# Inert Model and the evolution of the Universe

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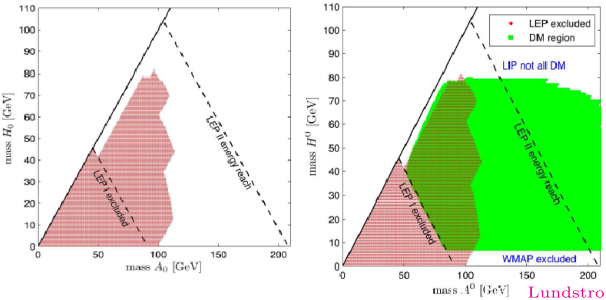
# Motivation

T.D. Lee, ’73

Deshpande, Ma, ’78

Two Higgs Doublet Model (2HDM):

- two scalar $SU(2)_W$ doublets $\Phi_S, \Phi_D$ with the same hypercharge $Y = 1$
- CP violation in the scalar sector (explicit or spontaneous violation)
- different types of extrema (possible violation of $U(1)_{EM}$)
- 2HDM with an exact $Z_2$ symmetry
  $\rightarrow$ candidate for the dark matter (Inert Model)




Lundstrom et al. ’08

# 2HDM

T.D. Lee, '73

Higgs potential $V$ with an explicit $Z_2$ symmetry:

$$Z_2 : \quad \Phi_S \to \Phi_S, \quad \Phi_D \to -\Phi_D$$

$$V = -\frac{1}{2}\left[m_{11}^2 \Phi_S^\dagger \Phi_S + m_{22}^2 \Phi_D^\dagger \Phi_D\right] + \frac{1}{2}\left[\lambda_1\left(\Phi_S^\dagger \Phi_S\right)^2 + \lambda_2\left(\Phi_D^\dagger \Phi_D\right)^2\right]$$
$$+\lambda_3\left(\Phi_S^\dagger \Phi_S\right)\left(\Phi_D^\dagger \Phi_D\right) + \lambda_4\left(\Phi_S^\dagger \Phi_D\right)\left(\Phi_D^\dagger \Phi_S\right) + \frac{1}{2}\lambda_5\left[\left(\Phi_S^\dagger \Phi_D\right)^2 + \left(\Phi_D^\dagger \Phi_S\right)^2\right]$$

- All parameters $\in \mathbf{R}$ – no CP violation
- Model I – only $\Phi_S$ couples to fermions

The positivity constrains are required to have a stable vacuum:

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad R + 1 > 0, \quad R_3 + 1 > 0$$

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad R = \lambda_{345}/\sqrt{\lambda_1 \lambda_2}, \quad R_3 = \lambda_3/\sqrt{\lambda_1 \lambda_2}$$

Positivity constrains → extremum with the lowest energy is the global minimum (vacuum).

# Spontaneous Symmetry Breaking

The EW symmetric extremum:

$$\langle \Phi_S \rangle = \langle \Phi_D \rangle = 0$$

local minimum if $m^2_{11,22} < 0$.

Barroso et al., '05

The general type of EWSB VEV:

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_D \end{pmatrix}$$

$u \neq 0 \Longrightarrow U(1)_{EM}$ broken:

- charge breaking $(Ch)$

$u = 0 \Longrightarrow U(1)_{EM}$ conserved:

- $v_{S,D} \neq 0$ neutral mixed $(M)$
- $v_D = 0$ neutral Inert $(I_1)$

  Deshpande, Ma, '78; Barbieri, Hall, Rychkov, '06
- $v_S = 0$ neutral Inert-like $(I_2)$

# Charge breaking and Mixed extrema

**Charge breaking extremum $Ch$:**

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ 0 \end{pmatrix}$$

- $U(1)_{EM}$ symmetry broken by $u \neq 0$ – massive photon
- not a case that is realized now, a possible vacuum in the past if

$$\lambda_4 \pm \lambda_5 > 0, \quad R_3 < 1$$

**Mixed extremum $M$:**

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_D \end{pmatrix}$$

- CP conserving, $\tan\beta = v_D/v_S$
- massive $Z^0, W^\pm$, massless photon, 5 physical Higgs bosons $H^\pm, A, H, h$

$$R < 1, \quad \lambda_4 + \lambda_5 < 0, \quad \lambda_5 < 0$$

# Inert and Inert-like

Deshpande, Ma, '78, Barbieri et al., '06

**Inert extremum $I_1$:**

$$\langle\Phi_S\rangle = \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle\Phi_D\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- exact $Z_2$ symmetry – both in Lagrangian and in the extremum
- only $\Phi_D$ has odd $Z_2$-parity

