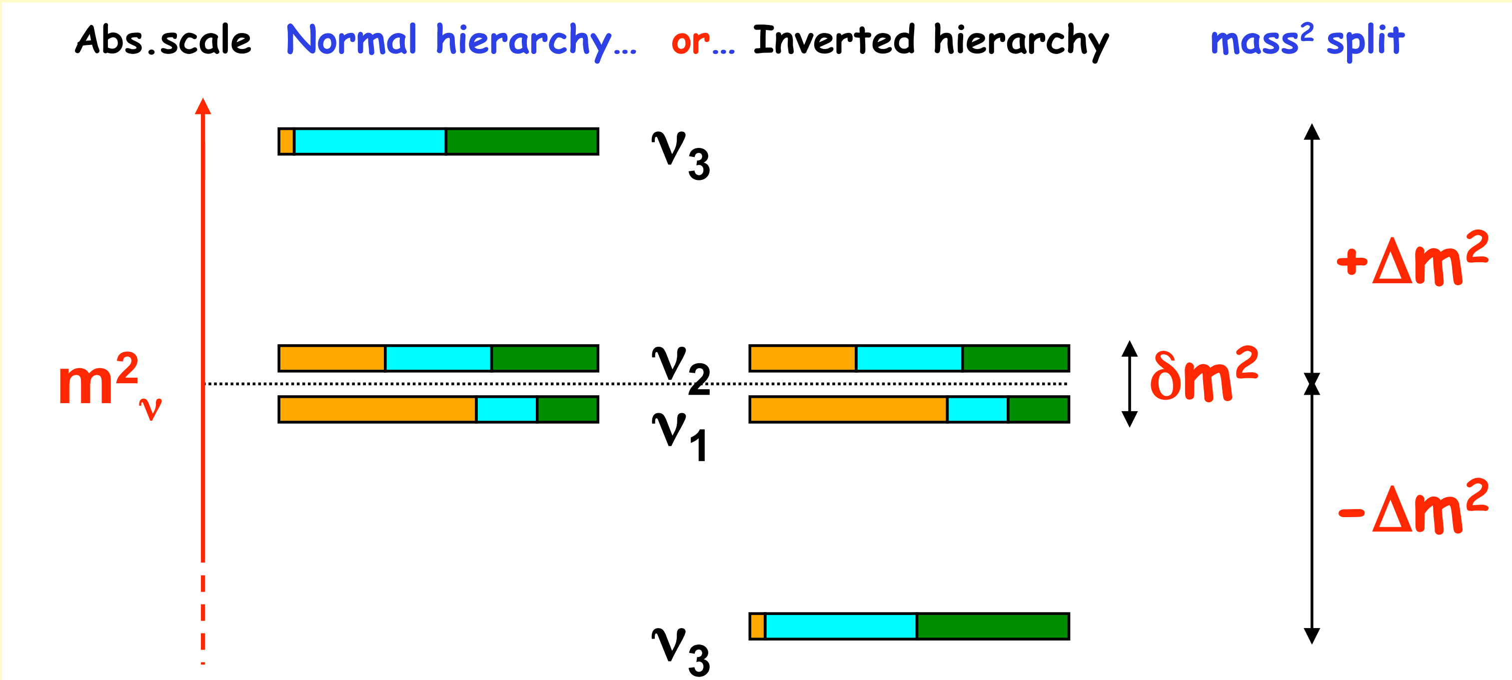
Interesting (small) hints emerge... [See arXiv:0806.2649].



Hint of θ_{13} >0 ? Time will tell.

3ν mass-mixing overview in just one slide

(here, with 1 digit accuracy). Flavors = e μ τ



$\delta m^2 \sim 8 \times 10^{-5}\ \text{eV}^2$ $\quad$ $\sin^2\theta_{12} \sim 0.3$

$\Delta m^2 \sim 3 \times 10^{-3}\ \text{eV}^2$ $\quad$ $\sin^2\theta_{23} \sim 0.5$

$m_\nu < O(1)\ \text{eV}$ $\quad$ $\sin^2\theta_{13} < \text{few}\%$

$\text{sign}(\pm\Delta m^2)$ unknown $\quad$ δ (CP) unknown

More digits from global 3ν analysis:
Synopsis of neutrino mass² and mixing parameters

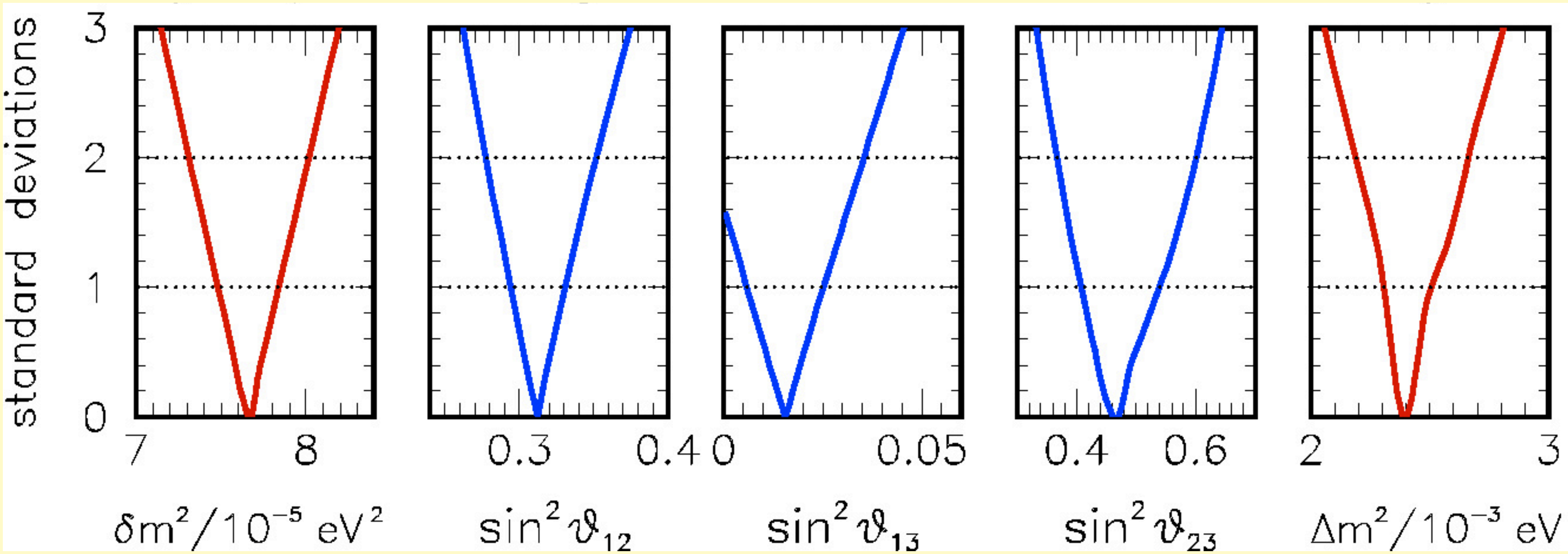
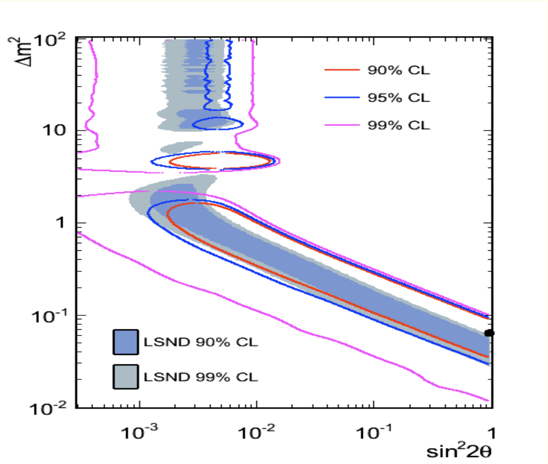


TABLE I: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges for the mass-mixing parameters.

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

arXiv:0805.2517

Evidence for new (sterile) states? LSND & MiniBooNE



The LSND experiment found a signal of possible $\nu_\mu \rightarrow \nu_e$ oscillations at a relatively high **ΔM^2 scale of $O(0.1-1)$ eV^2**

MiniBoone was designed to test LSND, but results are –so far- inconclusive…

ν_S oscillation interpr… remains difficult after latest anti-ν results (2010)

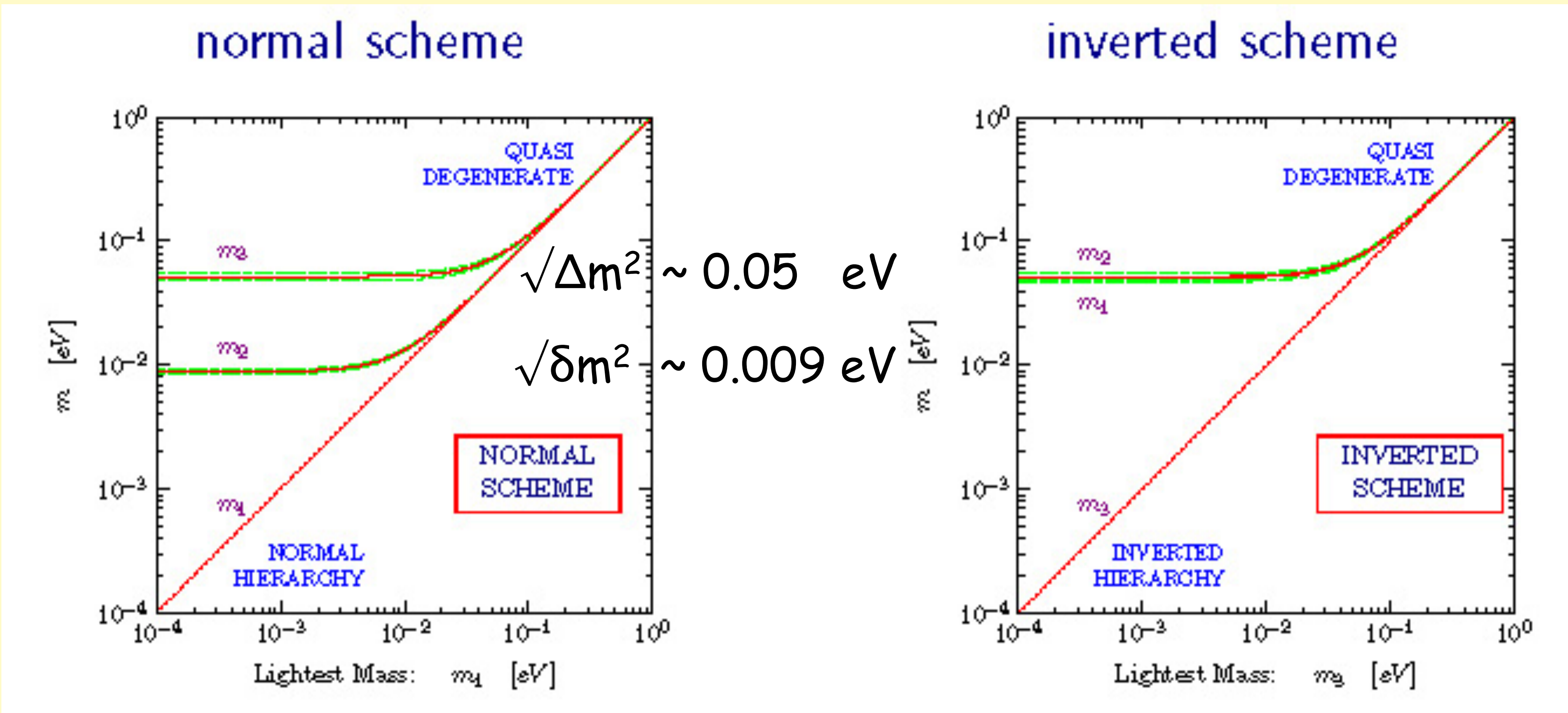
Analysis reveals tension between different datasets:
Low/high E, ν/antiν, appearance/disappear., SBL/atm…
Can be mitigated by selective choice/adjustment of data sets/errors, and/or by exotic new physics (CPTV?)

