noncommutative spectral geometry as an approach to unification

mairi sakellariadou

UNIVERSENET
MRTN-CT-2006-035863

king's college london

the 4th universenet school

frontiers of particle cosmology

13-18 september 2010, lecce

outline

- motivation
- noncommutative geometry (NCG) spectral action
- cosmological consequences

  corrections to einstein's equations

  nelson, sakellariadou, PRD 81 (2010) 085038

  gravitational waves

  nelson, ochoa, sakellariadou, arXiv:1005.4276

  nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

  inflation

  nelson, sakellariadou, PLB 680 (2009) 263

  buck, fairbairn, sakellariadou, PRD 82 (2010) 043509
- conclusions

motivation

cosmology

early universe cosmological models can be tested with very accurate astrophysical data, while high energy experiments (LHC) will test some of the theoretical pillars of these models

cosmological models have been built upon:

- string theory
- LQG, SF, WdW, CDT, CS,...

- noncommutative geometry spectral action

particle physics

laws of physics at low energies:

$$S_{\text{Einstein}-\text{Hilbert}} + S_{\text{Standard Model}}$$

depends on geometry of manifold $(\mathcal{M}, g)$

GR is governed by diffeomorphism invariance

(outer automorphism)

depends on internal symmetries of a gauge group G

gauge symmetries are based on local gauge invariance

(inner automorphism)

the difference between these two kinds of symmetries is responsible for not finding a unified theory of all interactions including gravity

noncommutative geometry
spectral action

ali chamseddine, alain connes

SM of electroweak and strong interactions:

a phenomenological model, which dictates geometry of space-time, so that the maxwell-dirac action functional produces the SM

a geometric space defined by the product $\mathcal{M} \times \mathcal{F}$ of a continuum compact riemannian manifold $\mathcal{M}$ and a tiny discrete finite noncommutative space $\mathcal{F}$ composed of 2 points

the geometry is the tensor product of an internal geometry for the SM and a continuous geometry for space-time

NCG approach is based on 3 ansatz:

1. at some energy level, ST is the product $\mathcal{M} \times \mathcal{F}$ of a continuous 4dim manifold $\mathcal{M}$ times a discrete noncommutative space $\mathcal{F}$

the noncommutative nature of $\mathcal{F}$ is given by a spectral triple

$$\mathcal{F} = (\mathcal{A}, \mathcal{H}, D)$$

associative algebra

self-adjoint operator in $\mathcal{H}$

complex Hilbert space carrying a representation of the algebra

- the fermions of the SM provide the hilbert space $\mathcal{H}$ of a spectral triple for the algebra $\mathcal{A}$

- the bosons of the SM (including the higgs boson) are obtained through inner fluctuations of the dirac operator of the product $\mathcal{M} \times \mathcal{F}$ geometry

11. the finite dimensional algebra is (main input):

$$\mathcal{A} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})$$

$k = 2a$

algebra of quaternions

algebra of complex $k \times k$ matrices

$k = 4$ is the first value that produces the correct number of fermions in each generation; $k^2 = 16$ in each of 3 generations

chamseddine, connes (2007)

III. Dirac operator connects the two pieces of product geometry nontrivially

<u>spectral</u> action principal

the action functional depends only on the <u>spectrum</u> of the Dirac operator and is of the form:

$$Tr(f(D/\Lambda))$$

it only accounts for the bosonic part of the model

cut-off function

fixes the energy scale

physical dim of a mass; no absolute scale on which they can be measured for 4dim riemannian geometry, the asymptotic expansion of the trace in terms of the geometrical seeley de witt coefficients is:

$$\mathrm{Tr}\left(f\left(\frac{D}{\Lambda}\right)\right) \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4 + \Lambda^{-2k} f_{-2k} a_{4+2k} + \cdots$$

the smooth even function f, which decays fast at infinity, only enters at the multiplicative factors:

$$\begin{aligned}
f_4 &= \int_0^\infty f(u) u^3 du \,, \\
f_2 &= \int_0^\infty f(u) u du \,, \\
f_0 &= f(0) \,, \\
f_{-2k} &= (-1)^k \frac{k!}{(2k)!} f^{(2k)}(0)
\end{aligned}$$

since f is a cut-off function, its taylor expansion at zero vanishes, so the asymptotic expansion of the trace reduces to:

$$\mathrm{Tr}\left(f\left(\frac{D}{\Lambda}\right)\right) \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4$$

f plays a role through its momenta f_0, f_2, f_4

these 3 additional real parameters are physically related to the coupling constants at unification, the gravitational constant, and the cosmological constant the full lagrangian of SM, minimally coupled to gravity in euclidean form, is obtained* as the asymptotic expansion (in inverse powers of Λ) of the spectral action for the product ST:

chamseddine, connes, marcolli (2007)

* the discussion of phenomenological aspects of the theory relies on a wick rotation to imaginary time

$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu\partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu\partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu -$$
$$\frac{1}{2}\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu\partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu -$$
$$W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu -$$
$$W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)) - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu +$$
$$g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2}\partial_\mu H\partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 -$$
$$\beta_h\left(\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h - g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) -$$
$$\frac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - gMW^+_\mu W^-_\mu H -$$
$$\frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) +$$
$$\frac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) +$$
$$M\left(\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) -$$

the NCG spectral action offers an elegant geometric interpretation of the SM

$$g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^2s_w^2A_\mu A_\mu\phi^+\phi^- + \frac{1}{2}ig_s\,\lambda^a_{ij}(\bar q^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar e^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar\nu^\lambda(\gamma\partial +$$
$$m^\lambda_\nu)\nu^\lambda - \bar u^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j - \bar d^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_wA_\mu\left(-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar d^\lambda_j\gamma^\mu d^\lambda_j)\right) +$$
$$\frac{ig}{4c_w}Z^0_\mu\{(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar d^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + (\bar u^\lambda_j\gamma^\mu(1-\frac{8}{3}s_w^2 +$$
$$\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar\nu^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) +$$
$$\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar e^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar e^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar e^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa)\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar\nu^\lambda\nu^\lambda) - \frac{g}{2}\frac{m^\lambda_e}{M}H(\bar e^\lambda e^\lambda) +$$
$$\frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar\nu^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar e^\lambda\gamma^5 e^\lambda) - \frac{1}{4}\bar\nu_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa - \frac{1}{4}\overline{\bar\nu_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa} +$$
$$\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j)\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar u^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar d^\lambda_j d^\lambda_j) +$$
$$\frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar u^\lambda_j\gamma^5u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j)$$

- full SM lagrangian
- Majorana mass terms for right-handed neutrinos
- gravitational & cosmological terms coupled to matter

> - EH action with a cosmological term
> - topological term related to euler characteristic of ST manifold

> - conformal gravity term with the weyl curvature tensor
> - conformal coupling of higgs to gravity

the coefficients of the gravitational terms depend upon the yukawa parameters of the particle physics content

spectral action is taken at unification scale;
it fixes the boundary conditions at unification scale

the model lives naturally at unification scale

⇒ the NCG spectral action provides early universe models

extrapolations to lower energies: via renormalisation group analysis

extensions to recent universe: considering nonperturbative effects in the spectral action

phenomenology

- number of fundamental fermions is 16
- algebra of the finite space is $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$
- correct representations of fermions w.r.t $SU(3) \times SU(2) \times U(1)$
- higgs doublet and SSB mechanism
- mass of top quark of around 179 GeV
- see-saw mechanism to give very light left-handed ν 's
- relations between gauge couplings: $g_2^2 = g_3^2 = \frac{5}{3} g_1^2$
- $\sin^2 \theta_W = \frac{3}{8}$
- mass of higgs field in zeroth order approximation of spectral action is around 170 GeV