  $\rightarrow$ the lightest scalar is a candidate for the dark matter
- $\Phi_S$ as in SM (SM-like Higgs boson $h$)

  $\Phi_D$ – "dark" or inert doublet with 4 dark scalars $(H, A, H^{\pm})$, no interaction with fermions

**Inert-like extremum $I_2$:**

$$\langle\Phi_S\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle\Phi_D\rangle = \begin{pmatrix} 0 \\ v_D \end{pmatrix}$$

- $\Phi_S$ and $\Phi_D$ exchange roles
- fermions massless at tree-level (Model I)

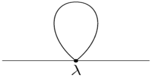
# Evolution of the Universe

Ivanov '08

- We assume that today Inert Model is realized, however, in the past some other extrema could have been lower.
- We consider evolution of the Universe due to the thermal corrections to the potential.
- At finite $T$ ground state is given by minimum of Gibbs potential:

$$V_G(T) = Tr(Ve^{-H/T})/Tr(e^{-H/T}) \equiv V(T=0) + \Delta V(T)$$

- $\Delta V(T)$ – leading corrections $\propto T^2$ given by diagrams:




$\Longrightarrow$ fixed quartic terms, quadratic (mass) terms change with T

$$\Delta V(T) = \frac{1}{2} c_1 T^2 \Phi_S^\dagger \Phi_S + \frac{1}{2} c_2 T^2 \Phi_D^\dagger \Phi_D$$

# Evolution of the Universe

From scalar, bosonic and fermionic contributions to $\Delta V$:

$$m_{11}^2(T) = m_{11}^2 - c_1 T^2\,, \quad m_{22}^2(T) = m_{22}^2 - c_2 T^2$$

$$c_1 = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{32} + \frac{g_t^2 + g_b^2}{8}, \quad c_2 = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{32}$$

- fermionic contribution in $c_1$ because of Model I
- $c_1$ and $c_2$ positive to restore EW symmetry in the past
- $c_1 + c_2 > 0$ from positivity constrains

For a given $T$ we determine:

- sign of $m_{ii}^2$ → possible existence of given extremum
- values of $\lambda_i$ (fixed) → existence of a given local minimum
- value of extremum energy → global minimum

⇒ **sequences of possible phase transitions**

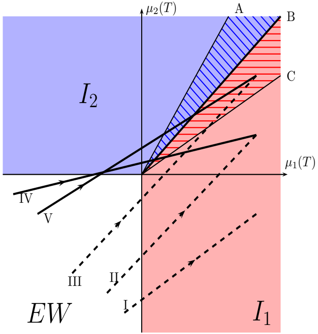
The evolution of vacuum states and phase transitions in 2HDM during cooling of Universe, I.F. Ginzburg, I.P. Ivanov, K.A. Kanishev, Phys.Rev.D81:085031,2010

– 2HDM with soft $Z_2$ violation

# Possible sequences I

The possible sequences of phase transitions on $(\mu_1, \mu_2)$ plane:

$$\mu_1(T) = m_{11}^2(T)/\sqrt{\lambda_1}, \quad \mu_2(T) = m_{22}^2(T)/\sqrt{\lambda_2}.$$




$R > 1$

red hatch – $I_1$ global and $I_2$ local min,
blue hatch – $I_2$ global and $I_1$ local min;

$$A: \ \mu_2(T) = \mu_1(T)R,$$
$$B: \ \mu_2(T) = \mu_1(T),$$
$$C: \ \mu_2(T) = \mu_1(T)/R$$