No obvious "single" theor. explanation. Possibly: several underlying effects of different origin (including cross sections)

Absolute neutrino masses and cosmology

Oscillations constrain neutrino mixings and mass splittings but not the absolute mass scale.

E.g., can take the lightest neutrino mass as free parameter:



$\sqrt{\Delta m^2} \sim 0.05$ eV

$\sqrt{\delta m^2} \sim 0.009$ eV

However, the lightest neutrino mass is not really an "observable"

We know only three realistic observables to attack ν masses →

(m_β, $m_{\beta\beta}$, Σ)

1) β decay: $m^2_i \neq 0$ can affect spectrum endpoint. Sensitive to the "effective electron neutrino mass":

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

2) $0\nu\beta\beta$ decay: Can occur if $m^2_i \neq 0$ and $\overline{\nu}=\nu$ (Majorana, not Dirac) Sensitive to the "effective Majorana mass" (and phases):

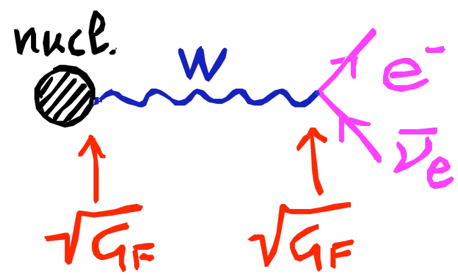
$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

3) Cosmology: $m^2_i \neq 0$ can affect large scale structures in (standard) cosmology constrained by CMB + other data. Sensitive to:

$$\Sigma = m_1 + m_2 + m_3$$

β-decay

rate: $d\Gamma \propto G_F^2 \times (\text{phase sp.})$

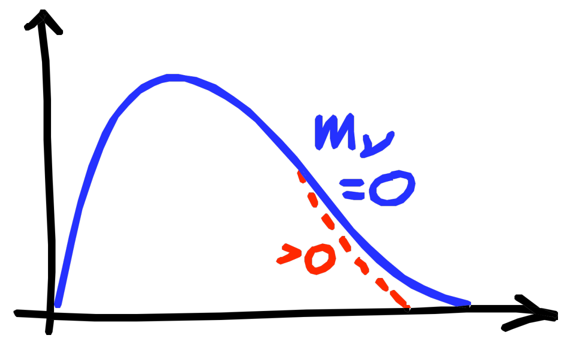


$|\ \cdots\ |^2 \propto G_F^2$

energy spectrum:

$$\frac{d\Gamma}{dE_e} \propto G_F^2\, p_e E_e (Q-E_e)^2 \qquad (m_\nu \equiv 0)$$

$$G_F^2\, p_e E_e (Q-E_e)\sqrt{(Q-E_e)^2+m_\nu^2} \qquad (>0)$$



For just one (electron) neutrino family: sensitivity to $m^2(\nu_e)$ (obsolete)

For three neutrino families ν_i, and individual masses experimentally <u>unresolved</u> in beta decay: sensitivity to the sum of $m^2(\nu_i)$, weighted by squared mixings $|U_{ei}|^2$ with the electron neutrino. Observable:

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

(so-called "effective electron neutrino mass")

In construction: KATRIN.

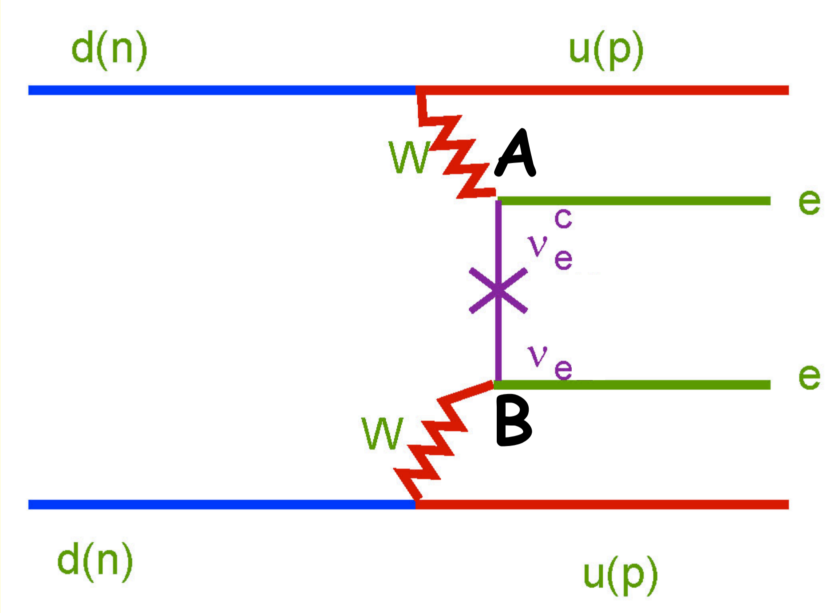
Sensitivity: $m_\beta \sim 0.2$ eV

(x10 better than current limits)

...Probably the "ultimate" spectrometer of this kind!

$0\nu\beta\beta$ decay and 3ν mass-mixing

For each mass state ν_i, $0\nu\beta\beta$ amplitude proportional to:



... mixing of ν_e with ν_i

... mass of ν_i

... mixing of ν_i with ν_e

(times an unknown ν_i phase)

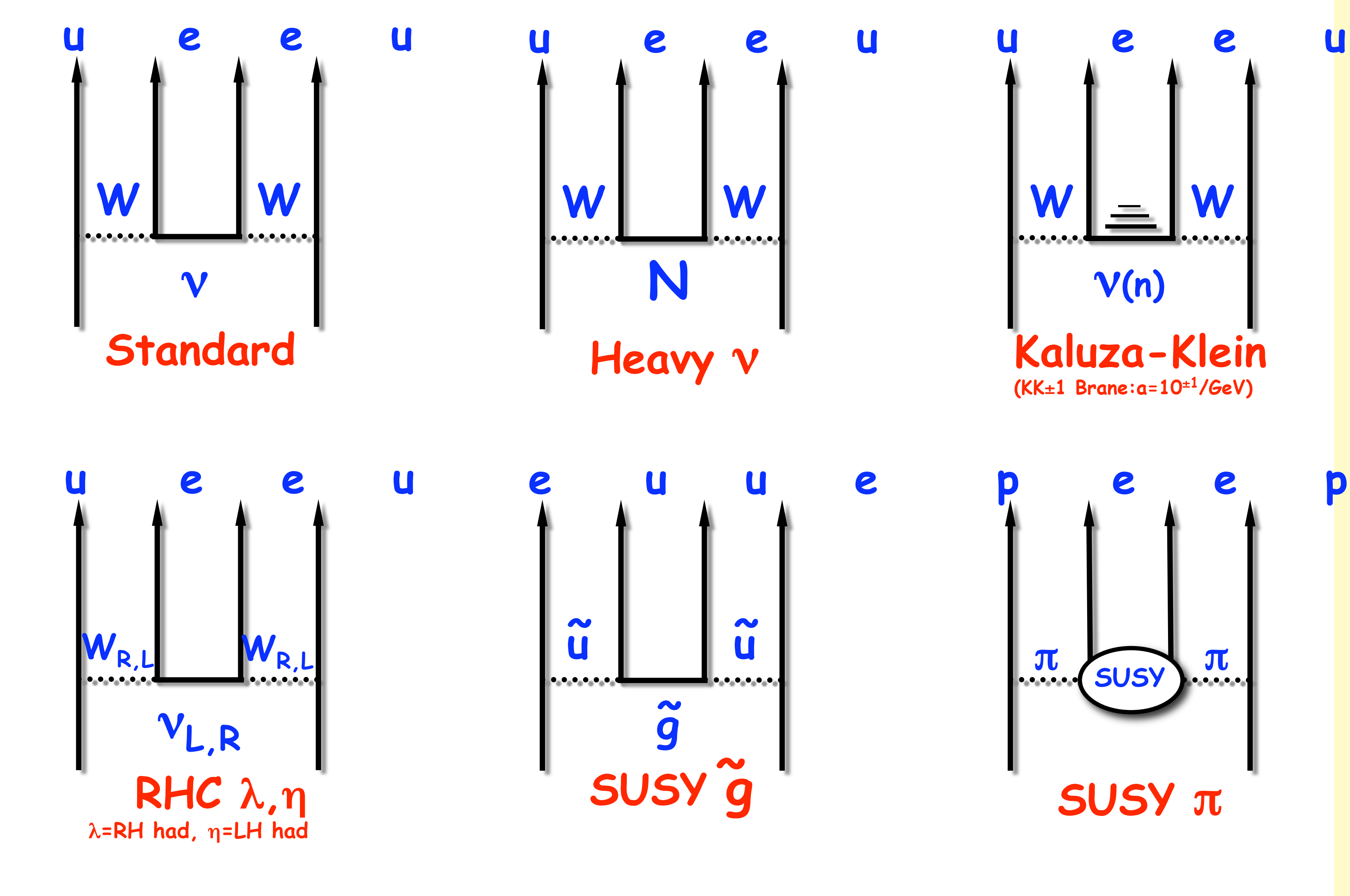
Summing up for three massive neutrinos:

Amplitude ~ "effective Majorana mass"

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right|$$

[complex linear combination of masses; $c_{ij} = \cos\theta_{ij}$ etc.]

Warning: $0\nu\beta\beta$ decay might also arise from new physics!



Cosmology

Standard big bang cosmology predicts a relic neutrino background with total number density $336/cm^3$ and temper. $T_\nu \sim 2$ K $\sim 1.7 \times 10^{-4}$ eV $\ll \sqrt{\delta m^2}, \sqrt{\Delta m^2}$.

