Holographic Superconductors in Gauss-Bonnet Gravity

Luke Barclay[1]

Department of Mathematics
University of Durham
luke.barclay@durham.ac.uk

17th September 2010

[1]R.Gregory, S. Kanno, P. Sutcliffe: arXiv:1009.1991v1

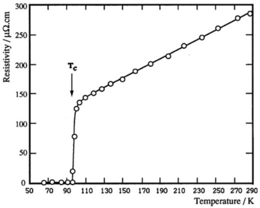
Outline

- Superconductivity
- Holographic Superconductors
- Holographic Superconductors in Gauss-Bonnet Gravity
- Summary

- Superconductivity
- Holographic Superconductors
- Holographic Superconductors in Gauss-Bonnet Gravity
- Summary

Superconductivity

Superconductivity is a phenomenon whereby the electrical resistance of some materials abruptly falls to precisely zero below some characteristic temperature T_c.



- Discovered in 1911
- Ginzburg-Landau theory and BCS theory (Bardeen, Cooper, and Schrieffer) are two theories that could describe the phenomenon fairly well, using the idea of a field theory undergoing a phase transition with a field condensing out of its vacuum state.
- in 1980's high temperature superconductors were discovered ($T_c \approx 90$K instead of $T_c \approx 10$K).
- This is not well explained by current theories.
- It is thought to be explained by a strongly coupled gauge theory meaning perturbative calculations are impossible.

- Superconductivity
- Holographic Superconductors
- Holographic Superconductors in Gauss-Bonnet Gravity
- Summary

Gauge/Gravity Correspondence

General Idea:

- Duality between Gravity theory in D dimensions and a field theory in $D - 1$ dimensions which lives on the boundary of the spacetime.
- Idea is to find a gravity theory dual to a field theory on the boundary that exhibits superconducting behaviour.
- Strong/Weak duality - meaning a weakly coupled gravity theory that can be studied with relative ease can be used to study a strongly coupled gauge theory.
- Gravity theory is a low energy limit of some (unspecified) string theory and the field theory is an SU(N) gauge theory where $N \to \infty$

The Gravity Theory - Einstein Gravity

Gubser (2008) - further developed by Hartnoll, Herzog, Horowitz, Roberts:
- AdS black hole with complex scalar field, ψ coupled to a U(1) gauge field A_μ.

- AdS space - as required by the correspondence
- Black hole to provide temperature for the boundary system
- U(1) gauge symmetry
- Scalar field which can spontaneously break the U(1) and form scalar hair
- Negative scalar mass^2 which allows scalar hair to form.

The Gravity Theory - Einstein Gravity

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left[-R+\frac{12}{L^2}\right] \\
&\quad + \int d^D x\sqrt{-g}\left[-\frac{1}{4}F^{ab}F_{ab}+|\nabla_a\psi - iqA_a\psi|^2 - m^2|\psi|^2\right].
\end{aligned}
$$

Where L is the AdS length scale. set $L=1$
We use a static ansatz for the scalar and gauge fields

$$
A_\mu = \phi(r)\delta_{0\mu}, \qquad \psi = \psi(r).
$$

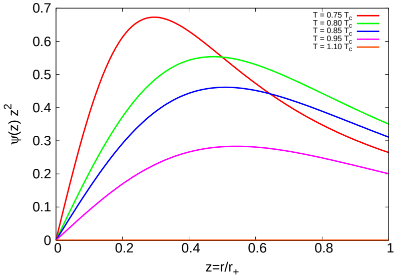
To examine super conductivity we look for plane symmetric black hole solutions with or without scalar hair.