$EW \to I_1$

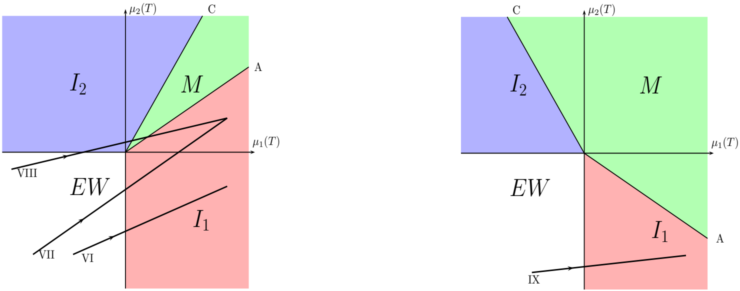
- ray **I** – $I_2$ is not an extremum
- ray **II** – $I_2$ is an extremum, but never was a (local) minimum
- ray **III** – $I_2$ is a local minimum, but never was a global minimum

$EW \to I_2 \to I_1$

- ray **IV** – $I_2$ is not a local minimum, but was a global minimum in the past
- ray **V** – $I_2$ is a local minimum, it was a global minimum in the past

# Possible sequences II

$0 < R < 1$ $\quad\quad$ $-1 < R < 0$



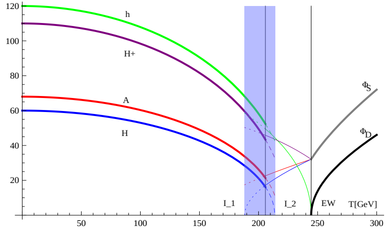

$$EW \to I_1$$

- rays **VI, IX** – $I_2$ is not an extremum
- rays **VII** – $I_2$ is an extremum, but never was a (local) minimum

$$EW \to I_2 \to M \to I_1$$

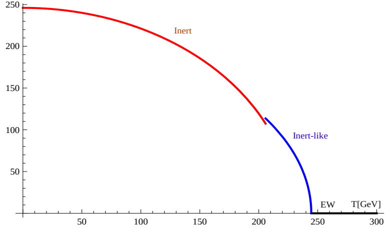
- ray **VIII** – $I_2, M$ were global minima in the past

# Ray IV: $EW \rightarrow I_2 \rightarrow I_1$

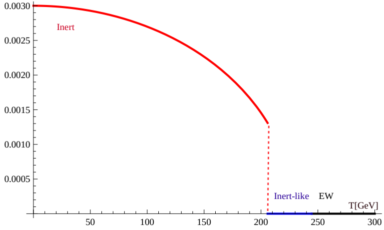



1st order phase transition between $I_2$ and $I_1$ – two coexisting minima (shaded region), discontinuity in physical parameters.

$$M_h = 120 \text{ GeV}, \quad M_H = 60 \text{ GeV},$$
$$M_A = 68 \text{ GeV}, \quad M_{H\pm} = 110 \text{ GeV},$$
$$\lambda_2 = 0.19, \quad \lambda_{345} = 0.26.$$

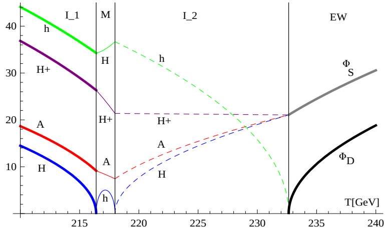



Value of $v(T)$



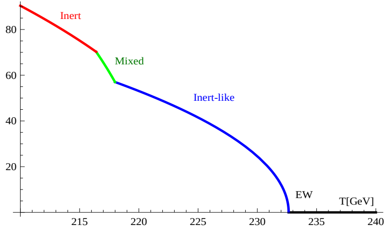

Value of $m_u(T)$

# Ray VIII: $EW \to I_2 \to M \to I_1$

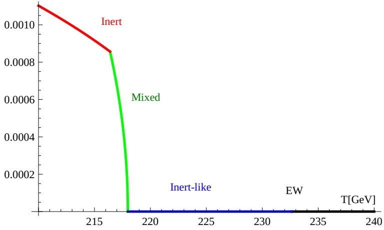



Sequence of three 2nd order phase transitions – no coexisting minima

$$M_h = 120 \text{ GeV}, \quad M_H = 60 \text{ GeV},$$
$$M_A = 68 \text{ GeV}, \quad M_{H\pm} = 110 \text{ GeV},$$
$$\lambda_2 = 0.41, \quad \lambda_2 = 0.30.$$