→At least two relic neutrino species are nonrelativistic today (we can't exclude the lightest to be ~ massless)

→Their total mass contributes to the normalized energy density as $\Omega_\nu \approx \Sigma / 50$ eV, where

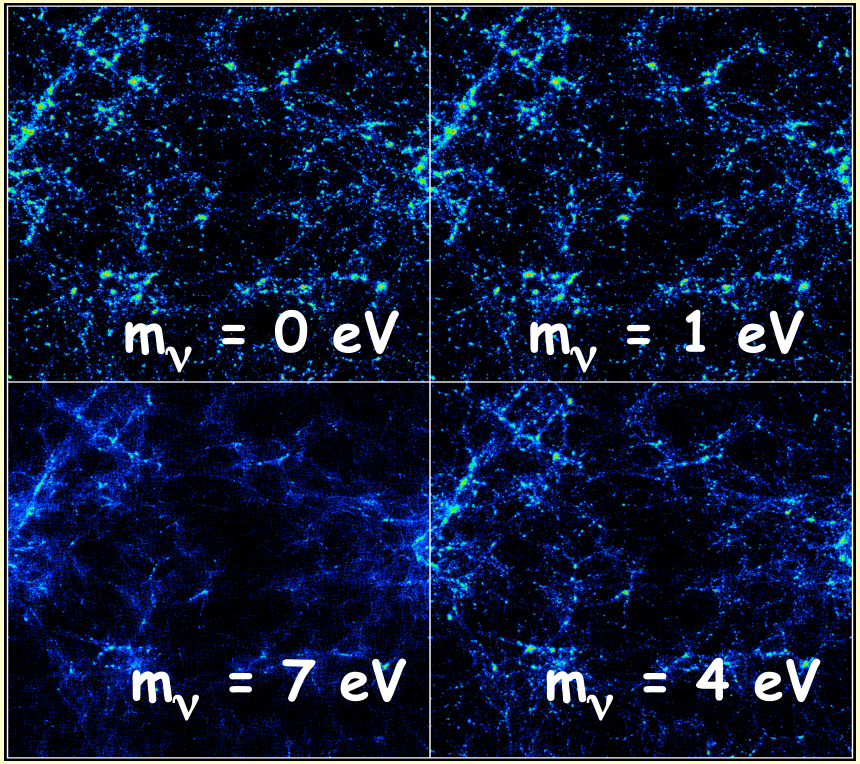
$$\Sigma = m_1 + m_2 + m_3$$

→So, if we just impose that neutrinos do not saturate the total matter density, $\Omega_\nu < \Omega_m \approx 0.25$, we get

$m_i < 4$ eV – not bad!

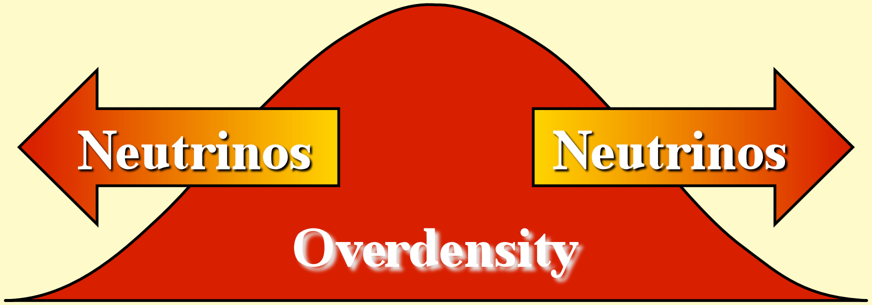
Much better bounds can be derived from neutrino effects on structure formation.

Massive neutrinos are difficult to cluster because of their relatively high velocities: they suppress matter fluctuations on scales smaller than their free-streaming scale



(E..g., Ma 1996)

→ Get mass-dependent suppression of small scale structures

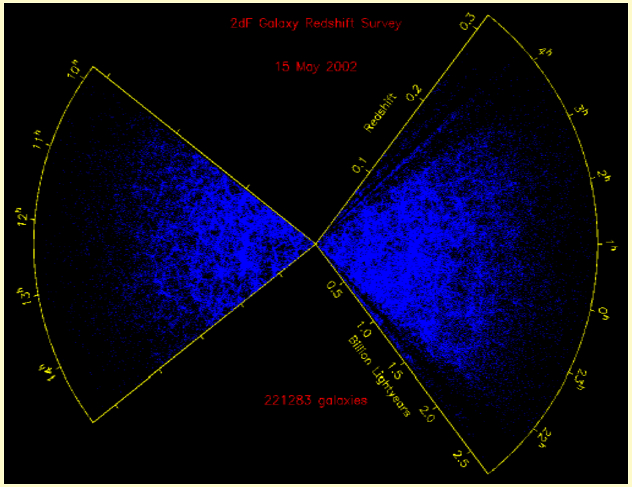


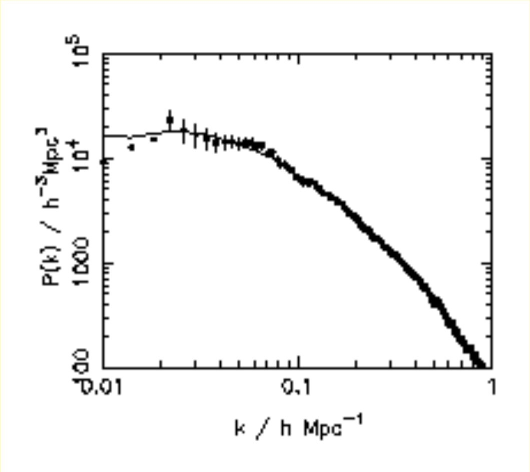
Constraints from CMB crucial to remove degeneracies.

Observations:

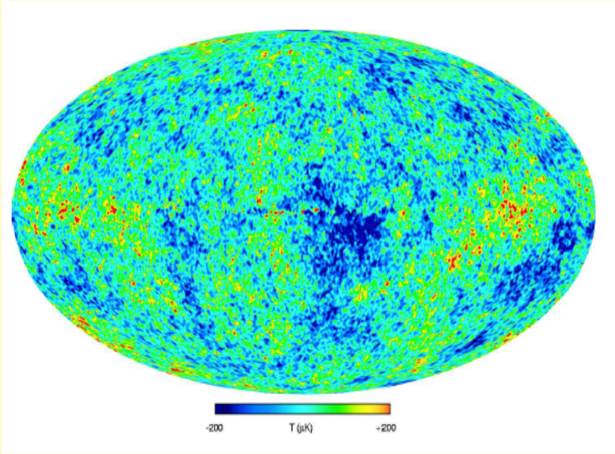
Spectra:

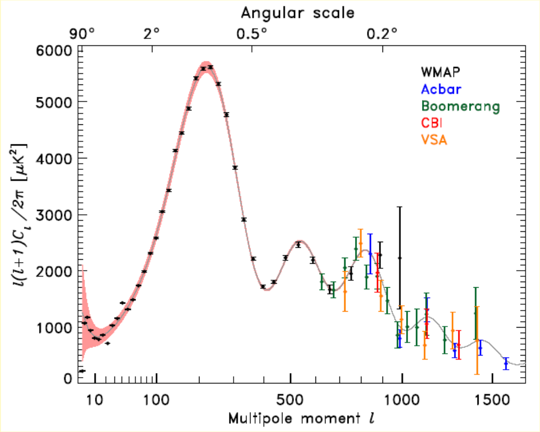
LSS





CMB





Spectral effect of massive neutrinos (e.g., from Lesgourgues & Pastor)

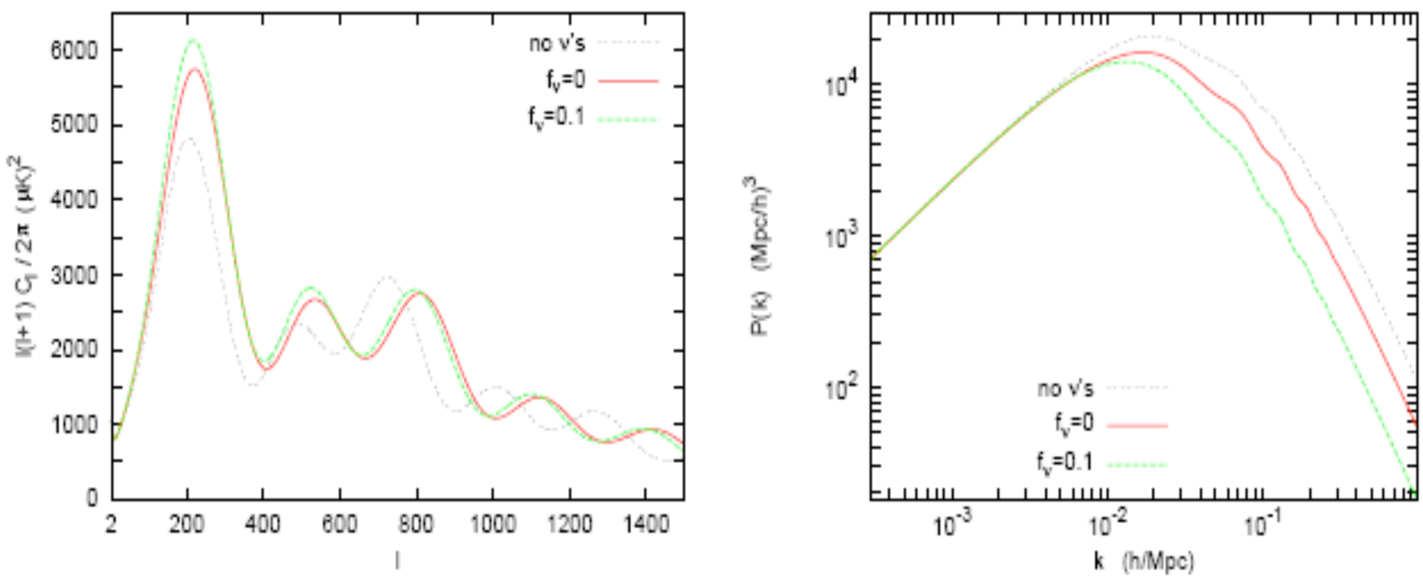


Fig. 14. CMB temperature anisotropy spectrum C_l^T and matter power spectrum $P(k)$ for three models: the neutrinoless ΛCDM model of section 4.4.6, a more realistic ΛCDM model with three massless neutrinos ($f_\nu \simeq 0$), and finally a ΛMDM model with three massive degenerate neutrinos and a total density fraction $f_\nu = 0.1$. In all models, the values of $(\omega_{\rm b}, \omega_{\rm m}, \Omega_\Lambda, A_s, n, \tau)$ have been kept fixed.

Significant progress after WMAP

Smaller scales probed by Ly-alpha

Just an example of recent limits on the sum of ν masses from various data sets (assuming the "flat ΛCDM model"): [arXiv:0805.2517]

TABLE II: Representative cosmological data sets and corresponding 2σ (95% C.L.) constraints on the sum of ν masses Σ.

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

Case 1: "conservative" (only CMB data, dominated by WMAP 5y)
Case 5: "aggressive" (all relevant cosmological data)

Upper limits in the range $\Sigma < 0.6$-1.2 eV have gained large consensus.
More stringent limits require more "faith" in current control of syst.'s.

Updated values: see talks by Cooray and Melchiorri at NOW 2010 for discussions of ~0.3 eV as "safe" upper limit.