cosmological consequences

corrections to einstein's equations

nelson, sakellariadou, PRD 81 (2010) 085038

bosonic action in euclidean signature:

$$\mathcal{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \,,$$

Yang-Mills

scalar minimal coupling

EH term

Weyl curvature term

cosmological term

scalar mass term

toological, thus nondynamical

$$R^\star R^\star = \tfrac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} \; R^{\alpha\beta}_{\mu\nu} R^{\gamma\delta}_{\rho\sigma}$$

$$\mathbf{H} = (\sqrt{a f_0}/\pi)\phi$$

coupling gravity with matter

scalar quartic potential

from bosonic action, consider the gravitational part including coupling between Higgs field and Ricci curvature

equations of motion

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \frac{1}{\beta^2} \delta_{cc} \left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa} R_{\lambda\kappa} \right] = \kappa_0^2 \delta_{cc} T^{\mu\nu}_{\mathrm{matter}}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2 \alpha_0}$$

$$\delta_{cc} \equiv [1 - 2\kappa_0^2 \xi_0 \mathbf{H}^2]^{-1}$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2}$$

nelson, sakellariadou, PRD 81 (2010) 085038

neglect nonminimal coupling between geometry and higgs

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \frac{1}{\beta^2} \delta_{cc} \left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa} R_{\lambda\kappa} \right] = \kappa_0^2 \delta_{cc} T^{\mu\nu}_{\mathrm{matter}}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2 \alpha_0}$$

$$\delta_{cc} \equiv [1 - 2\kappa_0^2 \xi_0 \mathbf{H}^2]^{-1}$$

$$\delta_{cc} = 1$$

FLRW:
weyl tensor vanishes, so NCG corrections to einstein eq. vanish

corrections to einstein's eqs. will be apparent at leading order, only in anisotropic models

bianchi V

integer

$$g_{\mu\nu} = \mathrm{diag}\left[-1, \{a_1(t)\}^2 e^{-2nz}, \{a_2(t)\}^2 e^{-2nz}, \{a_3(t)\}^2\right]$$

arbitrary functions

$$\kappa_0^2 T_{00} =$$

$$-\dot{A}_3\left(\dot{A}_1+\dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right)$$

$$+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\left[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2 - \dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\right]$$

$$-\frac{4\alpha_0\kappa_0^2}{3}\sum_i \Big\{ \dot{A}_1\dot{A}_2\dot{A}_3\dot{A}_i + \dot{A}_i\dot{A}_{i+1}\left(\left(\dot{A}_i - \dot{A}_{i+1}\right)^2 - \dot{A}_i\dot{A}_{i+1}\right)$$

$$+\left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left[-\ddot{A}_i - \left(\dot{A}_i\right)^2 + \frac{1}{2}\left(\ddot{A}_{i+1} + \ddot{A}_{i+2}\right) + \frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\right]$$

$$+\left[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\right] \times \left[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right]\Big\}$$

same order as standard EH term, but $\propto n^2$ so it vanishes for homogeneous types of bianchi V

$$A_i(t) = \ln a_i(t)$$

for slowly varying functions: small corrections

nelson, sakellariadou, PRD 81 (2010) 085038

$$\kappa_0^2 T_{00} =$$

$$-\dot{A}_3\left(\dot{A}_1+\dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right)$$

$$+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\left[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2 - \dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\right]$$

$$+\frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\Big]$$

$$+\left[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\right]$$

$$\times\left[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right]\Big\}$$

same order as standard EH term, but $\propto n^2$ so it vanishes for homogeneous types of bianchi V

$$A_i(t) = \ln a_i(t)$$

for slowly varying functions: small corrections

neglecting nonminimal coupling between geometry and higgs field, NCG corrections to einstein's eqs. are present only in inhomogeneous and anisotropic space-times

nelson, sakellariadou, PRD 81 (2010) 085038

at energies approaching higgs scale, the nonminimal coupling of higgs field to curvature cannot be neglected

e.o.m. (neglecting conformal term, for simplicity):