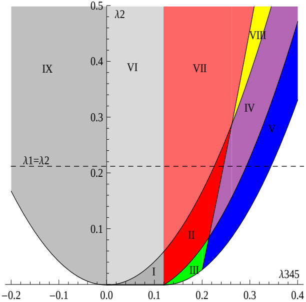

Value of $v(T)$



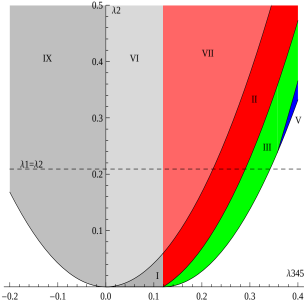

Value of $m_u(T)$

# $\lambda_2$ dependance

- HHHH coupling $\propto \lambda_2$ – does not affect the relict density, but important for the evolution
- Fermionic contribution in $c_1$ important.
- Parameter range for different rays:




fermionic contr. included




without fermions

$M_h = 120$ GeV, $M_H = 60$ GeV, $M_A = 68$ GeV, $M_{H\pm} = 110$ GeV

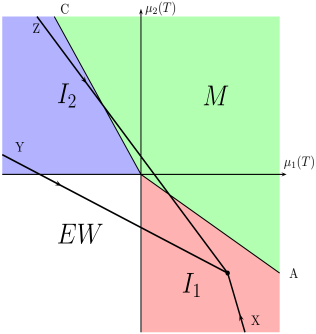
# Conclusions

- Today – Inert Model (dark matter).
- Different types of extrema can be realized in the past.
- Possible sequences of phase transitions:

$$EW \to I_2 \to M \to I_1$$
$$EW \to I_2 \to I_1$$
$$EW \to I_1$$

- It is possible to have no DM for high $T$ (going through $I_2$).
- $\lambda_2$ important for the evolution.
- Not covered in this talk: transition through charge breaking vacuum, non-restoration of EW symmetry

# Non-restoration of EW symmetry




$-1 < R < 0$

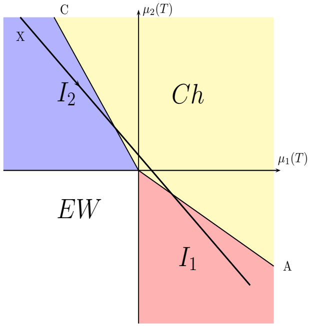
$M \to Ch$ for $R \to R_3$

- $c_1 + c_2 > 0$ required from positivity constrains
- if $c_1 < 0$ or $c_2 < 0$ – non-restoration of EW symmetry in the past
- possible only if $R < 0$ (possiblity of $M$ vacuum) or $R_3 < 0$ (possiblity of $Ch$ vacuum, next slide)
- **ray X**– intial state is $I_1$, no phase transition, DM always existed ($c_1 < 0$)
- **ray Y**– initial state is $I_2$, temporary apperance of $EW$ state, DM after last transition to $I_1$ ($c_2 < 0$)
- **ray Z**– initial state is $I_2$, sequence $I_2 \to M \to I_1$, no EW symmetric state, DM after last transition ($c_2 < 0$)

Not ruled out, but contradicts the modern approach.

# $EW \to I_2 \to Ch \to I_1$

For $|R_3| < 1$ possibility of transition through $Ch$ vacuum.




$-1 < R_3 < 0$

- possible if $\lambda_4 \pm \lambda_5 > 0 \Rightarrow M^2_{H^\pm} < M^2_A, M^2_H$ (charged DM particle)
- allowed values of $M_{H^\pm} \approx 100$ TeV give

$$m_{22}^2 < 0, |m_{22}^2|/m_{11}^2 \approx 10^5$$

(perturbativity and unitarity fulfiled)

- ray X realized if:
  $c_2 < 0$ and $|c_2|/c_1 > |m_{22}^2|/m_{11}^2 \gtrsim 10^6 \Rightarrow$
  in contradiction with positivity constrains

Conclusion: **excluded in our model**.