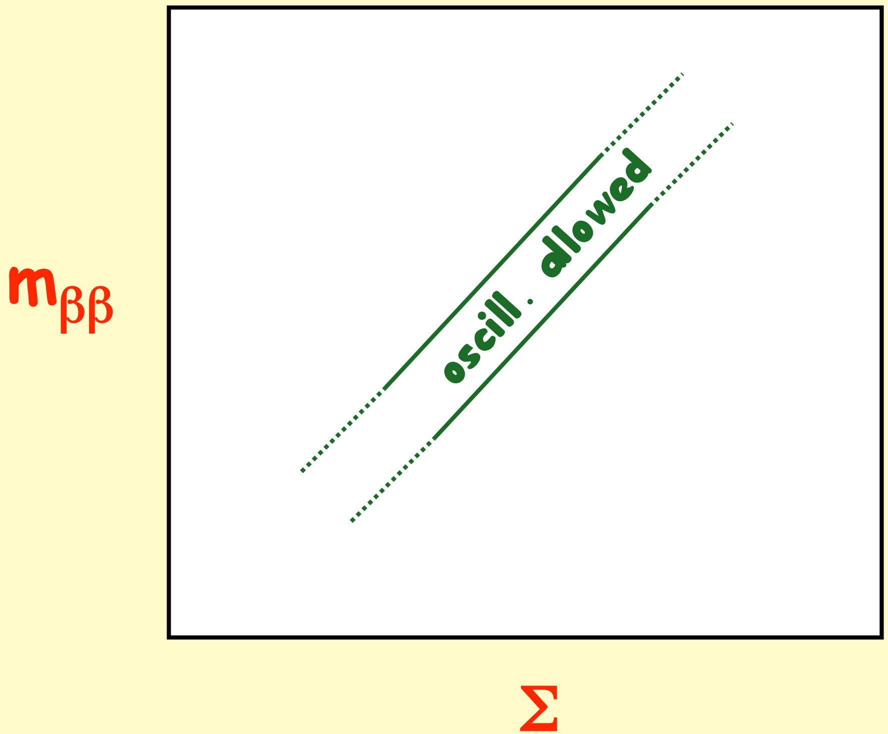
Hunting absolute masses... with a trident

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

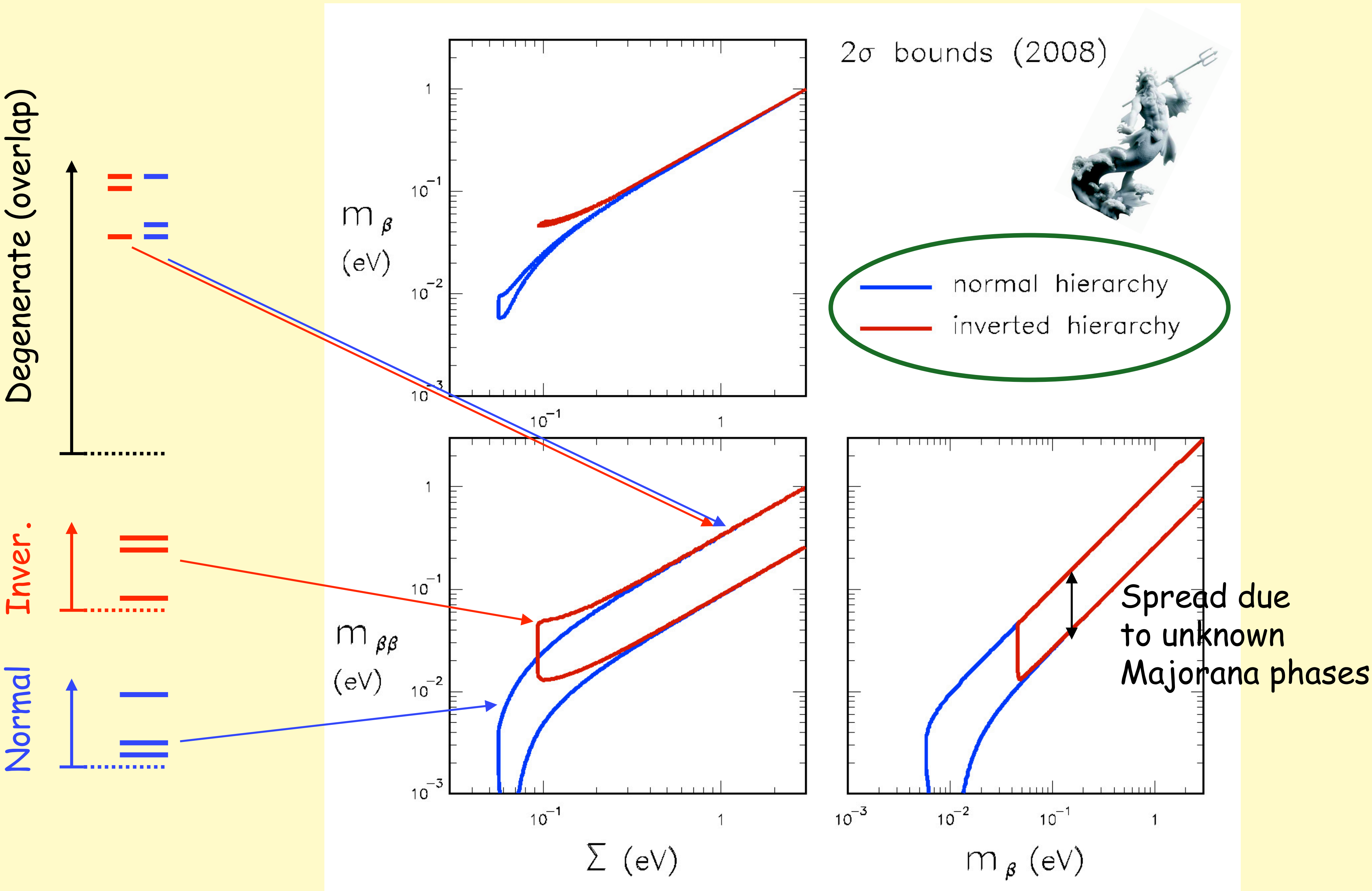
$$\Sigma = m_1 + m_2 + m_3$$

Interplay: Oscillations fix the mass^2 splittings, and thus induce positive correlations between any pair of the three observables (m_β, $m_{\beta\beta}$, Σ), e.g.:

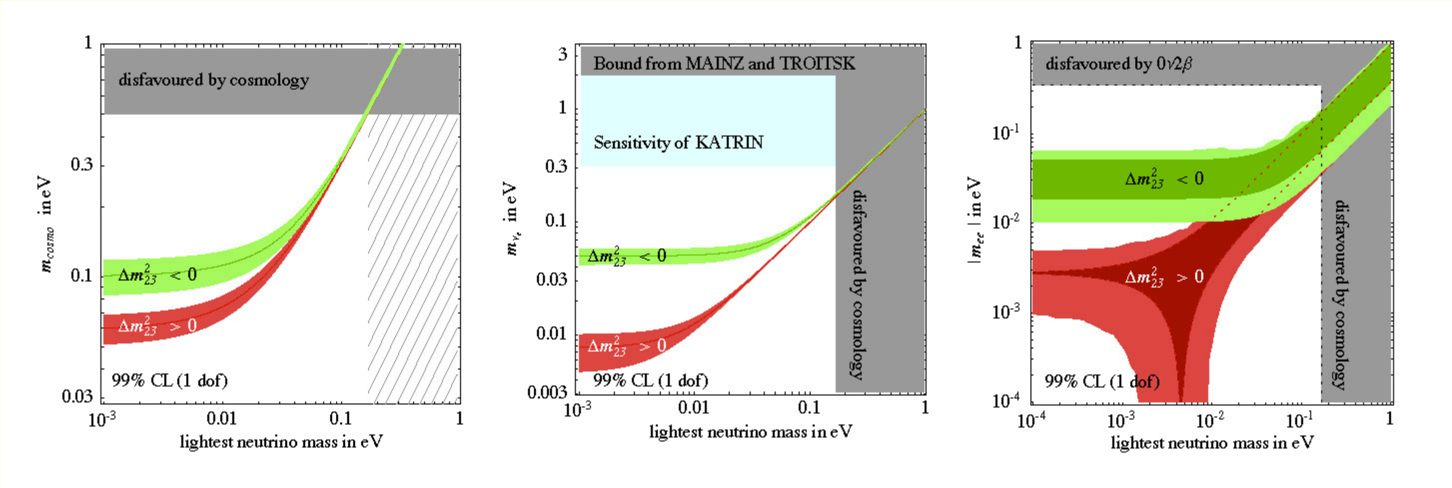


i.e., if one observable increases, the other one (typically) must increase to match mass splitting

The “spear” (oscill. data) sets the “hunting direction” in the $(m_\beta,\ m_{\beta\beta},\ \Sigma)$ parameter space:

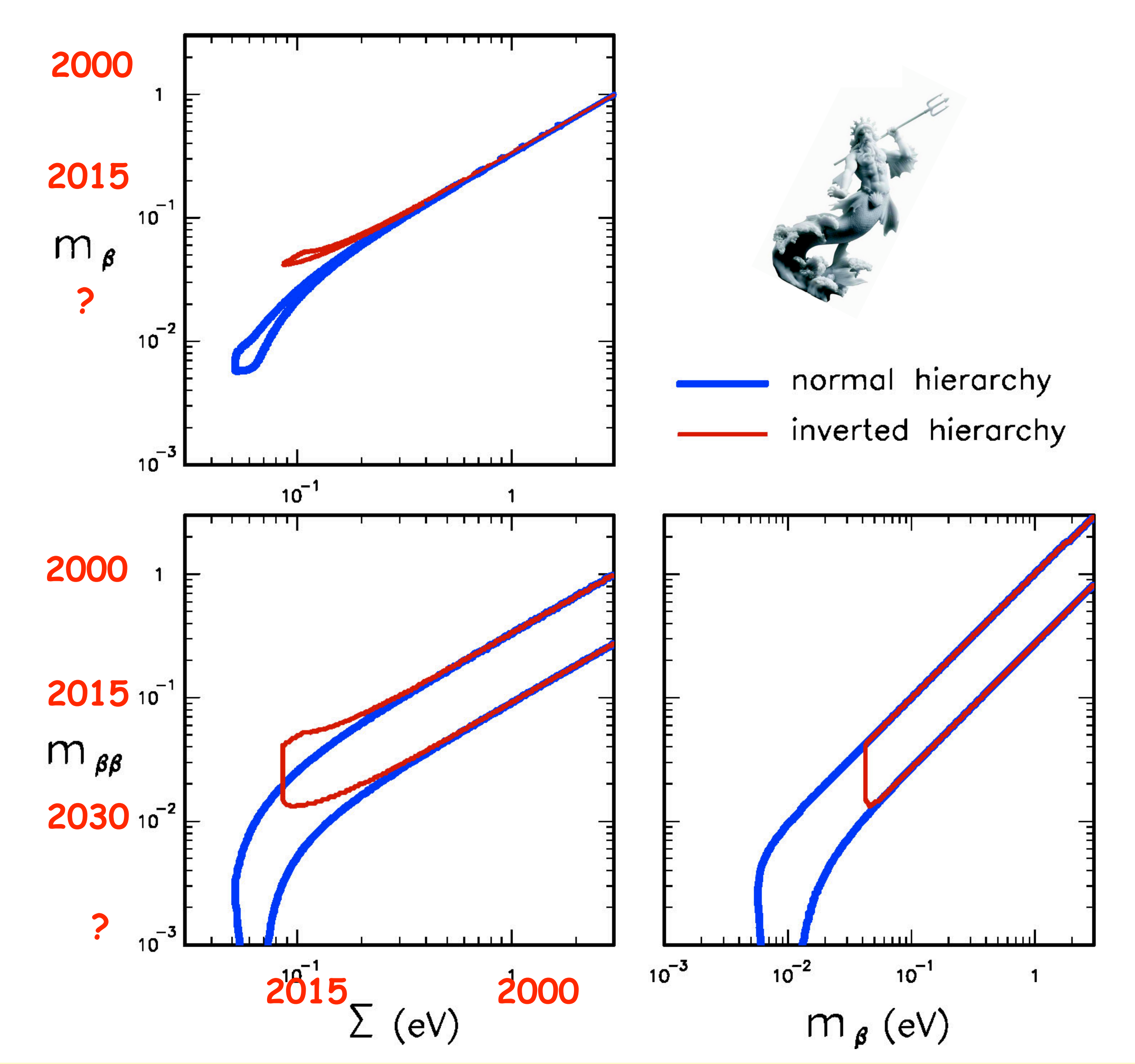


Footnote - Previous plots project away the "unobservable" lightest neutrino mass from graphs like:



Taken from Strumia and Vissani, 2006

History plots → "Moore's law": factor of ~10 improvement every ~15 years



Such "logarithmic progress" seems to be:

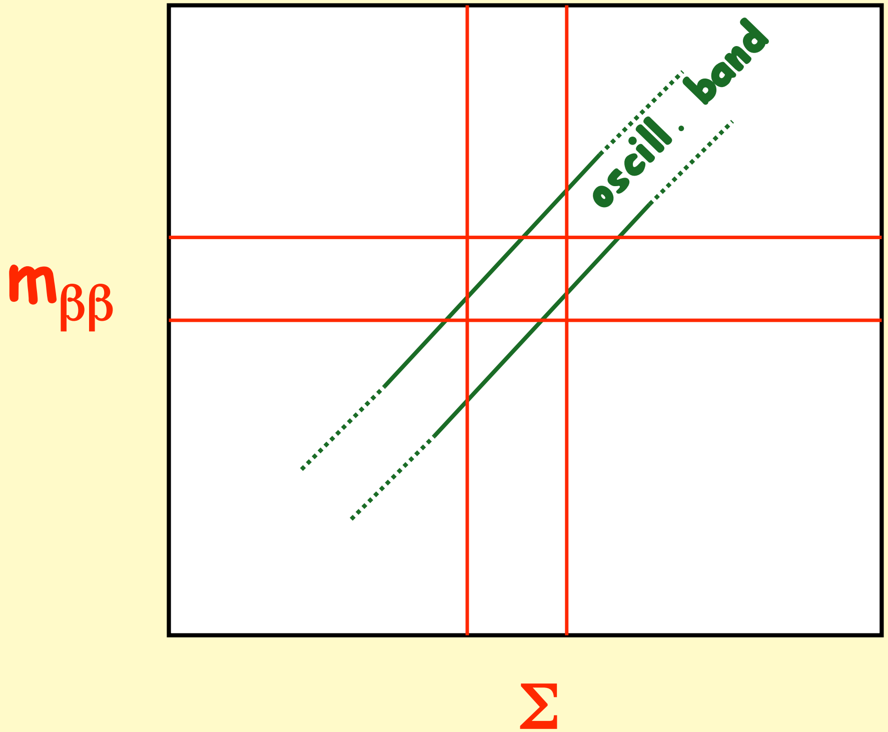
- maybe slowing for β decay (after KATRIN)

- continuing for $0\nu2\beta$ decay

- "accelerating" for cosmology: the only probe where the ultimate goal **(Σ_{min} = $\sqrt{\Delta m^2} \approx$ 0.05 eV)** is **claimed to be reachable**

You have good chances to see first successful results within your career!

Generic expectations: **In the absence of new physics** (beyond 3ν masses and mixing), any two data among **(m_β, $m_{\beta\beta}$, Σ)** are expected to **cross the oscillation band**



This requirement provides either an important consistency check or, if not realized, an indication for new physics (barring expt mistakes)

⇒ Data accuracy/reliability/redundance are crucial

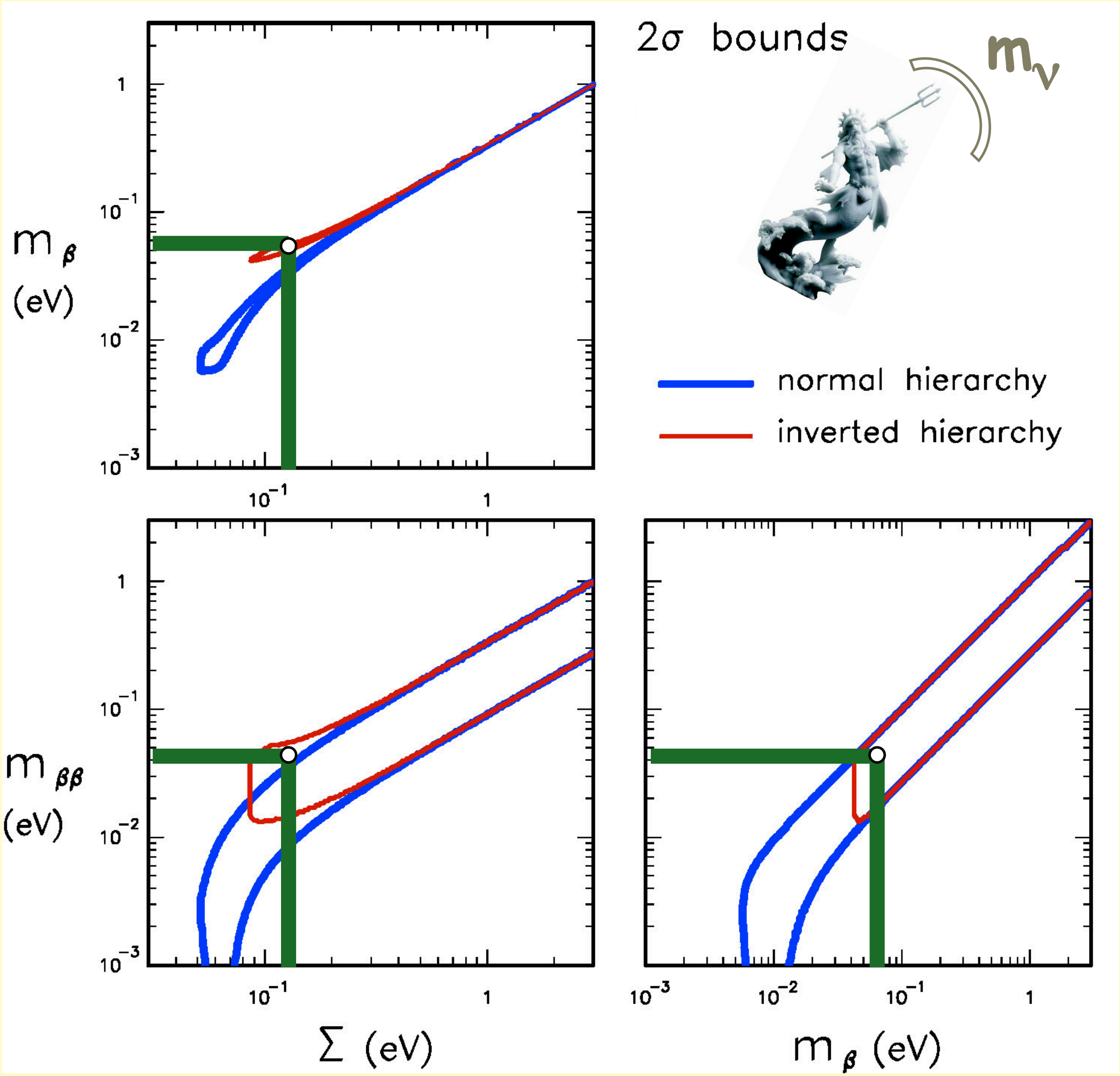
With “dreamlike” nonoscillation data one could, e.g.