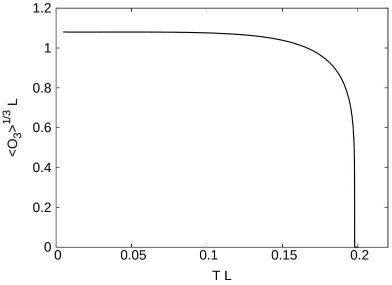
$$
ds^2 = f(r)e^{2\nu(r)}dt^2 - \frac{dr^2}{f(r)} - r^2(dx^2+dy^2+....+dz^2)
$$

$$
\text{with} \qquad T = \frac{1}{4\pi}f'(r)e^{\nu(r)}|_{r=r_+}
$$

- In general there is no scalar solution, but for low T, a scalar condensate forms
- the fall off of the scalar gives the expectation value of an operator $<\mathcal{O}>$, in the boundary theory.

The Condensate

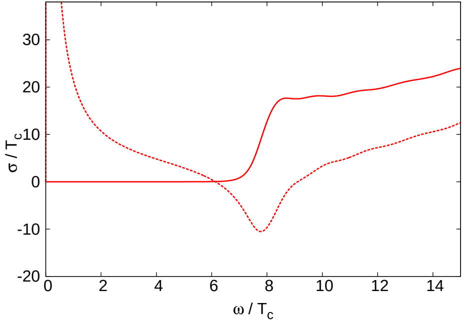




Conductivity

The conductivity σ, of the system can be calculated from perturbations of the gauge field A_μ in the bulk.

$$A + \delta A = (\phi(r), 0, A(r)e^{-i\omega t}\mathbf{e}_i).$$



A "universal" frequency gap $\omega_g \approx 8\,T_c$ was observed by HHH

ARE THESE RESULTS STABLE TO HIGHER ORDER CORRECTIONS?

- Superconductivity
- Holographic Superconductors
- Holographic Superconductors in Gauss-Bonnet Gravity
- Summary

Gauss-Bonnet Gravity

Add Gauss-Bonnet terms in to the gravitational action

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[-R+\frac{12}{L^2}+\frac{\alpha}{2}\left(R^{abcd}R_{abcd}-4R^{ab}R_{ab}+R^2\right)\right] \\
&\quad+\int d^5x\sqrt{-g}\left[-\frac{1}{4}F^{ab}F_{ab}+|\nabla_a\psi-iqA_a\psi|^2-m^2|\psi|^2\right].
\end{aligned}
$$

- α is Gauss Bonnet coupling and, $0\le\alpha\le\frac{L^2}{4}$
- This is the unique curvature squared term that gives rise to 2^{nd} order equations of motion.
- Believed to be the next order term in a full string theory expansion.

Gauss Bonnet Black Holes

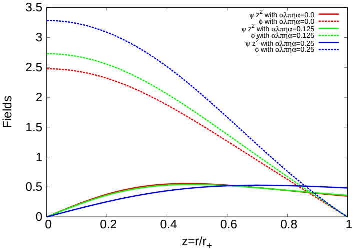
The Gauss Bonnet black hole has a potential

$$f(r) = \frac{r^2}{2\alpha}\left[1 - \sqrt{1 - \frac{4\alpha}{L^2}\left(1 - \frac{ML^2}{r^4}\right)}\right] \rightarrow \frac{r^2}{L_e^2}, \quad \text{as} \quad r \rightarrow \infty$$

$$L_e^2 \rightarrow \begin{cases} L^2 & \alpha \rightarrow 0 \\ \frac{L^2}{2} & \alpha \rightarrow \frac{L^2}{4} \end{cases}$$

- The GB term alters the relation between the cosmological constant and the AdS length scale.
- In this work we have chosen to fix the scalar mass to the AdS length scale $m^2 = -3/L_e^2$.

- The GB potential is qualitatively the same as that of Einstein gravity and the fields have a similar behaviour.



