Anisotropic stress and stability in modified gravity models

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Goal & Motivation

- Modified gravity models provide an alternative explanation to the late time acceleration of the universe - No need for cosmological constant or scalar fields.

[2] S. F. Daniel and E. V. Linder (2010), arXiv: 1008.0397

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- **Smoking gun**: The detection of non-zero anisotropic stress.
- **Question**: *Is it possible to build a viable modified gravity scenario with a signature sufficiently close to that of GR?*

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What is the significance of anisotropic stress?

- Scalar perturbations around flat FRW metric:

$$\boxed{ds^2 = -(1+2\psi)dt^2 + a(t)^2(1+2\phi)d\mathbf{x}^2}$$

- Anisotropy equation describes the *scalar anisotropic stress* Π:

$$\boxed{\phi - \psi \equiv \Pi(\mathbf{x}, t)}$$

- The ratio ϕ/ψ *is a signature of any gravity model at perturbation level.* (Future surveys, ex. Euclid, Planck: Weak lensing + Galaxy power spectrum + CMB.)

- In GR, *at all times* $\phi/\psi = 1$, unless relativistic species present (negligible at late times).

Anisotropic stress in modified gravity

- The higher order nature of modified gravity models contributes an *effective anisotropic stress* contribution *of geometrical origin*.

$$\boxed{\phi - \psi = \Pi^{(\mathrm{eff})}(g)}$$

- As a result, $\phi/\psi \neq 1 \leftrightarrow$ non zero (effective) anisotropic stress.
- Typical behavior for all higher order gravity models, $f(R)$, $f(R, G)$, DGP, e.t.c.

So, can we have a modified gravity model with a *sufficiently small effective anisotropic stress contribution*, such that it could escape observations?

$\longrightarrow$ Try to attempt a *qualitative analysis* in de Sitter space.

Special case I: Anisotropic stress in $f(R)$ gravity

The most straightforward generalization of GR:

$$S = \int d^4x\sqrt{-g}f(R)$$

- Exhibits an extra scalar degree of freedom ("scalaron") with

$$m_{eff}^2 \propto \frac{1}{f_{RR}}.$$

- Anisotropy equation:

$$\boxed{\phi - \psi = \Pi_R^{(\text{eff})} \equiv \frac{f_{RR}}{f_R}\delta R}$$

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- If we would like $\Pi_R^{(\mathrm{eff})} = 0$ at all times we should require that $f_{RR} = 0$. GR is the only solution $f(R) = R + \Lambda$!

Special case II: Anisotropic stress in $f(G)$ gravity

$$S = \int d^4x \sqrt{-g}\,(R + f(G))$$

- The Gauss–Bonnet term: $G \equiv R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\kappa\lambda}R_{\mu\nu\kappa\lambda}$. [3]
- Gauss–Bonnet term correction to the Einstein–Hilbert term motivated by string theory. [4]
- Extra scalar degree of freedom with $m^2_{eff} \propto \frac{1}{f_{GG}}$.
- The anisotropy equation is similar to $f(R)$ case:

$$\boxed{\phi - \psi \equiv \Pi_G^{(\mathrm{eff})} = 4H^2 f_{GG}\,\delta G}$$

[3] D. Lovelock (1971) *J. Math. Phys.* **12** 3 498–501

[4] B. Zwiebach *Phys. Lett.* B **156** 315 (1985)

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- $\Pi^{(\mathrm{eff})}_G = 0$ at all times $\leftrightarrow S \propto R + G + \Lambda$.

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The more general case: Anisotropic stress in $f(R, G)$ gravity

$$S = \int d^4x \sqrt{-g} f(R, G)$$

- Characterized by *two different contributions*: R and G contribution.
- Anisotropy equation:

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- In order for $\Pi^{(\mathrm{eff})}_{\mathrm{total}} = 0$: $\Pi^{(\mathrm{eff})}_{R} = -\Pi^{(\mathrm{eff})}_{G}$

$$\frac{\partial^2 f}{\partial R^2} + 4H_0^2\frac{\partial^2 f}{\partial R\partial G} + 16H_0^4\frac{\partial^2 f}{\partial G^2} = 0.$$

Solution: $f(R, G) = f_1\left(R - \frac{G}{4M^2}\right) + Rf_2\left(R - \frac{G}{4M^2}\right)$, with $M^2 = H_0^2$

Evolution of gauge invariant potential

$$\boxed{\ddot{\Phi} + 3H\dot{\Phi} + \left(\frac{k^2}{a^2} + m_{eff}^2\right)\Phi = 0}$$

- Effective mass of scalaron:

$$m_{eff}^2 \equiv \frac{f_R}{3\left[f_{RR} + 4H_0^2\left(2f_{GG} + 4H_0^2 f_{GG}\right)\right]} - 4H_0^2 \geq 0.$$

[5] A. A. Starobinsky JETP Lett. **86**, 157-163 (2007)

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- For $f(R, G) = f_1\left(R - \frac{G}{4M^2}\right) + Rf_2\left(R - \frac{G}{4M^2}\right)$, when $M^2 \to H_0^2 \quad \Rightarrow \quad m_{eff} \to \infty$.

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- WKB approximation: $\Phi(t) \approx \sum_{\pm} C_{\pm} e^{(-H_0 \pm 2im_{\rm eff})t}$.

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- $\delta R(t) \propto m_{eff}^2 \Phi(t) \quad \Rightarrow$ large curvature oscillations. (Overproduction of scalarons? [5])

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(Overproduction of scalarons? [5])
- Upper bound: $m_{eff}^2 < M_P^2$.

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The sound speed

The sound speed for $f(R, G) = f_1\left(R - \frac{G}{4M^2}\right) + Rf_2\left(R - \frac{G}{4M^2}\right)$ and *a general expansion* $H \equiv H(t)$:

$$c_s^2 = 1 + \left(\frac{1}{1 - \frac{M^2}{4H^2}}\right)\frac{8\dot{H}}{4H^2}$$

- $\frac{M^2}{4H^2} > 1 \quad \Rightarrow c_s^2 < 0$, $(f(G)$ regime)
- $M \to H_{t=t_0} \Rightarrow c_s^2 \to \infty$, (limiting case)
- $\frac{M^2}{4H^2} < 1 \quad \Rightarrow c_s^2 > 0$, $(f(R)$ regime)

Above three regimes are a *general feature* of all $f(R, G)$ theories. [6]

[6] A. De Felice and T. Suyama, JCAP06, 034 (2009)

Conclusions & future work

- The *effective anisotropic stress* in modified gravity theories is directly linked with the extra scalar degree of freedom that characterizes them.
- If the theory is to be viable, the higher order effects *cannot be arbitrarily suppressed* $\rightarrow$ lower bound will be crucial for ruling out modified gravity observationally.
- Although demonstrated qualitatively for de Sitter spacetime, the same is expected for other cases (as inclusion of matter) $\rightarrow$ numerical investigation needed.[7]
- *Future work* includes putting a lower limit on ϕ/ψ for viable modified gravity models, considering the more general case with the inclusion of matter.

[7] L. Pogosian and A. Silvestri Phys. Rev. D **77**, 023503 (2008)

Thank you !