Determine the mass scale…

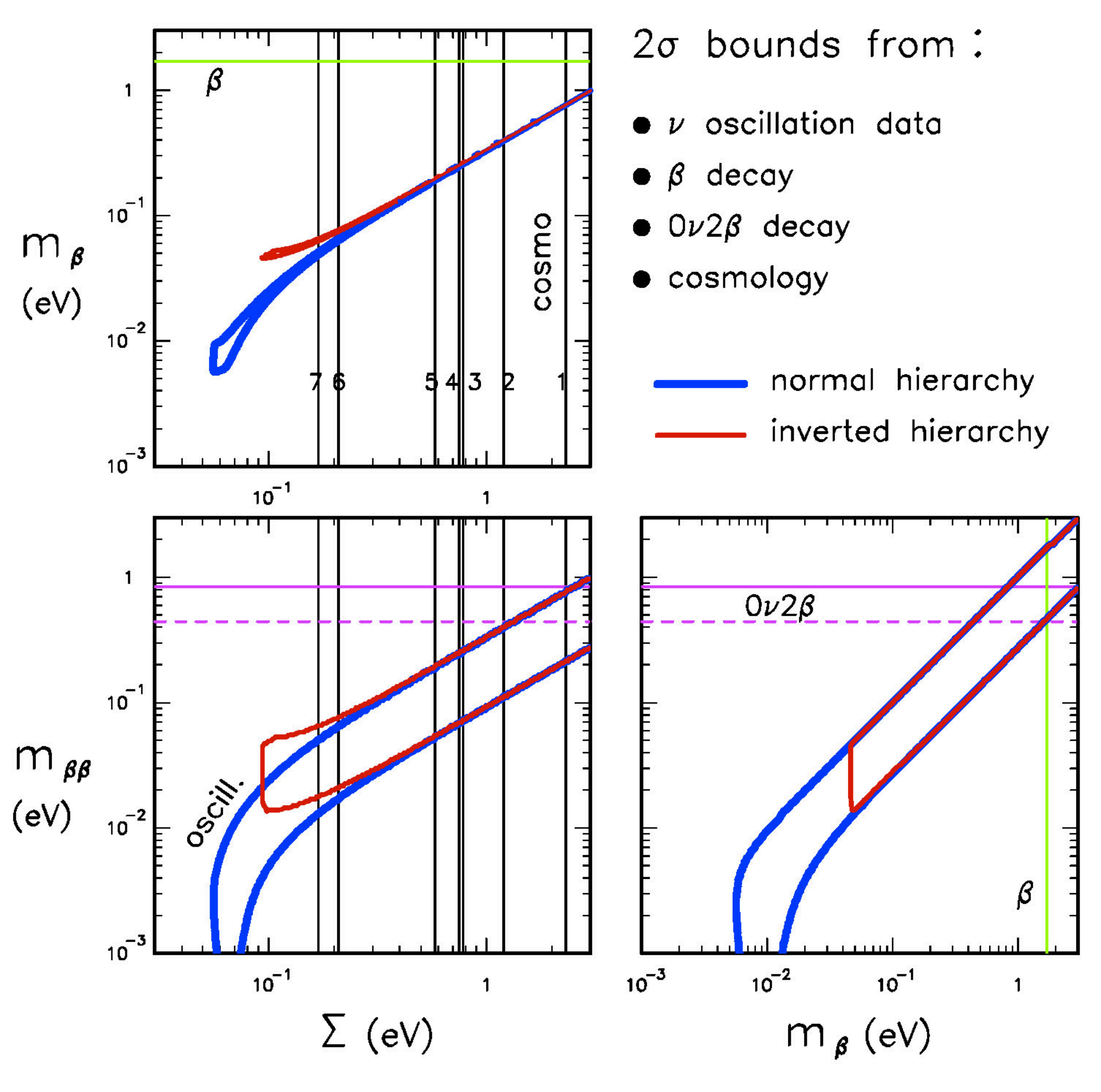
Check 3ν consistency …

Identify the hierarchy …

Probe the Majorana phase(s) …



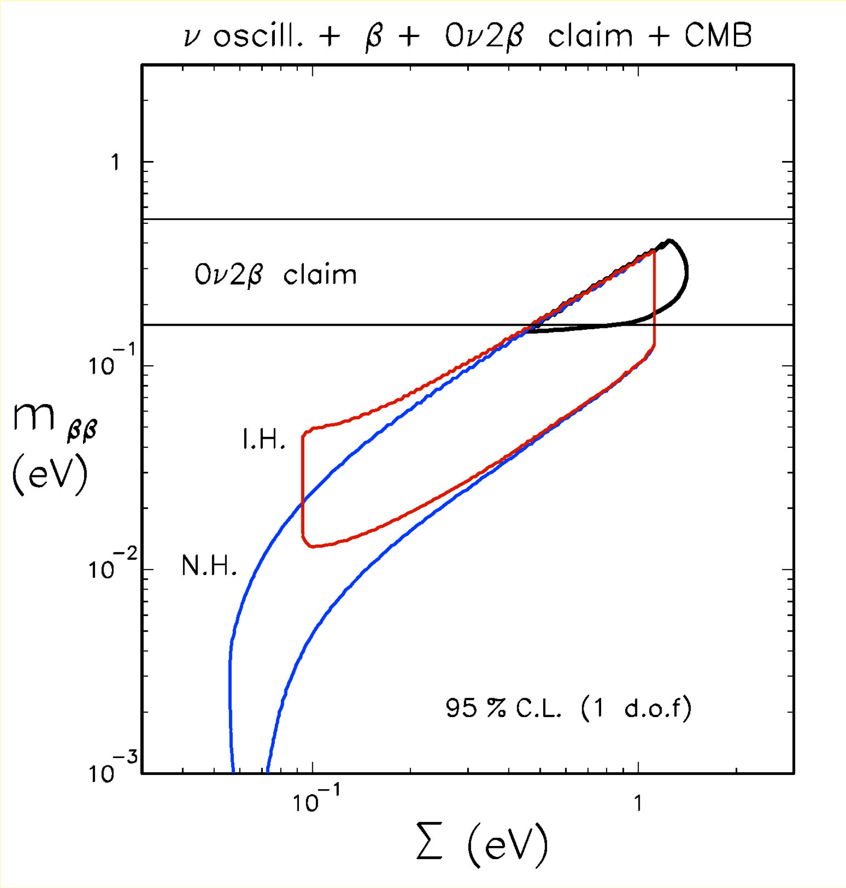
We are still far from this situation (an example with data a few yrs ago):



Different choices ⇒ Different possible combinations (and implications)

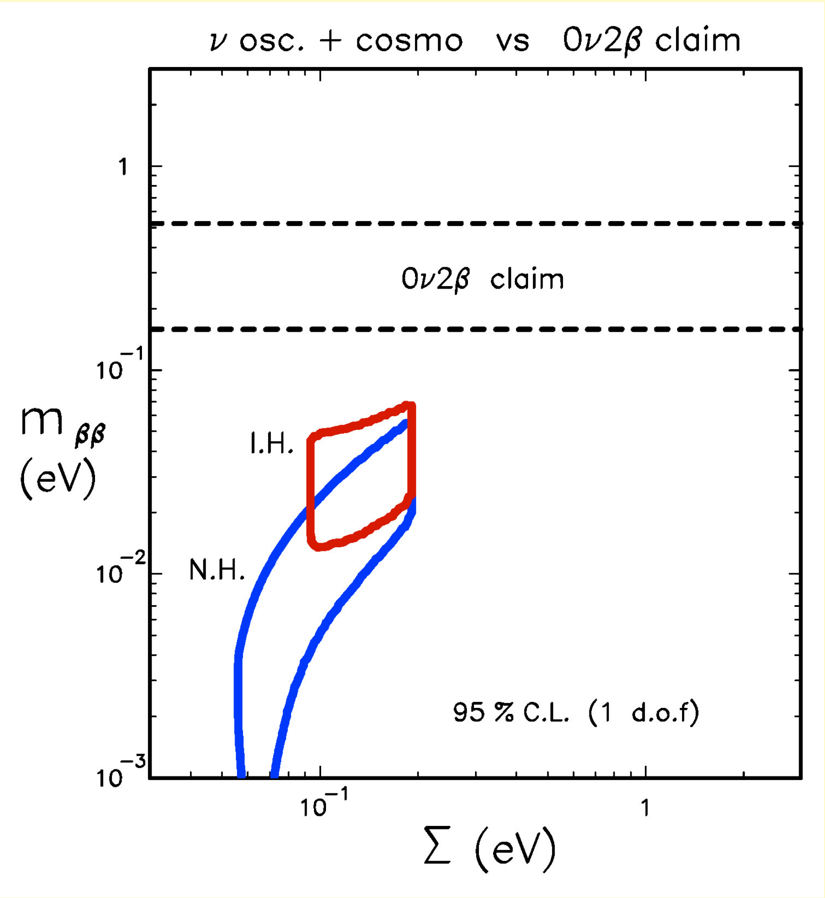
Current situation inconclusive, e.g., wrt to a disputed $0\nu2\beta$ claim

"Conservative" cosmo limits:



limits can match…

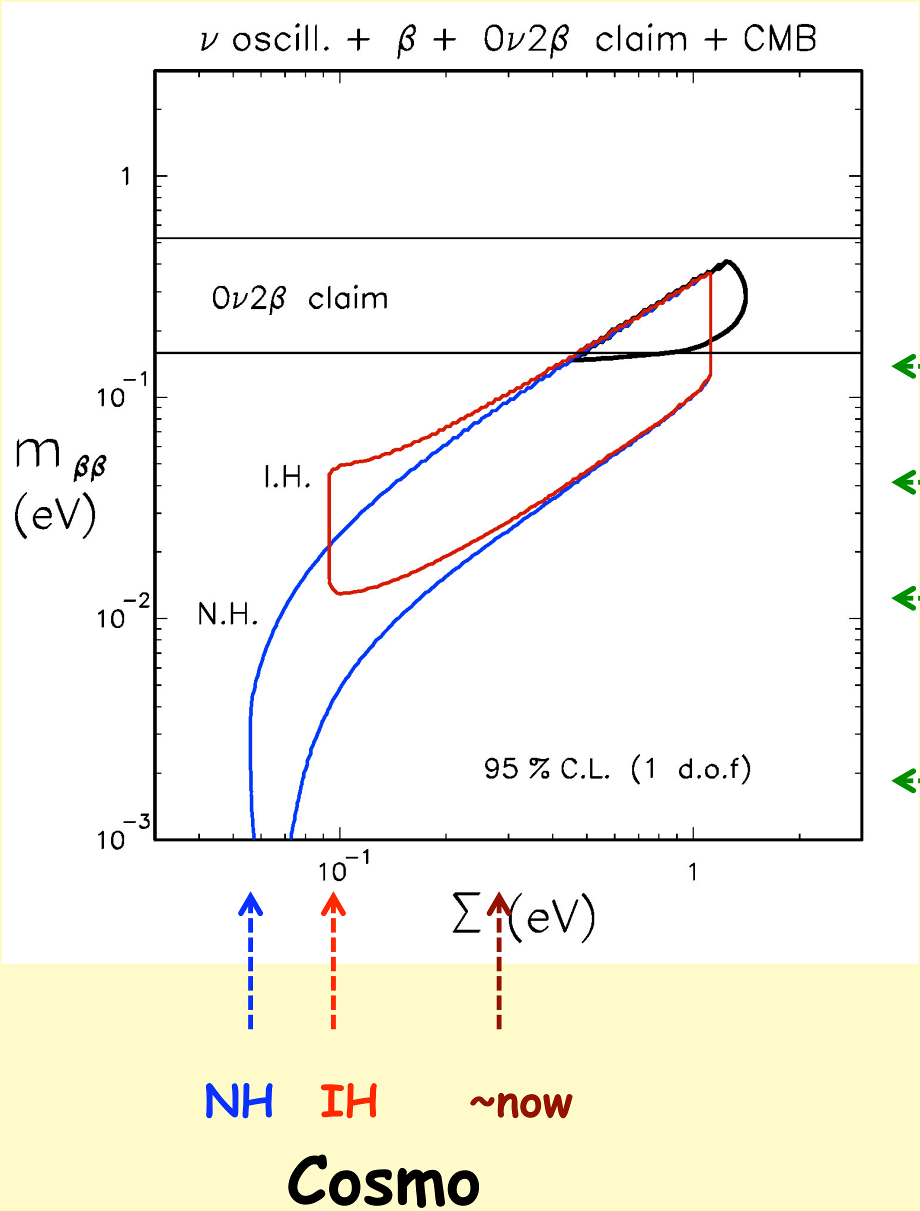
"Aggressive" cosmo limits:



limits don't match…

[Note: consider also that the "standard" cosmological model might require revision: extra radiation, dynamical DE, DE-DM interactions…]