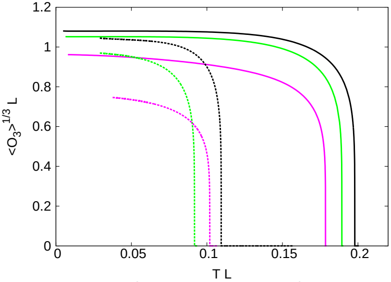
$$\phi'' + \left(\frac{3}{r} - \nu'\right)\phi' - 2q^2\frac{\psi^2}{f}\phi = 0$$

$$\psi'' + \left(\frac{3}{r} + \nu' + \frac{f'}{f}\right)\psi' + \left(\frac{q^2\phi^2}{f^2 e^{2\nu}} - \frac{m^2}{f}\right)\psi = 0$$

$$\left(1 - \frac{2\alpha f}{r^2}\right)\nu' = \frac{2\kappa^2}{3}r\left(\psi'^2 + \frac{q^2\phi^2\psi^2}{f^2 e^{2\nu}}\right)$$

$$\left(1 - \frac{2\alpha f}{r^2}\right)f' + \frac{2}{r}f - \frac{4r}{L^2} = -\frac{2\kappa^2}{3}r\left[\frac{\phi'^2}{2e^{2\nu}} + m^2\psi^2 + f\psi'^2 + \frac{q^2\phi^2\psi^2}{fe^{2\nu}}\right]$$

Condensate and Critical Temperature

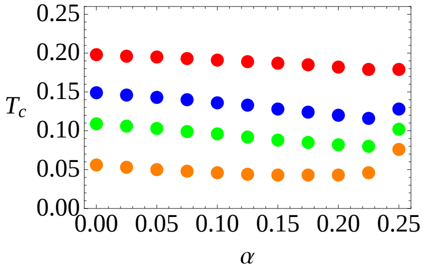


$\kappa^2 = 0.05$ $\kappa^2 = 0.0$

black: $\alpha = 0.0$

green: $\alpha = 0.125$

pink: $\alpha = 0.25$

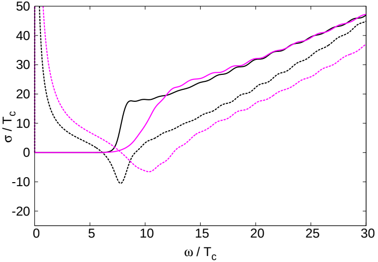


red: $\kappa^2 = 0$, blue: $\kappa^2 = 0.05$

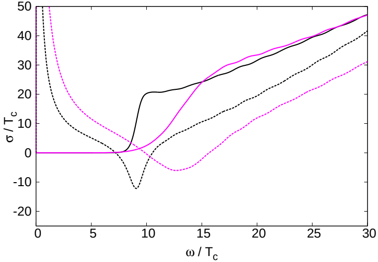
green: $\kappa^2 = 0.1$, orange: $\kappa^2 = 0.2$

Conductivity

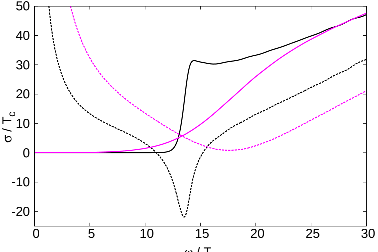
$\alpha = 0.0$



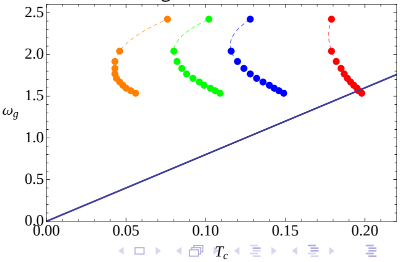
$\alpha = 0.125$



$\alpha = 0.25$



ω_g vs T_c



Summary

- Gauss Bonnet corrections change the details, but not the basic physics of the holographic superconductor
- Back reaction makes condensation harder
- GB corrections seem to initially make condensation harder, but as it is turned up it does become slightly easier -effect is amplified by increased back reaction
- 'Universal' frequency gap $\omega_g \approx 8T_c$, unstable to back reaction and GB corrections.