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \kappa_0^2 \left[\frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2/6} \right] T^{\mu\nu}_{\mathrm{matter}}$$

the effect of a nonzero higgs field is to create an effective gravitational constant

nelson, sakellariadou, PRD 81 (2010) 085038

gravitational waves in NCG

nelson, ochoa, sakellariadou, arXiv:1005.4276

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602 weak limit of NCG gravitational theory shows:

- GW are only sourced from systems with nontrivial quadrupole moment (as in GR)

  (due to energy momentum conservation)

- propagation of GW is significantly altered, by the presence of additional massive modes linear perturbations:

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2\left[-1, \left(\delta_{ij} + h_{ij}\left(x\right)\right)\right]\right)$$

energy lost to gravitational radiation by orbiting circular binaries:

$$-\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \approx \frac{c^2}{20G}|\mathbf{r}|^2 \dot{h}_{ij}\dot{h}^{ij}$$

in terms of quadrupole moments

strong deviations from GR at $2\omega_c = \beta c$

the theory has a natural frequency scale $\beta c \sim c(-\alpha_0 G)^{-1}$

scale at which NCG effects become dominant

$$\beta^2 = -\frac{1}{32\pi G\alpha_0}$$

$$\alpha_0 = \frac{-3f_0}{10\pi^2}$$

binaries must have $\omega < \omega_c$

for $\alpha_0 \to 0$ GR cannot be reproduced energy lost to gravitational radiation should agree with GR prediction within observational uncertainties

PSR J0737-3039	$\beta > 7.55 \times 10^{-13}\ \mathrm{m}^{-1}$
PSR J1012-5307	$\beta > 7.94 \times 10^{-14}\ \mathrm{m}^{-1}$
PSR J1141-6545	$\beta > 3.90 \times 10^{-13}\ \mathrm{m}^{-1}$
PSR B1913+16	$\beta > 2.39 \times 10^{-13}\ \mathrm{m}^{-1}$
PSR B1534+12	$\beta > 1.83 \times 10^{-13}\ \mathrm{m}^{-1}$
PSR B2127+11C	$\beta > 2.30 \times 10^{-13}\ \mathrm{m}^{-1}$

future observations of rapidly orbiting binaries, relatively close to the earth, could improve this constraint by many orders of magnitude

amplitude of effects is proportional $(1 - 2\omega/c\beta)^{-1}$

inflation through the nonminimal coupling between the geometry and the higgs field

nelson, sakellariadou, PLB 680 (2009) 263

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509 proposal: the scalar field of the SM, the higgs field, could play the role of the inflaton