Staged approach - sensitivity goals



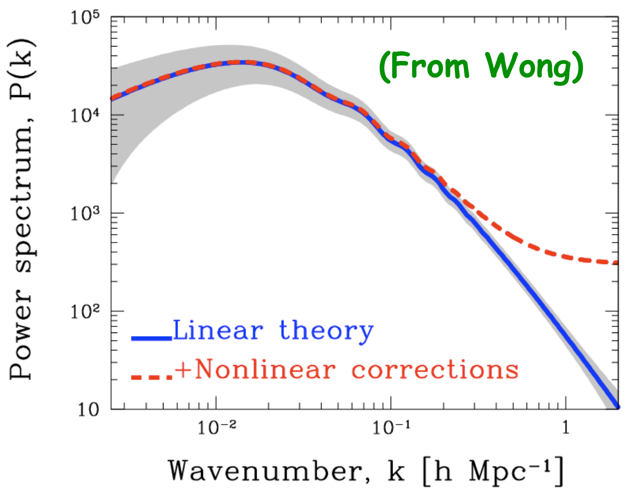
$0\nu2\beta$

1: (Dis)prove current claim

2: Approach inverted hierarchy

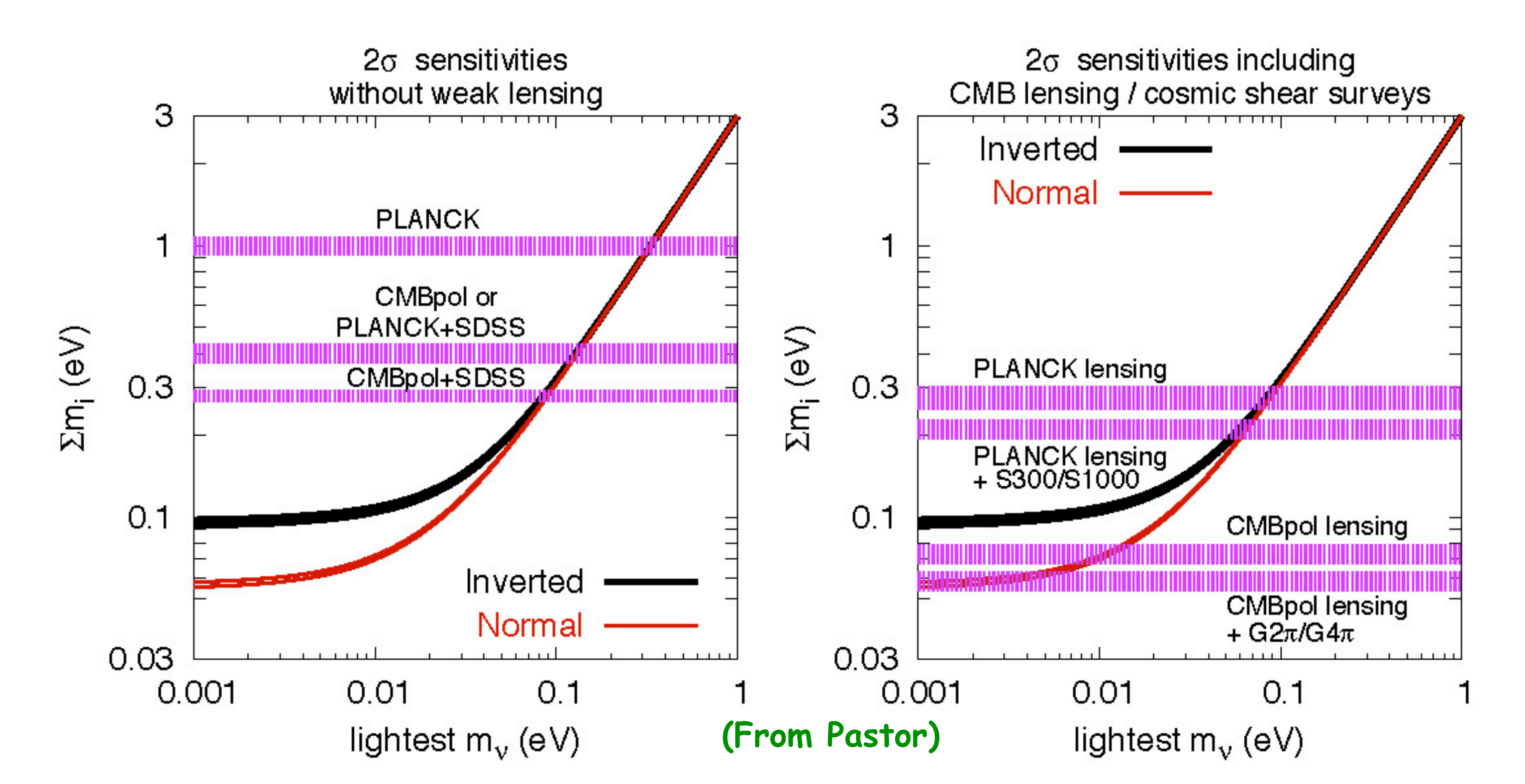
3: Cover inverted hierarchy

(Need new ideas to get there...)



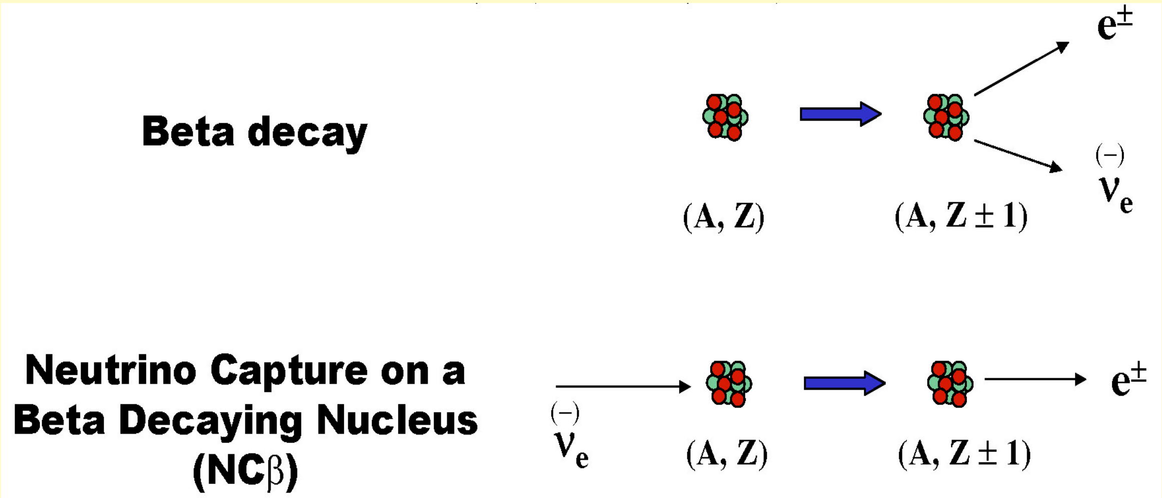
Slicing in redshift bins via lensing data will allow sensitivities close to $\sqrt{\Delta m^2}$ and thus relevant to probe the hierarchy ….

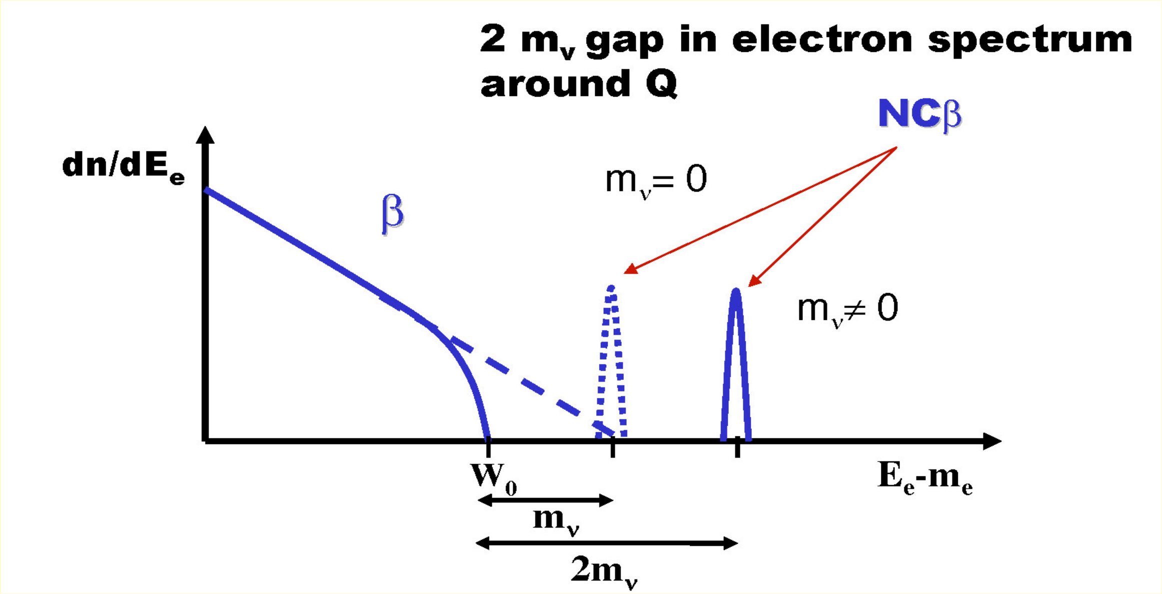
… provided that numerical or semianalytical calculations can reach the 1% level of accuracy
→ next challenge for precision cosmology



Ultimate dreams about β decay and BBN neutrinos..

Very far future … a possible observation of the relic neutrino bkgd ?





(Cocco, Mangano & Messina)

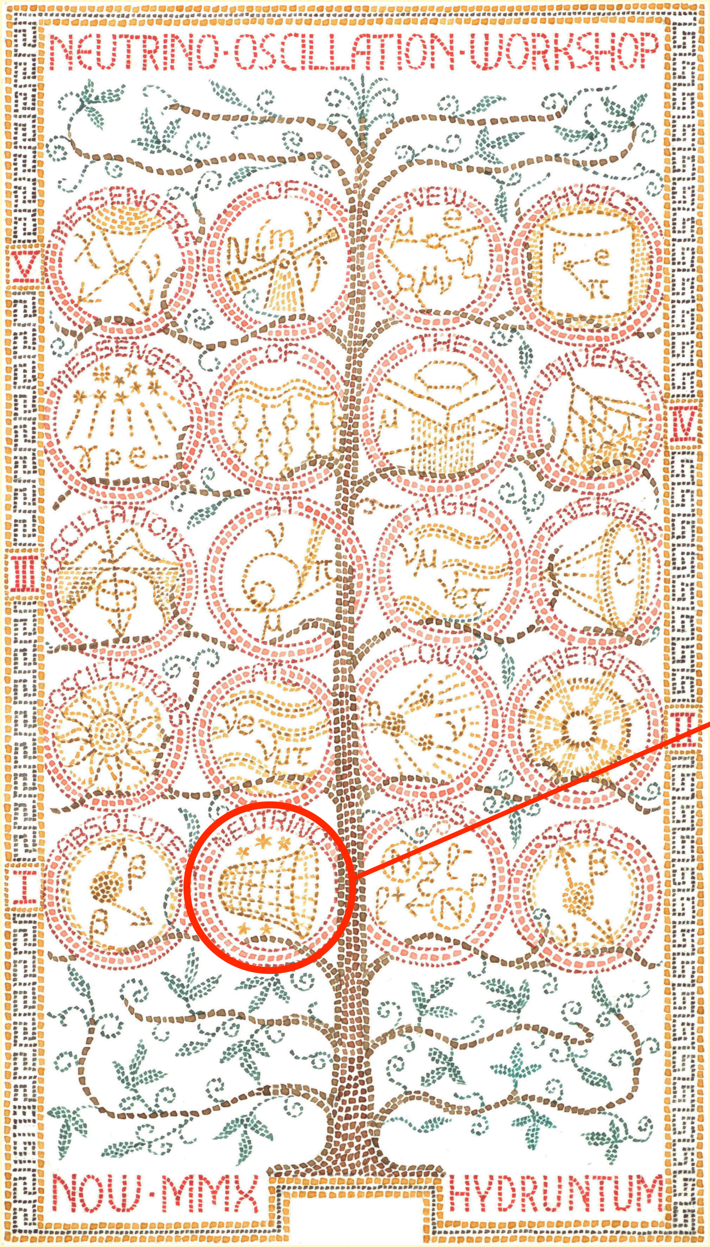
Conclusions and Open Problems

Great progress in recent years ...

Neutrino mass & mixing: established fact
Determination of $(\delta m^2, \theta_{12})$ and $(\Delta m^2, \theta_{23})$
Upper bounds on θ_{13}
Observation of (half)-period of oscillations
Direct evidence for solar ν flavor change
Evidence for matter effects in the Sun
Upper bounds on ν masses in (sub)eV range
…………

Determination of θ_{13}
Flavor appearance searches
Leptonic CP violation
Absolute m_ν from (2)β-decay and cosmology
Test of $0\nu2\beta$ claim and of Dirac/Majorana ν
Matter effects in the Earth, Supernovae…
Normal vs inverted hierarchy
(Dis)confirmation of standard 3ν scenario
Deeper theoretical understanding
Neutrino geo- and astro-physics
…………

… and great challenges for the future!