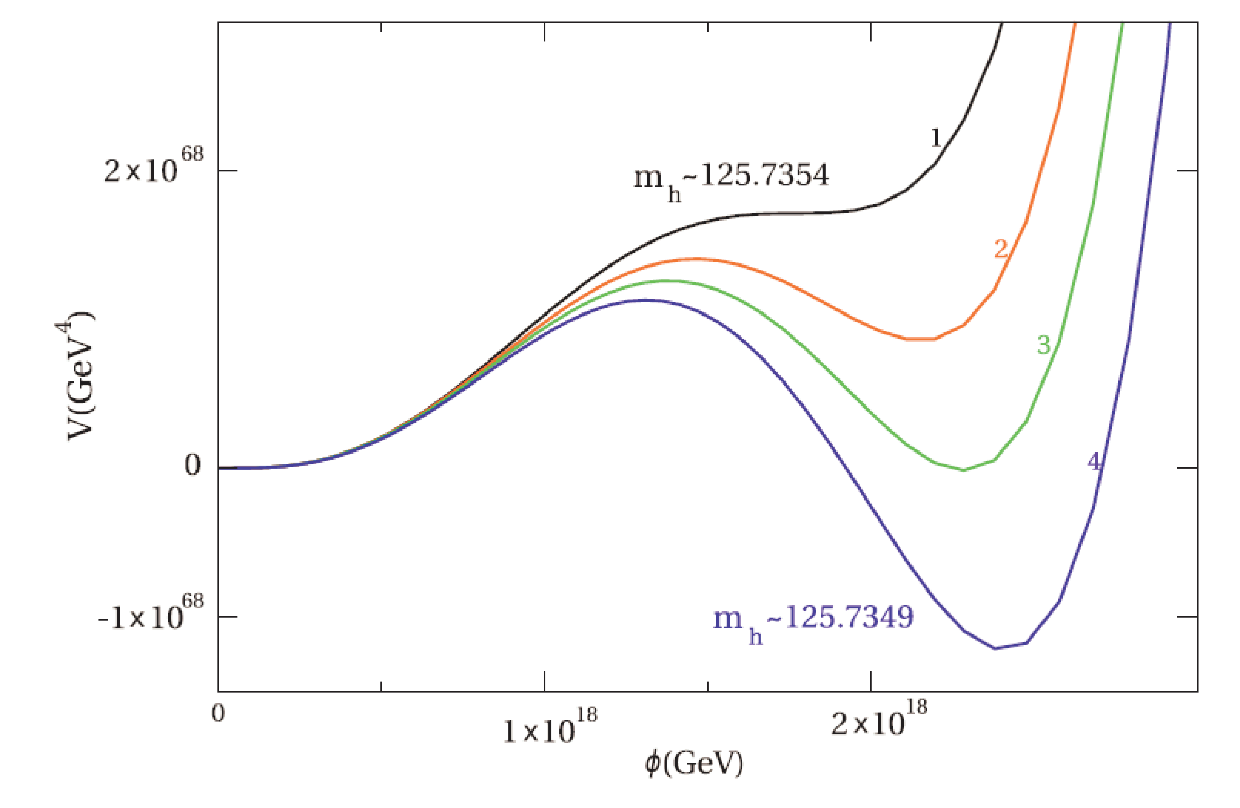
but

within GR cosmology, to get the correct amplitude of density perturbations, the higgs mass would have to be 11 orders of magnitude **higher** than its particle physics value

re-examine the validity of this statement within NCG flat potential through 2-loop quantum corrections of SM

for large values of the field, calculate renormalised higgs coupling

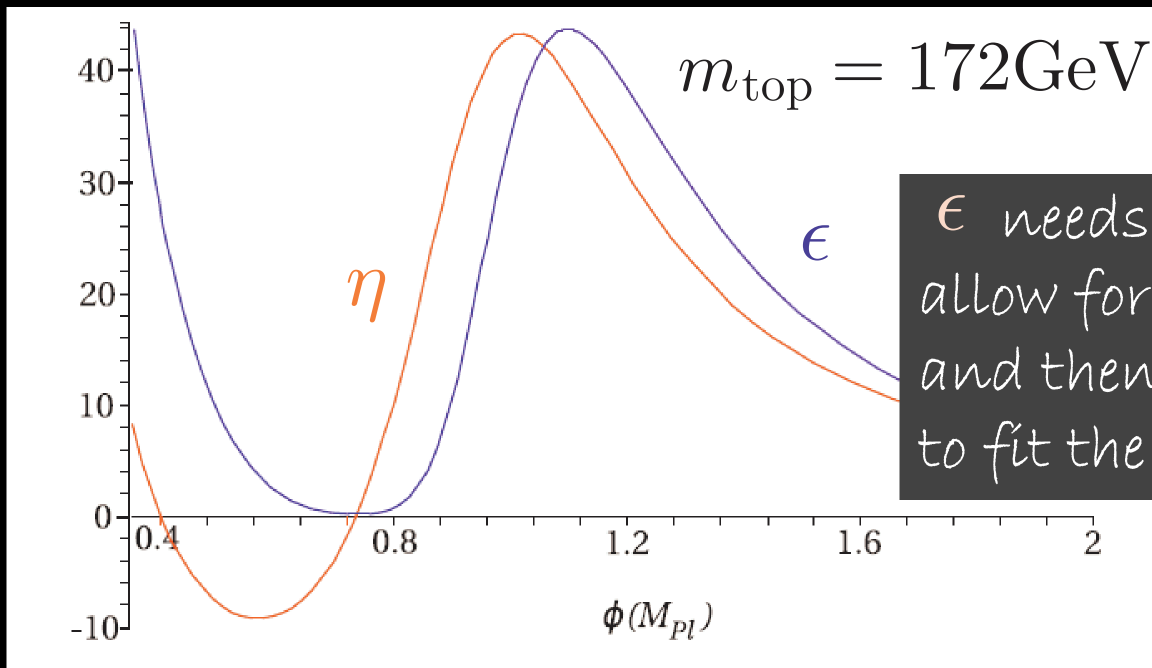
effective potential at high energies: $V^{\text{eff}} = \lambda_0^{\text{eff}}(H) H^4$