The neutrino tree continues to grow…

Many opportunities open for your research activity

Be protagonist of ν mass searches in cosmology!

Thank you for your attention.

NOW 2010 Poster: www.ba.infn.it/now

HOMEWORK

Solution 1

- If ν's are Dirac, then $\nu_e \neq \bar{\nu}_e$, and one can attach a leptonic number to the doublets (ν_e, e^-) and $(\bar{\nu}_e, e^+)$, which is conserved in the observed reactions $(\Delta L = 0)$ and would be violated in the other two $(\Delta L = 2)$.

- If ν's are Majorana, then $\nu_e = \bar{\nu}_e$, and we are just naming:

  "ν_e" = LH component of ν state

  "$\bar{\nu}_e$" = RH component of ν state

  The initial "ν_e" is LH, being produced in a weak (β^+) decay. While propagating, it remains dominantly LH, but can develop a small RH component ("$\bar{\nu}_e$") at $\mathcal{O}(m/E)$. Then also the reaction $\bar{\nu}_e + n \to p + e^-$ can take place in principle, but is so suppressed to be practically unobservable. Lepton number violation $(\Delta L = 2)$ is allowed in principle, but suppressed at $\mathcal{O}(m/E)$ in practice.

Solution 2

- Mass basis $\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$ and flavor basis $\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}$ are related by:

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad \text{with } \Delta m^2 = m_2^2 - m_1^2$$

- Evolution equation in mass basis (mb):

$$i \frac{d}{dt} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \mathcal{H}_{mb} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad \leftarrow \text{Schrödinger eq. in natural units } (\hbar = c = 1)$$

where the Hamiltonian is simply

$$\mathcal{H}_{mb} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \simeq \begin{pmatrix} p + \frac{m_1^2}{2E} & 0 \\ 0 & p + \frac{m_2^2}{2E} \end{pmatrix} = \underbrace{\left(p + \frac{m_1^2 + m_2^2}{4E} \right) \mathbb{1}}_{\propto \mathbb{1}} + \underbrace{\begin{pmatrix} -\frac{\Delta m^2}{4E} & 0 \\ 0 & +\frac{\Delta m^2}{4E} \end{pmatrix}}_{\text{Traceless}}$$

Final results do not depend on the part proportional to $\mathbb{1}$ – check it.
(Reason: it gives an overall phase which disappears in observable real quantities). So we take:

$$\mathcal{H}_{mb} = \frac{\Delta m^2}{4E} \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}$$

Solution 2 (ctd)

- Evolution operator in mass basis:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_t = S_{mb} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_0 \quad \text{where}$$

$$S_{mb} = e^{-i\mathcal{H}_{mb}t} \simeq e^{-i\mathcal{H}_{mb}x} = \begin{pmatrix} e^{i\frac{\Delta m^2}{4E}x} & 0 \\ 0 & e^{-i\frac{\Delta m^2}{4E}x} \end{pmatrix}$$

$\leftarrow$ $x \simeq t$ for ultrarelativistic neutrinos

- Evolution operator in flavor basis (fb):

$$S_{fb} = U\, S_{mb}\, U^T$$

$$= \cos\left(\frac{\Delta m^2 x}{4E}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i\sin\left(\frac{\Delta m^2 x}{4E}\right)\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

- Amplitudes for flavor transitions:

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_t = S_{fb} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_0 \rightarrow$$

Off-diagonal elements of S_{fb} give amplitudes for $\nu_\alpha \to \nu_\beta$ and $\nu_\beta \to \nu_\alpha$

Solution 2 (ctd)

- Probability of flavor transition is the square modulus of the amplitude:

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha) = \left| -i \sin 2\theta \sin\left(\frac{\Delta m^2 x}{4E}\right)\right|^2$$

$$= \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

($x = L$ in the lecture).

- The diagonal elements of the evolution operator would give the "flavor survival" amplitude. Check that

$$1 - P(\nu_\alpha \to \nu_\beta) = P(\nu_\alpha \to \nu_\alpha) = P(\nu_\beta \to \nu_\beta).$$

$$\to \quad P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

Solution 3

Oscillations depend only on the difference of phases, and thus of neutrino energies. Indeed, the results do not change by an overall shift of the Hamiltonian:

$$H \rightarrow H + \text{const} \cdot \mathbb{1}$$

Since the zero-point energy is irrelevant in this context, the absolute neutrino mass scale is unobservable (in oscillation searches).

Solution 4

$\hbar c = 197.327$ MeV $\cdot$ fm $= 1$ in natural units.

Therefore: $1\ \mathrm{MeV} \cdot 1\ \mathrm{m} = 5.0677 \times 10^{12}$

Then: $$\frac{\Delta m^2 L}{4E} = \frac{1}{4}\left(\frac{\Delta m^2}{eV^2}\, eV^2\right)\left(\frac{L}{m}\cdot m\right)\left(\frac{MeV}{E}\cdot\frac{1}{MeV}\right)$$

$$= \frac{1}{4}\left(\frac{1 eV^2 \cdot 1m}{1\ MeV}\right)\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{m}\right)\left(\frac{MeV}{E}\right)$$

$$\frac{1}{4}\frac{eV^2 m}{MeV} = \frac{1}{4}\times 10^{-12}\ \frac{MeV^{\cancel{2}}\cdot 1m}{1\ \cancel{MeV}} = \frac{10^{-12}}{4}(MeV.m) = 0.25\times 10^{-12}\times 5.0677\times 10^{12} = 1.267$$

$$\frac{\Delta m^2 L}{4E} = 1.267\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{m}\right)\left(\frac{MeV}{E}\right) = 1.267\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{km}\right)\left(\frac{GeV}{E}\right)$$

Solution 5

- Mass basis: $\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$, $\Delta m^2 = m_2^2 - m_1^2$
- Flavor basis: $\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$; $\nu_x = \nu_{\mu,\tau}$; $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

- Hamiltonian in vacuum, flavor basis (see Exercise 2):

$$H = U \begin{pmatrix} -\frac{\Delta m^2}{4E} & 0 \\ 0 & +\frac{\Delta m^2}{4E} \end{pmatrix} U^T$$

- Hamiltonian in matter, flavor basis:

$H \to \tilde{H} = H + \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix}$ with $V = \sqrt{2} G_F N_e$ (extra ν_e energy in matter)

- It is convenient to put $\tilde{H}$ in traceless form (extract $\mathrm{tr}(\tilde{H}) \cdot \mathbb{1}$):

$$\tilde{H} = \frac{1}{4E} \begin{bmatrix} A - \cos 2\theta \, \Delta m^2 & \sin 2\theta \, \Delta m^2 \\ \sin 2\theta \, \Delta m^2 & -A + \cos 2\theta \, \Delta m^2 \end{bmatrix} , \quad A = 2VE$$

(diagonalization becomes easier)

Solution 5 (ctd)

- Eigenvalues of $\tilde{H}$: $\pm \frac{\Delta\tilde{m}^2}{4E}$ with $\Delta\tilde{m}^2 = \Delta m^2 \sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}$

- Diagonalizing rotation:

$$\tilde{H} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} \begin{pmatrix} -\frac{\Delta\tilde{m}^2}{4E} & 0 \\ 0 & +\frac{\Delta\tilde{m}^2}{4E} \end{pmatrix} \begin{pmatrix} \cos\tilde{\theta} & -\sin\tilde{\theta} \\ \sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix}$$

with $\sin 2\tilde{\theta} = \dfrac{\sin 2\theta}{\sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}}$; $\cos 2\tilde{\theta} = \dfrac{\cos 2\theta - \frac{A}{\Delta m^2}}{\sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}}$

This is analogous to the vacuum case, with the replacement $\theta \to \tilde{\theta}$ and $\Delta m^2 \to \Delta\tilde{m}^2$. [Note that $\Delta\tilde{m}^2 \sin 2\tilde{\theta} = \Delta m^2 \sin 2\theta$].

- If $A = \text{const}$ (i.e., $\tilde{\theta}$ is constant), then the evolution operator can be obtained by exponentiation as in Exercise 2. Then one gets in a similar way:

$$P(\nu_e \to \nu_x) = \sin^2 2\tilde{\theta} \sin^2\left(\frac{\Delta\tilde{m}^2 L}{4E}\right).$$

Solution 6

Let's prove first that $1\ \frac{\text{mol}}{\text{cm}^3} = 4.267 \times 10^{-9}\ \text{MeV}^3$

with 1 mol = 6.022×10^{23} particles (Avogadro number):

$$1\ \frac{\text{mol}}{\text{cm}^3} = \frac{6.022 \times 10^{23}}{10^{-6}\ \text{m}^3}\left(\frac{\text{MeV}^3}{\text{MeV}^3}\right) = 6.022 \times 10^{29}\ \frac{1}{(\text{m}\cdot\text{MeV})^3}\ \text{MeV}^3 = \frac{6.022 \times 10^{29}}{(5.0677 \times 10^{12})^3}\ \text{MeV}^3$$

$$= 4.627 \times 10^{-9}\ \text{MeV}^3$$

see Exercise 4

Then: $A = 2\sqrt{2}\, G_F N_e E$ with $G_F = 1.16637 \times 10^{-5}\ \text{GeV}^{-2} = 1.16637 \times 10^{-11}\ \text{MeV}^{-2}$ (Fermi const.)

$$\frac{A}{\Delta m^2} = \frac{2\sqrt{2}\, G_F N_e E}{\Delta m^2} = 2\sqrt{2}\,(1.16637 \times 10^{-11}\ \text{MeV}^{-2})\left(\frac{N_e}{\text{mol/cm}^3}\cdot \text{mol/cm}^3\right)\left(\frac{E}{\text{MeV}}\cdot\text{MeV}\right)\left(\frac{\text{eV}^2}{\Delta m^2}\cdot\frac{1}{\text{eV}^2}\right)$$

$$= 3.299 \times 10^{-11}\ \frac{\text{MeV}^{-2}\,\text{MeV}}{\text{eV}^2}\ \frac{\text{mol}}{\text{cm}^3}\left(\frac{N_e}{\text{mol/cm}^3}\right)\left(\frac{E}{\text{MeV}}\right)\left(\frac{\text{eV}^2}{\Delta m^2}\right)$$

$$3.299 \times 10^{-11}\ \frac{\text{MeV}^{-2}\,\text{MeV}}{\text{eV}^2}\ \frac{\text{mol}}{\text{cm}^3} = 3.299 \times 10^{-11}\ \frac{10^{12}}{\cancel{\text{MeV}^3}} \times 4.627 \times 10^{-9}\ \cancel{\text{MeV}^3} = 1.526 \times 10^{-7}$$

$$\frac{A}{\Delta m^2} = 1.526 \times 10^{-7}\left(\frac{N_e}{\text{mol/cm}^3}\right)\left(\frac{E}{\text{MeV}}\right)\left(\frac{\text{eV}^2}{\Delta m^2}\right)$$