for each value of m_{top} there is a value of m_{higgs} where V_{eff} is on the verge of developing a metastable minimum at large values of ϕ and V_{higgs} is locally flattened approach:

- calculate the renormalisation of the higgs self-coupling
- construct an effective potential which fits the renormalisation group improved potential around the flat region

for inflation to occur via the higgs field, the top quark mass fixes the higgs mass extremely accurately

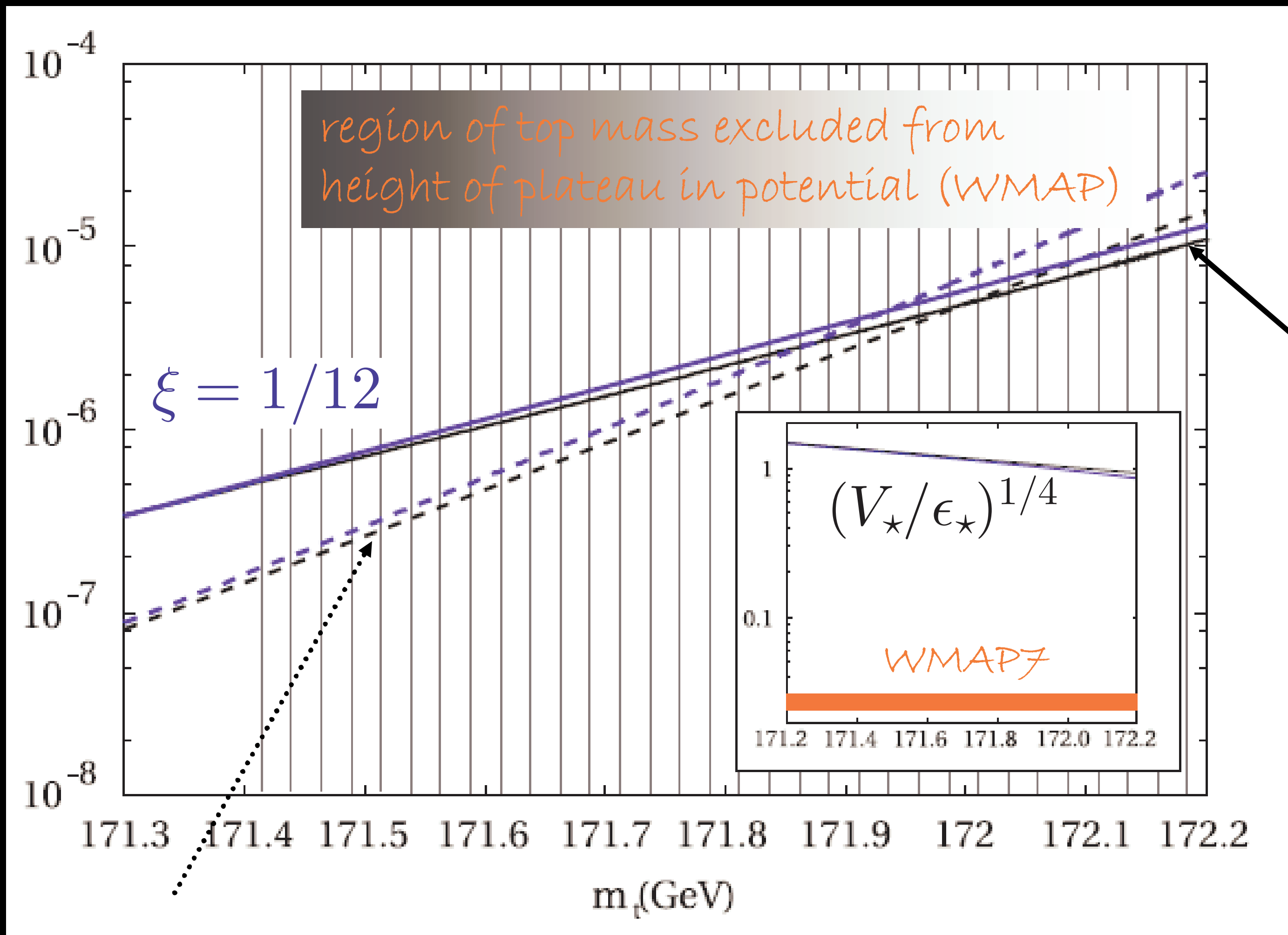


ϵ needs to be too small to allow for sufficient e-folds, and then V becomes too large to fit the CMB constraint

$$\left(\frac{V_*}{\epsilon_*}\right)^{\frac{1}{4}} = (2.75 \pm 0.30) \times 10^{-2}\, m_{\rm Pl}$$

$$\epsilon_* \leq 1$$

$$N \sim \epsilon^{-1/2} \mathrm{d}\phi$$



potential in units of κ^{-4} for minimal coupling

maximum value of the first slow-roll parameter at horizon crossing for minimal coupling while the higgs field potential can lead to the slow-roll conditions being satisfied once the running of the self-coupling at two-loops is included, the constraints imposed from the CMB data make the predictions incompatible with the measured value of the top quark

could ξ be away from its conformal value?

there are no nonconformal values for the coupling ξ for which there is a renormalisation group flow towards the conformal value as one runs the SM parameters up in the energy scale

buchbinder, odintsov, lichtzier (1989)

youngsoo yoon, yongsung yoon (1997)

↓

there are no quantum corrections to ξ, if it is exactly conformal at some energy scale remark

what about conventional cosmological models with $\xi \sim 10^4$?

bezrukov, shaposhnikov (2008)

- effective theory ceases to be valid beyond cut-off scale $m_{\rm Pl}/\xi$ while one should know the higgs potential profile for field values relevant for inflation, namely $m_{\rm Pl}/\sqrt{\xi}$

burgess, lee, trott (2009)
barbon, espinosa (2009)

- models with large nonminimal coupling are also ruled out because of unitarity violation

atkins, calmet (2010)

can we accommodate an inflationary era without introducing (by hand) a scalar field?

the arbitrary mass scale in the spectral action for the Dirac operator can be made dynamical by introducing a dilaton field, which guarantees the scale invariance of the SM interactions, and provides a mechanism to generate mass hierarchies

chamseddine and connes (2006)

could this dilaton field play the role of the inflaton?

buck, sakellariadou (in progress)

conclusions

NCG spectral action extends notion of commutative spaces, using data encoded in a spectral triple on a space composed by the product of a 4-dim smooth compact riemannian manifold and a finite noncommutative space

- offers simple and elegant explanation for SM phenomenology compatible with right-handed neutrinos and neutrino masses

- offers a framework for cosmological applications, while astrophysics may be used to determine its free parameters